Hindawi Publishing Corporation
Mathematical Problems in Engineering
Volume 2013, Article ID 163565, 5 pages
http://dx.doi.org/10.1155/2013/163565

Research Article

Wind Farm Power Forecasting

Nabiha Haouas[1,2] and Pierre R. Bertrand[1]

[1] *Laboratoire de Mathématiques (UMR CNRS 6620), Université Blaise Pascal (Clermont-Ferrand II), 63177 Aubière Cedex, France*
[2] *Computational Mathematics Laboratory, Monastir, Tunisia*

Correspondence should be addressed to Nabiha Haouas; nabiha.haouas@math.univ-bpclermont.fr

Received 19 April 2013; Accepted 4 July 2013

Academic Editor: Wei-Jen Lee

Forecasting annual wind power production is useful for the energy industry. Until recently, attention has only been paid to the mean annual wind power energy and statistical uncertainties on this forecasting. Recently, Bensoussan et al. (2012) have pointed that the annual wind power produced by one wind turbine is a Gaussian random variable under a reasonable set of assumptions. Moreover, they can derive both mean and quantiles of annual wind power produced by one wind turbine. The novelty of this work is the obtainment of similar results for estimating the annual wind farm power production. Eventually, we study the relationship between the power production for each turbine of the farm in order to avoid interaction between them.

1. Introduction

The energy industry is one of the most important types of modern industries. In recent years, wind power has become increasingly popular as a renewable energy source that can both develop the economy and protect the environment. Thereby, the development of suitable mathematical models becomes necessary. A detailed review on the existing tools used in wind power prediction is provided by [1], which proposes a perspective of future developments. A more recent update can be found in [2] where they used two alternative numerical prediction models: an empirical one and a computational one, in order to forecast the power output of two Greek wind farms before their installation. Different models for monitoring and forecasting the turbine output are considered such as those in the studies by the authors of [3–7] or [8] and recently in [9]. But, to our knowledge, the performance of a wind power farm has not been adequately studied. In this work, we suggest a statistical analysis based on central limit theorem as in [9]. Firstly, by using the wind speed, as input variables, we can forecast the annual energy production and its quantiles. Secondly, we study the relationship between the power outputs for each turbine in the farm to avoid the effect of interaction between them.

The rest of this paper is organized as follows. In Section 2, we present the dataset. In Section 3, we first recall the state on the art on the wind farm power forecasting, then we give the theoretical results for the forecasting of the annual wind power production, and we apply these results to real datasets. Moreover, we study the relationship between the power outputs for each turbine in the farm. Finally, discussions are available in Section 4.

2. Data Presentation

In this case study, we have processed ten-minute wind speed and ten-minute wind power production corresponding to a wind farm with four turbines. The duration of observation is 29 months leading to large series. The wind farm is located in a flat area close to the sea.

The wind farm power production depends on wind speed. Thus we begin with data representation. The wind speed series is intermittent; that is, it presents very irregular variations, as shown in Figure 1. This intermittency induces forecasting difficulties. A different approach has been considered, but also with a different horizon of time; see, for example, [10, 11] or [12]. Let us point out that the previous methods take only into account the absolute value of wind speed, leading to 1D time series. Other modern methods include the 2D or 3D behavior of wind; see, for example, [13] or [14].

By a simple calculation, the average wind speed over the period is equal to 7.25 m/s, which is enough to guarantee a

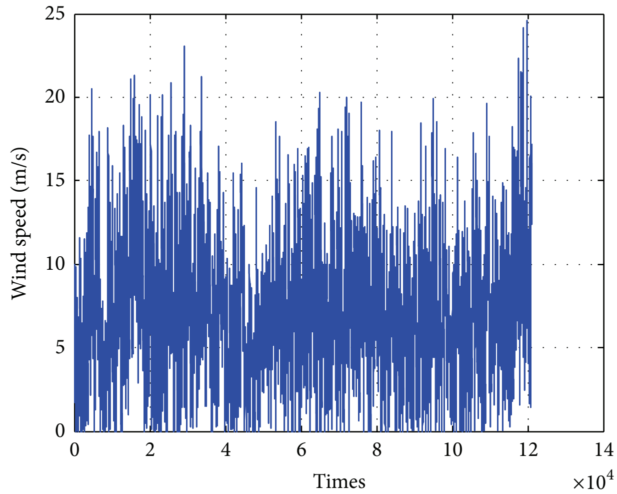

FIGURE 1: Mean wind speed in the farm.

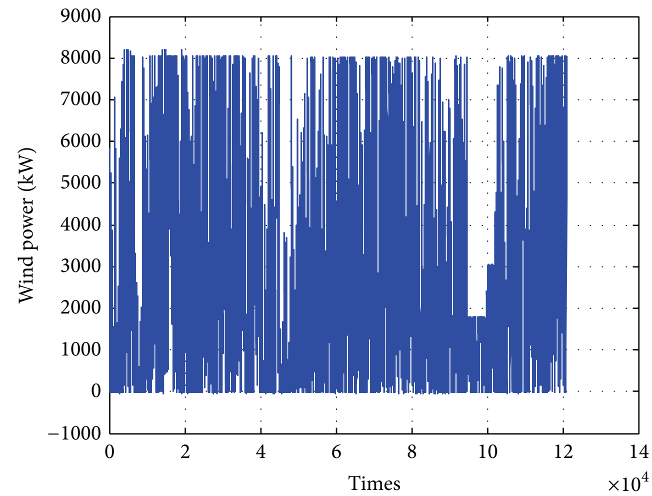

FIGURE 2: Average wind farm power every ten minutes.

good profitability of the project but does not allow a detailed forecast.

Regarding the energy output, we can note the large amount generated by this wind farm which reaches 2870 kW on average every ten minutes. This perfectly fits the building of a wind farm on a flat area close to a windy sea. We can also plot the ten-minute wind speed versus the ten-minute power; see Figure 3. We then get a cloud of points around the nominal power law. Let us recall that the nominal power law is provided by the manufacturer and indicates the power P produced for a given wind speed v, which corresponds to the map $v \mapsto P(v)$ (see Figure 2).

Moreover, the turbine is cut for wind speed outside the interval $(V_{\text{cut.in}}, V_{\text{cut.off}})$. The large dispersion in Figure 3 around the nominal power law is due to outliers and error of measurement. We see that the power

(i) is null if the wind speed is less than the starting speed ($V_{\text{cut.in}} = 3.5$ m/s) and beyond the cut out speed $V_{\text{cut.off}} = 25$ m/s,

(ii) is proportional to the wind speed rise between $V_{\text{cut.in}}$ and the rated speed (about 13 m/s),

(iii) is constant between the rated speed and the cut out speed.

Let us remark that the energy output is not the same for each of the four turbines. Indeed, there is a turbine that produces on average less than the others although the four turbines have the same power law, being of the same type. A possible explanation is the wake effect. In this frame, modeling by 2D wind series could enhance wind power forecasting (see Table 1).

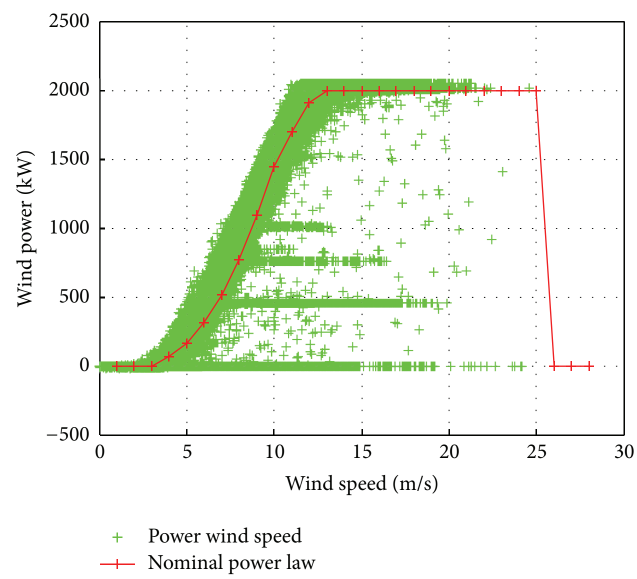

FIGURE 3: Wind power versus wind speed for turbine 1.

TABLE 1: Mean wind power for ten-minute by each turbine.

Turbine	T_1	T_2	T_3	T_4
Mean wind power (kW)	756	712	684	717

3. Wind Farm Power Forecasting

3.1. State of the Art. The traditional method is based on modeling ten-minute wind speed probability density function (pdf) and then calculating the average ten-minute wind power production as $\mathscr{P} = (1 - p_0)\int_{V_{\text{cut.in}}}^{V_{\text{cut.off}}} P(v) f(v) dv$, where p_0 denotes the probability of zero wind; $V_{\text{cut.in}}, V_{\text{cut.off}}$ denote the wind speed for cutting in and cutting off the turbine; $v \mapsto P(v)$ denotes the nominal power law; $f(v)$ denotes the wind speed probability density function (pdf). Most often the ten-minute wind speed is assumed to follow a Weibull distribution, or a hybrid Weibull law; see, for example, [15, 16], or [7]. This model gives a good estimation of both mean ten-minute wind speed and mean ten-minute wind power production and after that provides us with a good forecasting of the mean annual wind power production. However, this traditional method discards the time structure of wind speed and wind power and thus it does not allow to forecast the variance, nor the quantiles of annual wind power production.

Recently, the authors of [9] have proposed a new method which takes into account the dynamical structure of the annual wind power production. This method provides forecasting not only of the mean annual wind power production, but also of its variance and the quantiles of annual wind power production. The techniques rely on the central limit theorem (CLT) which asserts that the pdf of annual wind power production is almost Gaussian under natural assumptions. In the next section, we slightly adapt this method to the case of a wind farm.

3.2. Forecasting Wind Farm Power. The annual wind power production of the farm is defined by

$$\mathscr{P}^F_{\text{annual}} = \sum_{t=1}^{T} P^F_t, \quad (1)$$

where $T = 52,560$ denotes the number of ten-minute periods during one year. The farm production is the sum of the individual production of the J turbines, that is, $P^F_t = \sum_{j=1}^{J} P^{(j)}_t$. The most simple but quite large model is to assume that each series P_t is wide sense stationary. Wide sense stationary means that on the one hand both mean $\mathscr{P}^{(j)}$ and variance $\mathscr{V}^{(j)}$ are not depending on time and that on the other hand the dynamical structure read on the covariance does not depend on the time t. As a corollary, the farm production series P^F_t is also wide sense stationary with mean $\mathscr{P}^F = \mathbb{E}(P^F_t) = \sum_{j=1}^{J} \mathscr{P}^{(j)}$ and variance as follows:

$$\mathscr{V}^F = \operatorname{Var}\left(P^F_t\right) = \sum_{i,j=1}^{J} \operatorname{cov}\left(P^{(i)}_t, P^{(j)}_t\right) \quad (2)$$

and correlation coefficient $\rho^F_P(k) = \operatorname{cov}(P^F_t, P^F_{t+k}) / \operatorname{Var}(P^F_t)$ which does not depend on the time t. Moreover, the family of random variables P^F_t is weakly dependent, and it admits a finite second-order moment (i.e., $\mathbb{E}(\mathscr{P}^F_t) < \infty$ for each $t \geq 1$). In addition, the annual wind production $\mathscr{P}^F_{\text{annual}}$ is also random with mean $\mathbb{E}(\mathscr{P}^F_{\text{annual}}) = T \times \mathscr{P}^F$ and variance as follows:

$$\operatorname{var}\left(\mathscr{P}^F_{\text{annual}}\right) = \operatorname{var}\left(\sum_{t=1}^{T} P^F_t\right) = T \times \mathscr{V}^F \times \left(\Gamma^F_T\right)^2, \quad (3)$$

where $\Gamma^F_T = \{1 + 2\sum_{k=1}^{T} [\rho^F_p(k) \times (1 - k/T)]\}^{1/2}$ with $\rho^F_P(k) = \operatorname{cov}(P^F_t, P^F_{t+k}) / \operatorname{Var}(P^F_t)$ which was introduced to variance analysis for characterizing wind energy conversion as in [17].

Let us stress that the variance of annual production of the farm depends both on the variance of ten-minute wind power $\mathscr{V}^F$ and its correlogram $\rho^F_p(k)$, which corresponds to the time structure of the series. We will also need the two following assumptions.

(A1) The second-order moment of P_t is finite, that is, size $\mathscr{V} = \mathbb{E}((P_t)^2) < \infty$;

(A2) The family of random variables P_t is weakly dependent.

After having made precise assumptions and notation, by using the same tricks as in Proposition 3.1 in [9], we can deduce the following CLT.

Theorem 1 (CLT for wind farm annual production).

(i) *If the family of r.v. P_t is wide sense stationary, and assumptions (A1) and (A2) are fulfilled, then one has*

$$\mathscr{P}^F_{\text{annual}} = T \times \mathscr{P}^F + T^{1/2} \times \left(\mathscr{V}^F\right)^{1/2} \cdot \Gamma^F_T \times \varepsilon_T, \quad (4)$$

where

$$\mathscr{P}^F = \sum_{j=1}^{J} \mathscr{P}^{(j)},$$

$$\mathscr{V}^F = \operatorname{Var}\left(P^F_t\right) = \sum_{i,j=1}^{J} \operatorname{cov}\left(P^{(i)}_t, P^{(j)}_t\right), \quad (5)$$

$$\Gamma^F_T = \left\{1 + 2\sum_{k=1}^{T} \left[\rho^F_p(k) \times \left(1 - \frac{k}{T}\right)\right]\right\}^{1/2}$$

and ε_T is a zero mean r.v. with variance 1 which converges towards a standard Gaussian law; that is, $\varepsilon_T \rightarrow \mathscr{N}(0,1)$ when $T \rightarrow \infty$.

(ii) *Moreover, one can deduce the quantiles of annual production as follows:*

$$Q_{0.05} = T \times \mathscr{P}^F - 1.65 \times T^{1/2} \times \left(\mathscr{V}^F\right)^{1/2} \times \Gamma^F_T,$$
$$Q_{0.95} = T \times \mathscr{P}^F + 1.65 \times T^{1/2} \times \left(\mathscr{V}^F\right)^{1/2} \times \Gamma^F_T. \quad (6)$$

Remark 2. Theorem 1 is warrant under the assumptions (A1) and (A2), which correspond to the stationarity of wind series and consequently of power series P_t. However, this result can be enhanced by taking into account the wind direction as in [13, 14] or by replacing the stationarity assumption by seasonality.

To sum up, the quantiles and the mean annual wind power production depend on three parameters, that is $\mathscr{P}^F$ the mean of ten-minute wind power of the farm, $\mathscr{V}^F$ the variance of ten-minute wind power, and Γ^F_T which depends on the correlation structure of the wind power time series. Before starting the estimation procedures, we check the correlation coefficient $\rho_X(k)$ as described previously, which proves that wind speed and the average wind farm power of four turbines are strongly correlated at a time scale smaller than ten hours and become uncorrelated at scale 48 hours (confirmed by Ljung-Box test; see e.g., [18]) as shown in the following Figure 4.

3.3. Results. In order to carry out the overestimation of the mean wind farm power production, first, we take again

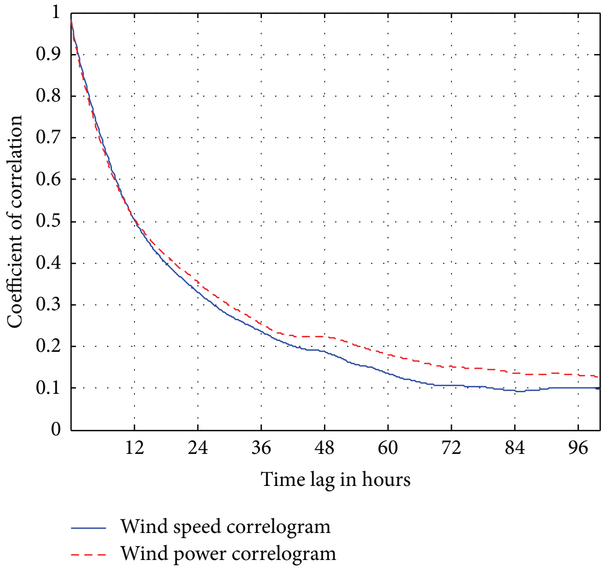

Figure 4: Correlogram of mean wind speed and mean wind farm power, duration 19 months.

the nominal power law proposed in [9] based on the use of a numerical sample of the ten-minute wind power derived from the sample of wind speed. Secondly, we use the traditional method which is modeled by a Weibull distribution law described in Section 3.1. We compare the overestimation rate for each turbine and for all the farm as illustrated in Table 2.

We should note that the overestimation of the mean power farm production based on a Weibull model is 10.43% and with the numerical wind power series it becomes 1.8%. The sources of this overestimation are out of the scope of this article. However, starting from the estimation of the autocorrelation coefficient, defined in Section 3.2, we proceed to calculate the mean and the quantiles of the annual power production for the whole farm. It is based on a sample of ten-minute wind power as a postproduction approach. Then, we use the new method, that is, the preproduction approach which has already been used in the computed precedents.

3.3.1. The Postproduction Approach. For the measurements P_t^F of the existing turbines, we calculate the quantity Γ_T^F provided by the entire farm and $\Gamma_T^{E_j}$ for each turbine, that is, $j = 1, \ldots, 4$, and we get the values stored in Table 3.

Then, we can deduce the mean and the quantiles of the annual power production (see Table 4).

3.3.2. The Preproduction Approach. First, we calculate $P_t^{E_1}$, $P_t^{E_2}$, $P_t^{E_3}$, and $P_t^{E_4}$ based on the wind speed measurements provided by each turbine through power law. Thereafter, we follow the same procedure as mentioned previously. We get Table 5.

Then, from these values, we obtained the mean and the quantiles by using Theorem 1 for each turbine and for the farm, for the annual wind power production (see Table 6).

We can also forecast the twenty-year wind farm power production (20yPP), which corresponds to the lifespan of some turbines (see Table 7).

Table 2: The overestimation rate.

	E_1	E_2	E_3	E_4	Farm
Power law	−4.4	−3.8	−0.7	+2.1	−1.8
Weibull law	−1.8	−0.2	−46.3	+4.7	−10.43

Table 3: Estimated values of Γ for existing turbines.

Γ_T^F	$\Gamma_T^{E_1}$	$\Gamma_T^{E_2}$	$\Gamma_T^{E_3}$	$\Gamma_T^{E_4}$
9.159	10.068	10.015	9.891	10.008

Table 4: The annual mean power production and its quantiles (GWH).

	T_1	T_2	T_3	T_4	Farm
$E(\mathscr{P}_{\text{annual}})$	6.62	6.24	6.0	6.29	25.14
$Q_{0.05}$	6.18	5.82	5.58	5.86	23.47
$Q_{0.95}$	7.06	6.67	6.41	6.71	26.82
Spread (%)	13.4	13.6	13.9	13.5	13.3

Table 5: Estimated values of Γ obtained by the preproduction approach.

Γ_T^F	$\Gamma_T^{E_1}$	$\Gamma_T^{E_2}$	$\Gamma_T^{E_3}$	$\Gamma_T^{E_4}$
10.095	10.085	9.995	9.904	10.026

Table 6: Forecasting power production for one year and its quantiles (GWH).

	T_1	T_2	T_3	T_4	Farm
$\mathbb{E}(\mathscr{P}_{\text{annual}})$	6.33	6.0	5.95	6.42	24.70
$Q_{0.05}$	5.89	5.58	5.53	5.58	23
$Q_{0.95}$	6.77	6.42	6.37	6.85	26.41
Spread (%)	14	14	14.1	13.5	13.8

Table 7: Forecasting of twenty-year power production and its quantiles (GWH).

	T_1	T_2	T_3	T_4	Farm
20yPP	127	120	119	128	494
$Q_{0.05}$	125	118	117	126	486
$Q_{0.95}$	129	122	121	130	502
Spread (%)	3.1	3.1	3.1	3	3

We find that the uncertainties decrease with the length of the forecast period, rising from 13.8% for one year to 3% for twenty years on total wind farm power, whereas it is decreased from 14% for one year to 3% for twenty years at each turbine.

3.4. The Relationship between the Measurements of Each Turbine in the Farm. To achieve a good characterization of the relationship between the power output and the wind speed for the four turbines, we calculate the correlation coefficients of wind power production (denoted by P_1, P_2, P_3, and P_4) and the correlation coefficients of wind speed (denoted by S_1, S_2, S_3, and S_4) of the four turbines. Let us recall that the correlation coefficient for two series X and Y is

Table 8: Correlation coefficients for P_1, P_2, P_3, and P_4.

	P_1	P_2	P_3	P_4
P_1	1	0.91	0.90	0.91
P_2	0.91	1	0.92	0.90
P_3	0.90	0.92	1	0.90
P_4	0.91	0.90	0.90	1

Table 9: Correlation coefficients using numerical sample for P_1, P_2, P_3, and P_4.

	P_1	P_2	P_3	P_4
P_1	1	0.96	0.94	0.96
P_2	0.96	1	0.97	0.96
P_3	0.94	0.97	1	0.97
P_4	0.96	0.96	0.97	1

Table 10: Correlation coefficients for S_1, S_2, S_3, and S_4.

	S_1	S_2	S_3	S_4
S_1	1	0.97	0.91	0.97
S_2	0.97	1	0.92	0.97
S_3	0.91	0.92	1	0.92
S_4	0.97	0.97	0.92	1

calculated as $\rho(X,Y) = \mathrm{cov}(X,Y)/\sigma_X\sigma_Y$. The correlations for the existing turbines can be summarized in Table 8.

Let us point out that the coefficient correlation is always superior to 0.9 for the wind power production of the four turbines. This assessment is of importance to appreciate the quality of the location of the turbines. We should take into account the effect of interactions between neighbour turbines which can eventually present loss in wind power production. The results obtained by the data generated through the power law are very close to this one. This once again underlines the contribution of the proposed approach to study the relationship that may exist between the energy outputs of wind turbines on a farm before their installation.

Similarly, regarding the correlation between the four wind speeds (denoted by S_1, S_2, S_3, and S_4), we can distinguish the strong correlation between all the turbines which is nearly equal to 1 (see Tables 9 and 10).

4. Discussion

The grouping of several wind turbines on the same site reduces the investment costs. However, it is important to make an optimal configuration of the turbines locations. For this wind farm site, there will be certain directions for which other turbines affect the production of single wind turbine. A more detailed analysis of this dependence will be made in further work.

References

[1] I. Colak, S. Sagiroglu, and M. Yesilbudak, "Data mining and wind power prediction: a literature review," *Renewable Energy*, vol. 46, pp. 241–247, 2012.

[2] C. Chourpouliadis, E. Ioannou, A. Koras, and A. I. Kalfas, "Comparative study of the power production and noise emissions impact from two wind farms," *Energy Conversion and Management*, vol. 60, pp. 233–242, 2012.

[3] L. Landberg, "Short-term prediction of the power production from wind farms," *Journal of Wind Engineering and Industrial Aerodynamics*, vol. 80, no. 1-2, pp. 207–220, 1999.

[4] G. Giebel, L. Landberg, G. Kariniotakis, and R. Brownsword, "Stateof- the-art methods and software tools for short-term prediction of wind energy production," in *Proceedings of European Wind Energy Conference and Exhibition (EWEC 03)*, Madrid, Spain, 2003, CD-ROM.

[5] P. Ramírez and J. A. Carta, "The use of wind probability distributions derived from the maximum entropy principle in the analysis of wind energy. A case study," *Energy Conversion and Management*, vol. 47, no. 15-16, pp. 2564–2577, 2006.

[6] A. Costa, A. Crespo, J. Navarro, G. Lizcano, H. Madsen, and E. Feitosa, "A review on the young history of the wind power short-term prediction," *Renewable and Sustainable Energy Reviews*, vol. 12, no. 6, pp. 1725–1744, 2008.

[7] J. A. Carta, P. Ramírez, and S. Velázquez, "A review of wind speed probability distributions used in wind energy analysis. Case studies in the Canary Islands," *Renewable and Sustainable Energy Reviews*, vol. 13, no. 5, pp. 933–955, 2009.

[8] A. Kusiak, H. Zheng, and Z. Song, "Wind farm power prediction: a data-mining approach," *Wind Energy*, vol. 12, no. 3, pp. 275–293, 2009.

[9] A. Bensoussan, P. R. Bertrand, and A. Brouste, "Forecasting the energy produced by a windmill on a yearly basis," *Stochastic Environmental Research and Risk Assessment*, vol. 26, pp. 1109–1122, 2012.

[10] A. Sfetsos, "A comparison of various forecasting techniques applied to mean hourly wind speed time series," *Renewable Energy*, vol. 21, no. 1, pp. 23–35, 2000.

[11] J. M. Sloughter, T. Gneiting, and A. E. Raftery, "Probabilistic wind speed forecasting using ensembles and Bayesian model averaging," *Journal of the American Statistical Association*, vol. 105, no. 489, pp. 25–35, 2010.

[12] P. Pinson, "Very-short-term probabilistic forecasting of wind power with generalized logit-normal distributions," *Journal of the Royal Statistical Society. Series C*, vol. 61, no. 4, pp. 555–576, 2012.

[13] D. P. Mandic, S. Javidi, S. L. Goh, A. Kuh, and K. Aihara, "Complex-valued prediction of wind profile using augmented complex statistics," *Renewable Energy*, vol. 34, no. 1, pp. 196–201, 2009.

[14] C. C. Took, G. Strbac, K. Aihara, and D. P. Mandic, "Quaternion-valued short-term joint forecasting of three-dimensional wind and atmospheric parameters," *Renewable Energy*, vol. 36, no. 6, pp. 1754–1760, 2011.

[15] O. A. Jaramillo and M. A. Borja, "Wind speed analysis in La Ventosa, Mexico: a bimodal probability distribution case," *Renewable Energy*, vol. 29, no. 10, pp. 1613–1630, 2004.

[16] M. Li and X. Li, "MEP-type distribution function: a better alternative to Weibull function for wind speed distributions," *Renewable Energy*, vol. 30, no. 8, pp. 1221–1240, 2005.

[17] R. B. Corotis, A. B. Sigl, and M. P. Cohen, "On a measure of lack of fit in time series models," *Journal of Applied Meteorology*, vol. 16, no. 11, pp. 1149–1157, 1977.

[18] G. M. Ljung and G. E. P. Box, "On a measure of lack of fit in time series models," *Biometrika*, vol. 65, no. 2, pp. 297–303, 1978.