Couplings varying on cosmological scales and Lorentz breaking

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Abstract. In the context of $N = 4$ supergravity in four dimensions, we present an exact classical solution that leads to spacetime-dependent electromagnetic couplings and discuss the ensuing Lorentz-violating effects. We comment briefly on experimental bounds.

1. INTRODUCTION

From a theoretical point of view, our current understanding of nature at the fundamental level leaves unresolved a variety of issues, so that present-day physical theories are believed to be the low-energy limit of some underlying framework. Any effects from an underlying theory involving gravity are expected to be minuscule due to the likely suppression by at least one power of the Planck mass. In such a situation, it appears practical to consider violations of symmetries that hold exactly in our present fundamental laws, might be violated in approaches to underlying physics, and are amenable to high-precision experiments.

Although spacetime symmetries are a cornerstone of all known physics, they might be violated at a more fundamental level: in the context of string field theory, an explicit mechanism for the spontaneous breaking of Lorentz invariance exists [1]. Other examples of Lorentz-violating frameworks include spacetime foam [2], nontrivial spacetime topology [3], realistic noncommutative field theories [4], and loop quantum gravity [5]. Moreover, Lorentz tests are currently among the most precise null experiments available. Thus, spacetime-symmetry investigations provide a promising tool in the search for underlying physics [6].

The low-energy effects of Lorentz breaking are described by a general Standard-Model Extension [7], which has been constructed to contain all coordinate-invariant lagrangian terms formed by combining conventional field operators and coefficients carrying Lorentz indices. Although these terms are observer Lorentz symmetric, they explicitly break invariance under boosts and rotations of particles [8]. The Standard-Model Extension has provided the theoretical framework for the analysis of numerous Lorentz-symmetry tests involving hadrons [9, 10], protons and neutrons [11], leptons [12, 13, 14], photons [15, 16, 17, 18], muons [19], and neutrinos [20].

In the present work, we investigate the relation between Lorentz breaking and violations of translation invariance [21]. More specifically, we argue that scalar couplings varying on cosmological scales also lead to the type of Lorentz violation described by the Standard-Model Extension. Since both Lorentz transformations and spacetime trans-

lations are interwoven in the Poincaré group, such a result does not come as a surprise. Intuitively, the behavior of the vacuum is that of a spacetime-varying medium so that isotropy, for example, can be lost in certain local inertial frames.

Early work in the field of spacetime-dependent couplings includes Dirac's large-number hypothesis [22]. More recently, it has been realized that varying couplings are natural in many fundamental theories [23, 24], which provides an additional example for the fact that spacetime-symmetry violations are a promising candidate experimental signature for more fundamental physics. Investigations in this field are further motivated by current claims of observational evidence for a time-varying electromagnetic coupling [25]. The experimental status and theoretical ideas are reviewed in Ref. [26].

In the context of $N = 4$ supergravity in four dimensions, we demonstrate how smoothly varying couplings can naturally be obtained from a classical cosmological solution. In particular, the fine-structure parameter $\alpha = e^2/4\pi$ and the electromagnetic angle acquire related spacetime dependences leading to the aforementioned Lorentz-violating effects. Although the employed supergravity framework is known to be unrealistic in detail, it is contained in the $N = 1$ supergravity in 11 spacetime dimensions, which is a limit of M theory. Our approach can therefore yield some insight into generic features of a candidate fundamental theory.

This talk is organized as follows. In Sec. 2, we set up our supergravity model in the context of cosmology. In particular, we obtain an analytical solution to the equations of motion. Section 3 discusses the emergent time-varying couplings and comments on experimental constraints. Aspects of the associated Lorentz violation are investigated in Sec. 4. A short summary is contained in Sec. 5.

2. SUPERGRAVITY COSMOLOGY

The $N = 4$ supergravity in four spacetime dimensions contains in its spectrum a simple graviton represented by the metric $g_{\mu\nu}$, four gravitinos ψ_μ^j, six abelian graviphotons A_μ^{jk}, four fermions χ^j, and a complex scalar Z. Latin indices $j, k, \ldots$ transform under the internal SO(4) symmetry group, and the A_μ^{jk} lie in the adjoint representation. In Planck units, the bosonic part $\mathscr{L}$ of the lagrangian takes the form [27]

$$\mathscr{L} = \sqrt{g}\left(-\tfrac{1}{2}R - \tfrac{1}{4}M_{jklm}F_{\mu\nu}^{jk}F^{lm\mu\nu} - \tfrac{1}{8}N_{jklm}\varepsilon^{\mu\nu\rho\sigma}F_{\mu\nu}^{jk}F_{\rho\sigma}^{lm} + \frac{\partial_\mu Z \partial^\mu \overline{Z}}{(1 - Z\overline{Z})^2}\right). \quad (1)$$

The complex scalar Z determines the generalized electromagnetic coupling constant M_{jklm} and the generalized -term coupling N_{jklm}, which are both real:

$$M_{jklm} + iN_{jklm} = \tfrac{1}{2}(\delta_{jl}\delta_{km} - \delta_{jm}\delta_{kl})\frac{1 - Z^2}{1 + Z^2} - i\varepsilon_{jklm}\frac{Z}{1 + Z^2}. \quad (2)$$

Note also that Z contains an axion and a dilaton. It is convenient to isolate the dilaton piece B via a field redefinition. Employing the Cayley map $W = -i(Z-1)/(Z+1)$ and defining real fields A and B such that $W = A + iB$ yields the following expression for the

scalar kinetic term: $\mathscr{L}_b = \sqrt{g}(\partial_\mu A \partial^\mu A + \partial_\mu B \partial^\mu B)/4B^2$. The couplings M_{jklm} and N_{jklm} transform accordingly. The fermion kinetic terms are just

$$\mathscr{L}_{\text{fermion}} = \sqrt{g}\delta^{jk}\left(\overline{\psi}_j^\mu \gamma_{\mu\nu\rho} D^\nu \psi_k^\rho + \overline{\chi}_j \gamma_\mu D^\mu \chi_k\right). \tag{3}$$

Note that these are independent of the scalars A and B. There are also higher-order terms in the fermions coupled to the gauge fields and pieces that are quartic in the fermions.

In what follows, we look at situations in which only one graviphoton, $F_{\mu\nu}^{12} \equiv F_{\mu\nu}$, is excited. The bosonic lagrangian then takes the form

$$\mathscr{L} = \sqrt{g}\left(-\tfrac{1}{2}R - \tfrac{1}{4}MF_{\mu\nu}F^{\mu\nu} - \tfrac{1}{4}NF_{\mu\nu}\tilde{F}^{\mu\nu} + \frac{\partial_\mu A \partial^\mu A + \partial_\mu B \partial^\mu B}{4B^2}\right), \tag{4}$$

where we have abbreviated $\tilde{F}^{\mu\nu} = \varepsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}/2$, as usual. The electromagnetic and -term couplings become

$$M = \frac{B(A^2 + B^2 + 1)}{(1 + A^2 + B^2)^2 - 4A^2}, \qquad N = \frac{A(A^2 + B^2 - 1)}{(1 + A^2 + B^2)^2 - 4A^2}. \tag{5}$$

Next, we construct an exact classical solution within the model described by lagrangian (4). To this end, we consider a homogeneous and isotropic Universe, with a flat ($k = 0$) Friedmann-Robertson-Walker (FRW) line element given by $ds^2 = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2)$. With these assumptions, the scalars A and B and the scale factor a can only depend on the comoving time t. In the absence of energy-momentum sources other than the scalar fields, one of the Einstein equations reads $a\ddot{a} + 2\dot{a}^2 = 0$. Besides the trivial solution $a = \text{const.}$, this equation is solved by

$$a(t) = c\sqrt[3]{t}, \tag{6}$$

where c is an integration constant. This time evolution of the scale factor is far slower than the observed one. This is a consequence of the fact that the above approach fails to model the matter content of the Universe.

To describe a more realistic situation we refine our model by including the energy-momentum tensor of dust given by $T_{\mu\nu} = \rho u_\mu u_\nu$. Here, u^μ is a unit timelike vector orthogonal to the spatial hypersurfaces and $\rho(t)$ is the average energy density of galaxies and other matter. In the present context, this energy-momentum tensor is associated with the fermions in our model. Note that the fermionic sector does not couple directly to the scalar fields A and B, so that we take $T_{\mu\nu}$ as conserved separately:

$$\frac{d(\rho a^3)}{dt} = 0. \tag{7}$$

For the moment, we set $F_{\mu\nu}$ to zero and consider the equations of motion for our model. Variation of the action with respect to A and B yields:

$$\frac{d}{dt}\left(\frac{a^3\dot{A}}{B^2}\right) = 0, \quad \frac{d}{dt}\left(\frac{a^3\dot{B}}{B^2}\right) + \frac{a^3}{B^3}(\dot{A}^2 + \dot{B}^2) = 0, \tag{8}$$

where the dot indicates a derivative with respect to the comoving time t. Our supergravity model is also governed by the Einstein equations. Varying with respect to the metric and incorporating the energy-momentum tensor of dust gives:

$$G_{\mu\nu} = T_{\mu\nu} + \frac{1}{2B^2}(\partial_\mu A \partial_\nu A + \partial_\mu B \partial_\nu B) - \frac{1}{4B^2} g_{\mu\nu}(\partial_\lambda A \partial^\lambda A + \partial_\lambda B \partial^\lambda B). \qquad (9)$$

In the present context, the ten equations (9) contain only two independent ones:

$$-3\frac{\ddot{a}}{a} = \tfrac{1}{2}\rho + \frac{1}{2B^2}(\dot{A}^2 + \dot{B}^2), \qquad \frac{\ddot{a}}{a} + 2\frac{\dot{a}^2}{a^2} = \tfrac{1}{2}\rho. \qquad (10)$$

The energy-conservation equation (7) yields $\rho(t) = c_n/a^3(t)$. The integration constant c_n describes the amount of fermionic matter in our model. If the Universe has matter density ρ_n and scale size $a_n = a(t_n)$ at the present time t_n, then c_n obeys $c_n = \rho_n a_n^3$. This result can be used to integrate the second one of the Einstein equations (10), which determines the time evolution of the scale factor:

$$a(t) = \sqrt[3]{\tfrac{3}{4}c_n(t + t_0)^2 - c_1}, \qquad (11)$$

where c_1 and t_0 are integration constants with the following physical interpretations: c_1 controls the amount of energy stored in the scalar fields A and B, and t_0 sets the value of the comoving time t at the initial singularity. Our choice is $t_0 = \sqrt{4c_1/3c_n}$, which corresponds to $t = 0$ when $a(t) = 0$. Note that in this refined version of our supergravity model including the dust, the time dependence of the scale factor is $a(t) \sim t^{2/3}$ at late times $t \gg t_0$, as anticipated for a $k = 0$ matter-dominated Universe.

The equation of motion for A in (8) yields $\dot{A} = c_2 B^2/a^3$, where the integration constant c_2 has been introduced. The complete solution can most easily be obtained in terms of a parameter time τ defined by

$$t = t_0 \left(\coth \frac{\sqrt{3}}{4\tau} - 1 \right). \qquad (12)$$

Note that $\tau = 0$ at the initial singularity when $t = 0$, and τ increases when t increases. In terms of this parametric time, the fields A and B evolve according to

$$A = \pm\lambda \tanh(\frac{1}{\tau} + c_3) + A_0, \quad B = \lambda \operatorname{sech}(\frac{1}{\tau} + c_3). \qquad (13)$$

Here $\lambda \equiv \mp 4c_1/\sqrt{3}c_2 t_0$, and c_3 and A_0 are integration constants. In the remaining part of this work we take c_3 to be zero for simplicity. One can then verify that at late times on a scale set by t_0, the parameter time obeys $\tau \approx \sqrt{3}t/4t_0$. This implies that the late-time values of A and B are given by $\pm 4\lambda t_0/\sqrt{3}t + A_0$ and $\lambda(1 - 8t_0^2/3t^2)$, respectively. Thus, the axion and the dilaton tend to constant values. Note in particular, that this feature occurs for the string-theory dilaton B, despite the absence of a dilaton potential. This is basically a consequence of energy conservation.

3. SPACETIME-VARYING COUPLINGS

The next step is to allow small fluctuations of $F_{\mu\nu}$. For the moment, we take the axion-dilaton background determined by (13) as nondynamical. Many experiments are confined to spacetime regions small on cosmological scales. We will therefore continue our analysis in a local inertial frame.

The values of the couplings associated with the dynamics of the field $F_{\mu\nu}$ are most easily extracted by comparison with the conventional electrodynamics lagrangian in the presence of a nontrivial angle. In a local inertial frame, this lagrangian can be taken as

$$\mathscr{L}_{\text{em}} = -\frac{1}{4e^2}F_{\mu\nu}F^{\mu\nu} - \frac{}{16\pi^2}F_{\mu\nu}\tilde{F}^{\mu\nu}. \tag{14}$$

Then, inspection shows that in our supergravity model we can identify

$$e^2 \equiv 1/M(t), \qquad \equiv 4\pi^2 N(t). \tag{15}$$

Note that M and N are functions of the comoving time t via Eq. (13). It follows that in an arbitrary local inertial frame, e and acquire related spacetime dependences.

We continue with a few considerations regarding experimental estimates. Our simplifying assumption is $c_3 = 0$, as mentioned before. Matching the asymptotic electromagnetic coupling as determined from the background (13) with the observed present-day value yields the boundary condition $e^2(t \to \infty) \simeq 4\pi/137$. It follows that $|A_0| \simeq 1$ and $\lambda \lesssim 2\pi/137$. Within this restricted range of parameters, we take

$$\lambda = 2\pi/137, \qquad A_0 = \sqrt{1-\lambda^2}. \tag{16}$$

This special case simplifies the analysis further because it leads to a zero asymptotic value for . Note, however, that the above values are sufficiently general in the sense that the estimates determined below remain valid or improve for other parameter choices in more than 98% of the allowed range.

In what follows, we can replace the time coordinates t' in comoving local inertial frames with the comoving time t because t' and t agree to first order. At late times $t \gg t_0$, one can verify that $e^2 \sim 2\lambda \mp 8\lambda^2 t_0/\sqrt{3}t$ and thus $\dot{\alpha}/\alpha \sim \pm 4\lambda t_0/\sqrt{3}t^2$. Observational constraints on $\dot{\alpha}/\alpha$ at late times, i.e., in a recent cosmological epoch, have been determined through various analyses of data from the Oklo fossil reactor [28]. The bounds obtained are roughly $|\dot{\alpha}/\alpha| \lesssim 10^{-16}$ yr^{-1}. If the present age of the Universe is taken to be $t_n \simeq 10^{10}$ yr, the Oklo constraint yields the estimate $t_0 \lesssim 10^6$ yr, which is consistent with our previous late-times assumption.

In our supergravity model, the variation of the electromagnetic coupling α with time can be relatively complicated, and qualitative features of this variation can depend on the integration constants determining the background [29]. A sample time dependence of α is depicted in Fig. 1. The solid line represents the relative variation of α for the case $t_n/t_0 = 2000$ as a function of the fractional look-back time $1 - t/t_n$ to the initial singularity. To provide an approximate match to the recently reported data favoring a
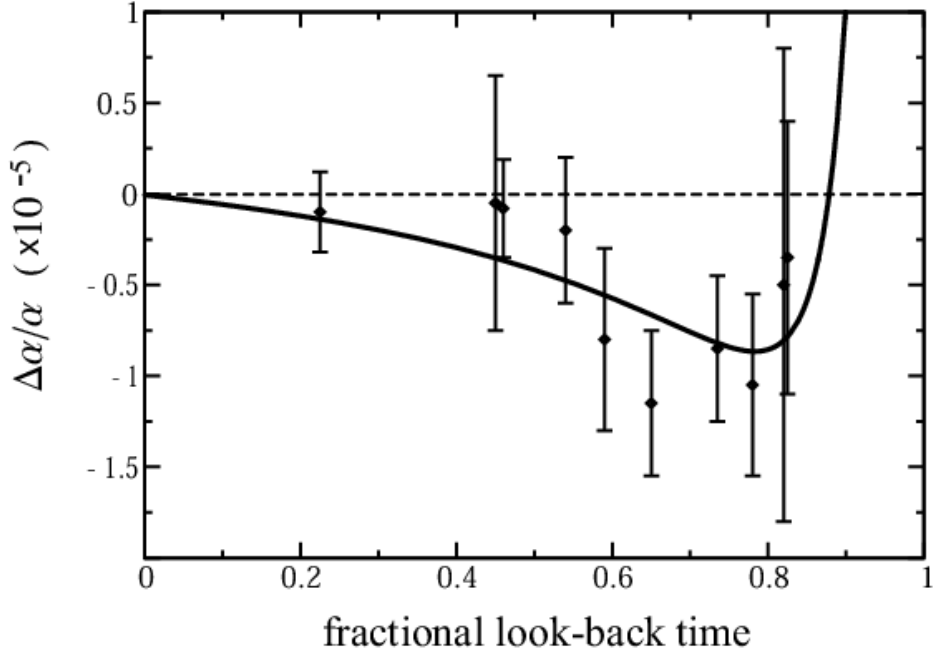
FIGURE 1. Sample relative variation of the fine-structure constant α as function of the fractional look-back time $1 - t/t_n$ to the Big Bang. The solid line corresponds to a parameter choice in the close vicinity of that given in (16). The dotted line represents a constant α.

varying fine-structure parameter,[1] the integration constants λ and A_0 have been changed fractionally by parts in 10^3 from the values (16). Although these choices of parameters have no overlap with the Oklo data set, they lie within the constraints for a variation of to be discussed in the next section. Note that the solid line reflects both nonlinearities and non-monotonic features of $\alpha(t)$.

4. LORENTZ VIOLATION

The Lorentz-violating effects in our supergravity cosmology can be seen explicitly at the level of the equations of motion for $F_{\mu\nu}$:

$$\frac{1}{e^2}\partial_\mu F^{\mu\nu} - \frac{2}{e^3}(\partial_\mu e)F^{\mu\nu} + \frac{1}{4\pi^2}(\partial_\mu)\tilde{F}^{\mu\nu} = j^\nu. \qquad (17)$$

Here, we have introduced charged matter described by a 4-current j^ν for completeness. When e and are constant, their derivatives in (17) would vanish and the usual inhomogeneous Maxwell equations would emerge. However, in the present case of varying M and N, the dynamics of the electromagnetic field is modified. Restricting attention to

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[1] The data, also plotted in Fig. 1, were obtained from measurements of high-redshift spectra over periods of approximately $0.6t_n$ to $0.8t_n$ assuming $H_0 = 65$ km/s/Mpc and $(\Omega_m, \Omega_\Lambda) = (0.3, 0.7)$ [25].

spacetime regions small on cosmological scales, the gradients $\partial_\mu M$ and $\partial_\mu N$ must be approximately constant. Although these gradients do not spoil coordinate invariance, they do select a preferred 4-direction in any local inertial frame. For example, in a comoving inertial frame $\partial_\mu M$ and $\partial_\mu N$ are both purely timelike. It follows that particle Lorentz symmetry, i.e., symmetry under boosts, rotations, or both of localized electromagnetic fields, is violated.

The above type of Lorentz breaking is to be distinguished from the usual violation of global Lorentz symmetry in textbook FRW cosmologies: in a conventional situation without varying scalars any local inertial frame is Lorentz symmetric, whereas in the present case the variation of e and results in particle Lorentz breaking in all local inertial frames. Note also that the Lorentz violation in our supergravity cosmology is independent of the details of the model as long as e and vary on cosmological scales. This suggests particle Lorentz violation could be a common feature of models incorporating couplings with a sufficiently smooth and slow spacetime dependence.

An integration by parts of the action yields an equivalent form of our modified electrodynamics lagrangian:

$$\mathscr{L}'_{\mathrm{em}} = -\frac{1}{4e^2}F_{\mu\nu}F^{\mu\nu} + \frac{1}{8\pi^2}(\partial_\mu)A_\nu\tilde{F}^{\mu\nu}. \tag{18}$$

Since varies in the present supergravity model, particle Lorentz violation and CPT breaking are apparent already at the lagrangian level. Again, in most practical situations it suffices to consider small spacetime regions, so that the gradient of can be taken as a constant 4-vector. We can then identify $e^2\partial_\mu/8\pi^2$ with the Lorentz- and CPT-violating $(k_{AF})_\mu$ parameter in the Standard-Model Extension.

In addition to a constant $(k_{AF})_\mu$, consider now the special situation in which e does not vary. This case has recently received a lot of attention in the literature [15, 7, 30, 31, 32]. Then, the lagrangian (18) becomes translationally invariant and energy-momentum conservation holds. Note, however, that the conserved energy fails to be positive definite, so that instabilities can occur [15, 7, 34]. On the other hand, the lagrangian (18) is associated with a positive-definite supergravity theory[2] and the question arises how this difficulty is avoided in the present context.

Although $(k_{AF})_\mu$ has been treated thus far as constant and nondynamical, it is associated with the dynamical degrees of freedom A and B in the present context. In the full theory, excitations of the field $F_{\mu\nu}$ will lead to deformations δA and δB in the background solution (13), such that $A \to A + \delta A$ and $B \to B + \delta B$. Thus, in the presence of a nonzero $F_{\mu\nu}$ the energy-momentum tensor $(T^{\mathrm{b}})^{\mu\nu}$ of the background receives an additional contribution from the perturbations δA and δB, so that $(T^{\mathrm{b}})^{\mu\nu} \to (T^{\mathrm{b}}_F)^{\mu\nu} = (T^{\mathrm{b}})^{\mu\nu} + \delta(T^{\mathrm{b}})^{\mu\nu}$. This contribution compensates the negative-energy ones arising from the $(k_{AF})_\mu$ term.

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[2] The N term in the lagrangian (4) is independent of the metric and so does not contribute to the conserved symmetric energy-momentum tensor. The remaining terms have conventional structure and it is straightforward to verify that they are positive definite.

The compensation mechanism can be illustrated explicitly at the classical level[3] in the lagrangian

$$\mathscr{L} = \mathscr{L}'_{\text{em}} + \mathscr{L}_{\text{b}}, \qquad (19)$$

where $\mathscr{L}_{\text{b}}$ has been defined in Sec. 2. In what follows we concentrate on the A- and B-dependence of , and take as e as constant for simplicity. It can be checked that incorporating the spacetime variation of e leaves the conclusions unchanged. We begin by considering the total conserved energy-momentum tensor $(T_F^{\text{t}})^{\mu\nu}$ and isolating the piece $(T^{\text{em}})^{\mu\nu}$ associated with $F_{\mu\nu}$: $(T_F^{\text{t}})^{\mu\nu} = (T^{\text{em}})^{\mu\nu} + (T_F^{\text{b}})^{\mu\nu}$, where

$$(T^{\text{em}})^{\mu\nu} = \frac{\partial\mathscr{L}}{\partial(\partial_\mu A^\lambda)}\partial^\nu A^\lambda - \eta^{\mu\nu}\mathscr{L}'_{\text{em}},$$

$$(T_F^{\text{b}})^{\mu\nu} = \frac{\partial\mathscr{L}}{\partial(\partial_\mu A)}\partial^\nu A + \frac{\partial\mathscr{L}}{\partial(\partial_\mu B)}\partial^\nu B - \eta^{\mu\nu}\mathscr{L}_{\text{b}}. \qquad (20)$$

With these definitions we obtain explicitly:

$$(T^{\text{em}})^{\mu\nu} = \frac{1}{e^2}F^\mu{}_\lambda F^{\lambda\nu} + \frac{1}{4e^2}\eta^{\mu\nu}F^{\rho\sigma}F_{\rho\sigma} + \frac{1}{8\pi^2}(\partial^\nu)A_\lambda\tilde{F}^{\lambda\mu}. \qquad (21)$$

Note that only the last term in (21) can lead to negative energies. Similarly, we find for the piece associated with the background

$$(T_F^{\text{b}})^{\mu\nu} = \frac{\partial^\mu A\partial^\nu A}{2B^2} - \frac{\eta^{\mu\nu}}{4B^2}(\partial_\lambda A\partial^\lambda A + \partial_\lambda B\partial^\lambda B) + \frac{\partial^\mu B\partial^\nu B}{2B^2} - \frac{1}{8\pi^2}(\partial^\nu)A_\lambda\tilde{F}^{\lambda\mu}, \quad (22)$$

where again negative-energy contributions can arise only from the last term. Equations (21) and (22) show that $(T_F^{\text{t}})^{\mu\nu}$ is free from unsatisfactory terms, so that the total conserved energy is positive definite, even in the presence of a nonzero $(k_{AF})_\mu$. The apparent paradox lies in the fact that the two pieces $(T_F^{\text{em}})^{\mu\nu}$ and $(T_F^{\text{b}})^{\mu\nu}$, each containing the term with the positivity difficulty, become separately conserved in the limit of a constant ∂^ν.[4]

In the present supergravity cosmology, the spacetime dependences of both e and follow from the background (13) and are therefore related. This fact can be exploited in the context of experimental estimates. In our model, we obtain $\dot{N} \sim \mp 2t_0/\sqrt{3}\lambda t^2$ for the time variation of N at late times. The direct observational limit of $(k_{AF})_0 \lesssim 10^{-42}$ GeV [15] then bounds the variation of α in time to be $|\dot{\alpha}/\alpha| \lesssim 10^{-12}$ yr^{-1}, consistent with the Oklo data [28]. Reversing the analysis, the Oklo bounds constrain $(k_{AF})_\mu$ to be less than $\sim 10^{-46}$ GeV, which compares favorably with the above direct limit.

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[3] In quantum field theory, radiative corrections mix these terms [33].

[4] A constant timelike $(k_{AF})_\mu$ violates microscopic causality [15, 7, 3, 34]. Our supergravity model may circumvent this, but a complete analysis of this lies outside our present scope.

5. SUMMARY

In a cosmological context, we have determined an analytical solution within a simple supergravity model. This classical solutions describes a situation with varying electro-magnetic couplings e and . The functional dependence of these couplings on spacetime is highly nonlinear. We have demonstrated within this model and argued in the general case that spacetime-dependent couplings lead to particle Lorentz violation. Our super-gravity cosmology avoids the usual positivity problems associated with a varying angle.

ACKNOWLEDGMENTS

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