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Research Article

Delay-Dependent Guaranteed Cost Controller Design for Uncertain Neural Networks with Interval Time-Varying Delay

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This paper studies the problem of guaranteed cost control for a class of uncertain delayed neural networks. The time delay is a continuous function belonging to a given interval but not necessary to be differentiable. A cost function is considered as a nonlinear performance measure for the closed-loop system. The stabilizing controllers to be designed must satisfy some exponential stability constraints on the closed-loop poles. By constructing a set of augmented Lyapunov-Krasovskii functionals combined with Newton-Leibniz formula, a guaranteed cost controller is designed via memoryless state feedback control, and new sufficient conditions for the existence of the guaranteed cost state feedback for the system are given in terms of linear matrix inequalities (LMIs). Numerical examples are given to illustrate the effectiveness of the obtained result.

1. Introduction

The last few decades have witnessed the use of artificial neural networks (ANNs) in many real-world applications and have offered an attractive paradigm for a broad range of adaptive complex systems. In recent years, ANNs have enjoyed a great deal of success and have proven useful in wide variety pattern recognition feature-extraction tasks. Examples include optical character recognition, speech recognition, and adaptive control, to name a few. To keep the pace with the huge demand in diversified application areas, many different kinds of ANN architecture and learning types have been proposed to meet varying needs as robustness and stability. Stability and control of neural networks with time delay have attracted considerable attention in recent years [1–8]. In many practical systems, it is desirable to design neural networks which are not only asymptotically or exponentially stable but can also guarantee an adequate level of system performance. In the area of control, signal processing, pattern recognition, and image processing, delayed neural networks have many useful applications. Some of these applications require that the equilibrium points of the designed network be stable. In both biological and artificial neural systems, time delays due to integration and communication are ubiquitous and often become a source of instability. The time delays in electronic neural networks are usually time varying, and sometimes vary violently with respect to time due to the finite switching speed of amplifiers and faults in the electrical circuitry. Guaranteed cost control problem [9–12] has the advantage of providing an upper bound on a given system performance index and thus the system performance degradation incurred by the uncertainties or time delays is guaranteed to be less than this bound. The Lyapunov-Krasovskii functional technique has been among the popular and effective tool in the design of guaranteed cost controls for neural networks with time delay. Nevertheless, despite such diversity of results available, most existing work either assumed that the time delays are constant or differentiable [13–16]. Although, in some cases, delay-dependent guaranteed cost control for systems with time-varying delays was considered in [12, 13, 15], the approach used there can not be applied to systems with interval, nondifferentiable time-varying delays. To the best of our knowledge, the guaranteed cost control and state feedback stabilization for uncertain neural networks with interval, non-differentiable time-varying delays have not been fully studied yet (see, e.g., [4–26] and the references therein), which are important in both theories and applications. This motivates our research.

In this paper, we investigate the guaranteed cost control for uncertain delayed neural networks problem. The novel features here are that the delayed neural network under consideration is with various globally Lipschitz continuous activation functions, and the time-varying delay function is interval, non-differentiable. A nonlinear cost function is considered as a performance measure for the closed-loop system. The stabilizing controllers to be designed must satisfy some exponential stability constraints on the closed-loop poles. Based on constructing a set of augmented Lyapunov-Krasovskii functionals combined with Newton-Leibniz formula, new delay-dependent criteria for guaranteed cost control via memoryless feedback control are established in terms of LMIs, which allow simultaneous computation of two bounds that characterize the exponential stability rate of the solution and can be easily determined by utilizing Matlabs LMI control toolbox.

The outline of the paper is as follows. Section 2 presents definitions and some well-known technical propositions needed for the proof of the main result. LMI delay-dependent criteria for guaraneed cost control and a numerical examples showing the effectiveness of the result are presented in Section 3. The paper ends with conclusions and cited references.

2. Preliminaries

The following notation will be used in this paper. $\mathbb{R}^+$ denotes the set of all real nonnegative numbers; $\mathbb{R}^n$ denotes the n-dimensional space with the scalar product $\langle x, y\rangle$ or x^Ty of two vectors x, y, and the vector norm $\|\cdot\|$; $M^{n\times r}$ denotes the space of all matrices of $(n \times r)$-dimensions. A^T denotes the transpose of matrix A; A is symmetric if $A = A^T$; I denotes the identity matrix; $\lambda(A)$ denotes the set of all eigenvalues of A; $\lambda_{\max}(A) = \max\{\mathrm{Re}\,\lambda; \lambda \in \lambda(A)\}$. $x_t := \{x(t+s) : s \in [-h, 0]\}$, $\|x_t\| = \sup_{s\in[-h,0]}\|x(t+s)\|$; $C^1([0,t], \mathbb{R}^n)$ denotes the set of all $\mathbb{R}^n$-valued continuously differentiable functions on $[0,t]$; $L_2([0,t], \mathbb{R}^m)$ denotes the set of all the $\mathbb{R}^m$-valued square integrable functions on $[0,t]$.

Matrix A is called semipositive definite ($A \geq 0$) if $\langle Ax, x\rangle \geq 0$, for all $x \in \mathbb{R}^n$; A is positive definite ($A > 0$) if $\langle Ax, x\rangle > 0$ for all $x \neq 0$; $A > B$ means $A - B > 0$. The notation $\text{diag}\{\cdots\}$ stands for a block-diagonal matrix. The symmetric term in a matrix is denoted by $*$.

Consider the following uncertain neural networks with interval time-varying delay:

$$\begin{aligned}\dot{x}(t) = -(A + \Delta A(t))x(t) + (W_0 + \Delta W_0(t))W_0 f(x(t)) + (W_1 + \Delta W_1(t))g(x(t-h(t))) \\ +Bu(t), \quad t \geq 0,\ x(t) = \phi(t),\ t \in [-h_1, 0],\end{aligned} \tag{2.1}$$

where $x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^T \in \mathbb{R}^n$ is the state of the neural; $u(\cdot) \in L_2([0,t], \mathbb{R}^m)$ is the control; n is the number of neurals, and

$$\begin{aligned}f(x(t)) &= \left[f_1(x_1(t)), f_2(x_2(t)), \ldots, f_n(x_n(t))\right]^T, \\ g(x(t-h(t))) &= \left[g_1(x_1(t-h(t))(t)), g_2(x_2(t-h(t))(t)), \ldots, g_n(x_n(t-h(t)))\right]^T,\end{aligned} \tag{2.2}$$

are the activation functions; $A = \text{diag}(\overline{a}_1, \overline{a}_2, \ldots, \overline{a}_n)$, $\overline{a}_i > 0$ represents the self-feedback term; $B \in R^{n\times m}$ is control input matrix; W_0, W_1 denote the connection weights, the discretely delayed connection weights and the distributively delayed connection weight, respectively; the time-varying uncertain matrices $\Delta A(t)$, $\Delta W_0(t)$, and $\Delta W_1(t)$ are defined by

$$\Delta A(t) = E_a F_a(t) H_a, \qquad \Delta W_0(t) = E_{w_0} F_{w_0}(t) H_{w_0}, \qquad \Delta W_1(t) = E_{w_1} F_{w_1}(t) H_{w_1}, \tag{2.3}$$

where E_a, E_{w_0}, E_{w_1}, H_a, H_{w_0}, and H_{w_1} are known constant real matrices with appropriate dimensions. $F_a(t), F_{w_0}(t)$, and $F_{w_1}(t)$ are unknown uncertain matrices satisfying

$$F_a^T(t) F_a(t) \leq I, \qquad F_{w_0}^T(t) F_{w_0}(t) \leq I, \qquad F_{w_1}^T(t) F_{w_1}(t) \leq I, \quad t \in R^+. \tag{2.4}$$

The time-varying delay function $h(t)$ satisfies the condition

$$0 \leq h_0 \leq h(t) \leq h_1. \tag{2.5}$$

The initial functions $\phi(t) \in C^1([-h_1, 0], R^n)$, with the norm

$$\|\phi\| = \sup_{t \in [-h_1, 0]} \sqrt{\|\phi(t)\|^2 + \|\dot{\phi}(t)\|^2}. \tag{2.6}$$

In this paper we consider various activation functions and assume that the activation functions $f(\cdot)$, $g(\cdot)$ are Lipschitzian with the Lipschitz constants f_i, $e_i > 0$:

$$\begin{aligned}\left|f_i(\xi_1) - f_i(\xi_2)\right| &\leq f_i|\xi_1 - \xi_2|, \quad i = 1, 2, \ldots, n,\ \forall \xi_1, \xi_2 \in \mathbb{R}, \\ \left|g_i(\xi_1) - g_i(\xi_2)\right| &\leq e_i|\xi_1 - \xi_2|, \quad i = 1, 2, \ldots, n,\ \forall \xi_1, \xi_2 \in \mathbb{R}.\end{aligned} \tag{2.7}$$

The performance index associated with the system (2.1) is the following function:

$$J = \int_0^{\infty} f^0(t, x(t), x(t-h(t)), u(t))dt, \tag{2.8}$$

where $f^0(t, x(t), x(t-h(t)), u(t)) : R^+ \times R^n \times R^n \times R^m \to R^+$ is a nonlinear cost function that satisfies

$$\exists Q_1, Q_2, R : f^0(t, x, y, u) \leq \langle Q_1 x, x\rangle + \langle Q_2 y, y\rangle + \langle Ru, u\rangle, \tag{2.9}$$

for all $(t, x, u) \in R^+ \times R^n \times R^m$ and $Q_1, Q_2 \in R^{n\times n}$, $R \in R^{m\times m}$ are given symmetric positive definite matrices. The objective of this paper is to design a memoryless state feedback controller $u(t) = Kx(t)$ for system (2.1) and the cost function (2.8) such that the resulting closed-loop system

$$\begin{aligned} \dot{x}(t) = &- [(A + E_a F_a(t) H_a) - BK]x(t) + (W_0 + E_{w_0} F_{w_0}(t) H_{w_0}) f(x(t)) \\ &+ (W_1 + E_{w_1} F_{w_1}(t) H_{w_1}) g(x(t-h(t))) \end{aligned} \tag{2.10}$$

is exponentially stable and the closed-loop value of the cost function (2.10) is minimized.

Definition 2.1. Given $\alpha > 0$. The zero solution of closed-loop system (2.8) is α-exponentially stabilizable if there exists a positive number $N > 0$ such that every solution $x(t, \phi)$ satisfies the following condition:

$$\|x(t, \phi)\| \leq N e^{-\alpha t} \|\phi\|, \quad \forall t \geq 0. \tag{2.11}$$

Definition 2.2. Consider the control system (2.1). If there exists a memoryless state feedback control law $u^*(t) = Kx(t)$ and a positive number J^* such that the zero solution of the closed-loop system (2.10) is exponentially stable and the cost function (2.8) satisfies $J \leq J^*$, then the value J^* is a guaranteed constant and $u^*(t)$ is a guaranteed cost control law of the system and its corresponding cost function.

We introduce the following technical well-known propositions, which will be used in the proof of our results.

Proposition 2.3 (Schur complement lemma [27]). *Given constant matrices $X, Y,$ and Z with appropriate dimensions satisfying $X = X^T, Y = Y^T > 0$, then $X + Z^T Y^{-1} Z < 0$ if and only if*

$$\begin{pmatrix} X & Z^T \\ Z & -Y \end{pmatrix} < 0. \tag{2.12}$$

Proposition 2.4 (integral matrix inequality [28]). *For any symmetric positive definite matrix $M > 0$, scalar $\gamma > 0$ and vector function $\omega : [0,\gamma] \to R^n$ such that the integrations concerned are well defined, the following inequality holds*

$$\left(\int_0^\gamma \omega(s)ds\right)^T M\left(\int_0^\gamma \omega(s)ds\right) \leq \gamma\left(\int_0^\gamma \omega^T(s)M\omega(s)ds\right). \tag{2.13}$$

3. Design of Guaranteed Cost Controller

In this section, we give a design of memoryless guaranteed feedback cost control for uncertain neural networks (2.1). Let us set

$$\begin{aligned}
W_{11} &= -PA^T - AP - 2\alpha P + 0.25BRB^T + \sum_{i=0}^{1} G_i + 2\epsilon_1 E_a^T E_a + 6\epsilon_1^{-1} P H_a^T H_a P \\
&\quad + 4\epsilon_2^{-1} PFH_{w_0}^T H_{w_0} FP, \\
W_{12} &= P + AP - 0.5BB^T, \qquad W_{13} = e^{-2\alpha h_0} H_0 + 0.5BB^T + AP, \\
W_{14} &= e^{-2\alpha h_1} H_1 + 0.5BB^T + AP, \qquad W_{15} = P + 0.5BB^T + AP, \\
W_{22} &= \sum_{i=0}^{1} W_i D_i W_i^T + \sum_{i=0}^{1} h_i^2 H_i + (h_1 - h_0)U - 2P - BB^T + \epsilon_1 E_a^T E_a + \epsilon_2 E_{w_0}^T E_{w_0} + \epsilon_3 E_{w_1}^T E_{w_1}, \\
W_{23} &= P, \qquad W_{24} = P, \qquad W_{25} = P, \\
W_{33} &= -e^{-2\alpha h_0} G_0 - e^{-2\alpha h_0} H_0 - e^{-2\alpha h_1} U + \sum_{i=0}^{1} W_i D_i W_i^T + \epsilon_1 E_a^T E_a + \epsilon_2 E_{w_0}^T E_{w_0} + \epsilon_3 E_{w_1}^T E_{w_1}, \\
W_{34} &= 0, \qquad W_{35} = e^{-2\alpha h_1} U, \\
W_{44} &= \sum_{i=0}^{1} W_i D_i W_i^T - e^{-2\alpha h_1} U - e^{-2\alpha h_1} G_1 - e^{-2\alpha h_1} H_1 + \epsilon_1 E_a^T E_a + \epsilon_2 E_{w_0}^T E_{w_0} + \epsilon_3 E_{w_1}^T E_{w_1}, \\
W_{45} &= e^{-2\alpha h_1} U, \\
W_{55} &= -e^{-2\alpha h_1} U + W_0 D_0 W_0^T + 4\epsilon_3^{-1} PEH_{w_1}^T H_{w_1} EP + \epsilon_1 E_a^T E_a + \epsilon_2 E_{w_0}^T E_{w_0} + \epsilon_3 E_{w_1}^T E_{w_1}, \\
E &= \operatorname{diag}\{e_i,\ i = 1,\dots,n\}, \qquad F = \operatorname{diag}\{f_i,\ i = 1,\dots,n\}, \\
\lambda_1 &= \lambda_{\min}\left(P^{-1}\right), \\
\lambda_2 &= \lambda_{\max}\left(P^{-1}\right) + h_0 \lambda_{\max}\left[P^{-1}\left(\sum_{i=0}^{1} G_i\right)P^{-1}\right] \\
&\quad + h_1^2 \lambda_{\max}\left[P^{-1}\left(\sum_{i=0}^{1} H_i\right)P^{-1}\right] + (h_1 - h_0)\lambda_{\max}\left(P^{-1} U P^{-1}\right).
\end{aligned} \tag{3.1}$$

Theorem 3.1. *Consider control system* (2.1) *and the cost function* (2.8)*. If there exist symmetric positive definite matrices* P, U, G_0, G_1, H_0, *and* H_1, *and diagonal positive definite matrices* D_i, $i = 0, 1$, *and* $\epsilon_i > 0$, $i = 1, 2, 3$ *satisfying the following LMIs*

$$\mathcal{E} = \begin{bmatrix} W_{11} & W_{12} & W_{13} & W_{14} & W_{15} \\ * & W_{22} & W_{23} & W_{24} & W_{25} \\ * & * & W_{33} & W_{34} & W_{35} \\ * & * & * & W_{44} & W_{45} \\ * & * & * & * & W_{55} \end{bmatrix} < 0, \tag{3.2}$$

$$\mathcal{S}_1 = \begin{bmatrix} -PA - A^TP - \sum_{i=0}^{1} e^{-2\alpha h_i} H_i & 2PF & PQ_1 \\ * & -D_0 & 0 \\ * & * & -Q_1^{-1} \end{bmatrix} < 0, \tag{3.3}$$

$$\mathcal{S}_2 = \begin{bmatrix} W_1 D_1 W_1^T - e^{-2\alpha h_1} U & 2PE & PQ_2 \\ * & -D_1 & 0 \\ * & * & -Q_2^{-1} \end{bmatrix} < 0, \tag{3.4}$$

then

$$u(t) = -\frac{1}{2} B^T P^{-1} x(t), \quad t \geq 0 \tag{3.5}$$

is a guaranteed cost control and the guaranteed cost value is given by

$$J^* = \lambda_2 \|\phi\|^2. \tag{3.6}$$

Moreover, the solution $x(t, \phi)$ *of the system satisfies*

$$\|x(t, \phi)\| \leq \sqrt{\frac{\lambda_2}{\lambda_1}} e^{-\alpha t} \|\phi\|, \quad \forall t \geq 0. \tag{3.7}$$

Proof. Let $Y = P^{-1}$, $y(t) = Yx(t)$. Using the feedback control (2.8) we consider the following Lyapunov-Krasovskii functional:

$$\begin{aligned} V(t, x_t) &= \sum_{i=1}^{6} V_i(t, x_t), \\ V_1 &= x^T(t) Y x(t), \\ V_2 &= \int_{t-h_0}^{t} e^{2\alpha(s-t)} x^T(s) Y G_0 Y x(s) ds, \\ V_3 &= \int_{t-h_1}^{t} e^{2\alpha(s-t)} x^T(s) Y G_1 Y x(s) ds, \end{aligned}$$

$$V_4 = h_0 \int_{-h_0}^{0} \int_{t+s}^{t} e^{2\alpha(\tau-t)} \dot{x}^T(\tau) Y H_0 Y \dot{x}(\tau) d\tau\, ds,$$
$$V_5 = h_1 \int_{-h_1}^{0} \int_{t+s}^{t} e^{2\alpha(\tau-t)} \dot{x}^T(\tau) Y H_1 Y \dot{x}(\tau) d\tau\, ds,$$
$$V_6 = (h_1 - h_0) \int_{t-h_1}^{t-h_0} \int_{t+s}^{t} e^{2\alpha(\tau-t)} \dot{x}^T(\tau) Y U Y \dot{x}(\tau) d\tau\, ds. \tag{3.8}$$

It is easy to check that

$$\lambda_1 \|x(t)\|^2 \le V(t, x_t) \le \lambda_2 \|x_t\|^2, \quad \forall t \ge 0. \tag{3.9}$$

Taking the derivative of V_1 we have

$$\begin{aligned}
\dot{V}_1 &= 2x^T(t) Y \dot{x}(t) \\
&= y^T(t)\Big[-P(A + E_a F_a(t) H_a)^T - (A + E_a F_a(t) H_a) P\Big] y(t) - y^T(t) B B^T y(t) \\
&\quad + 2y^T(t)(W_0 + E_{w_0} F_{w_0}(t) H_{w_0}) f(\cdot) y(t) + 2y^T(t)(W_1 + E_{w_1} F_{w_1}(t) H_{w_1}) g(\cdot) y(t), \\
\dot{V}_2 &= y^T(t) G_0 y(t) - e^{-2\alpha h_0} y^T(t - h_0) G_0 y(t - h_0) - 2\alpha V_2, \\
\dot{V}_3 &= y^T(t) G_1 y(t) - e^{-2\alpha h_1} y^T(t - h_1) G_1 y(t - h_1) - 2\alpha V_3, \\
\dot{V}_4 &= h_0^2 \dot{y}^T(t) H_0 \dot{y}(t) - h_1 e^{-2\alpha h_0} \int_{t-h_0}^{t} \dot{x}^T(s) H_0 \dot{x}(s)\ ds - 2\alpha V_4, \\
\dot{V}_5 &= h_1^2 \dot{y}^T(t) H_1 \dot{y}(t) - h_1 e^{-2\alpha h_1} \int_{t-h_1}^{t} \dot{y}^T(s) H_1 \dot{y}(s) ds - 2\alpha V_4, \\
\dot{V}_6 &= (h_1 - h_0)^2 \dot{y}^T(t) U \dot{y}(t) - (h_1 - h_0) e^{-2\alpha h_1} \int_{t-h_1}^{t-h_0} \dot{y}^T(s) U \dot{y}(s) ds - 2\alpha V_6,
\end{aligned} \tag{3.10}$$

Applying Proposition 2.4 and the Leibniz-Newton formula

$$\int_{s}^{t} \dot{y}(\tau) d\tau = y(t) - y(s). \tag{3.11}$$

We have for $j = 1, 2, i = 0, 1$

$$
\begin{aligned}
-h_i \int_{t-h_i}^{t} \dot{y}^T(s) H_j \dot{y}(s) ds &\le -\left[\int_{t-h_i}^{t} \dot{y}(s) ds\right]^T H_j \left[\int_{t-h_i}^{t} \dot{y}(s) ds\right] \\
&\le -[y(t) - y(t-h(t))]^T H_j [y(t) - y(t-h(t))] \\
&= -y^T(t) H_i y(t) + 2x^T(t) H_j y(t-h(t)) \\
&\quad - y^T(t-h_i) H_j y(t-h_i).
\end{aligned}
\tag{3.12}
$$

Note that

$$
\int_{t-h_1}^{t-h_0} \dot{y}^T(s) U \dot{y}(s) ds = \int_{t-h_1}^{t-h(t)} \dot{y}^T(s) U \dot{y}(s) ds + \int_{t-h(t)}^{t-h_0} \dot{y}^T(s) U \dot{y}(s) ds. \tag{3.13}
$$

Applying Proposition 2.4 gives

$$
\begin{aligned}
[h_1 - h(t)] \int_{t-h_1}^{t-h(t)} \dot{y}^T(s) U \dot{y}(s) ds &\ge \left[\int_{t-h_1}^{t-h(t)} \dot{y}(s) ds\right]^T U \left[\int_{t-h_1}^{t-h(t)} \dot{y}(s) ds\right] \\
&\ge \left[y\left(t-h(t) - y(t-h_1)\right)\right]^T U \left[y\left(t-h(t) - y(t-h_1)\right)\right].
\end{aligned}
\tag{3.14}
$$

Since $h_1 - h(t) \le h_1 - h_0$, we have

$$
[h_1 - h_0] \int_{t-h_1}^{t-h(t)} \dot{y}^T(s) U \dot{y}(s) ds \ge \left[y\left(t-h(t) - y(t-h_1)\right)\right]^T U \left[y\left(t-h(t) - y(t-h_1)\right)\right], \tag{3.15}
$$

then

$$
-[h_1 - h_0] \int_{t-h_1}^{t-h(t)} \dot{y}^T(s) U \dot{y}(s) ds \le -\left[y\left(t-h(t) - y(t-h_1)\right)\right]^T U \left[y\left(t-h(t) - y(t-h_1)\right)\right]. \tag{3.16}
$$

Similarly, we have

$$
-(h_1 - h_0) \int_{t-h(t)}^{t-h_0} \dot{y}^T(s) U \dot{y}(s) ds \le -[y(t-h_0) - y(t-h(t))]^T U [y(t-h_0) - y(t-h(t))]. \tag{3.17}
$$

Then, we have

$$\begin{aligned}
\dot{V}(\cdot) + 2\alpha V(\cdot) \le\, & y^T(t)\left[-P(A + E_aF_a(t)H_a)^T - (A + E_aF_a(t)H_a)P\right]y(t) - y^T(t)BB^Ty(t) \\
& + 2y^T(t)(W_0 + E_{w_0}F_{w_0}(t)H_{w_0})f(\cdot) + 2y^T(t)(W_1 + E_{w_1}F_{w_1}(t)H_{w_1})g(\cdot) \\
& + y^T(t)\left(\sum_{i=0}^{1} G_i\right)y(t) + 2\alpha\langle Py(t), y(t)\rangle \\
& + \dot{y}^T(t)\left(\sum_{i=0}^{1} h_i^2 H_i\right)\dot{y}(t) + (h_1 - h_0)\dot{y}^T(t)U\dot{y}(t) \\
& - \sum_{i=0}^{1} e^{-2\alpha h_i} y^T(t - h_i)G_i y(t - h_i) \\
& - e^{-2\alpha h_0}\left[y(t) - y(t - h_0)\right]^T H_0\left[y(t) - y(t - h_0)\right] \\
& - e^{-2\alpha h_1}\left[y(t) - y(t - h_1)\right]^T H_1\left[y(t) - y(t - h_1)\right] \\
& - e^{-2\alpha h_1}\left[y(t - h(t)) - y(t - h_1)\right]^T U\left[y(t - h(t)) - y(t - h_1)\right] \\
& - e^{-2\alpha h_1}\left[y(t - h_0) - y(t - h(t))\right]^T U\left[y(t - h_0) - y(t - h(t))\right].
\end{aligned} \tag{3.18}$$

Using (2.8)

$$\begin{aligned}
& P\dot{y}(t) + (A + E_aF_a(t)H_a)Py(t) - (W_0 + E_{w_0}F_{w_0}(t)H_{w_0})f(\cdot) - (W_1 + E_{w_1}F_{w_1}(t)H_{w_1})g(\cdot) \\
& \quad + 0.5BB^Ty(t) = 0,
\end{aligned} \tag{3.19}$$

and multiplying both sides with $[2y(t), -2\dot{y}(t), 2y(t - h_0), 2y(t - h_1), 2y(t - h(t))]^T$, we have

$$\begin{aligned}
& 2y^T(t)P\dot{y}(t) + 2y^T(t)(A + E_aF_a(t)H_a)Py(t) - 2y^T(t)(W_0 + E_{w_0}F_{w_0}(t)H_{w_0})f(\cdot) \\
& \quad - 2y^T(t)(W_1 + E_{w_1}F_{w_1}(t)H_{w_1})g(\cdot) + y^T(t)BB^Ty(t) = 0, \\
& - 2\dot{y}^T(t)P\dot{y}(t) - 2\dot{y}^T(t)(A + E_aF_a(t)H_a)Py(t) + 2\dot{y}^T(t)(W_0 + E_{w_0}F_{w_0}(t)H_{w_0})f(\cdot) \\
& \quad + 2\dot{y}^T(t)(W_1 + E_{w_1}F_{w_1}(t)H_{w_1})g(\cdot) - \dot{y}^T(t)BB^Ty(t) = 0, \\
& 2y^T(t - h_0)P\dot{y}(t) + 2y^T(t - h_0)(A + E_aF_a(t)H_a)Py(t) - 2y^T(t - h_0)(W_0 + E_{w_0}F_{w_0}(t)H_{w_0}) \\
& \quad \times f(\cdot) - 2y^T(t - h_0)(W_1 + E_{w_1}F_{w_1}(t)H_{w_1})g(\cdot) + y^T(t - h_0)BB^Ty(t) = 0,
\end{aligned}$$

$$\begin{aligned}
&2y^T(t-h_1)P\dot{y}(t) + 2y^T(t-h_1)(A + E_aF_a(t)H_a)Py(t) - 2y^T(t-h_1)(W_0 + E_{w_0}F_{w_0}(t)H_{w_0}) \\
&\quad \times f(\cdot) - 2y^T(t-h_1)(W_1 + E_{w_1}F_{w_1}(t)H_{w_1})g(\cdot) + y^T(t-h_1)BB^Ty(t) = 0, \\
&2y^T(t-h(t))P\dot{y}(t) + 2y^T(t-h(t))(A + E_aF_a(t)H_a)Py(t) - 2y^T(t-h(t)) \\
&\quad \times (W_0 + E_{w_0}F_{w_0}(t)H_{w_0})f(\cdot) - 2y^T(t-h(t))(W_1 + E_{w_1}F_{w_1}(t)H_{w_1})g(\cdot) \\
&\quad + y^T(t-h(t))BB^Ty(t) = 0.
\end{aligned} \tag{3.20}$$

Adding all the zero items of (3.20) and $f^0(t,x(t),x(t-h(t)),u(t)) - f^0(t,x(t),x(t-h(t)),u(t)) = 0$, respectively, into (3.18) and using the condition (2.7) for the following estimations:

$$\begin{aligned}
f^0(t,x(t),x(t-h(t)),u(t)) &\le \langle Q_1x(t),x(t)\rangle + \langle Q_2x(t-h(t)),x(t-h(t))\rangle + \langle Ru(t),u(t)\rangle \\
&= \langle PQ_1Py(t),y(t)\rangle + \langle PQ_2Py(t-h(t)),y(t-h(t))\rangle \\
&\quad + 0.25\left\langle BRB^Ty(t),y(t)\right\rangle, \\
2\langle W_0f(x),y\rangle &\le \left\langle W_0D_0W_0^Ty,y\right\rangle + \left\langle D_0^{-1}f(x),f(x)\right\rangle, \\
2\langle W_1g(z),y\rangle &\le \left\langle W_1D_1W_1^Ty,y\right\rangle + \left\langle D_1^{-1}g(z),g(z)\right\rangle, \\
2\left\langle D_0^{-1}f(x),f(x)\right\rangle &\le \left\langle FD_0^{-1}Fx,x\right\rangle, \\
2\left\langle D_1^{-1}g(z),g(z)\right\rangle &\le \left\langle ED_1^{-1}Ez,z\right\rangle, \\
2\langle E_aF_a(t)H_aPy,y\rangle &\le \left\langle \epsilon_1E_a^TE_ay,y\right\rangle + \left\langle \epsilon_1^{-1}PH_a^TH_aPy,y\right\rangle, \quad \epsilon_1 > 0, \\
2\langle E_{w_0}F_{w_0}(t)H_{w_0}Pf(x),y\rangle &\le \left\langle \epsilon_2E_{w_0}^TE_{w_0}y,y\right\rangle + \left\langle \epsilon_2^{-1}PD_0H_{w_0}^TH_{w_0}D_0Py,y\right\rangle, \quad \epsilon_2 > 0, \\
2\langle E_{w_1}F_{w_1}(t)H_{w_1}Pg(z),y\rangle &\le \left\langle \epsilon_3E_{w_1}^TE_{w_1}y,y\right\rangle + \left\langle \epsilon_3^{-1}PD_1H_{w_1}^TH_{w_1}D_1Pz,z\right\rangle, \quad \epsilon_3 > 0,
\end{aligned} \tag{3.21}$$

we obtain

$$\begin{aligned}
\dot{V}(\cdot) + 2\alpha V(\cdot) &\le \zeta^T(t)\mathcal{E}\zeta(t) + y^T(t)S_1y(t) + y^T(t-h(t))S_2y(t-h(t)) \\
&\quad - f^0(t,x(t),x(t-h(t)),u(t)),
\end{aligned} \tag{3.22}$$

where $\zeta(t) = [y(t), \dot{y}(t), y(t-h_0), y(t-h_1), y(t-h(t))]$, and

$$\mathcal{E} = \begin{bmatrix} W_{11} & W_{12} & W_{13} & W_{14} & W_{15} \\ * & W_{22} & W_{23} & W_{24} & W_{25} \\ * & * & W_{33} & W_{34} & W_{35} \\ * & * & * & W_{44} & W_{45} \\ * & * & * & * & W_{55} \end{bmatrix},$$

$$S_1 = -PA - A^T P - \sum_{i=0}^{1} e^{-2\alpha h_i} H_i + 4PFD_0^{-1}FP + PQ_1P,$$

$$S_2 = W_1 D_1 W_1^T - e^{-2\alpha h_2} U + 4PED_1^{-1}EP + PQ_2P. \tag{3.23}$$

Note that by the Schur complement lemma, Proposition 2.3, the conditions $S_1 < 0$ and $S_2 < 0$ are equivalent to the conditions (3.3) and (3.4), respectively. Therefore, by condition (3.2), (3.3), and (3.4), we obtain from (3.22) that

$$\dot{V}(t, x_t) \leq -2\alpha V(t, x_t), \quad \forall t \geq 0. \tag{3.24}$$

Integrating both sides of (3.24) from 0 to t, we obtain

$$V(t, x_t) \leq V(\phi) e^{-2\alpha t}, \quad \forall t \geq 0. \tag{3.25}$$

Furthermore, taking condition (3.9) into account, we have

$$\lambda_1 \|x(t,\phi)\|^2 \leq V(x_t) \leq V(\phi) e^{-2\alpha t} \leq \lambda_2 e^{-2\alpha t} \|\phi\|^2, \tag{3.26}$$

then

$$\|x(t,\phi)\| \leq \sqrt{\frac{\lambda_2}{\lambda_1}} e^{-\alpha t} \|\phi\|, \quad t \geq 0, \tag{3.27}$$

which concludes the exponential stability of the closed-loop system (2.8). To prove the optimal level of the cost function (2.4), we derive from (3.22) and (3.2)–(3.4) that

$$\dot{V}(t, z_t) \leq -f^0(t, x(t), x(t-h(t)), u(t)), \quad t \geq 0. \tag{3.28}$$

Integrating both sides of (3.28) from 0 to t leads to

$$\int_0^t f^0(t, x(t), x(t-h(t)), u(t)) dt \leq V(0, z_0) - V(t, z_t) \leq V(0, z_0), \tag{3.29}$$

dute to $V(t, z_t) \geq 0$. Hence, letting $t \to +\infty$, we have

$$J = \int_0^{\infty} f^0(t, x(t), x(t-h(t)), u(t))dt \leq V(0, z_0) \leq \lambda_2 \|\phi\|^2 = J^*. \tag{3.30}$$

This completes the proof of the theorem. □

Remark 3.2. Note that $h(t)$ is non-differentiable and interval time-varying delay; therefore, the stability criteria proposed in [5–8, 12, 15–26] are not applicable to this system.

Example 3.3. Consider the uncertain neural networks with interval time-varying delays (2.1), where

$$\begin{gathered}
A = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.3 \end{bmatrix}, \quad W_0 = \begin{bmatrix} 0.1 & 0.1 \\ 0.2 & 0.3 \end{bmatrix}, \quad W_1 = \begin{bmatrix} 0.2 & 0.2 \\ 0.1 & 0.4 \end{bmatrix}, \quad B = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}, \\
E = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.4 \end{bmatrix}, \quad F = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.3 \end{bmatrix}, \quad Q_1 = \begin{bmatrix} 0.2 & 0.1 \\ 0.1 & 0.4 \end{bmatrix}, \quad Q_2 = \begin{bmatrix} 0.3 & 0.2 \\ 0.2 & 0.5 \end{bmatrix}, \\
R = \begin{bmatrix} 0.1 & 0.1 \\ 0.1 & 0.3 \end{bmatrix}, \quad E_a = \begin{bmatrix} 0.1 & 0.1 \\ 0.1 & 0.3 \end{bmatrix}, \quad E_{w_0} = \begin{bmatrix} 0.1 & 0.1 \\ 0.1 & 0.2 \end{bmatrix}, \quad E_{w_1} = \begin{bmatrix} 0.2 & 0.1 \\ 0.1 & 0.3 \end{bmatrix}, \\
H_a = \begin{bmatrix} 0.3 & 0.2 \\ 0.2 & 0.2 \end{bmatrix}, \quad H_{w_0} = \begin{bmatrix} 0.2 & 0.1 \\ 0.1 & 0.2 \end{bmatrix}, \quad H_{w_1} = \begin{bmatrix} 0.3 & 0.1 \\ 0.1 & 0.3 \end{bmatrix}, \\
h(t) = 0.1 + 1.3 \sin^2 t \quad \text{if } t \in \mathcal{I} = \bigcup_{k \geq 0} [2k\pi, (2k+1)\pi] \\
h(t) = 0 \quad \text{if } t \in R^+ \setminus \mathcal{I}.
\end{gathered} \tag{3.31}$$

Note that $h(t)$ is non-differentiable; therefore, the stability criteria proposed in [4–8, 12, 15–26] are not applicable to this system. Given $\alpha = 0.1$, $h_0 = 0.1$, and $h_1 = 1.4$, by using the Matlab LMI toolbox, we can solve for P, U, G_0, G_1, H_0, H_1, D_0, and D_1 which satisfy the conditions (3.2)–(3.4) in Theorem 3.1. A set of solutions are $\epsilon_1 = 0.0017$, $\epsilon_2 = 0.0013$, $\epsilon_3 = 0.0012$,

$$\begin{gathered}
P = \begin{bmatrix} 1.1578 & -0.1128 \\ -0.1128 & 1.0597 \end{bmatrix}, \quad U = \begin{bmatrix} 2.3269 & -0.3820 \\ -0.3820 & 2.6681 \end{bmatrix}, \\
G_0 = \begin{bmatrix} 1.4596 & 0.1397 \\ 0.1397 & 1.2369 \end{bmatrix}, \quad G_1 = \begin{bmatrix} 2.2694 & 0.8114 \\ 0.8114 & 1.0125 \end{bmatrix}, \\
H_0 = \begin{bmatrix} 0.6455 & 0.0452 \\ 0.0452 & 0.5104 \end{bmatrix}, \quad H_1 = \begin{bmatrix} 0.3005 & 0.0233 \\ 0.0233 & 0.2306 \end{bmatrix}, \\
D_0 = \begin{bmatrix} 0.0011 & 0 \\ 0 & 0.0011 \end{bmatrix}, \quad D_1 = \begin{bmatrix} 0.7809 & 0 \\ 0 & 0.7809 \end{bmatrix}.
\end{gathered} \tag{3.32}$$

Then

$$u(t) = -0.2292 x_1(t) - 0.1816 x_2(t), \quad t \geq 0 \tag{3.33}$$

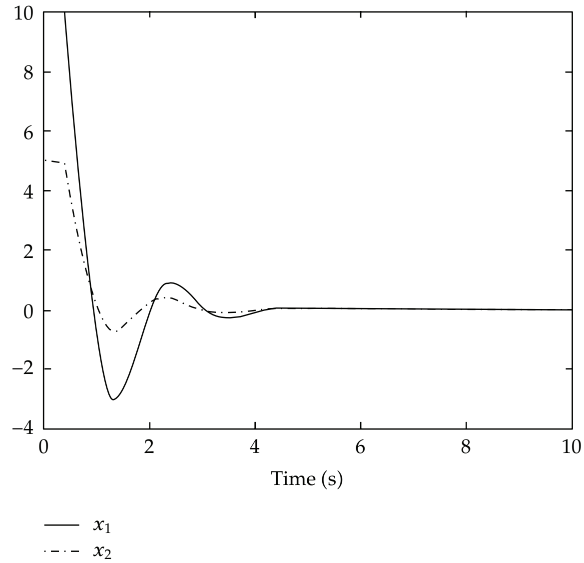

Figure 1: The simulation of the solutions $x_1(t)$ and $x_2(t)$ with the initial condition $\phi(t[10\ 5]^T,\ t \in [-0.4, 0]$.

is a guaranteed cost control law and the cost given by

$$J^* = 5.4631\|\phi\|^2. \tag{3.34}$$

Moreover, the solution $x(t,\phi)$ of the system satisfies

$$\|x(t,\phi)\| \leq 3.6984e^{-0.1t}\|\phi\|, \quad \forall t \geq 0. \tag{3.35}$$

The exponential convergence dynamics of the network (2.1) are shown in Figure 1.

Example 3.4. Consider the uncertain neural networks with interval time-varying delays (2.1),where

$$\begin{gathered}
A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}, \qquad W_0 = \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}, \qquad W_1 = \begin{bmatrix} 2 & 2 \\ 1 & 4 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \\
E = \begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix}, \qquad F = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}, \qquad Q_1 = \begin{bmatrix} 2 & 1 \\ 1 & 4 \end{bmatrix}, \qquad Q_2 = \begin{bmatrix} 3 & 2 \\ 2 & 5 \end{bmatrix}, \\
R = \begin{bmatrix} 1 & 1 \\ 1 & 3 \end{bmatrix}, \qquad E_a = \begin{bmatrix} 1 & 1 \\ 1 & 3 \end{bmatrix}, \qquad E_{w_0} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}, \qquad E_{w_1} = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}, \\
H_a = \begin{bmatrix} 3 & 2 \\ 2 & 2 \end{bmatrix}, \qquad H_{w_0} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}, \qquad H_{w_1} = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}, \\
h(t) = 0.1 + 0.7\sin^2 t \quad \text{if } t \in \mathcal{I} = \bigcup_{k \geq 0} [2k\pi, (2k+1)\pi] \\
h(t) = 0 \quad \text{if } t \in R^+ \setminus \mathcal{I}.
\end{gathered} \tag{3.36}$$

Note that $h(t)$ is non-differentiable; therefore, the stability criteria proposed in [5–8, 12, 15–26] are not applicable to this system. Given $\alpha = 0.3$, $h_0 = 0.1, h_1 = 0.8$, by using the Matlab LMI toolbox, we can solve for P, U, G_0, G_1, H_0, H_1, D_0, and D_1 which satisfy the conditions (3.2)–(3.4) in Theorem 3.1. A set of solutions are $\epsilon_1 = 0.9$, $\epsilon_2 = 0.8$, $\epsilon_3 = 0.7$,

$$\begin{aligned} P &= \begin{bmatrix} 0.7832 & -0.0213 \\ -0.0213 & 0.0011 \end{bmatrix}, & U &= \begin{bmatrix} 0.1297 & -0.0019 \\ -0.0019 & 0.0197 \end{bmatrix}, \\ G_0 &= \begin{bmatrix} 0.1795 & 0.0137 \\ 0.0137 & 0.2211 \end{bmatrix}, & G_1 &= \begin{bmatrix} 1.2197 & 0.9648 \\ 0.9648 & 0.7391 \end{bmatrix}, \\ H_0 &= \begin{bmatrix} 0.8931 & 0.1183 \\ 0.1183 & 0.7197 \end{bmatrix}, & H_1 &= \begin{bmatrix} 0.6851 & 0.1297 \\ 0.1297 & 0.5726 \end{bmatrix}, \\ D_0 &= \begin{bmatrix} 0.1397 & 0 \\ 0 & 0.2278 \end{bmatrix}, & D_1 &= \begin{bmatrix} 0.6812 & 0 \\ 0 & 0.6813 \end{bmatrix}. \end{aligned} \tag{3.37}$$

Then

$$u(t) = -0.7314x_1(t) - 0.0196x_2(t), \quad t \geq 0, \tag{3.38}$$

is a guaranteed cost control law and the cost given by

$$J^* = 24.3219\|\phi\|^2. \tag{3.39}$$

Moreover, the solution $x(t,\phi)$ of the system satisfies

$$\|x(t,\phi)\| \leq 12.3690e^{-0.3t}\|\phi\|, \quad \forall t \geq 0. \tag{3.40}$$

The exponential convergence dynamics of the network (2.1) are shown in Figure 2.

4. Conclusions

In this paper, the problem of guaranteed cost control for uncertain neural networks with interval nondifferentiable time-varying delay has been studied. A nonlinear quadratic cost function is considered as a performance measure for the closed-loop system. The stabilizing controllers to be designed must satisfy some exponential stability constraints on the closed-loop poles. By constructing a set of time-varying Lyapunov-Krasovskii functionals combined with Newton-Leibniz formula, a memoryless state feedback guaranteed cost controller design has been presented, and sufficient conditions for the existence of the guaranteed cost state-feedback for the system have been derived in terms of LMIs.

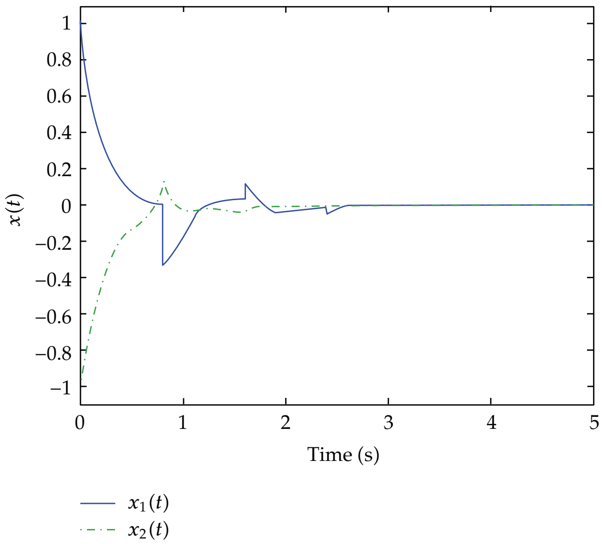

Figure 2: The simulation of the solutions $x_1(t)$ and $x_2(t)$ with the initial condition $\phi(t) = [1\ -1]^T$, $t \in [-1, 0.8]$.

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