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Research Article

Dynamic Traffic Network Model and Time-Dependent Braess' Paradox

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We propose a dynamic traffic network model and give the equilibrium condition and the equivalent variational inequality of the network. In this model, instead of the influence of inflow rate and output rate on the link congestion, the influence of the adjacent links at the same paths is considered; in this case, the equivalence between the equilibrium condition and the variational inequality is proved. Then we take an example about the paradox using the variational inequality and find that the probability and the severity that Braess' paradox occurs change with the influence of other links changing. Subsequently, we discuss the influence of other links on whether the adding link works under the dynamic system optimal. At last, we give the relationship between the total congestion under dynamic user equilibrium and that under dynamic system optimal. The results imply that we should take some methods and adjust the interaction between links rationally with the dynamic change of traffic situations.

1. Introduction

Dynamic traffic assignment (DTA) is one of the most important technologies of the intelligent transportation system (ITS), which has received extensive attention of researchers and practitioners. During the process of investigating DTA, mathematical programming [1–4], optimal control [5–7], and variational inequality [8–10] are labeled as three main analytical approaches. About the optimal control methods, dynamic user equilibrium (DUE), dynamic user optimal (DUO), and dynamic system optimal (DSO) are proposed; the variational inequality methods equivalent to DUO include the models based on the path congestion and link congestion. In the previous dynamic traffic models, the congestion of the link at time t is dependent on the link flow, the inflow rate, and outflow rate of the link at time t. In this paper, we assume that the link congestion at time t is related to the flow of this link and the adjacent links at the same paths at time t, which simplifies the traffic assignment model owing to only considering the influence of the link flow.

The well-known Braess' paradox has been considerably investigated in the scientific literatures [11–16] since it is proposed by Braess [17]. For example, Yang and Bell [18] gave a capacity paradox design and studied how to avoid it. Pas and Principio [19] gained the specific range that the paradox occurs. In recent years, Hallefjord et al. [20] analyzed the traffic paradox when travel demand is elastic; Arnott et al. [21] discussed the properties of dynamic traffic equilibrium including a paradox; Nagurney et al. [22] investigated the time-dependent Braess paradox using evolutionary variational inequalities. In this paper, we assume that the link congestion is also influenced by the flow of other links and investigate the paradox of the dynamic traffic network.

As we know, many distribution methods are proposed during the process of investigating the traffic assignment, such as the static methods including UE, UO, and SO and the dynamic methods including DUE and DSO. The relationship between UE and SO has been investigate by large number of researchers, from which [23] we know that the solution of UE and SO is similar in the free flow state and the difference becomes greater in the congested state. In this paper, we investigate the relationship between DUE and DSO of the dynamic traffic network.

The paper is organized as follows. We first construct the evolution model and prove the equivalence between the dynamic network equilibrium and the variational inequality; then we give an example about simple networks, discuss the paradox and whether the adding link makes sense under DSO using the variational inequality method when the influence of other links is considered, and investigate the relationship of the total congestion between different assignments; at last, the conclusions about the results of the paper are given.

2. The Model Construction

We consider the network $G = [N, L]$, where N, L denote the sets of nodes and links, respectively. Let W with n_W elements represent the set of origin/destination (O/D) and let P_W represent the set of paths joining the O/D pair w; P with n_P elements denotes the set of all paths connecting all the O/D pairs in this network. Let $d_w(t)$ denote the demand at time t between O/D pair w; $f_a(t)$, $x_r(t)$ stand for the flow on link a and path r at time t, respectively. $[0, T]$ denotes the time interval under consideration. $c_a(t)$ is the congestion of link a at time t; $C_r(t)$ is the congestion of path r at time t.

In this model, we assume that the congestion of the link is dependent on the flow of this link and the adjacent links on the same path with it at time t; that is,

$$c_a(t) = c_a(f_a(t), f_1(t), f_2(t), \ldots, f_\Lambda(t)), \quad \forall a \in L, \tag{1}$$

where $\{1, 2, \ldots, \Lambda\}$ is the set of links which is adjacent and at the same path with link a. The link flows and the route flows satisfy the following conservation of flow equations:

$$f_a(t) = \sum_{r \in P} x_r(t)\,\delta_{ar}, \quad \forall a \in L, \tag{2}$$

where $\delta_{ar} = 1$ if link a is contained in route r, and $\delta_{ar} = 0$, otherwise.

Then we have $c_a(t) = c_a(x_1(t), x_2(t), \ldots, x_\Gamma(t))$, where $\{1, 2, \ldots, \Gamma\}$ is the set of paths containing link a or the links which are adjacent to and appear on the same path with link a. The path congestion and the link congestion satisfy the following equations:

$$C_r(t) = \sum_{a \in L} c_a(x_1(t), x_2(t), \ldots, x_\Gamma(t))\,\delta_{ar}, \quad \forall r \in P. \tag{3}$$

The traffic demand at time t must satisfy the following conservation of flow:

$$d_w(t) = \sum_{r \in P_w} x_r(t), \quad \forall w \in W. \tag{4}$$

In addition, the model meets the following nonnegative constraint and boundary initial condition:

$$\begin{gathered} x_a(t) \geq 0, \\ x_a(0) = 0. \end{gathered} \tag{5}$$

We denote the vector grouping the demands at time t of all the O/D pairs by $\overrightarrow{d(t)}$, denote the vector grouping all route flows at time t by $\overrightarrow{x(t)}$, and denote the vector grouping all routes congestion at time t by $\overrightarrow{C(t)}$. In the following, we give the definition of dynamic network equilibrium satisfying (1)–(5).

Definition 1 (dynamic network equilibrium). A path flow pattern $\overrightarrow{x^*(t)}$ is defined as a dynamic network equilibrium if, at each time t, only the minimum congestion routes are used for each O/D pair, whose mathematical expression is given as follows: $\lambda_w(t)$ is the minimal path congestion at time t; that is, $\lambda_w(t) = \min_{p \in P}\{C_p(t)\}$.

Theorem 2. *$\overrightarrow{x^*(t)}$ is an equilibrium flow if and only if it satisfies the following variational inequality:*

$$\int_0^T \left\langle \overrightarrow{C\left(\overrightarrow{x^*(t)}\right)}, \overrightarrow{x(t)} - \overrightarrow{x^*(t)} \right\rangle dt \geq 0. \tag{6}$$

Proof.
(i) Proof of Necessity. Assume that (6) holds; then

$$\begin{aligned}
&\left\langle \overrightarrow{C\left(\overrightarrow{x^*(t)}\right)}, \overrightarrow{x(t)} - \overrightarrow{x^*(t)} \right\rangle \\
&= \sum_{w \in W} \sum_{p \in P_w} C_p\left(\overrightarrow{x^*(t)}\right)\left(x_p(t) - x_p^*(t)\right) \\
&= \sum_{w \in W} \left\{ \sum_{p \in P_w, C_p(\overrightarrow{x^*(t)}) > \lambda_w(t)} C_p\left(\overrightarrow{x^*(t)}\right)\left(x_p(t) - x^*(t)\right) \right. \\
&\qquad \left. + \sum_{p \in P_w, C_p(\overrightarrow{x^*(t)}) = \lambda_w(t)} C_p\left(\overrightarrow{x^*(t)}\right)\left(x_p(t) - x_p^*(t)\right) \right\} \\
&\geq \sum_{w \in W} \left\{ \sum_{p \in P_w, C_p(\overrightarrow{x^*(t)}) > \lambda_w(t)} \lambda_w(t)\left(x_p(t) - x^*(t)\right) \right. \\
&\qquad \left. + \sum_{p \in P_w, C_p(\overrightarrow{x^*(t)}) = \lambda_w(t)} \lambda_w(t)\left(x_p(t) - x_p^*(t)\right) \right\} \\
&= \sum_{w \in W} \lambda_w(t) \sum_{p \in P_w} \left(x_p(t) - x_p^*(t)\right) \\
&= 0.
\end{aligned} \tag{7}$$

Hence, the necessity is proved.

(ii) Proof of Sufficiency. It is known that (6) is equivalent to

$$\begin{gathered}
C_p\left(\overrightarrow{x^*(t)}\right) - \lambda_w(t) \geq 0, \\
x_p^*(t) \geq 0, \\
x_p^*(t)\left(C_p\left(\overrightarrow{x^*(t)}\right) - \lambda_w(t)\right) = 0.
\end{gathered} \tag{8}$$

We know the first two inequalities are satisfied; in the following, we prove

$$x_p^*(t)\left(C_p\left(\overrightarrow{x^*(t)}\right)-\lambda_w(t)\right)=0. \quad (9)$$

We can get the solution $\overrightarrow{x^*(t)}$ of the variational inequality, according to (2); $\overrightarrow{f^*(t)}$ can be given, owing to (1) and (3); we have the link congestion on each link and the path congestion on each path; then we search the minimum congestion path and put all flows on the path. For the minimum path, we have $C_p(\overrightarrow{x^*(t)}) = \lambda_w(t)$; then $x_p(t)(C_p(\overrightarrow{x^*(t)}) - \lambda_w(t)) = 0$. For other paths, no flow passes; then the flows on these paths are 0; thus $x_p(t)(C_p(\overrightarrow{x^*(t)}) - \lambda_w(t)) = 0$. In addition, because

$$\begin{aligned}&\sum_{p\in P_w}\left[\left(C_p^*\left(\overrightarrow{x^*(t)}\right)-\lambda_w(t)\right)\right]x_p(t)\\ &\quad\geq\sum_{p\in P_w}\left[\left(C_p^*\left(\overrightarrow{x^*(t)}\right)-\lambda_w(t)\right)\right]x_p^*(t),\end{aligned} \quad (10)$$

then $x_p^*(t)(C_p(\overrightarrow{x^*(t)}) - \lambda_w(t)) = 0$ is proved; hence, the necessity is satisfied. □

3. An Example

3.1. The Four-Link Dynamic Network Description and Its Equilibrium Solution. In the following, we consider a simple transportation network with a single origin node o and a single destination node r in Figure 1. Let the total demand for travel from origin o to destination r be $d_w(t) = t$. Further, assume that the problem is symmetric. Specifically, the link congestion functions of the four-link network in Figure 1 are

$$\begin{aligned}c_{op}(t)&=3\left(\gamma f_{op}(t)+f_{pr}(t)\right)+10,\\ c_{qr}(t)&=3\left(\gamma f_{qr}(t)+f_{oq}(t)\right)+10,\\ c_{oq}(t)&=\left(\gamma f_{oq}(t)+f_{qr}(t)\right)+20,\\ c_{pr}(t)&=\left(\gamma f_{pr}(t)+f_{op}(t)\right)+20,\end{aligned} \quad (11)$$

where $c_{ij}(t)$ is the travel congestion on link ij at time t, $f_{ij}(t)$ is the flow on link ij at time t, γ is the scaling parameter which differentiates the influence between the given link only and others, generally, $\gamma \geq 1$. In the four-link network, there are two paths from the origin o to the destination r, and the path flow satisfies the following relationship:

$$\begin{aligned}x_1(t)&=f_{op}(t)=f_{pr}(t),\\ x_2(t)&=f_{oq}(t)=f_{qr}(t),\end{aligned} \quad (12)$$

where x_k is the flow from o to r along path k at time t; in addition, the costs along the paths are given as follows:

$$\begin{aligned}C_1(t)&=c_{op}(t)+c_{pr}(t)=4(\gamma+1)x_1(t)+30,\\ C_2(t)&=c_{oq}(t)+c_{qr}(t)=4(\gamma+1)x_2(t)+30,\end{aligned} \quad (13)$$

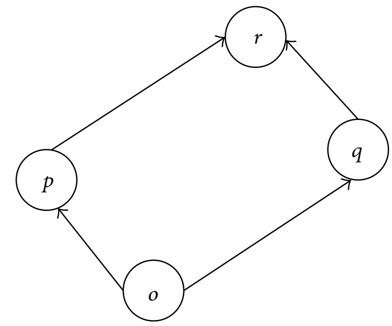

Figure 1: Four-link network.

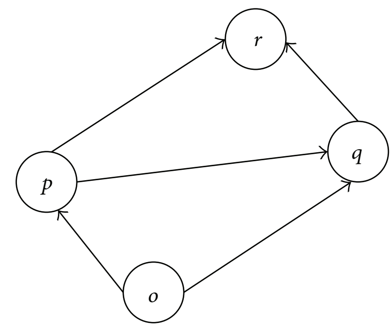

Figure 2: Five-link network.

where $C_k(t)$ is the travel congestion from o to r along path k at time t; the total demand satisfies the following conservation of the flow: $d_w(t) = f_1(t) + f_2(t)$.

The equilibrium solution of the four-link network is easily got as follows according to Definition 1:

$$\begin{aligned}x_1^*(t)&=x_2^*(t)=\frac{t}{2},\\ C_1(t)&=C_2(t)=2(\gamma+1)t+30,\\ T^4(t)&=2(\gamma+1)t^2+30t,\end{aligned} \quad (14)$$

where $T^4(t)$ is the total system travel congestion under dynamic network equilibrium at time t for the four-link network.

3.2. The Five-Link Dynamic Network Description and Its Equilibrium Solution. Add the link pq based on Figure 1; there appears a new path $opqr$ from o to r in Figure 2.

The link congestion functions are given as follows:

$$\begin{aligned}c_{op}(t)&=3\left(\gamma f_{op}(t)+f_{pr}(t)+f_{pq}(t)\right)+10,\\ c_{qr}(t)&=3\left(\gamma f_{qr}(t)+f_{oq}(t)+f_{pq}(t)\right)+10,\\ c_{oq}(t)&=\left(\gamma f_{oq}(t)+f_{qr}(t)\right)+20,\\ c_{pr}(t)&=\left(\gamma f_{pr}(t)+f_{op}(t)\right)+20,\\ c_{pq}(t)&=2\left(\gamma f_{pq}(t)+f_{op}(t)+f_{qr}(t)\right)+5.\end{aligned} \quad (15)$$

The traffic flow on each link is as follows:

$$\begin{aligned} f_{op}(t) &= x_1(t) + x_3(t), \\ f_{pr}(t) &= x_1(t), \\ f_{oq}(t) &= x_2(t), \\ f_{qr}(t) &= x_2(t) + x_3(t), \\ f_{pq}(t) &= x_3(t). \end{aligned} \tag{16}$$

The costs along the paths of five-link network are given as follows:

$$\begin{aligned} C_1(t) &= 4(\gamma+1)x_1(t) + (3\gamma+4)x_3(t) + 30, \\ C_2(t) &= 4(\gamma+1)x_2(t) + (3\gamma+4)x_3(t) + 30, \\ C_3(t) &= (3\gamma+5)(x_1(t)+x_2(t)) + 2(4\gamma+5)x_3(t) + 25. \end{aligned} \tag{17}$$

The total demand satisfies the conservation of the flow as follows:

$$d_w(t) = x_1(t) + x_2(t) + x_3(t). \tag{18}$$

According to Theorem 2, the variational inequality of the five-link dynamic traffic network over $t \in [0, T]$ is given as follows:

$$\begin{aligned} \int_0^T & (4(\gamma+1)x_1^*(t) \\ &+ (3\gamma+4)x_3^*(t) + 30)(x_1(t) - x_1^*(t)) \\ &+ (4(\gamma+1)x_2^*(t) + (3\gamma+4)x_3^*(t) + 30) \\ &\times (x_2(t) - x_2^*(t)) \\ &+ ((3\gamma+5)(x_1^*(t) + x_2^*(t)) \\ &+2(4\gamma+5)x_3^*(t) + 25) \\ &\times (x_3(t) - x_3^*(t))\, dt \geq 0. \end{aligned} \tag{19}$$

Because $d_w(t) = x_1(t) + x_2(t) + x_3(t)$, $d_w(t) = x_1^*(t) + x_2^*(t) + x_3^*(t)$, $x_1^*(t) = x_2^*(t)$, the above variational inequality implies that

$$\begin{aligned} \int_0^T & (2(4\gamma+3)x_1^*(t) - (5\gamma+6)t + 5) \\ &\times (x_1(t) + x_2(t) - 2x_1^*(t))\, dt \geq 0. \end{aligned} \tag{20}$$

We consider the term

$$(2(4\gamma+3)x_1^*(t) - (5\gamma+6)t + 5)(x_1(t) + x_2(t) - 2x_1^*(t)), \tag{21}$$

for the fixed t and analyze it when its value is greater than or equal to zero. It implies that if $x_1^*(t) = 0$, we have $-(5\gamma+6)t + 5 \geq 0$; then $t \leq 5/(5\gamma+6)$; that is, when $t \in [0, 5/(5\gamma+6)]$,

$$x_1^*(t) = x_2^*(t) = 0, \qquad x_3^*(t) = t. \tag{22}$$

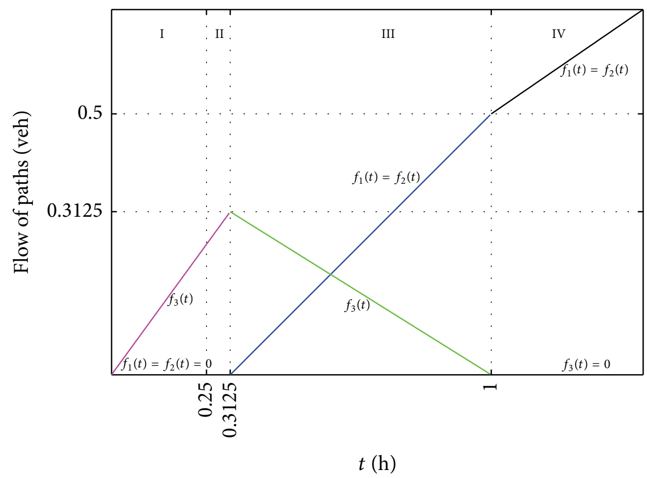

Figure 3: Equilibrium flow of the dynamic network (t(h) versus flow (veh)).

If $x_3^*(t) = 0$, we have $x_1^*(t) = t/2$, then $t > 5/(\gamma+3)$; that is, when $t \in (5/(\gamma+3), +\infty)$,

$$x_1^*(t) = x_2^*(t) = \frac{t}{2}, \qquad x_3^*(t) = 0. \tag{23}$$

When $t \in (5/(5\gamma+6), 5/(\gamma+3)]$,

$$\begin{aligned} x_1^*(t) = x_2^*(t) &= \frac{(5\gamma+6)t - 5}{2(4\gamma+3)}, \\ x_3^*(t) &= \frac{-(\gamma+3)t + 5}{4\gamma+3}. \end{aligned} \tag{24}$$

Assuming $\gamma = 2$, the equilibrium flow of the five-link network is pictured in Figure 3 (time (h) versus flow (veh)), in range I and II: $[0, 5/16]$, only the new path is used; in range III: $(5/16, 1)$, all three paths are used; in range IV: $[1, +\infty)$, only the first two paths are used; that is, the third path is never used when $t > 1$. Corresponding to different ranges, the total travel congestion of five-link network is given as follows:

$$\begin{aligned} T^5(t) &= 2(4\gamma+5)t^2 + 25t, \quad \text{if } t \leq \frac{5}{5\gamma+6}, \\ &= t\left[\frac{(7\gamma^2+9\gamma)t + 5(\gamma+2)}{4\gamma+3} + 30\right], \\ &\qquad \text{if } \frac{5}{5\gamma+6} < t < \frac{5}{\gamma+3}, \\ &= 2(\gamma+1)t^2 + 30t, \quad \text{if } t \geq \frac{5}{\gamma+3}, \end{aligned} \tag{25}$$

where $T^5(t)$ is the total system travel congestion under dynamic network equilibrium for the five-link network.

3.3. The Paradox under Dynamic Network Equilibrium. Letting $T^5 > T^4$, we get $t \in (5/2(3\gamma+4), 5/(\gamma+3))$; that is, the paradox occurs in the range. Letting $\gamma = 2$, we can find that the paradox occurs in ranges II and III in Figure 3.

In order to capture the trend of the paradox when we take different values of γ, we give the definition of the average difference of two functions F_1, F_2 in range $[a,b]((a,b][a,b),(a,b)), b > a$ as follows:

$$D(F_1, F_2) = \frac{|F_1 - F_2|}{b-a}. \quad (26)$$

We use the average difference of T^4, T^5 in range $(5/2(3\gamma + 4), 5/(\gamma+3))$ to represent the severity of the paradox; the greater the average difference is, the greater the severity of the paradox is. In Figure 4, we discuss the situations when $\gamma = 1, 2, 5$, respectively, and find that, in most ranges, the greater γ is, the smaller the average difference of T^4, T^5 is and the smaller the range of the traffic demand in which the paradox occurs is; that is, the severity and the range in which the paradox occurs increases with the influence of other links decreasing, which reminds us to let down the influence between links as much as possible if we wish to decrease the occurrence of the paradox. In addition, we find that the areas where the paradox occurs are different when the values of γ are different; thus we may take some measures to control the influence between the links as time and traffic demand change in order to avoid or decrease the occurrence of the paradox.

3.4. Does the Adding Link Make Sense under DSO? We have known, under the condition of the static traffic assignment, that adding a new link does not reduce the total system travel time even under system optimal [19]. Subsequently, we discuss whether the phenomenon occurs or not under DSO. As we have known, DSO is obtained by charging users the marginal cost of traveling; for the link congestion in this work, the marginal link congestion functions are given as follows:

$$\begin{aligned} c'_{op}(t) &= 3\left(2\gamma f_{op}(t) + f_{pr}(t) + f_{pq}(t)\right) + 10, \\ c'_{qr}(t) &= 3\left(2\gamma f_{qr}(t) + f_{oq}(t) + f_{pq}(t)\right) + 10, \\ c'_{oq}(t) &= \left(2\gamma f_{oq}(t) + f_{qr}(t)\right) + 20, \\ c'_{pr}(t) &= \left(2\gamma f_{pr}(t) + f_{op}(t)\right) + 20, \\ c'_{pq}(t) &= 2\left(2\gamma f_{pq}(t) + f_{op}(t) + f_{qr}(t)\right) + 5. \end{aligned} \quad (27)$$

The corresponding path marginal cost equations are

$$\begin{aligned} C'_1(t) &= 4(2\gamma+1)x_1(t) + 2(3\gamma+2)x_3(t) + 30, \\ C'_2(t) &= 4(2\gamma+1)x_2(t) + 2(3\gamma+2)x_3(t) + 30, \\ C'_3(t) &= (6\gamma+5)(x_1(t)+x_2(t)) + 2(8\gamma+5)x_3(t) + 25. \end{aligned} \quad (28)$$

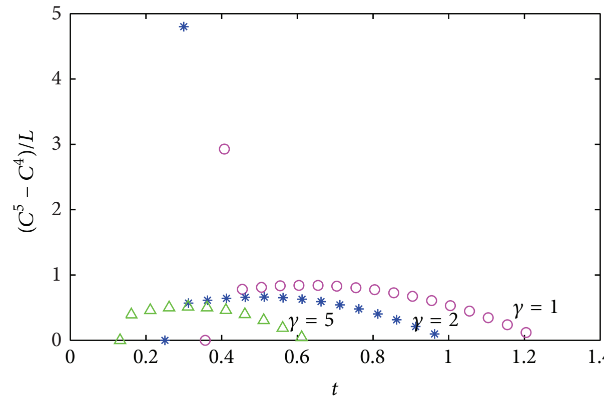

FIGURE 4: The severity in which the paradox occurs.

Then the variational inequality of the five-link dynamic traffic network over $t \in [0,T]$ under DSO is given as follows:

$$\begin{aligned} \int_0^T & \left(4(2\gamma+1)x_1^*(t) + 2(3\gamma+2)x_3^*(t) + 30\right) \\ & \times (x_1(t) - x_1^*(t)) \\ & + \left(4(2\gamma+1)x_2^*(t) + 2(3\gamma+2)x_3^*(t) + 30\right) \\ & \times (x_2(t) - x_2^*(t)) \\ & + ((6\gamma+5)(x_1^*(t) + x_2^*(t)) \\ & \quad + 2(8\gamma+5)x_3^*(t) + 25)(x_3(t) - x_3^*(t))\,dt \ge 0. \end{aligned} \quad (29)$$

Because $d_w(t) = x_1(t) + x_2(t) + x_3(t)$, $d_w(t) = x_1^*(t) + x_2^*(t) + x_3^*(t)$, $x_1^*(t) = x_2^*(t)$, we have

$$\begin{aligned} \int_0^T & \left(2(8\gamma+3)x_1^*(t) - 2(5\gamma+3)t + 5\right) \\ & \times (x_1(t) + x_2(t) - 2x_1^*(t))\,dt \ge 0. \end{aligned} \quad (30)$$

Letting $x_3^*(t) = 0$, then $x_1^*(t) = x_2^*(t) = t/2$; if the value of $\int_0^T (2(8\gamma+3)x_1^*(t) - 2(5\gamma+3)t + 5)(x_1(t) + x_2(t) - 2x_1^*(t))\,dt$ is greater than or equal to zero, we must have

$$2(8\gamma+3)\frac{t}{2} - 2(5\gamma+3)t + 5 \ge 0. \quad (31)$$

Then we obtain $t \le 5/(2\gamma+3)$; it implies that when $t \ge 5/(2\gamma+3)$, $x_3 = 0$; that is, the adding link is not used. Thus when $t \in (0, 5/(2\gamma+3))$, the adding link makes sense under DSO. In the following, we give the trend of the upper bound under which the adding link works under DSO as the parameter γ changes in Figure 5 and find that the bound becomes smaller as γ increases, which explains that the less the influence of the other links is, the less the possibility that the adding link works under DSO, which warns us of improving the influence between the links appropriately if we want to make the adding link work under DSO.

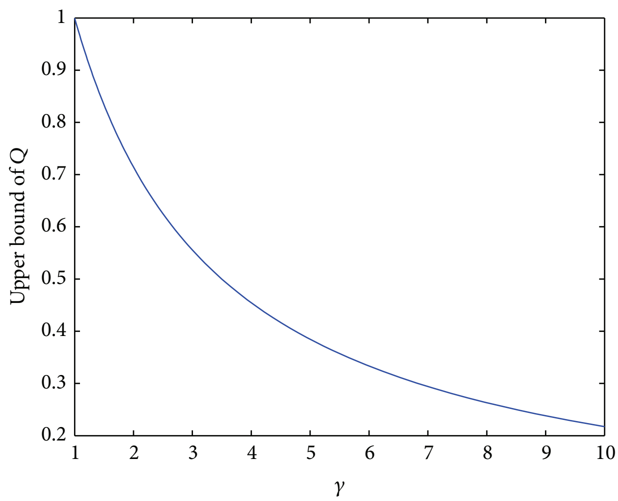

FIGURE 5: The bound above which an adding link cannot make the total costs increase under DSO.

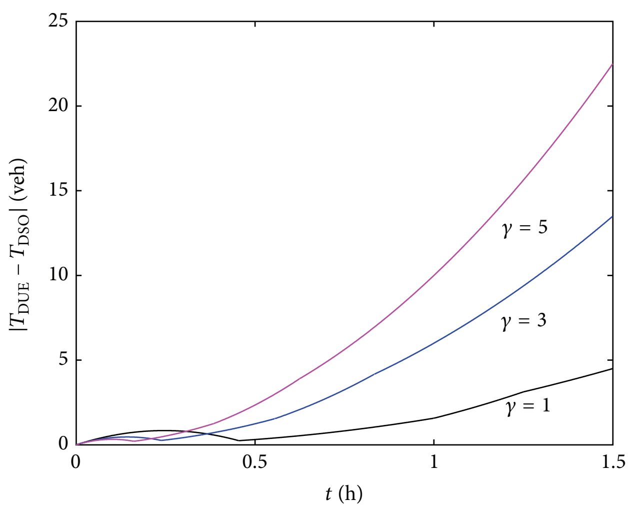

FIGURE 6: The distance between T_{DUE} and T_{DSO}.

3.5. Relationship between the Cost under DUE and That under DSO. About the relationship between total costs under different kinds distributions, researchers have done a large number of studies [23], where we know that, under the static traffic assignment, the solution between UE and SO is approximative in the free flow state; as the traffic becomes more congested, the difference between the solution under UE and that under SO becomes greater. In this work, it shows the influence of another link to relationship between cost under DUE and DSO with a plot Figure 6, in which $T_{\mathrm{DUE}} - T_{\mathrm{DSO}}$ means the distance between total congestion. In Figure 6, let γ be $1, 3, 5$, respectively, and find that the distance between different assignments under congested state is greater than that under free flow state, which is the same as the situation under the static assignments; in addition, $|T_{\mathrm{DUE}} - T_{\mathrm{DSO}}|$ becomes larger with γ increasing under congested state, which is explained as follows: with γ increasing, the influence of other links to the congestion on the given link decreases; that is, the influence of other links is ignored, which is contrary to the choice principle of DSO.

4. Conclusions

In this work, we construct a dynamic traffic network model; it is different from the previous model, in which, the link congestion is dependent on the flow, the inflow rate, and the outflow rate of the link; that is, $c_a(t) = c_a(f_a(t), u_a(t), v_a(t))$, where $c_a(t)$, $f_a(t)$, $u_a(t)$, $v_a(t)$ are the congestion, link flow, inflow rate, and outflow rate on link a, respectively. In our model, the link congestion is related to the flow of the link and the adjacent links at the same path with this link, which can solve the situation where there does not appear new travelers at the intersection (node) and greatly let down the computational load of large-scale traffic network because of reducing the number of vectors. Then we prove equivalence between the equilibrium condition and variational inequality and investigate the paradox phenomenon of the traffic network using the variational inequality. Using the simple four-link network and the five-link network, we find that the possibility and the severity that the paradox occurs become greater and greater with the influence of other links increasing, but the possibility that the adding link makes no sense decreases under DSO, which reminds us of adjusting the influence between the links correctly according to the different purposes in the traffic assignment; in addition, we find the difference between the total congestion under DUE and that under DSO increases with γ increasing, which further explains the essential difference between DUE and DSO. The mechanisms for the dynamic traffic network which are closer to the reality need further study.

Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

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