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Research Article

A New Wavelet Threshold Determination Method Considering Interscale Correlation in Signal Denoising

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Due to simple calculation and good denoising effect, wavelet threshold denoising method has been widely used in signal denoising. In this method, the threshold is an important parameter that affects the denoising effect. In order to improve the denoising effect of the existing methods, a new threshold considering interscale correlation is presented. Firstly, a new correlation index is proposed based on the propagation characteristics of the wavelet coefficients. Then, a threshold determination strategy is obtained using the new index. At the end of the paper, a simulation experiment is given to verify the effectiveness of the proposed method. In the experiment, four benchmark signals are used as test signals. Simulation results show that the proposed method can achieve a good denoising effect under various signal types, noise intensities, and thresholding functions.

1. Introduction

Due to the combined impacts of the internal measurement system and the external environmental factors, measured signals are often contaminated by noise [1]. Therefore, signal denoising technology has been a hot topic in the field of signal processing. In a noisy signal, noise energy is generally concentrated in the high frequency region, and the spectrum of useful signal is distributed in the low frequency region [2]. According to this theory, a variety of signal denoising methods were put forward, such as mean filtering [3], median filtering [4] and Wiener filtering [5]. Most of these methods can be considered as low-pass filters essentially. However, in some signals, the high frequency region not only contains noise but also possesses a lot of useful information. Therefore, directly filtering out high frequency information is unreasonable. Recently, because of multiresolution and low entropy, wavelet transform has become a popular research topic in the signal denoising field. A number of methods based on wavelet theory have been proposed, such as wavelet coefficient modulus maxima method [6], wavelet correlation method [7], and wavelet threshold method [8]. The essence of these methods is nonlinear processing on the wavelet coefficients and then using the processed coefficients to reconstruct signals. Among these methods, wavelet threshold method has been used the most widely because of its simple calculation and good effect.

Wavelet threshold method was proposed by Donoho and Johnstone [8], whose main idea is to reconstruct signal on the basis of thresholding coefficients. The denoising effect of wavelet threshold method depends on threshold determination. If the selected threshold is too large, then some useful information is filtered out, and if the threshold is too small, then a certain amount of noise is retained. In order to solve this problem, many researchers studied the threshold determination methods. Donoho and Johnstone [8] presented a universal threshold by analyzing normal Gaussian noise model; Tao et al. [9] improved the universal threshold and indicated that the threshold should be adaptively changed with the scale alter. The defect in this kind of methods is that universal threshold is often set too large, which may lead to overkill the useful information. Chang et al. [10] assumed that the wavelet coefficients obeyed generalized Gaussian distribution and proposed a Bayesian threshold method; Lu and Loizou [11] considered the coefficients obeyed Gaussian distribution and presented a new threshold based on maximum a posteriori probability; Li et al. [12] supposed the coefficients obeyed generalized Gamma distribution and put forward a threshold method based on Bayesian shrinkage. All of these methods are based on a particular coefficient distribution, but it may not satisfy the distribution to a specific signal. Donoho and Johnstone [13] proposed a new threshold method based on minimax criterion. However, this method requires the prior knowledge of the original signal, while the information of original signal is difficult to obtain in reality. Stein's unbiased risk estimate (SURE) criterion [8] and generalized cross validation (GCV) criterion [14] were presented based on the idea of parameter estimates, in which SURE criterion is unbiased estimate of the minimized mean square error (MSE) criterion, and GCV criterion is biased estimates of the minimized MSE criterion. Cai and Zhou [15] proposed a data-driven threshold determination method based on SURE criterion. Autin et al. [16] put forward a new idea by combining different threshold rules.

Although the wavelet threshold method has developed significantly, some deficiencies remain. The primary defect is that most of these methods ignore the relationship between the wavelet coefficients. According to wavelet correlation theory [7], the wavelet coefficients of a useful signal have a strong correlation in various decomposition scales, whereas the wavelet coefficients of noise are weakly correlated or uncorrelated. Therefore, analysis of the coefficients correlation can help distinguish useful information or noise. However, the existing threshold methods are mostly from the minimum error criterion or other optimization criteria that do not consider the coefficient correlation, and this defect may diminish the signal denoising effect.

The goal of this paper is to propose a new wavelet threshold determination method that considers the interscale correlation of coefficients. The method firstly adopts the universal threshold as the basic threshold. And then, a new correlation index is presented based on the wavelet correlation theory. At last, the new threshold is obtained using the correlation index to improve the basic threshold.

The remaining parts of the paper are organized as follows. Section 2 introduces the wavelet threshold denoising theory. Section 3 proposes a new correlation index and presents a new threshold determination method based on the new index and basic threshold. Section 4 provides a simulated experiment using four benchmark signals to verify the effectiveness of the new threshold. The conclusion is given in Section 5.

2. Wavelet Threshold Denoising Theory

In one-dimension noisy signals, noise acts on the original signal through linear superposition:

$$X(k) = S(k) + E(k), \tag{1}$$

where $X(k)$ is the noisy signal, $S(k)$ is the original signal, and $E(k)$ is white Gaussian noise, subject to $N\ (0, \sigma^2)$ distribution.

Wavelet transform is a linear transform. Therefore, wavelet coefficients obtained through wavelet transform of $X(k)$ still contain two parts. One part is from the original signal $S(k)$, and the other part is brought from the noise $E(k)$. Wavelet transform can concentrate signal energy on some large wavelet coefficients and distribute the noise energy throughout the whole wavelet domain. Thus, large amplitude wavelet coefficients may be produced by the useful signal, and the small amplitude is likely to represent the noise. According to this characteristics of wavelet coefficients, Donoho and Johnstone [8] proposed wavelet threshold method, which can be divided into three steps:

(1) choosing the appropriate wavelets basis and decomposition scale and computing the corresponding wavelet coefficients,

(2) selecting the proper threshold and thresholding function and obtaining the estimated values of the wavelet coefficients,

(3) reconstructing the signal based on the estimated values of wavelet coefficients by inverse wavelet transform.

In the wavelet threshold denoising method, a core issue is to determine the optimal threshold. Threshold can make a great influence on the denoising effect. If the threshold value is too small, then considerable noise will still exist, and if the threshold value is too large, then some important feature of signal may be filtered out. As mentioned in the introduction, many existing methods can determine the threshold. Among these methods, universal threshold is the most widely used because of its simpleness and effectiveness. The formula for the universal threshold is expressed as follows:

$$\lambda = \sigma \sqrt{2 \ln (N)}, \tag{2}$$

where σ is the average variance of the noise and N is the signal length.

σ is calculated using median estimate method. The formula is as follows:

$$\sigma = \frac{\text{Median}\left(\left|W_{1,K}\right|\right)}{0.6745}, \tag{3}$$

where $W_{1,k}$ represent all the wavelet coefficients in scale 1.

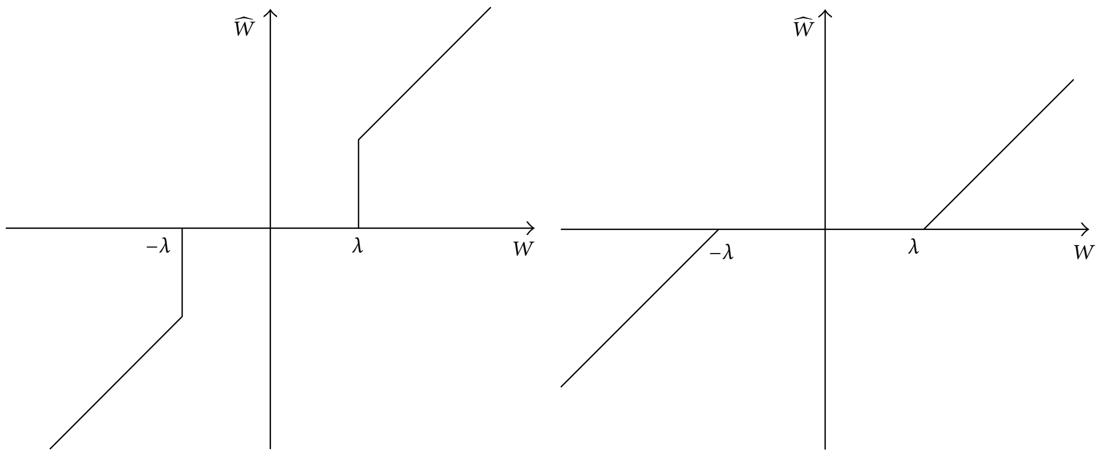
Because universal threshold is the most widely used threshold method, therefore it is selected as basis to construct the new threshold in this paper.

Thresholding function reflects different estimation strategies to the wavelet coefficients. There are two well-known thresholding functions named hard thresholding function and soft thresholding function. Their main ideas are to both remove small wavelet coefficients and shrink large wavelet coefficients. Hard thresholding function shown in Figure 1(a) is defined as

$$\widehat{W_{j,k}} = \begin{cases} W_{j,k}; & \left|W_{j,k}\right| \geq \lambda \\ 0; & \left|W_{j,k}\right| < \lambda. \end{cases} \tag{4}$$

Soft thresholding function shown in Figure 1(b) is defined as

$$\widehat{W_{j,k}} = \begin{cases} \operatorname{sgn}\left(W_{j,k}\right)\left(\left|W_{j,k}\right| - \lambda\right); & \left|W_{j,k}\right| \geq \lambda \\ 0; & \left|W_{j,k}\right| < \lambda. \end{cases} \tag{5}$$



(a) Hard thresholding function

(b) Soft thresholding function

FIGURE 1: Hard and Soft thresholding functions.

Some researchers focused on studying the thresholding function and proposed some new functions [17, 18]. Because the core issue of this study is to determine the threshold, the classical hard and soft thresholding functions are selected in the experimental section.

3. A New Wavelet Threshold Determination Method

3.1. Wavelet Correlation Denoising Theory. The propagation characteristics of wavelet coefficients show that signal has a strong correlation in various decomposition scales, while noise is weakly correlated or uncorrelated. Considering this feature, Xu et al. [7] proposed a classical wavelet correlation denoising method named spatially selective noise filtration. In this method, the correlation factors of different scales are calculated firstly. And then, the larger values are retained as useful information by comparing the correlation factors with original coefficients. Finally, the signal is reconstructed using the inverse wavelet transform.

In this method, the correlation of wavelet coefficients is measured using the normalised correlation index $N\mathrm{Cor}$:

$$N\mathrm{Cor}\left(j,n\right)=\mathrm{Cor}\left(j,n\right)\sqrt{\frac{PW\left(j\right)}{P\mathrm{Cor}\left(j\right)}}, \quad (6)$$

where $\mathrm{Cor}(j,n) = W_{j,n}W_{j+1,n}$ represents the coefficients correlation between $W_{j,n}$ and $W_{j+1,n}$; $W_{j,n}$ represents the coefficient in location point n and scale j; $P\mathrm{Cor}(j) = \sum_{n=1}^{N}\mathrm{Cor}(j,n)^2$ represents the correlation factor energy of scale j; $PW(j)=\sum_{n=1}^{N}W_{j,n}^2$ represents the coefficients energy of scale j.

$N\mathrm{Cor}$ can describe the correlation among the wavelet coefficients to a certain extent, but its calculation is complicated and inefficient. Therefore, it is necessary to propose a simple and efficient correlation index.

3.2. A New Interscale Correlation Index. Crouse et al. [19] found that the wavelet coefficients typically had the following distribution characteristics:

(1) clustering: if a particular wavelet coefficient is large/small, then adjacent coefficients are very likely to also be large/small;

(2) persistence across scale: large/small values of wavelet coefficients tend to propagate across scales.

According to this theory, a new index is proposed to measure the interscale correlation of wavelet coefficients in this paper:

$$K\left(n\right)=\frac{\max\left|W\left(:,n\right)\right|-\min\left|W\left(:,n\right)\right|}{\min\left|W\left(:,n\right)\right|}, \quad (7)$$

where $W(:,n)$ represents all the wavelet coefficients in location point n.

When $K(n)\in[0,r)$, the difference between maximum and minimum of the wavelet coefficients in location point n is little. It is considered that there is a strong correlation of wavelet coefficients in location point n. Therefore, the location point n is likely to be a signal point. A smaller value of the $K(n)$ means a bigger possibility to be a signal point for location point n. When $K(n)\in[r,+\infty)$, the difference between maximum and minimum of location point n is big. A weak correlation of wavelet coefficients is considered in location point n. Therefore, the location point n is likely to be a noise point.

r is an important parameter to measure the coefficients correlation. If the value is selected too small, it may result in overkilling some useful signal points. If the value is chosen too large, it may lead to retain some noise points. As a lot of experiments shown, when r takes 0.5 to 1.5, the denoising effect is good. Therefore, in the experiment of this paper, r is set 1.

3.3. A New Threshold Determination Strategy Considering Interscale Correlation. K is an index measuring the interscale correlation of wavelet coefficients, which can represent the possibility of a certain point belonging to the signal point. Because universal threshold often has the risk of overkilling the useful information, therefore, when $K(n) \in [0, r)$, the threshold needs to be shrunk to retain location point n, and when $K(n) \in [r, +\infty)$, the threshold of location point n remains invariant. Based on a lot of experiments, a new threshold determination method considering interscale correlation is presented as follows:

$$T_{\text{new}} = \begin{cases} 0.7\lambda & k(n) \in [0, 0.5r) \\ 0.8\lambda & k(n) \in [0.5r, 0.8r) \\ 0.9\lambda & k(n) \in [0.8r, r) \\ \lambda & k(n) \in [r, +\infty), \end{cases} \tag{8}$$

where λ is universal threshold.

If $K(n) \in [0, 0.5r)$, then the correlation of the wavelet coefficients is considered to be very large, and the location n is very likely to be a signal point. Therefore, we set the threshold value $T_{\text{new}} = 0.7\lambda$. If $K(n) \in [0.5r, 0.8r)$, then the correlation of the wavelet coefficients is considered large, and the location point n is likely to be a signal point. Therefore, we set the threshold value $T_{\text{new}} = 0.8\lambda$. If $K(n) \in [0.8r, r)$, then the correlation of the wavelet coefficients is considered to be slightly large, and there is slight possibility for location n to be a signal point. Therefore, we set the threshold value $T_{\text{new}} = 0.9\lambda$. If $K(n) \in [r, +\infty)$, the correlation of the wavelet coefficients is considered very little, and there is almost no possibility for location n to be a signal point. Therefore, we set the threshold value $T_{\text{new}} = \lambda$.

4. Simulation Experiment

4.1. Experimental Parameter Settings and Experiment Results. In order to verify the validity of the new threshold, four classic benchmark signals, namely, Blocks, Bumps, Heavy Sine, and Doppler, are used as test signals. The length of the signal is 512 and r is set 1. Two strengths of white Gaussian noise $\sigma = 3$ and $\sigma = 5$ are added to the test signals in the experiments. The calculation process of the correlation index K requires that the number of wavelet coefficients to be the same in different decompositions. Therefore, stationary wavelet transform is used in this paper [20]. Wavelet basis adopts sym6 wavelet, wavelet decomposition scale is set to be 3, and hard and soft thresholding functions are chosen as thresholding functions. The traditional universal threshold is selected as comparative experiment. In order to measure the denoising effect under different thresholds, MSE and signal-to-noise-ratio (SNR) are selected as comparative index:

$$\begin{aligned} \text{MSE} &= \frac{\sum_{k=1}^{N} [X(k) - S(k)]^2}{N}, \\ \text{SNR} &= 10 \ln \frac{\sum_{k=1}^{N} S(k)^2}{\sum_{k=1}^{N} [X(k) - S(k)]^2}. \end{aligned} \tag{9}$$

Figures 2 and 3 show the denoising effects of the four test signals with $\sigma = 5$ Gaussian white noise and hard thresholding function. In the figures, OS represents the original signal, NS is the noisy signal, DSUT is reconstructed signal using universal threshold, DSPT expresses the reconstructed signal using the proposed threshold.

The denoising results of soft thresholding function are given in Figures 4 and 5.

In order to exclude the effect of noise intensity, experiment is retested with $\sigma = 3$ white Gaussian noise.

4.2. Result Analysis. As shown in Figure 2, when $\sigma = 5$ and hard thresholding function is selected, denoising signal obtained by the proposed method is closer to the original signal than universal threshold for all the four different signals. As shown in Figure 3, the proposed method can achieve a smaller MSE and a larger SNR than traditional method. In order to avoid the interference of thresholding function, Figures 4 and 5 show the denoising results under soft thresholding function. Comparing Figure 2 with Figure 4, denoising signal obtained by our method is closer to the original signal than universal threshold, regardless of which hard threshold function or soft threshold function is selected. From Figures 3 and 5, we find that no matter which thresholding function is selected, denoising signal using our method can obtain a smaller MSE and greater SNR. Experiment is retested with $\sigma = 3$ white Gaussian noise to exclude the effect of noise intensity. The experiment results are shown in Figures 6, 7, 8, and 9, which indicate that the denoising effect and index of the proposed method are superior to those of the universal threshold with both heavy and light noise.

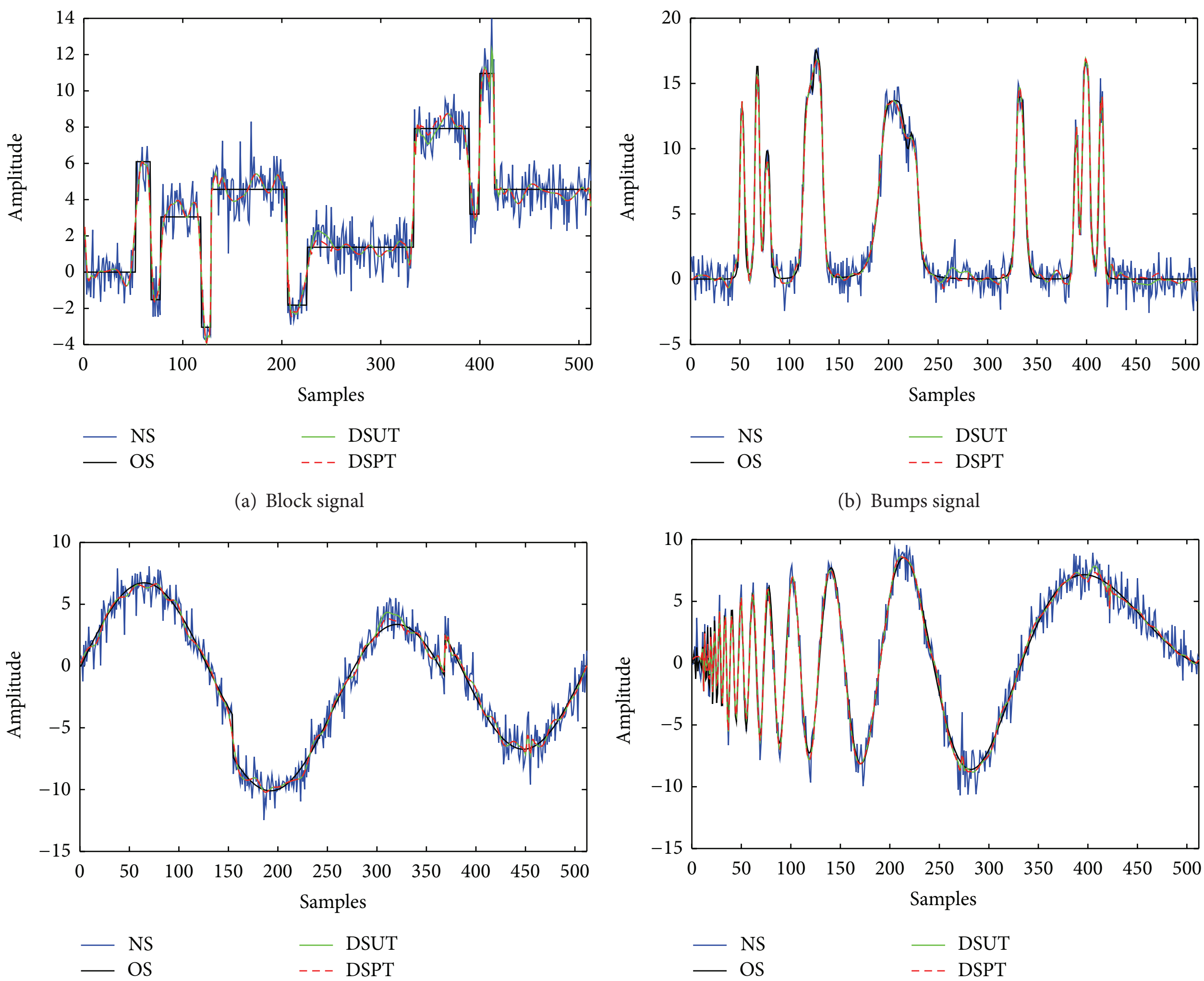
5. Conclusion

A new method considering interscale correlation is presented to solve the problem of wavelet threshold determination. Firstly, a new index is proposed to measure the coefficients correlation. Then, a new threshold determination strategy is obtained using new index to improve universal threshold. Some conclusions are summarized as follows.

(1) According to the propagation characteristics of the wavelet coefficients, a new index is proposed to measure the coefficients correlation. Compared with traditional index NCor, the new index has the advantage of a simple structure and convenient calculation.

(2) Universal threshold has the defect of overkilling the useful information. In order to address the issue, interscale correlation is used to shrink the universal threshold. Experimental results show that the proposed method can achieve optimum denoising effect under different signal types, noise intensities, and thresholding functions. Therefore, this method is effective and superior in signal denoising.

Universal threshold is selected as the basic threshold in this paper. In theory, the proposed idea is also applicable to the other threshold methods. Future research will be conducted on using the proposed idea to improve other thresholds.



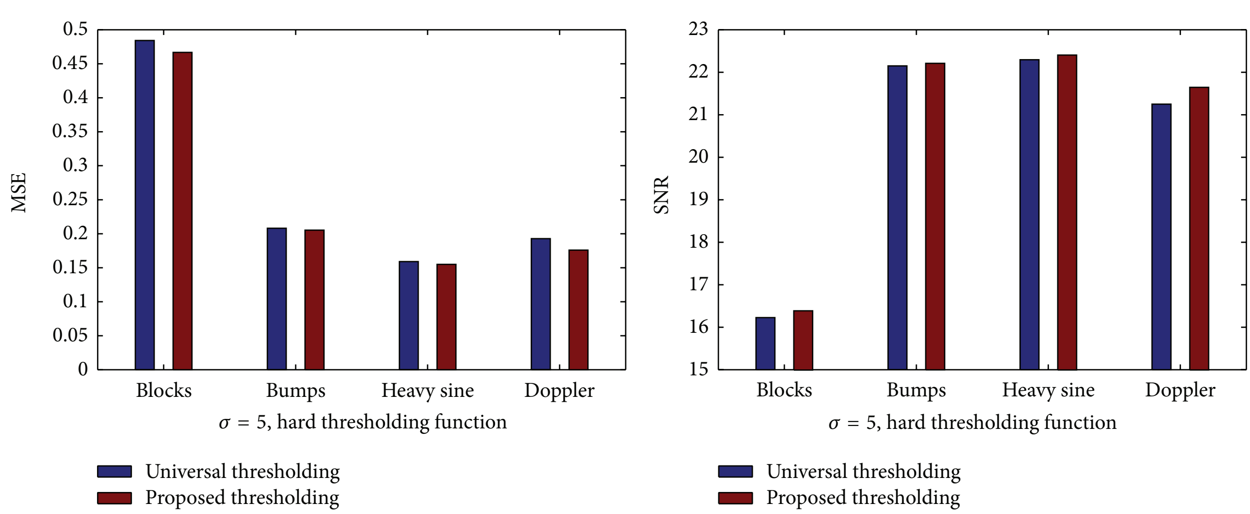
(a) Block signal

(b) Bumps signal

(c) Heavy sine signal

(d) Doppler signal

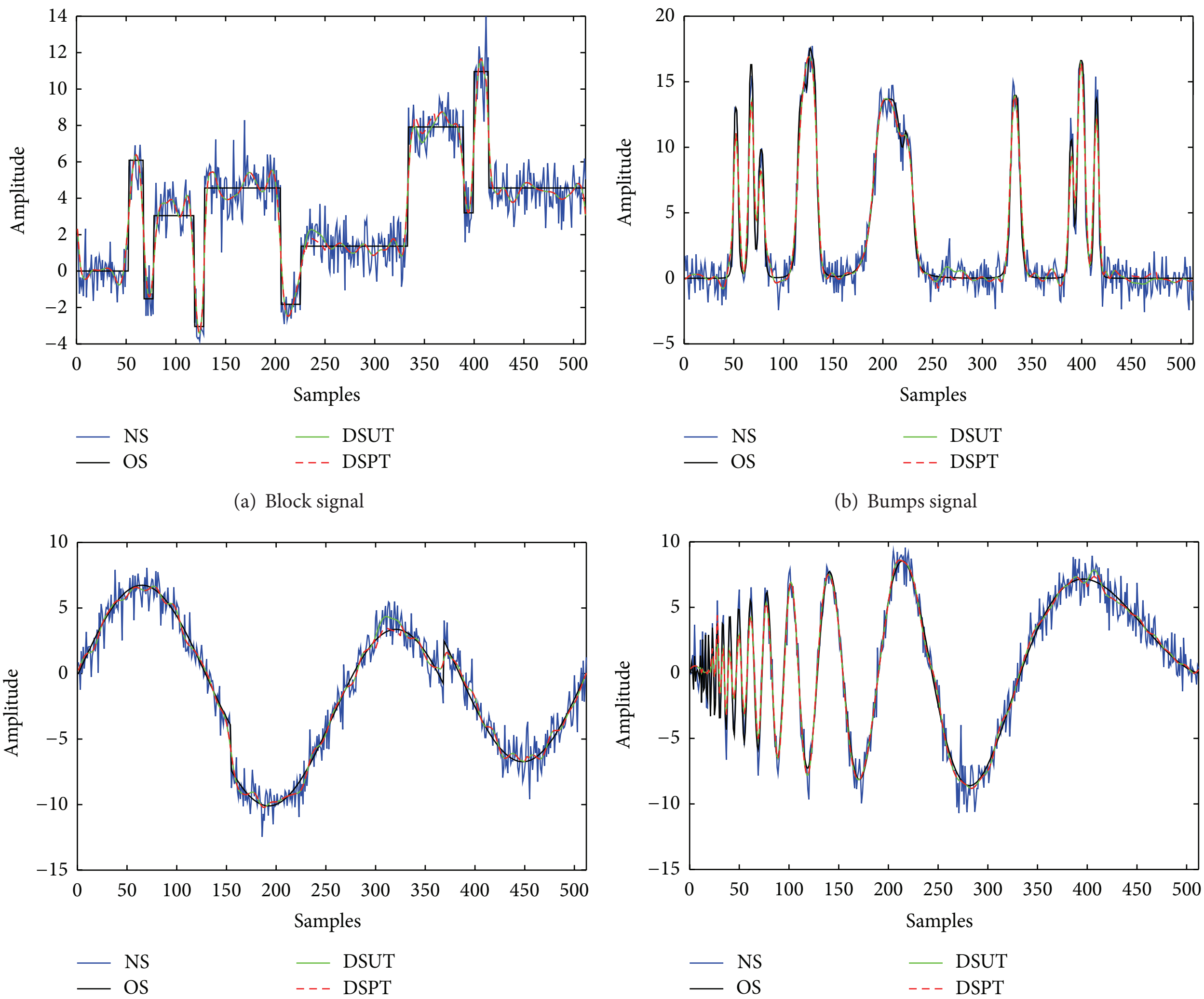
FIGURE 2: Denoising effect of $\sigma = 5$, hard thresholding function.



(a) MSE

(b) SNR

FIGURE 3: Denoising index of $\sigma = 5$, hard thresholding function.



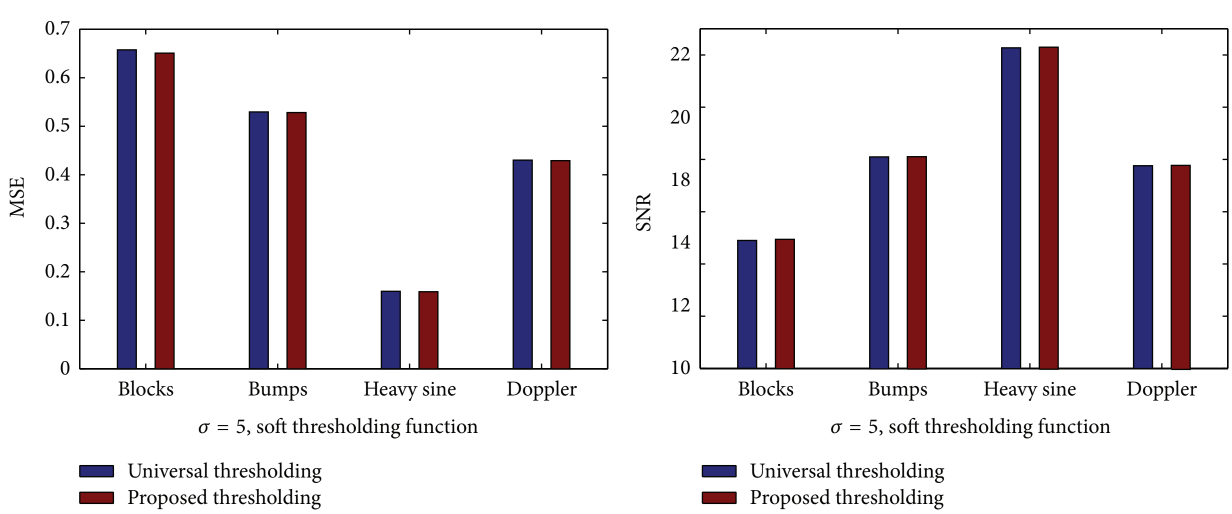
(a) Block signal

(b) Bumps signal

(c) Heavy sine signal

(d) Doppler signal

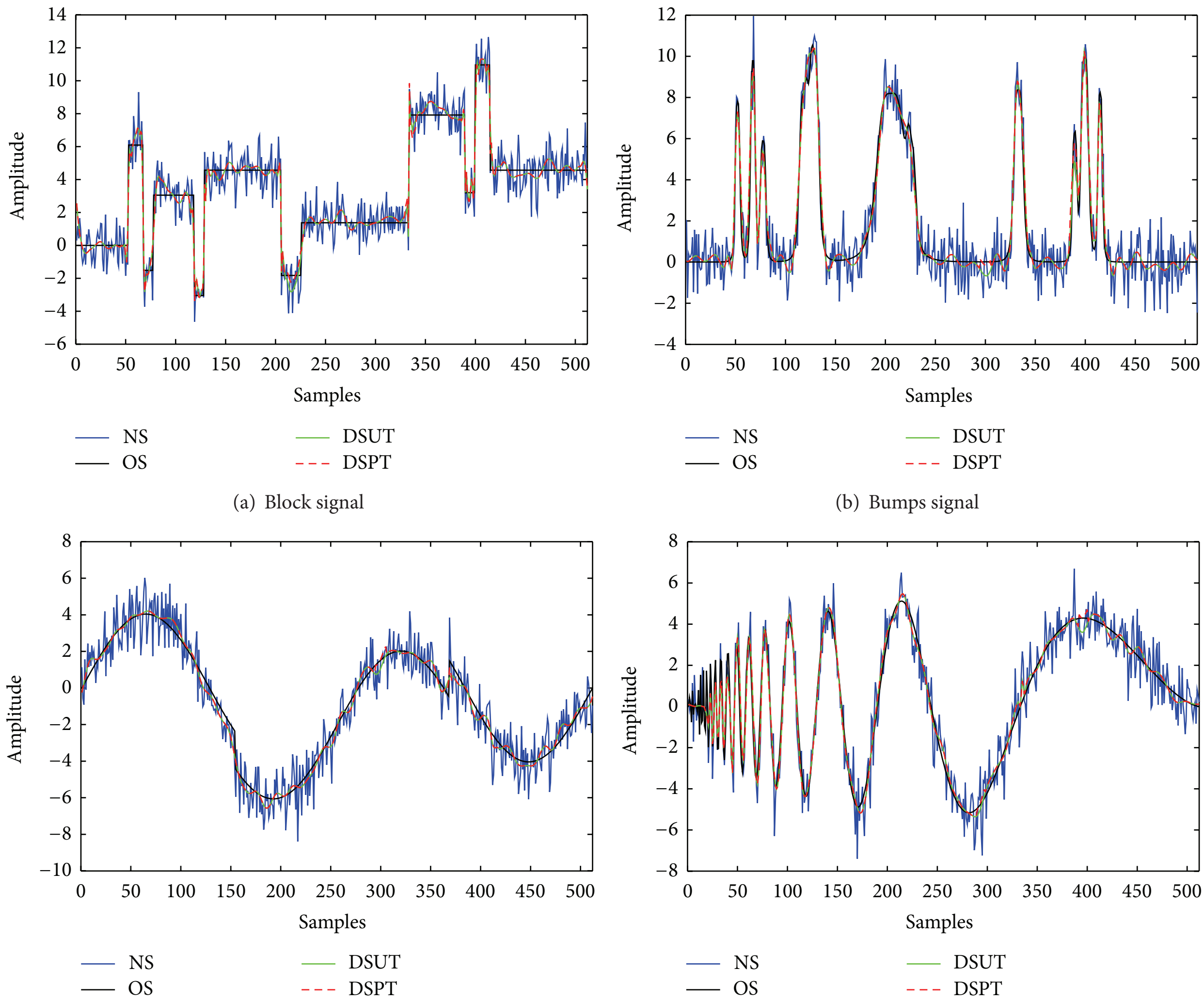
Figure 4: Denoising effect of $\sigma = 5$, soft thresholding function.



(a) MSE

(b) SNR

Figure 5: Denoising index of $\sigma = 5$, soft thresholding function.



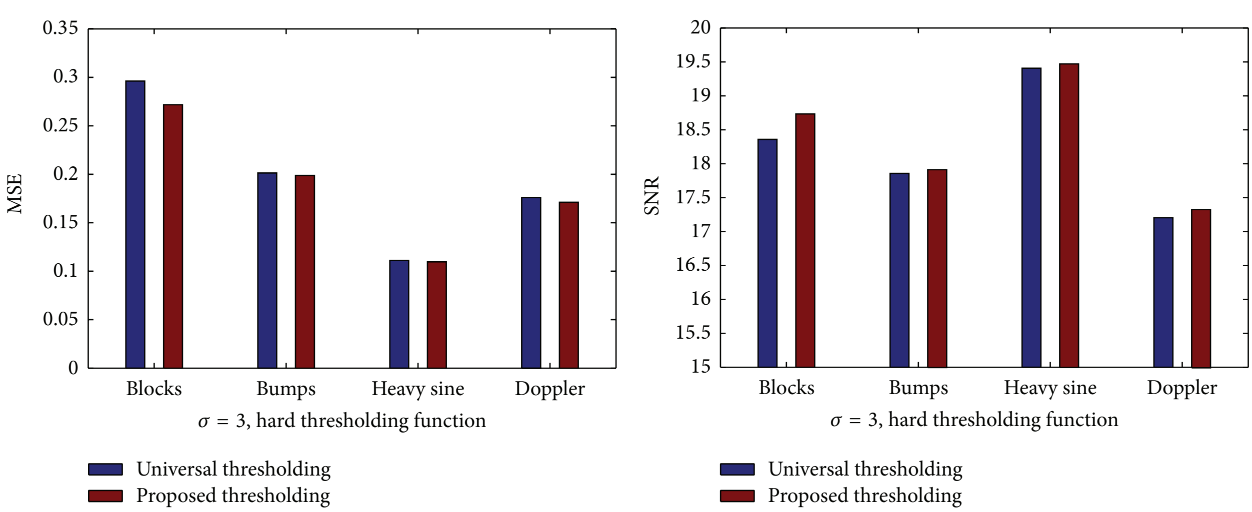
(a) Block signal

(b) Bumps signal

(c) Heavy sine signal

(d) Doppler signal

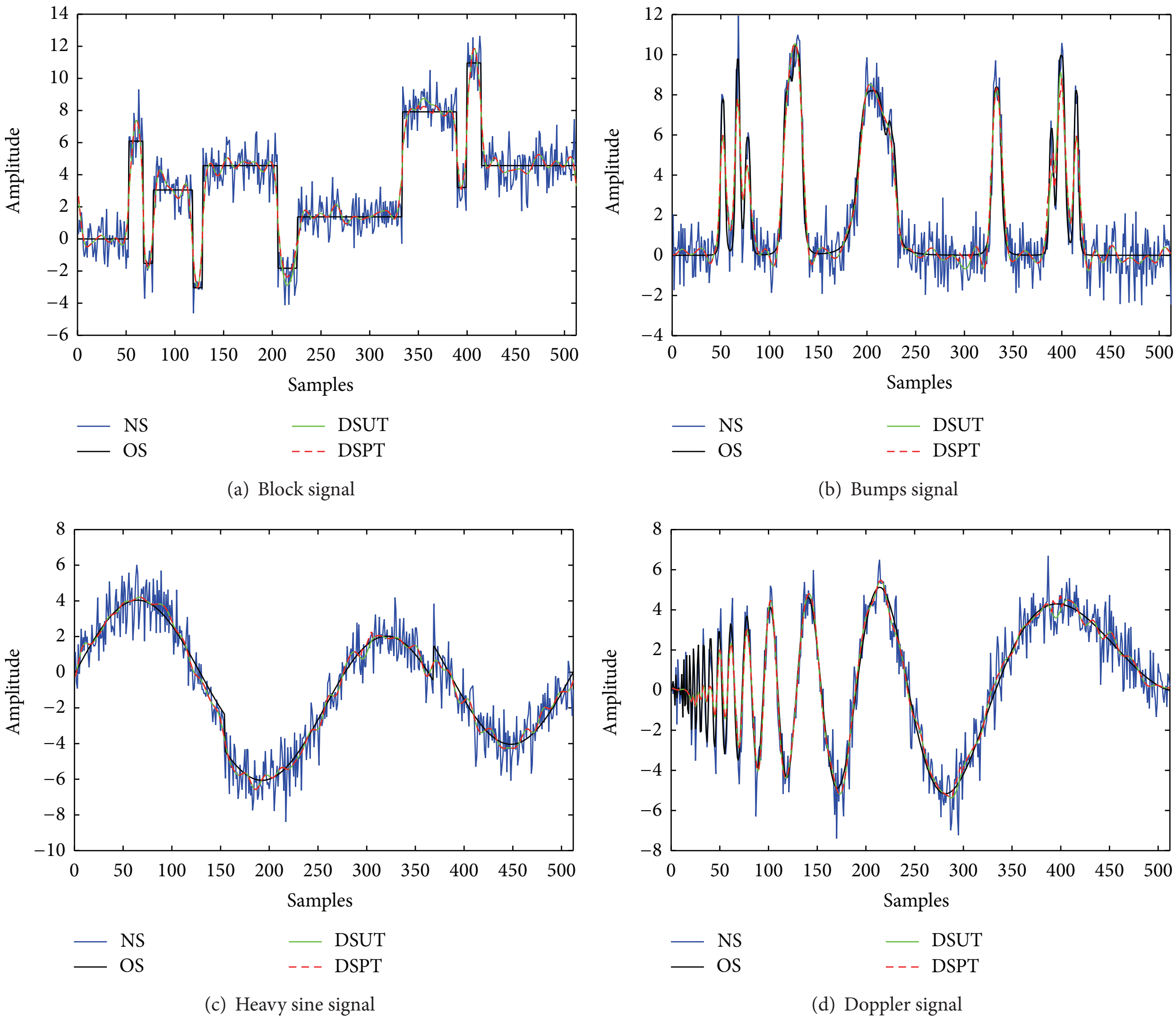
Figure 6: Denoising effect of $\sigma = 3$, hard thresholding function.



(a) MSE

(b) SNR

Figure 7: Denoising index of $\sigma = 3$, hard thresholding function.



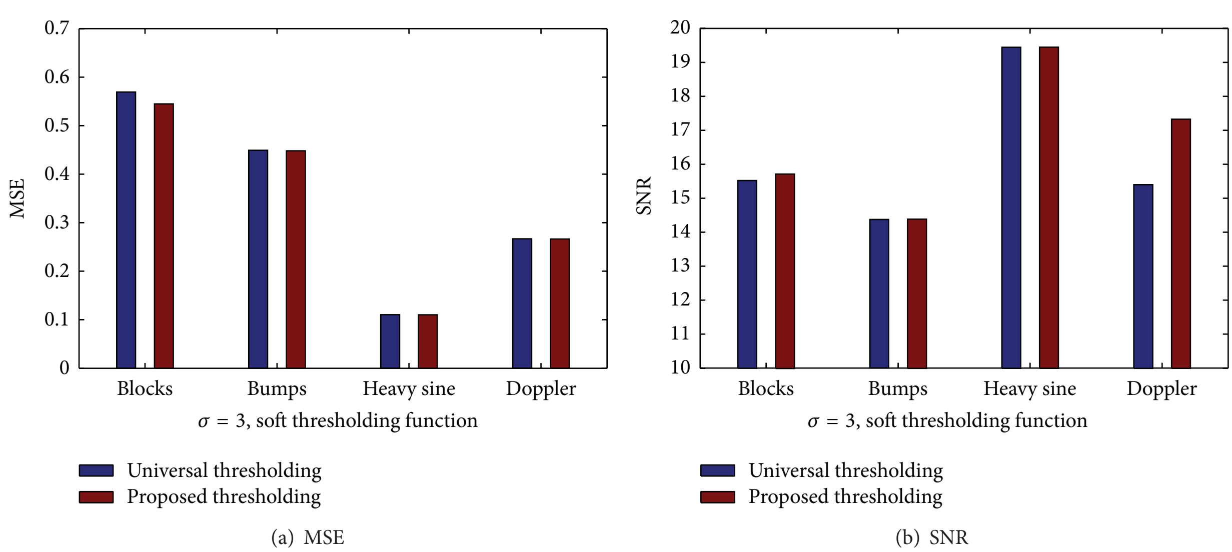
(a) Block signal

(b) Bumps signal

(c) Heavy sine signal

(d) Doppler signal

Figure 8: Denoising effect of $\sigma = 3$, soft thresholding function.



(a) MSE

(b) SNR

Figure 9: Denoising index of $\sigma = 3$, soft thresholding function

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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