Review

P. Seshu
Dept. of Mechanical Engineering
Indian Institute of Technology, Powai
Bombay 400 076, India

Substructuring and Component Mode Synthesis

Substructuring and component mode synthesis (CMS), is a very popular method of model reduction for large structural dynamics problems. Starting from the pioneering works on this technique in the early 1960s, many researchers have studied and used this technique in a variety of applications. Besides model reduction, CMS offers several other crucial advantages. The present work aims to provide a review of the available literature on this important technique. © *1997 John Wiley & Sons, Inc.*

INTRODUCTION

Finite element models of typical modern structures can involve a very large number of degrees of freedom (DOF) and this implies considerable computational effort. Significant research efforts have been focused toward *model reduction.* Substructuring and component mode synthesis (CMS) techniques involve partitioning of the entire structure into several *substructures* or *components.* The essential ideal is to derive the behavior of the entire assembly from its constituents. The individual substructure problems are first solved and then the coupling of interfaces is posed. The advantage of breaking up a single large problem into several reduced order problems is the expected gain in such an approach. Besides model reduction, the following advantages accrue from such a method of analysis:

- It is very useful in a design situation; for example, different teams of researchers design different components in an aircraft or nuclear reactor structure. Any design changes affect only the particular component concerned.
- In an optimization setting, recomputations need be done only for those substructures involving modifications.
- The substructure model need not be purely mathematical and it would be possible to include the experimental results too.
- If any nonlinearities such as plastic yielding are localized to a particular region of the entire structure, an elastic–plastic analysis based on substructuring becomes handy with *linear* and *nonlinear* substructures.
- Parallel processing of substructure analyses is possible.

Starting from the pioneering works on this technique in the early 1960s, many researchers studied and used this technique in a variety of applications. Craig (1977, 1987), Greif (1986), and Nelson (1979) reviewed the literature specifically related to substructuring and CMS. Noor (1994) discussed reduction methods in general wherein the main objective was posed as reducing a given problem in a large number of unknowns into a much smaller problem. This may or may not involve substructuring and CMS. The present work is an effort to

Received July 8, 1996; Accepted December 30, 1996

Shock and Vibration, Vol. 4, No. 3, pp. 199–210 (1997)

CCC 1070-9622/97/030199-12

summarize the state of the art in the field of substructuring and CMS. First, contributions to the fundamental methods of CMS are reviewed. Subsequently, modifications/refinements are discussed. Finally applications of the technique to real life problems are presented. However, certain aspects such as computational strategies for CMS on high performance computers and sensitivity analysis in conjunction with CMS are not discussed because they are beyond the scope of the present work.

BASIC METHODS OF CMS

Substructure Representation

Przemieniecki (1963), observing that ". . . some form of structural partitioning is usually necessary either because different methods are used on different structural components or because of the limitations imposed by the capacity of the digital computers," proposed the *substructure method* for *static analysis.* Treating the displacements as unknowns, the complete structure was initially partitioned into a number of substructures, whose boundaries could be specified arbitrarily. In the first step of the analysis, each substructure displacements under the external forces acting on its interior were obtained with all the generalized displacements on common boundaries completely arrested. Subsequently all the boundaries were simultaneously relaxed and the substructure boundary displacements were calculated. The total displacements of the structure were then obtained as the superposition of these displacements.

Hurty (1965) proposed the CMS technique based on substructure eigenproperties for *dynamic analysis.* In this method, the entire structure was partitioned into several substructures or components. In the first step, a set of suitably defined normal modes (called *component modes*) were set up to represent each such component. These were then coupled to get a reduced set of equations for the entire assemblage. From the eigenvectors of this reduced order problem, the mode shapes of the original assemblage were *synthesized.* Hurty proposed a *fixed interface* CMS technique. Three types of component modes were used: rigid body modes, constraint or attachment modes, and fixed interface normal modes. Craig and Bampton (1968) subsequently modified Hurty's approach by using only constraint modes and fixed interface normal modes. To obtain constraint modes, the mass matrix was neglected and unit displacements were imposed successively on all interface boundary DOF. Fixed interface normal modes were defined relative to the constrained system, in which all interface DOF were restrained. They showed that it was not necessary to separately consider the rigid body modes.

Goldman (1969) used *free interface* normal modes in the component vectors. Hou (1969) presented a variant of this free interface normal mode method, which included the possibility of mode truncation; i.e., not all the eigenmodes of a substructure might be used in its representation. The most suitable choice of modes could be made using an error index based on convergence of the eigenvalues. MacNeal (1971) subsequently proposed a *hybrid method* wherein fixed–free interface normal modes were used; i.e., some connection coordinates were free and others were restrained. Rubin (1975) extended MacNeal's approach in the sense of approximating the free–free modes not retained in the formulation. MacNeal's method only accounted for the static contribution of neglected modes. in this work, an inertial and possibly dissipative contribution to the residual effects were also included. Following these ideas, Craig and Chang (1976) presented a method wherein residual flexibility was incorporated into the free interface substructure coupling procedure. Significant improvement in the accuracy was demonstrated. Hintz (1975) used a statically complete interface mode set. Thus, the interface boundary conditions on component normal modes could be either free or fixed. Equivalence of the two statically complete interface mode sets, constraint modes and the attachment modes, was demonstrated.

While Przemieniecki (1963) and Hurty (1965) pioneered the development of the techniques of substructuring and CMS, Gladwell (1964) independently developed a technique called branched mode analysis, the basic process being again modal synthesis. Benfield and Hruda (1971) developed a method essentially similar to the branched mode method. Effects of loading by a component on its neighbors, called interface loading, was studied. Both stiffness and inertial loading were considered in what could be termed *loaded interface* CMS. Hurty et al. (1971) established several criteria for comparison. The different techniques were evaluated against these criteria. Curnier (1983) compared the accuracy of the three variants viz., fixed, free, and loaded interface CMS techniques. While presenting a unified formulation of the three methods, a formal proof of the exactness of these methods (in the absence of mode truncation at the substructure level) was also given.

Ookuma and Nagamatsu (1984a) developed a CMS technique called *multiple* CMS (MCMS), in which a structure was initially divided into a few "first-divided" components. Each first divided component was then subdivided into "second-divided" ones. Repeating this process n times, the structure was finally divided into "n-divided" components. The natural modes of an (n-1)th divided component were analyzed using the natural modes of the nth divided components by using the ordinary CMS method. This procedure was repeated for all system components and the vibration characteristics of the total structure were ultimately analyzed. This approach could be useful, particularly if the structure were made up of several "similar components."

Ookuma and Nagamatsu (1984b) compared the CPU time and accuracy of calculation by CMS and MCMS with those of MSC-NASTRAN concerning the natural frequencies and dynamic responses of two model structures. The first model structure was made up of several plates. Two kinds of division into components were used: one with nine components and the other with 13 components in the conventional CMS technique. In MCMS, the model structure was divided into seven first components and these first components were divided into 18 second components. Alternatively, the model structure was divided into five first components and these components were divided into 18 second ones. Also, three different numbers of adopted natural modes of the components were tried. It was demonstrated that it took much less CPU time (a factor of 6–10) to analyze the vibration by CMS and MCMS than that of MSC-NASTRAN. MCMS needed much less memory compared to CMS (about 65% in some cases). The division into the number of components or the number of modes used did not have much influence. An example of a cylinder block with six cylinders was also discussed.

All the methods discussed so far require the computation of eigenpairs of the substructures. Use of normal modes for mode superposition analysis of dynamic response of structures is well established. However, in the spirit of Rayleigh–Ritz, what is necessary is just a set of admissible functions and not necessarily the normal modes of vibration. This could mean a significant difference in the computational effort because the normal modes have to be extracted using an iterative eigenvalue algorithm. Thus, in general the component Ritz vectors could, but not necessarily, include the eigenvectors of the component under consideration. In this context, the term "mode" should be understood as a general Ritz-type displacement vector defined in some specific manner. Normal modes (eigenvectors) constitute a special case. Hale and Meirovitch (1980) proposed a *general method of CMS* wherein each substructure was modeled using *admissible functions.* This obviated the need for the solution of the substructure eigenvalue problem. Using a bracketing theorem, they discussed the nature of convergence of the eigenvalues.

The Wilson–Ritz vector algorithm was proposed and used by Wilson et al. (1982) and Arnold et al. (1985) as an alternative to normal mode superposition in the general dynamic response analysis of structures. Later on Wilson and Bayo (1986) and Abdallah and Huckelbridge (1990) used this in fixed interface CMS for each substructure. Craig and Hale (1988) used block Krylov Ritz vectors (generated from recurrence relations) in place of component normal modes. Accuracy was shown to be comparable to the conventional CMS while the computational effort was significantly less. Lou et al. (1993) modified the recursive approach of Ritz vector generation given in Wilson et al. (1982) and Arnold et al. (1985) and studied the formulation of fixed as well as free interface CMS using the Wilson–Ritz vector algorithm. The set of basis vectors for model reduction was generated based on successive reduction of residual errors in governing equilibrium equations by Haggblad and Eriksson (1993) in a process analogous to the Lanczos algorithm. A posteriori error bounds were also presented.

It might sometimes be very difficult to select suitable admissible vectors for a complicated structure. Wang and Chen (1990) proposed a method of calculating the expansion base vectors of each substructure. Their method was compared with the free interface and the fixed interface methods. For comparison purposes, they used the computer time required for each method to obtain the same accuracy in the end result. It was shown that their method required only about 40% of the computational time of the traditional methods. Meirovitch and Kwak (1991, 1992) observed that either admissible functions or comparison functions could be used in the Ritz vectors; the former satisfied only geometric boundary conditions while the latter satisfied all the boundary conditions. However, with reference to the case of a flexible multibody system, such as a flexible robotic manipulator, boundary conditions for a given substructure could not be defined independent of adjacent substructures; therefore, comparison functions could not be used. So they noted that only admissible functions could be used. Quasicomparison functions were defined as linear combinations of admissible functions capable of satisfying all the boundary conditions of the problem. It was shown that an inclusion principle existed for the Rayleigh–Ritz based substructure synthesis method; therefore, the eigenvalues so obtained would converge uniformly and from above. In this context, they felt that it was more appropriate to refer to this approach as *substructure synthesis* rather than CMS.

Coupling of Substructures

Another important contribution of Hale and Meirovitch (1980) lay in the *coupling of substructures.* In the conventional method of CMS, when the substructures were assembled together, the resulting structure might not be geometrically compatible in the physical space. This was pointed out and a technique was given for connecting the substructures together by imposing geometric compatibility conditions by means of the method of weighted residuals. Such a structure, whose internal boundary conditions were only approximations to the actual ones, was referred to as an *intermediate structure.*

The approximation of the eigenpairs of an intermediate structure obtained through CMS could normally be improved by taking more substructure trial vectors. Hale and Meirovitch (1982) proposed a scheme, based on subspace iteration, to improve upon the trial vectors until a desired accuracy was obtained. Arora and Nguyen (1980) also adapted the subspace iteration algorithm for use with substructures. However, unlike in the case of Arora and Nguyen (1980), it would be possible to choose each substructure trial vector independent of other substructures in the method of Hale and Meirovitch (1982). Jezequel (1985) presented a hybrid model to facilitate, in the context of CMS, assembly of substructures along a continuous boundary. The model utilized generalized boundary coordinates defined from branch modes obtained by introducing mass loading along the boundary.

In general, the finite element meshes associated with the substructures could have nonconforming discrete interfaces owing to the fact that the submeshes were designed by different analysts, or the corresponding substructures had different resolution requirements, etc. Bennighof (1987), using conforming finite element interpolation functions, constructed superelements to represent the substructures. This ensured exact interface compatibility when nodal DOF were shared between substructures.

Bourquin and d'Hennezel (1992a) observed that the convergence analysis of the different methods of CMS by strict mathematical considerations had not received due attention. Based on a nonconventional choice of constraint modes tied to the normal modes of the Poincaré–Steklov operator (a coupling operator used in domain decomposition methods) associated with the interface between the substructures, they presented a new fixed interface method. Certain error bounds were also given. Bourquin and d'Hennezel (1992b) applied this CMS technique to plate vibrations and error bounds were derived.

A hybrid modal synthesis method based on intermediate problem theory was proposed by Jezequel and Seito (1994) to deal with assembly of substructures along continuous boundaries. Using the integral operators associated with intermediate problems, two new methods of modal truncation were proposed. Farhat and Geradin (1994) developed a hybrid version of the Craig–Bampton approach based on a variational formulation (Lagrange multipliers, similar to Flashner (1986) for a compatible case) that enabled assembly of general incompatible finite element models of substructures. This would actually be a finite element refinement of the intermediate structure concept introduced by Hale and Meirovitch (1980).

Farhat and Geradin (1994) also presented a detailed derivation, based on an interface impedance operator and its spectral decomposition, to show that a given substructure could be exactly represented by a combination of the junction modes and fixed interface normal modes. Thus, they gave a mathematical justification of the Craig–Bampton approach of CMS (1968) as the most "natural" CMS method.

CMS for Damped/Gyroscopic Systems

Conventionally, the concepts of CMS were developed for the structural dynamics of lightly damped systems. For such systems the velocity dependent terms may be initially neglected and the coefficient matrices are real symmetric. Structures such as buildings, bridges, and airframes are of this type. However, other structures such as those with rotating parts or heavy damping fall under the category of nonconservative and/or gyroscopic systems that require more general analysis.

For a nongyroscopic–nonconservative system, Hasselman and Kaplan (1974) and Dowell (1980) used *complex* eigenvectors for the substructures. Hasselman and Kaplan (1974) used fixed interface complex modes. Glasgow and Nelson (1980) applied the method of component mode synthesis to the problem of dynamics of rotor-bearing systems, in particular, whirl mode and stability analysis. They used complex domain analysis and their approach required a general complex eigenvalue/eigenvector extraction methodology.

Craig and Chung (1982) used free interface complex modes. Wu and Greif (1983) used two successive transformations: one based on free interface undamped modes and the other on fixed interface damped complex modes. Nelson et al. (1983) extended the work of Glasgow and Nelson (1980) to use the CMS technique for simulating the nonlinear rotor dynamic behavior. Their approach involved modal decomposition of the linear rotating assembly using complex component modes with truncation; assembly of the reduced component equations were performed with the linear and/or nonlinear supports to form the system equations. Based on complex mode shapes rather than real normal modes of the substructures, Beliveau and Soucy (1985) developed a technique that involved only displacement transformation and not velocities as in a state space formulation. Kubomura (1987) studied the use of CMS techniques for structures with a nonproportional symmetric damping matrix. He used complex free–free, cantilever, and hybrid modes.

For complicated substructure models, obtaining the complex eigenvectors could itself be a difficult task. Also, a method that avoided complex matrices would be preferable. Following these, Hale (1984) developed a general substructure synthesis method for nonconservative vibratory systems. Viscous damping and/or gyroscopic forces were included by adopting a state space formulation for each substructure. The substructure equations were in terms of one symmetric matrix and one asymmetric matrix, both twice the dimension of the original coefficient matrices. A number of *real* trial vectors having no relationship with substructure eigenproblem were used. He also extended Hale and Meirovitch's (1982) procedure for systematic improvement of a fixed number of substructure trial vectors that was originally proposed for conservative nongyroscopic systems. It was observed that the substructure synthesis method involving trial vectors defined piecewise over each substructure could actually be seen as a discrete substructure (superelement) counterpart to Galerkin's method applied to distributed-parameter finite elements.

For the case of a nonclassically damped system, Craig and Ni (1989) used left and right free interface eigenvectors and attachment modes. Qian and Hansen (1995) extended CMS for systems with viscoelastic damping in hereditary integral form, differential operator form, and steady-state form. de Kraker and van Campen (1996) modified Rubin's (1975) CMS technique for systems with general viscous damping using complex (residual) flexibility modes, state-space rigid body modes, and complex free–free dynamic modes.

Substructuring for Locally Nonlinear Problems

In many structural problems, the nature of the nonlinearity, if any, is found in only a few local regions whereas the rest of the structure remains entirely linear elastic during the whole dynamic analysis. For example, vehicle dynamics (frame, cabin, etc.) can be described by linear models, but the engine–suspension may behave nonlinearly. In such locally nonlinear cases where the structure is divided into *linear* and *nonlinear substructures*, significant computational efficiency can be obtained due to the restriction of the iterative process to the DOF of the nonlinear substructure.

Bathe and Gracewski (1981) provided a detailed theoretical formulation using the substructuring technique to solve large dynamic problems with small isolated areas of nonlinearities. Zhong et al. (1988) successfully employed the substructuring technique in the dynamic analysis of a fixed offshore platform involving a locally nonlinear system foundation. Sheu et al. (1989) used a multilevel substructuring technique for 2-dimensional nonlinear analysis. Numerical experiments showed that the execution time for each iteration with a reformation of the stiffness matrix was about 3–5 times that required for iteration without stiffness matrix reformation. During the incremental iterative analysis of the entire structure, only the stiffness matrix of the nonlinear substructure should be updated according to the effective stress level at the Gauss integration points of the element. Sheu et al. (1990) presented several illustrative examples of elastic-plastic dynamic analysis under plane stress and axisymmetric conditions.

MODIFICATIONS OF CMS

Research efforts were also directed at improving the CMS method by incorporating other techniques. While using selected normal modes for the substructures, the CMS techniques usually involved truncation of higher modes and hence accurate estimates of only the lower modes of the entire structure were obtained. Kuhar and Stahle (1974) observed that the fixed interface methods were well conditioned so that truncation errors resulting from omission of the higher modes were reduced. The free–free interface methods, on the other hand, eliminated the need for constraint modes but resulted in solutions that were more susceptible to truncation error. They proposed a *dynamic transformation* method that included the effects of modes not retained explicitly in the eigenvalue solution. A dynamic transformation that related the unused coordinates to the retained coordinates at a selected system frequency was obtained from the complete equations of motion. This transformation was then used to reduce the mass and stiffness matrices while retaining those coordinates of primary interest. Because the transformation was at any system frequency, the method was not restricted to low mode solutions but was also useful for modes in any frequency range. If the system frequency were selected to be zero, the transformation would become that of Guyan Bathe (1982).

Nagamatsu and Ookuma (1981) used a generalized reduction technique for the components in conjunction with CMS, introducing a parameter α in the transformation matrix for reduction. If $\alpha = 0$, it is pure static reduction; if $\alpha = 1$, it is pure dynamic condensation. It was shown that the lower modes were predicted very poorly for large values of α. Hence, the static reduction process was recommended for analysis by CMS. Ookuma and Nagamatsu (1984c) proposed a CMS technique wherein the substructure matrices were condensed by Guyan's reduction. The interface regions of these components were considered as another component whose displacement was represented as a linear combination of natural modes calculated from the reduced matrices of all other components. For the example of a plate, they considered eight different ways of division into components. It was shown that the method of component division did not have a serious influence on the accuracy of calculation. Also, it was indicated that it would be enough to adopt the component modes in the same number as the demanded total modes. However, the numbers given were so close and the errors so small, that the limitations of these conclusions were not obvious. While this work was developed as an improvement over Benfield and Hruda's (1971), comparison with the earlier method, which clearly demonstrated the limitations and advantages, was not given. A procedure for calculation of the residual compliance matrix for substructures with free–free boundary conditions was subsequently presented by Ookuma and Nagamatsu (1985).

Addressing the issue of which modes of the components need be retained, Kubomura (1982) used a parameter that was essentially based on the difference between the component eigenvalue and the system eigenvalue desired. He observed that all modes whose eigenvalues were below 2.25 (1.5^2) times the highest system eigenvalues of interest and above 40% of the lowest system eigenvalues, must be retained. A substructure synthesis method based on state space mathematics was proposed by Tavakoli and Singh (1989) for application to hermetic shell structures. It was observed that the state space method (SSM) had strengths such as systematic, building-block fashion substructuring, exact and comprehensive matching of all boundary variables at the junctions of substructures, and the capability to model the dynamics of various axisymmetric thin shell structures easily and accurately. The solution phase of the method was improved upon by adopting the Pade approximation for matrix exponentiation. An example of an actual refrigeration compressor shell was considered and the measured modal data compared with the proposed method. However, comparison with other CMS techniques was not presented.

A method for substructure modal synthesis that did not require the setting up of an entire system dynamic equation was proposed by Yee and Tsuei (1989). The eigenvalues of the system were instead directly obtained from the determinant of the modal force matrix. Thus, it was possible to get exact results. Yee and Tsuei (1990) subsequently extended their analysis to include transient response calculation. Using Lanczos vectors, Zeng and Xie (1992) obtained a modal transformation matrix. Their method attempted to unify the fixed, free, and hybrid interface methods. Three illustrative examples were given. Singh and Suarez (1992) presented a technique that combined the CMS with dynamic condensation (Guyan reduction) at the substructure level. Suarez and Singh (1992) discussed a new fixed interface CMS technique with improved accuracy by using higher order modal combination techniques (similar to the mode acceleration method). Bennighof (1987) proposed a technique called component mode iteration, in which a form of subspace iteration was carried out at the substructure level. El-Sayed and Hsiung (1990) presented an approach for parallel processing of finite elements using substructuring, each substructure being analyzed independently on a separate processor.

Bouhaddi and Fillod (1992a,b) used Guyan's condensation at each substructure level. In the first stage, only interface DOF as well as certain selected interior DOF were retained. Subsequently, the condensed matrices of the substructures were assembled and the resulting system solved according to standard procedures. Let ω_G be the lowest frequency of all substructures when the interface DOF were fixed and ω_g the frequency of the entire structure predicted based on the condensation and synthesis. It was shown that acceptable precision would be available in ω_g only when $\omega_g < 1/2\ \omega_G$.

Yee (1990) observed that the generalized variables used in CMS might not be measurable in a modal test. Hence, a method to *eliminate all nonmeasured variables* in terms of measured quantities was presented. Engels (1992) focused on the need for consistent improvement in the accuracy of system eigenpairs obtained from CMS when modal truncation was resorted to at the substructure level. A method to reduce the residual norm associated with each of the eigenpairs was presented. From the results obtained on example problems, it was noticed that the cutoff frequency need not be taken larger than twice the desired system frequency.

Yasui (1993) observed that, in general, complex space structures were frequently modified in the design stage. A convenient *reanalysis method* would therefore be useful. Flexible space structures could be modeled as dynamic components that interact with the launch vehicle in order to estimate the dynamic characteristics during the launch stage. Hence a CMS based reanalysis technique was proposed. A simple beam model of a launch vehicle/spacecraft coupling structure was used as a demonstrative example.

APPLICATIONS OF CMS

Flexible Robotic Manipulators

As higher and higher operating speeds become necessary to meet increasing demands, inertia effects become predominant in robotic manipulators. Lighter links would be preferable, but this introduces flexibility. It would therefore be necessary to realize controlled motion in an elastic world! Moreover, as the manipulator moves in its entire work space, the configuration changes and its dynamic characteristics could in general change too. Thus, for effective control based on elastic system dynamic response calculations, it may become necessary to determine the eigenpairs of the manipulator as it moves. In conventional analysis, the complete model will have to be recalculated. Recognition that dynamics of any given individual link does not change and only the relative configuration of links changes, leads to effective use of CMS in such an application. Thus, in CMS, only the reduced model of interlink coupling need be recomputed for different positions because individual link substructure models remain invariant.

Sunada and Dubowsky (1983) modeled the behavior of an industrial robotic manipulator with flexible links. They incorporated finite element representation of the robot links into a system model based on CMS. Shabana and Wehage (1983) developed a computationally efficient coupled dynamic analysis method for mechanical systems consisting of rigid and flexible members based on substructuring. An elastic slider crank mechanism with an elastic connecting rod was presented as an example to illustrate the approach. Moreover, the problem of a tracked vehicle with a flexible gun tube negotiating rough terrain was considered.

Smet et al. (1989) considered modeling of flexible robotic manipulators using CMS. Each separate link was modeled using the finite element method. These models were assembled into a global model using compatibility matrices. The possibility of defining flexible joints between two links was included. An example three link manipulator indicated that the CMS method with fixed–fixed modes gave the most reliable resonance frequencies. CMS results on an industrial manipulator compared well with experimental data. Reynoso and Seering (1991) developed a dynamic system model of a Cartesian robot as a connection of substructures that remain time invariant and whose modal characteristics and transfer functions could be readily determined experimentally. A strategy was presented for modeling the response of adjacent components as they experienced relative translational motion. The experiments conducted showed that each of the first 10 modes, and sometimes higher modes as well, contributed significantly to the system response. It was observed that the simulations required about 40 times as long to run on a dedicated DEC VAX 11/750 computer, as the robot required to make the corresponding move. This emphasized the need for further studies to realize real time control in a complex manipulator. Moon and Cho (1992) used the CMS technique of Hou (1969) to study the vibrations of articulated robotic manipulators. They first considered a simple four bar mechanism and subsequently a four bar type robotic manipulator. The modal data for each component required for synthesis was obtained experimentally. CMS predictions of system eigenvalues were shown to be in close agreement with experimental results.

Turbomachinery Applications

Bladed Disks. The critical structural element of a typical gas turbine engine is the compressor or turbine disk carrying several blades around its circumference. These units operate in severe environments characterized by high speeds of rotation and cycle temperature. The majority of the failures reported in turbines have been due to vibration induced fatigue of the bladed disk units. The failure of even a single blade in an engine can adversely affect the performance. Also, the resulting fragments, being carried around the circumference, can damage the other blades. The classical structural analysis problem has been to determine the steady-state and dynamic behavior of these critical units under operating conditions. While the whole bladed disk assembly needs to be modeled for an accurate prediction of its dynamics, to a first approximation the blades can be assumed to be identical. Thus, CMS appears attractive to divide the entire unit into a *disk substructure* and a *blade substructure.*

Perlman and Schaeffer (1979) used CMS to calculate the natural frequencies and mode shapes of groups of turbomachinery blades. It was noted that CMS was ideally suited to such problems involving identical blades, because accurate determination of eigenproperties of one blade leads to accurate predictions of entire systems. They adopted the Craig–Bampton (1968) approach. Considering the vibrations of bladed disk assemblies, Irretier and Schmidt (1982) and Irretier (1983) used the CMS technique for tuned (i.e., all blades alike) and mistuned systems. Finite elements were used to model the disk and the blade substructures. It was observed that CMS yielded high accuracy in relation to the computational effort, especially when several identical substructures existed, e.g., tuned bladed disk assembly.

Rotor-Bearing Systems. Analysis of dynamic response of rotors is crucial especially at high speeds. Glasgow and Nelson (1980) applied the method of CMS to the problem of dynamics of rotor bearing systems, in particular, whirl mode and stability analysis. They discussed examples of a single uniform shaft with internal damping supported by identical bearings and also a dual rotor system with an intershaft bearing. Numerical results indicated better accuracy in the whirl mode analysis of rotor bearing systems when constraint modes were included than for the analysis without constraint modes. Because the onset of instability is usually associated with the first forward mode, modal truncation did not alter the instability predictions substantially.

Nelson et al. (1983) extended the work of Glasgow and Nelson (1980) to use the CMS technique for simulating the nonlinear rotor dynamic behavior. Two example rotor systems were analyzed to determine their dynamic characteristics due to blade loss. The results indicated that a high accuracy simulation was possible with the retention of a small number of component modes. Subbaiah et al. (1989) used the MCMS technique of Ookuma and Nagamatsu (1984a) to study the dynamic response of rotor bearing systems. Parszewski and Krynicki (1989) considered fluid-film force representation in journal bearings under dynamic conditions and combined this with a rotor model using CMS to conduct a stability analysis of rotor bearing systems. A case study of a vertical pump was discussed. While Glasgow and Nelson (1980) used fixed interface component complex mode analysis, Li and Gunter (1982) used Hou's (1969) free interface CMS for large rotor systems. Wang and Kirkhope (1994a) discussed a free interface CMS method for damped rotor systems with flexible couplings. Wang and Kirkhope (1994b) focused attention on rotor systems with flexible and rigid interfaces such as a multispool rotor system with flexible bladed disks; the interfaces between shaft and disk could be rigid while those between shafts could more likely be flexible. Truncated normal modes of the components were augmented with residual flexibility modes and inertia relief modes. They discussed a case study of a two spool aircraft gas turbine engine. Xu and Marangoni (1994) studied the mechanical vibration resulting from shaft misalignment and imbalance. The generalized equations of motion for a complete motor flexible coupling rotor system were derived based on the CMS technique of Craig and Bamptom (1968).

Other Applications

Prakash and Prabhu (1986) studied the free vibration of Indian Remote Sensing (IRS) spacecraft. The effectiveness of different methods of reducing the number of interface DOF at the system level was discussed. Recursive substructuring was suggested. Farhat and Geradin (1994) applied their technique to a real life example of modal analysis of a high speed civil transport aircraft. Haggblad and Eriksson (1993) discussed the dynamic analysis of a bogie for traction vehicles. The complete model contained 78,000 DOF distributed over three substructures.

Problems of contact–impact frequently occur in structural analysis, e.g., the response of aircraft structures to impact by foreign objects. Wu and Haug (1990) studied such problems. Impacting bodies were divided into several substructures and contact–impact was then considered as occurring between substructures. Additional modes were suitably introduced into the representation of deformation fields in impacting substructures, after impact occurs. The Craig–Bampton (1968) approach was used and constraint and fixed interface modes were used for each substructure. An impact constraint mode could be defined as a constraint mode obtained by giving a unit displacement of a contact node in the impact direction. The disturbance generated at the contact point was approximated by a velocity jump in the impacting substructures. Wave propagation was defined by subsequent excitation of deformation modes in each of the other substructures of the bodies in contact. Based on examples of a long rod and a beam, the authors showed that their method correctly predicted the contact duration, contact forces, and displacements. Ginsberg and McDaniel (1991) used CMS to study the effects of circumferential ring stiffeners on the vibration and acoustic radiation of a submerged circular plate. They observed that treating the stiffeners as separate substructures in CMS enabled extraction of explicit information about the plate–stiffener interaction.

The vibration of floor systems caused by human movements can be reduced using tuned mass dampers (TMD). Setareh and Hanson (1992) used the CMS method to compute the response of the floor TMD system using only a few natural modes of the floor. It was found that using the CMS method could result in optimum parameters of the TMDs, regardless of the closeness of the natural frequencies of the system. Setareh et al. (1992) extended this approach by adding a number of static mode shapes to the formulation. These shapes were obtained as the static displacements of the system when a unit load was applied at the attachment point of the TMD to the structure. It was shown that the addition of these modes resulted in better accuracy of the predicted response.

SUMMARY

For structural systems that have a large number of DOF or have components designed by separate groups of organizations, the method of component mode synthesis has proven to be an accurate, efficient, and economical method of analysis. The primary merit of CMS is that the number of DOF in the total structure is far less than in direct use of the finite element method. Over the years, the basic formulation has been extended to damped/gyroscopic systems as well as systems with strongly local nonlinearities. A wide variety of applications have emerged: design of tuned mass dampers in civil structures; modeling of contact–impact problems; solution of large structural dynamics problems such as aircraft analyses; high speed rotor dynamic problems; flexible robotic manipulators; etc. A review of the available literature on all aspects has been presented in this work. However, certain aspects, such as computational strategies for CMS on high performance computers and sensitivity analysis in conjunction with CMS, have not been discussed because they are beyond the scope of the present work.

REFERENCES

Abdallah, A. A., and Huckelbridge, A. A., 1990, "Boundary Flexible Method of Component Mode Synthesis Using Static Ritz Vectors," *Computers and Structures,* Vol. 35, pp. 51–61.

Arnold, R. R., Citerley, R. L., Chargin, M., and Gallant, D., 1985, "Application of Ritz Vectors for Dynamic Analysis of Large Structures," *Computers and Structures,* Vol. 21, pp. 461–467.

Arora, J. S., and Nguyen, D. T., 1980, "Eigensolution for Large Structural Systems with Substructures," *International Journal for Numerical Methods in Engg.,* Vol. 15, pp. 333–341.

Bathe, K. J., 1982, Finite Element Procedures in Engineering Analysts, Prentice Hall, Englewood Cliffs, N.J.

Bathe, K. J., and Gracewski, S., 1981, "On Non-Linear Dynamic Analysis Using Substructuring and Mode Superposition," *Computers and Structures,* Vol. 13, pp. 699–707.

Beliveau, J.-G., and Soucy, Y., 1985, "Damping Synthesis Using Complex Substructure Modes and a Hermitian System Representation," *AIAA Journal,* Vol. 23, pp. 1952–1956.

Benfield, W. A., and Hruda, R. F., 1971, "Vibration Analysis of Structures by Component Mode Substitution," *AIAA Journal,* Vol. 9, pp. 1255–1261.

Bennighof, J. K., 1987, "Component Mode Iteration for Frequency Calculations," *AIAA Journal,* Vol. 25, pp. 996–1002.

Bouhaddi, N., and Fillod, R., 1992a, "Substructuring Using a Linearized Dynamic Condensation Method," *Computers and Structures,* Vol. 45, pp. 679–683.

Bouhaddi, N., and Fillod, R., 1992b, "A Method for Selecting Master DOF in Dynamic Substructuring Using the Guyan Condensation Method," *Computers and Structures,* Vol. 45, pp. 941–946.

Bourquin, F., and d'Hennezel, F., 1992a, "Intrinsic Component Mode Synthesis and Plate Vibrations," *Computers and Structures,* Vol. 44, pp. 315–324.

Bourquin, F., and d'Hennezel, F., 1992b, "Numerical Study of an Intrinsic Component Mode Synthesis Method," *Computer Methods in Applied Mechanics and Engineering,* Vol. 97, pp. 49–76.

Craig, R. R., Jr., and Bampton, M. C. C., 1968, "Coupling of Substructures for Dynamic Analysis," *AIAA Journal,* Vol. 6, pp. 1313–1319.

Craig, R. R., Jr., and Chang, C. J., 1976, "Free Interface Methods of Substructure Coupling for Dynamic Analysis," *AIAA Journal,* Vol. 14, pp. 1633–1635.

Craig, R. R., Jr., 1977, "Methods of Component Mode Synthesis," *Shock and Vibration Digest,* Vol. 9, No. 11, pp. 3–10.

Craig, R. R., Jr., and Chung, Y. T., 1982, "Generalised Substructure Coupling Procedure for Damped Systems," *AIAA Journal,* Vol. 20, pp. 442–444.

Craig, R. R., Jr., 1987, "A Review of Time Domain and Frequency Domain Component Mode Synthesis Methods," *International Journal of Analytical and Experimental Modal Analysis,* Vol. 2, No. 2, pp. 59–72.

Craig, R. R., Jr., and Hale, A. L., 1988, "Block-Krylov Component Synthesis Method for Structural Model Reduction, *Journal of Guidance, Control, Dynamics, AIAA,* Vol. 11, pp. 562–570.

Craig, R. R., Jr., and Ni, Z., 1989, "Component Mode Synthesis for Model Order Reduction of Nonclassically Damped Systems," *Journal of Guidance, Control and Dynamics,* Vol. 12, pp. 577–584.

Curnier, A., 1983, "On Three Modal Synthesis Variants," *Journal of Sound and Vibration,* Vol. 90, pp. 527–540.

de Kraker, A., and van Campen, D. H., 1996, "Rubin's CMS Reduction Methods for General State Space Models," *Computers and Structures,* Vol. 58, pp. 597–606.

Dowell, E. H., 1980, "Component Mode Analysis of Nonlinear and Nonconservative Systems," *ASME Journal of Applied Mechanics,* Vol. 47, pp. 172–176.

El-Sayed, M. E. M., and Hsiung, C. K., 1990, "Parallel Finite Element Computation with Separate Substructures," *Computers and Structures,* Vol. 36, pp. 261–265.

Engels, R. C., 1992, "Convergence Improvement for Component Mode Synthesis," *AIAA Jounal,* Vol. 30, pp. 490–495.

Farhat, C., and Geradin, M., 1994, "On a Component Mode Synthesis Method and Its Application to Incompatible Substructures," *Computers and Structures,* Vol. 51, pp. 459–473.

Flashner, H., 1986, "An Orthogonal Decomposition Approach to Modal Synthesis," *International Journal for Numerical Methods in Engineering,* Vol. 23, pp. 471–493.

Ginsberg, J. H., and McDaniel, J. G., 1991, "An Acoustic Variational Principle and Component Mode Synthesis Applied to the Analysis of Acoustic Radiation from a Concentrically Stiffened Plate," *ASME Journal of Vibration and Acoustics,* Vol. 113, pp. 401–408.

Gladwell, G. M. L., 1964, "Branch Mode Analysis of Vibrating Systems," *Journal of Sound and Vibration,* Vol. 1, pp. 41–59.

Glasgow, D. A., and Nelson, H. D., 1980, "Stability Analysis of Rotor-Bearing Systems Using Component Mode Synthesis," *ASME Journal of Mechanical Design,* Vol. 102, pp. 352–359.

Goldman, R. L., 1969, "Vibration Analysis by Dynamic Partitioning," *AIAA Journal,* Vol. 7, pp. 1152–1154.

Grief, R., 1986, "Substructuring and Component Mode Synthesis," *The Shock and Vibration Digest,* Vol. 18, pp. 3–8.

Haggblad, B., and Eriksson, L., 1993, "Model Reduction Methods for Dynamic Analysis of Large Structures," *Computers and Structures,* Vol. 47, pp. 735–749.

Hale, A. L., and Meirovitch, L., 1980, "A General Substructures Synthesis Method for Dynamic Simulation of Complex Structures," *Journal of Sound and Vibration,* Vol. 69, pp. 309–326.

Hale, A. L., and Meirovitch, L., 1982, "A Procedure for Improving Discrete Substructure Representation in Dynamic Synthesis," *AIAA Journal,* Vol. 20, pp. 1128–1136.

Hale, A. L., 1984, "Substructure Synthesis and Its Iterative Improvement for Large Nonconservative Vibratory Systems," *AIAA Journal,* Vol. 22, pp. 265–272.

Hasselman, T. K., and Kaplan, A., 1974, "Dynamic Analysis of Large Systems by Complex Mode Substitution," *ASME Journal of Dynamic Systems, Measurement and Control,* Vol. 96, pp. 327–333.

Hintz, R. M., 1975, "Analytical Methods in Component Mode Synthesis," *AIAA Journal,* Vol. 13, pp. 1007–1016.

Hou, S., 1969, "Review of Modal Synthesis Techniques and New Approach," *Shock and Vibration Bulletin,* Vol. 40, pp. 25–39.

Hurty, W. C., 1965, "Dynamic Analysis of Structural Systems Using Component Modes," *AIAA Journal,* Vol. 3, pp. 678–685.

Hurty, W. C., Collins, J. D., and Hart, G. C., 1971, "Dynamic Analysis of Large Structures by Modal Synthesis Technique," *Computers and Structures,* Vol. 1, pp. 535–563.

Irretier, H., and Schmidt, K. J., 1982, "Mistuned Bladed Disks—Dynamical Behavior and Computation," *International Conference on Rotor Dynamics Problems in Power Plants,* Sept. 28–Oct. 1, Rome, Italy.

Irretier, H., 1983, "Spectral Analysis of Mistuned Bladed Disk Assemblies by Component Mode Synthesis," *Vibrations of Blades and Bladed Disks, ASME,* pp. 115–120.

Jezequel, L., 1985, "A Hybrid Method of Modal Synthesis Using Vibration Tests," *Journal of Sound and Vibration,* Vol. 100, pp. 191–200.

Jezequel, L., and Seito, H. D., 1994, "Component Modal Synthesis Methods Based on Hybrid Models, Parts 1 and 2," *ASME Journal of Applied Mechanics,* Vol. 61, pp. 100–116.

Kubomura, K., 1982, "A Theory of Substructure Modal Synthesis," *ASME Journal of Applied Mechanics,* Vol. 49, pp. 903–909.

Kubomura, K., 1987, "Component Mode Synthesis for Damped Structures," *AIAA Journal,* Vol. 25, pp. 740–745.

Kuhar, E. J., and Stahle, C. V., 1974, "Dynamic Transformation Method for Modal Synthesis," *AIAA Journal,* Vol. 12, pp. 672–678.

Li, D. F., and Gunter, E. J., 1982, "A Study of the Modal Truncation Error in the Component Mode Analysis of a Dual-Rotor System," *ASME Journal of Engineering for Power,* Vol. 104, pp. 525–532.

Lou, M., Ghobarah, A., and Aziz, T. S., 1993, "Application of Wilson-Ritz Vectors in Dynamic Substructuring," *International Journal of Solids and Structures,* Vol. 30, pp. 3159–3170.

MacNeal, R. H., 1971, "A Hybrid Method of Component Mode Synthesis," *Computers and Structures,* Vol. 1, pp. 581–601.

Meirovitch, L., and Kwak, M. K., 1991, "Rayleigh–Ritz Based Substructure Synthesis for Flexible Multibody Systems," *AIAA Journal,* Vol. 29, pp. 1709–1719.

Meirovitch, L., and Kwak, M. K., 1992, "Inclusion Principle for the Rayleigh–Ritz Based Substructure Synthesis," *AIAA Journal,* Vol. 30, pp. 1344–1351.

Moon, J. D., and Cho, D. W., 1992, "Component Mode Synthesis Applied to Mechanisms for an Investigation of Vibration," *Journal of Sound and Vibration,* Vol. 157, pp. 67–79.

Nagamatsu, A., and Ookuma, M., 1981. "Analysis of Vibration by Component Mode Synthesis Method, (Part 1, Natural Frequency and Natural Mode)," *Bulletin of JSME,* Vol. 24, pp. 1448–1453.

Nelson, F. C., 1979, "A Review of Substructure Analysis of Vibrating Systems," *The Shock and Vibration Digest,* Vol. 11, pp. 3–9.

Nelson, H. D., Meacham, W. L., Fleming, D. P., and Kascak, A. F., 1983, "Nonlinear Analysis of Rotor Bearing Systems Using Component Mode Synthesis," *ASME Journal of Engineering for Power,* Vol. 105, pp. 606–614.

Noor, A. K., 1994, "Recent Advances and Applications of Reduction Methods," *Applied Mechanics Reviews,* Vol. 47, No. 5, pp. 125–146.

Ookuma, M., and Nagamatsu, A., 1984a, "Vibration Analysis by Multiple Component Mode Synthesis Method," *Bulletin of JSME,* Vol. 27, pp. 1288–1293.

Ookuma, M., and Nagamatsu, A., 1984b, "Comparison of Component Mode Synthesis with MSC-NASTRAN," *Bulletin of JSME,* Vol. 27, pp. 1294–1298.

Ookuma, M., and Nagamatsu, A., 1984c, "Analysis of Vibration by Component Mode Synthesis Method, (Part 4, Natural Frequency and Natural Mode)," *Bulletin of JSME,* Vol. 27, pp. 529–533.

Ookuma, M., and Nagamatsu, A., 1985, "Vibration Analysis by Substructure Synthesis Method, (Part 4, Calculation of Residual Compliance Matrix)," *Bulletin of JSME,* Vol. 28, pp. 905–910.

Parszewski, Z. A., and Krynicki, K., 1989, "Rotor-Bearing Systems Stability: Composition Approach with Bearing Shape Function Presentation," *Tribology Transactions,* Vol. 32, pp. 517–523.

Perlman, A. B., and Schaeffer, M. T., 1979, "The Application of Component Mode Synthesis to Covered Groups of Blades," ASME Design Engg. Technical Conf., St. Louis, Sept. 10–12.

Prakash, B. G., and Prabhu, M. S. S., 1986, "Reduction Techniques in Dynamic Substructures for Large Problems," *Computers and Structures,* Vol. 22, pp. 539–552.

Przemieniecki, J. S., 1963, "Matrix Structural Analysis of Substructures," *AIAA Journal,* Vol. 1, pp. 138–147.

Qian, D., and Hansen, J. S., 1995, "Substructure Synthesis Method for Frequency Response of Viscoelastic Structures," *AIAA Journal,* Vol. 33, pp. 520–527.

Reynoso, A. G., and Seering, W., 1991, "Dynamic Modeling of Systems with Translating Components," *ASME Journal of Mechanical Design,* Vol. 113, pp. 248–252.

Rubin, S., 1975, "Improved Component Mode Representation for Structural Dynamic Analysis," *AIAA Journal,* Vol. 13, pp. 995–1006.

Setareh, M., and Hanson, R. D., 1992, "Tuned Mass Dampers to Control Floor Vibration from Humans," *ASCE Journal of Structural Engineering,* Vol. 118, pp. 741–762.

Setareh, M., Hanson, R. D., and Peek, R., 1992, "Using Component Mode Synthesis and Static Shapes for Tuning TMDs," *ASCE Journal of Structural Engineering,* Vol. 118, pp. 763–782.

Shabana, A., and Wehage, R. A., 1983, "Variable Degree-of-Freedom Component Mode Analysis of Inertia Variant Flexible Mechanical Systems," *ASME Journal of Mechanisms, Transmissions and Automation in Design,* Vol. 105, pp. 371–378.

Sheu, C.-H., Roeck, G. D., Laethem, M. V., and Geyskens, P., 1989, "Multilevel Substructuring and an Experimental Self-Adaptive Newton–Raphson Method for Two-Dimensional Nonlinear Analysis," *Computers and Structures,* Vol. 33, pp. 489–497.

Sheu, C.-H., Roeck, G. D., Laethem, M. V., and Geyskens, P., 1990, "Application of the Substructuring Technique to Nonlinear Dynamic Structural Analysis," *Computers and Structures,* Vol. 35, pp. 593–601.

Singh, M. P., and Suarez, L. E., 1992, "Dynamic Condensation with Synthesis of Substructure Eigenproperties," *Journal of Sound and Vibration,* Vol. 159, pp. 139–155.

Smet, M. D., Liefooghe, C., Sas, P., and Snoeys, P., 1989, "Dynamic Analysis of Flexible Structures Using Component Mode Synthesis," ASME Journal of Applied Mechanics, Vol. 56, pp. 874–880.

Suarez, L. E., and Singh, M. P., 1992, "Improved Fixed Interface Method for Modal Synthesis," *AIAA Journal,* Vol. 30, pp. 2952–2958.

Subbiah, R., Bhat, R. B., and Sankar, T. S., 1989, "Dynamic Response of Rotors Using Modal Reduction Techniques," *ASME Journal of Vibration, Acoustics, Stress and Reliability in Design,* Vol. 111, pp. 360–365.

Sunada, W. H., and Dubowsky, S., 1983, "On the Dynamic Analysis and Behavior of Industrial Robotic Manipulators with Elastic Members," *ASME Journal of Mechanisms, Transmission and Automation in Design,* Vol. 105, pp. 42–51.

Tavakoli, M. S., and Singh, R., 1989, "Eigensolutions of Joined/Hermetic Shell Structures Using the State Space Method," *Journal of Sound and Vibration,* Vol. 100, pp. 97–123.

Wang, J. H., and Chen, H. R., 1990, "A Substructure Modal Synthesis Method with High Computational Efficiency," *Computer Methods in Applied Mechanics and Engineering,* Vol. 79, pp. 203–217.

Wang, W., and Kirkhope, J., 1994a, "Component Mode Synthesis for Multi-Shaft Rotors with Flexible Inter-Shaft Bearings," *Journal of Sound and Vibration,* Vol. 173, pp. 537–556.

Wang, W., and Kirkhope, J., 1994b, "Component Mode Synthesis for Damped Rotor Systems with Hybrid Interfaces," *Journal of Sound and Vibration,* Vol. 177, pp. 393–410.

Wilson, E. L., Yuan, M., and Dickens, J. M., 1982, "Dynamic Analysis by Direct Superposition of Ritz Vectors," *Earthquake Engineering and Structural Dynamics,* Vol. 10, pp. 813–821.

Wilson, E. L., and Bayo, E. P., 1986, "Use of Special Ritz Vectors in Dynamic Substructure Analysis," *ASCE Journal of Structural Engineering,* Vol. 112, pp. 1944–1954.

Wu, L., and Grief, R., 1983, "Substructuring and Modal Synthesis for Damped Systems," *Journal of Sound and Vibration,* Vol. 90, pp. 407–422.

Wu, S.-C., and Haug, E. J., 1990, "A Substructure Technique for Dynamics of Flexible Mechanical Systems with Contact–Impact," *ASME Journal of Mechanical Design,* Vol. 112, pp. 390–398.

Xu, M., and Marangoni, R. D., 1994, "Vibration Analysis of a Motor-Flexible Coupling-Rotor System Subject to Misalignment and Unbalance, Part 1: Theoretical Model and Analysis, *Journal of Sound and Vibration,* Vol. 176, pp. 663–679.

Yasui, Y., 1993, "Improved Mode Superposition Reanalysis Method," *Computers and Structures,* Vol. 48, pp. 711–717.

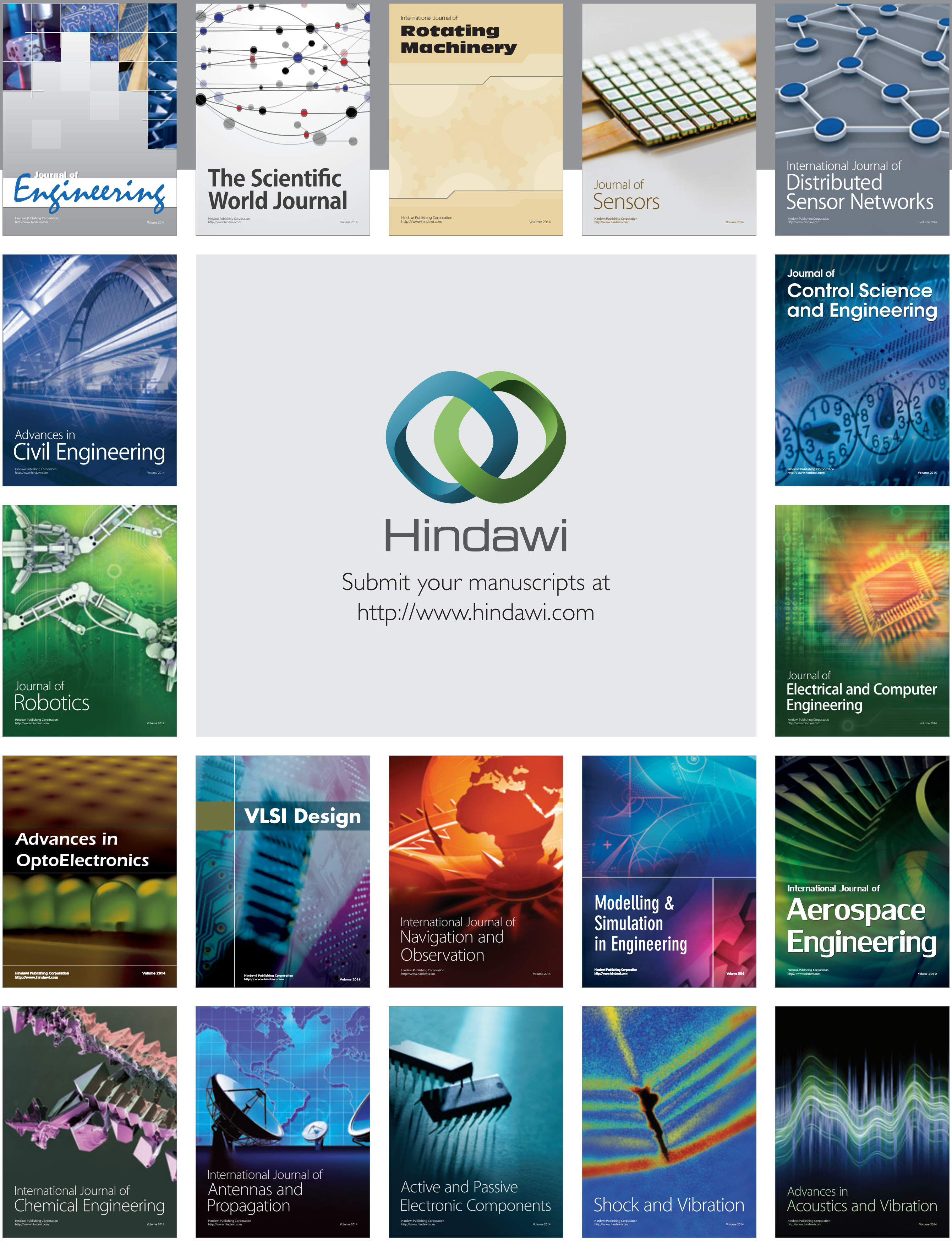
Yee, E. K. L., and Tsuei, Y. G., 1989, "Direct Component Modal Synthesis Technique for System Dynamic Analysis," *AIAA Journal,* Vol. 27, pp. 1083–1088.

Yee, E. K. L., and Tsuei, Y. G., 1990, "Transient Response by Component Modal Synthesis Method," *ASME Journal of Vibration and Acoustics,* Vol. 112, pp. 21–25.

Yee, V., 1990, "Reduction of Component Mode Synthesis Formulated Matrices for Correlation Studies," *AIAA Journal,* Vol. 28, pp. 1142–1143.

Zeng, Z. C., and Xie, G., 1992, "New Approach of the Dynamic Substructure Method," *Acta Mechanica Solida Sinica,* Vol. 5, pp. 407–418.

Zhong, W. X., Liu, Y. F., Lin, J. H., and Lu, X. 1988, "A 3D Analysis System of Jacket Platform DASOS-J," *Proceeding of the International Conference on Computer Modeling in Ocean Engineering,* Venice, pp. 81–88.


Journal of Engineering
The Scientific World Journal
International Journal of Rotating Machinery
Journal of Sensors
International Journal of Distributed Sensor Networks
Advances in Civil Engineering
Journal of Control Science and Engineering
Journal of Robotics
Journal of Electrical and Computer Engineering
Advances in OptoElectronics
VLSI Design
International Journal of Navigation and Observation
Modelling & Simulation in Engineering
International Journal of Aerospace Engineering
International Journal of Chemical Engineering
International Journal of Antennas and Propagation
Active and Passive Electronic Components
Shock and Vibration
Advances in Acoustics and Vibration