

Research Article

Determination of the Creep Parameters of Linear Viscoelastic Materials

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Creep process of linear viscoelastic materials is described by the integral equation of Boltzmann-Volterra in which creep kernel is approximated by Rabotnov's fractional exponential function. The creep equation contains four unknown parameters: α , singularity parameter; β , fading parameter; λ , rheological parameter; and ε_0 , conditionally instantaneous strain. Two-stage determination method of creep parameters is offered. At the first stage, taking into account weak singularity properties of Abel's function at the initial moment of loading, parameters ε_0 and α are determined. At the second stage, using already known parameters ε_0 and α , parameters β and λ are determined. Analytical expressions for calculating these parameters are obtained. An accuracy evaluation of the offered method with using experimentally determined creep strains of material Nylon 6 and asphalt concrete showed its high accuracy.

1. Introduction

Many materials used in technique and production (rocks, soils, metals, concretes, polymers, composites, etc.) under loading show viscoelastic properties. Currently a number of methods for quantitative evaluation of viscoelastic properties of materials are known [1–4]. One of the most frequently used methods is creep testing under uniaxial stress [5]. In the mathematical description of creep process of linear viscoelastic material the integral equation of Boltzmann-Volterra with corresponding creep kernel is widely used. A large number of analytical expressions for the creep kernel are known [1, 4, 5].

In testing samples of viscoelastic materials on creep, usually it is difficult to measure instantaneous elastic strain $\varepsilon_0 = \varepsilon(t = 0)$. This is due to the fact that in the initial time after application of load to sample the rate of creep is extremely high. Its accurate measurement is practically impossible due to the dynamic effects caused by inertia of loading devices and measuring equipment.

Creep process of many materials particularly at small loads and low temperatures proceeds for a very long time. At

high times of loading the rate of creep starts to fade and strain slowly approaching asymptotic value.

Thus, we can assume that creep kernel of integral equation must have the following two properties: firstly, having a weak singularity at the initial time point and, secondly, having a property of exponential function at high times of loading.

The most universal description of viscoelastic properties of materials satisfying the above requirements is a kernel in the form of fractional exponential function of Rabotnov [4–7]. Fractional exponential function is well studied and for simplification of calculations by using it, a special table has been developed [8]. Another advantage of the fractional exponential function is that for known creep kernel parameters constructed with using the fractional exponential function a kernel of relaxation becomes definite [9, 10].

Some methods for determining the parameters of Rabotnov's creep kernel are known: using Mittag-Leffler's function [11, 12], using Laplace-Carson's transform [9, 12–14], and a method of direct approximation [9, 12–15].

In this paper we offer a new double stage method for determining the creep kernel parameters of linear viscoelastic

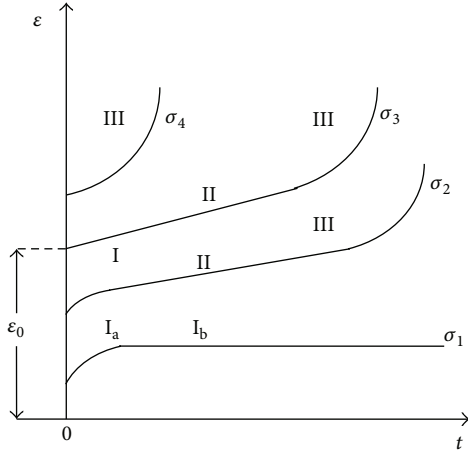


FIGURE 1: Various types of creep curves.

materials constructed with using Rabotnov's fractional exponential function. At the first stage, taking into account weak singularity properties of Abel's function at the initial moment of loading, parameters ϵ_0 and α are determined. At the second stage, using already known parameters ϵ_0 and α , parameters β and λ are determined.

Fractional derivative constitutive models for finite deformation of viscoelastic materials developed with using a kernel close to Abel's kernel are given in [16]. A new model for the linear theory of generalized thermoviscoelasticity using a relaxation kernel close to Abel's kernel is demonstrated in [17].

2. Creep Equation and Kernel

As is known generally creep curve of the viscoelastic material has three characteristic sites: I, site with a decreasing rate of creep (unstabilized creep), II, site with a constant (minimum) rate of creep (stabilized creep), and III, site of accelerating creep, preceding failure (Figure 1).

In the present work sites I and II of creep curves of linear viscoelastic materials under uniaxial tension are described by the integral equation of Boltzmann-Volterra:

$$\epsilon(t) = \frac{1}{E} \left[\sigma(t) + \int_0^t K(t-\tau) \sigma(\tau) d\tau \right], \quad (1)$$

where $\epsilon(t)$ is strain at time point t , $\sigma(t)$, $\sigma(\tau)$ are stresses at time points t and τ , $K(t-\tau)$ is creep kernel, E is instant modulus of elasticity, t is observation time, and τ is time, preceding observation time.

Taking into account that for creep process $\sigma_0 = \text{const}$, from (1) we will find

$$\epsilon(t) = \epsilon_0 \left[1 + \int_0^t K(t-\tau) d\tau \right], \quad (2)$$

where ϵ_0 is conditionally instantaneous strain.

In (2) it was accounted that on Hooke's law $\sigma_0 = E \cdot \epsilon_0$.

As mentioned above, creep kernel recorded using Rabotnov's fractional exponential function in the following form [6]:

$$\begin{aligned} K(t-\tau) &= \lambda \vartheta_{-\alpha}(-\beta, t-\tau) \\ &= \lambda (t-\tau)^{-\alpha} \sum_{n=0}^{\infty} \frac{(-\beta)^n (t-\tau)^{(1-\alpha)n}}{\Gamma[(1-\alpha)(1+n)]}, \end{aligned} \quad (3)$$

where $\vartheta_{-\alpha}(-\beta, t-\tau)$ is Rabotnov's fractional exponential function, α , β , and λ are creep kernel parameters ($\lambda > 0$, $0 < \alpha < 1$, and $\beta > 0$) and Γ is gamma-function.

Substituting expression (3) for creep kernel in the creep equation (2) we obtain the following equation:

$$\epsilon(t) = \epsilon_0 \left[1 + \lambda \sum_{n=0}^{\infty} \frac{(-\beta)^n t^{(1-\alpha)(1+n)}}{\Gamma[(1-\alpha)(1+n)+1]} \right]. \quad (4)$$

Equation (4) represents creep equation recorded using Rabotnov's fractional exponential function. It contains four unknown parameters: α is the singularity parameter; β is the attenuation parameter; λ is the rheological parameter; and ϵ_0 is conditionally instantaneous strain.

3. Method of Determining the Creep Parameters

3.1. Parameters ϵ_0 and α . Taking $n = 0$ from series (4) we find the first term:

$$\epsilon(t) = \epsilon_0 \left[1 + \frac{\delta}{1-\alpha} t^{1-\alpha} \right]. \quad (5)$$

As one can see, obtained expression contains well-known Abel's function with parameter $\delta > 0$. Abel's function at $t = 0$ has a singularity at order α . Based on this property of Abel's function, unknown parameters ϵ_0 and δ are determined by use of (5).

Knowing that parameter α takes value from interval $(0, 1)$ we will consider as unknown only parameters ϵ_0 and δ .

According to the least squares method the best values of parameters ϵ_0 and δ are those for which the following condition is met:

$$S(\epsilon_0, \delta) = \sum_{i=1}^m \left[\epsilon_0 \left(1 + \frac{\delta}{1-\alpha} t_i^{1-\alpha} \right) - \epsilon_{ei} \right]^2 \rightarrow \min, \quad (6)$$

where $S(\epsilon_0, \delta)$ is sum of squares of deviations, ϵ_{ei} is values of creep strain determined experimentally, and m is number of creep strains.

From two equations based on expressions $\partial S(\epsilon_0, \delta) / \partial \epsilon_0 = 0$ and $\partial S(\epsilon_0, \delta) / \partial \delta = 0$, we find expressions for determining the parameters ϵ_0 and δ :

$$\epsilon_0 = \frac{\sum_{i=1}^m \epsilon_i \sum_{i=1}^m t_i^{2(1-\alpha)} - \sum_{i=1}^m t_i^{(1-\alpha)} \sum_{i=1}^m \epsilon_i t_i^{(1-\alpha)}}{m \sum_{i=1}^m t_i^{2(1-\alpha)} - \left[\sum_{i=1}^m t_i^{(1-\alpha)} \right]^2}, \quad (7)$$

$$\delta = \frac{\sum_{i=1}^m (\epsilon_i / \epsilon_0 - 1) t_i^{(1-\alpha)}}{(1 / (1 - \alpha)) \sum_{i=1}^m t_i^{2(1-\alpha)}}. \quad (8)$$

Setting values of parameter α from interval (0, 1) with a certain step from expression (7) we will find values of parameter $\varepsilon_0 = \varepsilon_0(\alpha)$. Substituting the found values of parameter ε_0 and the corresponding values of singularity parameter α in expression (8) values of parameter $\delta = \delta(\varepsilon_0, \alpha)$ are determined.

Further, sequentially substituting values of singularity parameter α and calculated corresponding values of parameters ε_0 and δ in expression (5), values of creep strain $\varepsilon(t_i) = \varepsilon(t_i, \alpha, \varepsilon_0, \delta)$ are calculated.

If we designate an average deviation of calculated values of creep strains from experimental values through $\Delta\varepsilon_m(\lambda, \varepsilon_0, \delta)$, as a selecting criterion of the best values of parameters α, ε_0 , and δ the following condition can be accepted:

$$\Delta\varepsilon_{mi}(\alpha_i, \varepsilon_{0i}, \delta_i) \longrightarrow \min. \quad (9)$$

3.2. Parameters β and λ . Rewrite equation of creep (4) in the following form:

$$\varepsilon(t) = \varepsilon_0 \left(1 + \lambda t^{1-\alpha} F_{2i}\right), \quad (10)$$

$$\text{where } F_{2i} = \sum_{n=0}^{\infty} \frac{(-\beta)^n t^{(1-\alpha)n}}{\Gamma[(1-\alpha)(1+n)+1]}. \quad (11)$$

According to the least squares method analogically to condition (6) we write an extremum condition using the equation of creep (10):

$$S(\beta, \lambda) = \sum_{i=1}^m \left[\varepsilon_0 \left(1 + \lambda t_i^{1-\alpha} F_{2i}\right) - \varepsilon_{ei}\right]^2 \longrightarrow \min. \quad (12)$$

From two equations based on the expressions $\partial S(\beta, \lambda) / \partial \beta = 0$ and $\partial S(\beta, \lambda) / \partial \lambda = 0$, expressions for determining the parameters β and λ are found:

$$\sum_{i=1}^m \left[\left(1 - \frac{\varepsilon_i}{\varepsilon_0}\right) \sum_{i=1}^m \left(t_i^{1-\alpha} F_{2i}\right)^2 + t_i^{1-\alpha} F_{2i} \sum_{i=1}^m \left(\frac{\varepsilon_i}{\varepsilon_0} - 1\right) t_i^{1-\alpha} F_{2i} \right] \left(t_i^{1-\alpha}\right)^2 F_{3i} = 0, \quad (13)$$

$$\lambda = \frac{\sum_{i=1}^m (\varepsilon_i / \varepsilon_0 - 1) t_i^{1-\alpha} F_{2i}}{\sum_{i=1}^m \left(t_i^{1-\alpha} F_{2i}\right)^2}, \quad (14)$$

$$\text{where } F_{3i} = \sum_{n=1}^{\infty} (-1)^n \frac{n(\beta)^{n-1} t_i^{(1-\alpha)(n-1)}}{\Gamma[(1-\alpha)(n+1)+1]}, \quad (15)$$

but series F_{2i} is determined by expression (11).

Value of parameter of fading β is determined from (13) by substituting trial values. If (13) has a unique solution, it will be desired value of the parameter $\beta > 0$. Obviously, for determining the values of parameters β and λ from expressions (13) and (14) earlier calculated values of parameters ε_0 and α are used.

TABLE 1: Strains in site I of material Nylon 6 creep curve.

Time t, h	Creep strain (%) calculated by expression		Deviation, %
	(16)	(17)	
0,01	0,2919	0,2957	+1,30
0,03	0,3184	0,3186	+0,06
0,05	0,3315	0,3306	-0,27
0,07	0,3404	0,3389	-0,44
0,09	0,3472	0,3455	-0,49
0,10	0,3501	0,3483	-0,51
0,30	0,3819	0,3806	-0,34
0,50	0,3976	0,3975	-0,01
0,70	0,4083	0,4094	+0,27
0,90	0,4165	0,4187	+0,53

4. Experimental Approxobation

4.1. Material Nylon 6. In work [9] the creep curve of Nylon 6 at a stress of $\sigma_0 = 5$ MPa was approximated by expression

$$\varepsilon_e(t) = 0,42t^{0,079}, \quad (16)$$

where ε [%] is strain and t [h] is time.

Firstly, for determination of parameters ε_0 and α we calculate some strain values in time interval $t = 0,01 \dots 0,9$ h. Time values and corresponding creep strain values are given in the first and the second columns of Table 1.

Substituting values of time t , strain ε , and various values of singularity parameter α from interval $0,05 \dots 0,95$ with step of 0,05 in expressions (7) and (8) and taking into account condition (9) it has been found that $\alpha = 0,85$; $\varepsilon_0 = 0,1682$; $\delta = 0,2269$.

With consideration of found values of parameters α, ε_0 , and δ , (5) constructed using Abel's function for describing strain in site I of creep curve takes the form

$$\varepsilon(t) = 0,1682 \left(1 + 1,5127t^{0,15}\right). \quad (17)$$

Calculated by (17) values of creep strain and corresponding approximation errors are presented in the third and the fourth columns of Table 1.

As can be seen from Table 1, (17) with parameters ε_0 and α found by the offered method has high accuracy of approximation.

Since the parameter β characterizes rate of material creep strain at long times, for definition it and parameter λ consider creep process of material Nylon 6 in the time interval $t = 0,01 \dots 100$ h. Time values and corresponding values of creep strain calculated according to expression (16) are given in the first and the second columns of Table 2.

Using values of time t , strain ε , parameters $\alpha = 0,85, \varepsilon_0 = 0,1682$, and trial values of parameter $\beta > 0$, from expression (13) it has been found that $\beta = 0,18$. Further from expression (14) we will define $\lambda = 1,6682$.

TABLE 2: Strains in sites I_a and I_b of material Nylon 6 creep curve.

Time t, h	Creep strain (%) calculated by expression		Deviation, %
	(16)	(18)	
0,01	0,2919	0,3060	+4,83
0,03	0,3184	0,3282	+3,08
0,05	0,3315	0,3396	+2,44
0,07	0,3404	0,3474	+2,06
0,09	0,3472	0,3535	+1,82
0,1	0,3501	0,3561	+1,72
0,3	0,3819	0,3852	+0,86
0,5	0,3976	0,3999	+0,58
0,7	0,4083	0,4102	+0,46
0,9	0,4165	0,4180	+0,36
1	0,4200	0,4214	+0,33
3	0,4581	0,4584	+0,07
5	0,4769	0,4770	+0,02
7	0,4898	0,4898	0,00
9	0,4996	0,4996	0,00
10	0,5038	0,5038	0,00
20	0,5321	0,5321	0,00
30	0,5495	0,5495	0,00
40	0,5621	0,5621	0,00
50	0,5721	0,5721	0,00
60	0,5804	0,5804	0,00
70	0,5875	0,5875	0,00
80	0,5937	0,5937	0,00
90	0,5993	0,5992	0,00
100	0,6043	0,6042	0,00

After substitution values of the found parameters ε_0 , α , β , and λ (10) and auxiliary function F_{2i} (11) will have the next form:

$$\varepsilon(t) = 0,1682 \left(1 + 1,6682t^{0,15} F_{2i} \right), \quad (18)$$

$$F_{2i} = \sum_{n=0}^{\infty} \frac{(-0,18)^n t^{0,15n}}{\Gamma[0,15(1+n)+1]}. \quad (19)$$

Calculated by (18) values of creep strain and corresponding errors of approximation are given in the last two columns of Table 2.

As can be seen from Table 2 creep (18) with parameters ε_0 , α , β , and λ defined by the offered method has high accuracy of approximation.

4.2. Asphalt Concrete. The strains calculated by approximation formula (16) were taken for determination of the above creep parameters of material Nylon 6 instead of strains obtained experimentally. The reason for this was the fact that in work [9] the creep strains were given only graphically in the form of a creep curve and numerical values of the strain are not shown. It appeared almost impossible to find the strain

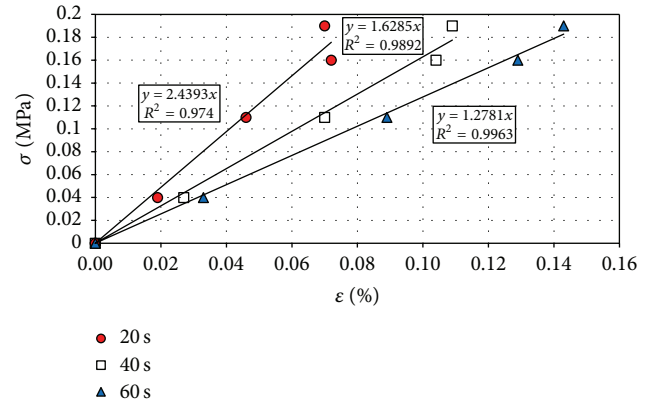


FIGURE 2: Isochronous lines of deformation of asphalt concrete.

values from the graphical creep curve with sufficient accuracy especially at short times of loading.

Possibly, in order to reduce the length of material in published works containing an approximation of creep curves of various materials, as a rule, information on creep strain is given graphically, for example, in works [9, 10, 12–14, 18–20].

Further, we will show determination of creep parameters ε_0 and α by the above offered method (expressions (7) and (8)) using experimentally determined values of strain at site I of creep of hot fine-grained asphalt concrete. This type of asphalt is used for application on top layer of road surface in many countries, including Kazakhstan. Asphalt concrete mix that meets the requirements of the Kazakhstan standard ST RK 1225 [21] was prepared with use of viscous road bitumen of grade BND 100/130 from Pavlodar petrochemical plant, which meets the standard requirements of ST RK 1373 [22]. Samples of asphalt concrete with rectangular cross-section dimensions of $5 \times 5 \times 15$ cm were prepared by means of Cooper compactor (UK, model CRT-RC2S) in accordance with European Union standard EN 12697-33 [23]. Detailed information on preparation of asphalt mixtures and samples, as well as standard properties of bitumen and asphalt concrete, is given in [24]. Samples of asphalt concrete were tested for creep according to the scheme of uniaxial tension at a temperature of $20 \pm 2^\circ\text{C}$ and different loads.

Isochronous lines of deformation for the asphalt concrete constructed using creep curves within site I at durations of loadings 20, 40, and 60 s are shown in Figure 2. As it can be seen dependence between stress and strain is linear; that is, asphalt concrete under these conditions is linearly deformable material and for description of its creep process the equation of Boltzmann-Volterra is applicable.

Figures 3 and 4 show site I of creep curves of asphalt concrete at stresses of 0.16 and 0.19 MPa, respectively. In these figures points designate the creep strain defined experimentally and lines designate its approximation by Abel's kernel (5). It is seen that compliance of approximating curves to the experimental data is quite good. Average deviations of calculated strains from experimental creep curves at 0,16 and 0,19 MPa are 1.5% and 0.5%, respectively. The values of parameters ε_0 and α found by the offered method are equal to

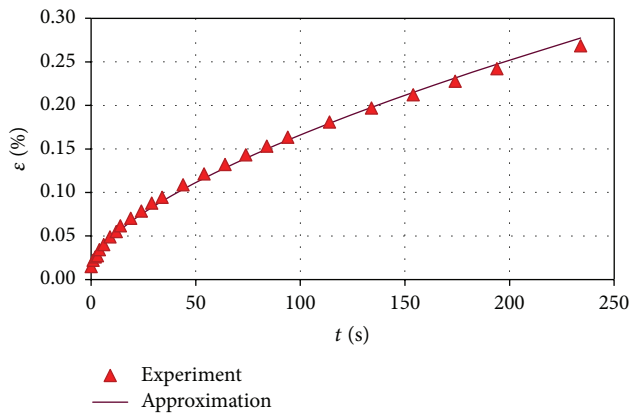


FIGURE 3: Creep curve of asphalt concrete at stress $\sigma_0 = 0,16$ MPa.

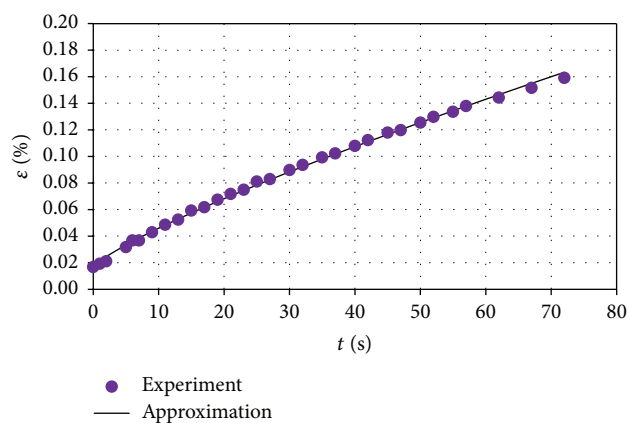


FIGURE 4: Creep curve of asphalt concrete at stress $\sigma_0 = 0,19$ MPa.

$\varepsilon_0 = 0,015217\%$ and $\alpha = 0,35$ at $\sigma_0 = 0,16$ MPa and $\varepsilon_0 = 0,016871\%$ and $\alpha = 0,18$ at $\sigma_0 = 0,19$ MPa.

Thus, the integral equation of Boltzmann-Volterra with creep kernel constructed using Rabotnov's fractional exponential function with parameters defined by the offered method describes the creep process of linear viscoelastic materials with high accuracy.

5. Conclusions

- (1) Describing the creep process of linear viscoelastic materials by the integral equation of Boltzmann-Volterra with creep kernel which is constructed using Rabotnov's fractional exponential function is suggested.
- (2) A new method of determining the creep kernel parameters of linear viscoelastic materials constructed using Rabotnov's fractional exponential function is offered. It is implemented in two stages. At the first stage based on the property of singularity of Abel's function at the initial moment of time parameters ε_0 and α are defined. At the second stage using already known parameters ε_0 and α parameters β and

λ are defined. Analytical expressions for calculating these parameters are obtained.

- (3) Applicability of the offered method is evaluated using data on creep strain of material Nylon 6 and hot fine-grained asphalt concrete obtained experimentally. The results showed high accuracy of the offered method.

Competing Interests

The authors declare that they have no competing interests.

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