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Research Article

An Iteration Scheme for Contraction Mappings with an Application to Synchronization of Discrete Logistic Maps

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This paper deals with designing a new iteration scheme associated with a given scheme for contraction mappings. This new scheme has a similar structure to that of the given scheme, in which those two iterative schemes converge to the same fixed point of the given contraction mapping. The positive influence of feedback parameters on the convergence rate of this new scheme is investigated. Moreover, the derived convergence and comparison results can be extended to nonexpansive mappings. As an application, the derived results are utilized to study the synchronization of logistic maps. Two illustrated examples are used to reveal the effectiveness of our results.

1. Introduction

Fixed point theory has achieved great progress since the last two decades. Various schemes have been constructed to approximate the fixed point of a contraction mapping (see, e.g., [1–30]).

For a contraction mapping, we can define an iteration scheme which converges to the fixed point of that mapping. Here is a question whether we can design another iteration scheme with a similar structure to that of given scheme to approximate the fixed point. Motivated by this question, we design a new iteration scheme which is associated with the given iteration scheme.

This new scheme has a similar structure to that of the given scheme. Those two schemes converge to the same fixed point of the given contraction mapping. The convergence rate of this new scheme can be accelerated by the increase of the feedback parameters. Those convergence and comparison criteria can be applied to nonexpansive mappings. Moreover, the derived results are utilized to study the synchronization of logistics maps. Two examples are used to reveal the effectiveness of our results.

2. Preliminaries

Let C be a nonempty convex subset of a normed linear space E. Let T be a contraction mapping of C into itself with the contraction constant μ; that is,

$$\|Tx - Ty\| \leq \mu \|x - y\|, \quad 0 < \mu < 1, \tag{1}$$

for any $x, y \in C$. The set of fixed points of T is denoted by $F(T) = \{x \in C : Tx = x\}$. The set of natural numbers is denoted by $\mathbb{N}$. $\{\alpha_n\}$ and $\{\beta_n\}$ are two sequences of real numbers such that $0 \leq \alpha_n$ and $0 \leq \beta_n$ for all $n \in \mathbb{N}$. Consider the following scheme:

$$\begin{aligned} &x_1 \in C, \\ &x_{n+1} = \alpha_n x_n + \beta_n T x_n, \quad n \in \mathbb{N}. \end{aligned} \tag{2}$$

Remark 1. It should be pointed out that scheme (2) is a general framework which includes the following well-known schemes as special cases.

(i) If $\alpha_n = 0$ and $\beta_n = 1$, scheme (2) reduces to Picard iteration.

(ii) If $0 < \alpha_n < 1$ and $\beta_n = 1 - \alpha_n$, scheme (2) reduces to Mann iteration.

(iii) If $0 < \alpha_n < 1$, $\beta_n = 1 - \alpha_n$, and $Tx_n = \widetilde{T}((1-\gamma_n)x_n + \gamma_n \widetilde{T}x_n)$, where $\widetilde{T}$ is a contraction mapping of C into itself and $0 < \gamma_n < 1$, scheme (2) reduces to Ishikawa iteration.

For the fixed point scheme described by (2), a question naturally arises whether we can design another iteration scheme with a similar structure to scheme (2) to approximate the fixed point. Moreover, this new scheme has a similar structure to that of the given scheme. Those two schemes converge to the same fixed point of the given contraction mapping.

Motivated by this question, we define the following scheme associated with scheme (2):

$$\begin{aligned} & y_1 \in C, \\ & y_{n+1} = \alpha_n y_n + \beta_n T y_n + k_n (x_n - y_n), \quad n \in \mathbb{N}, \end{aligned} \tag{3}$$

where $\{k_n\}$ is a scheme of feedback parameters which can be determined later. Let $e_n = x_n - y_n$ for $n \in \mathbb{N}$. Then we can construct the following scheme from schemes (2) and (3):

$$\begin{aligned} & e_1 = x_1 - y_1 \in C, \\ & e_{n+1} = (\alpha_n - k_n) e_n + \beta_n (T x_n - T y_n), \quad n \in \mathbb{N}. \end{aligned} \tag{4}$$

From [4, 31, 32], the fact $\lim_{n\to\infty}\|e_n\| = 0$ will ensure $\lim_{n\to\infty}\|x_n - y_n\| = 0$. The main purpose of this paper is to find the conditions to guarantee $\lim_{n\to\infty}\|e_n\| = 0$, which means that scheme (3) has a similar structure to scheme (2). Schemes (2) and (3) converge to the same fixed point of the given contraction mapping T.

3. Main Results

3.1. Convergence Results. Now, we give some convergence results for iteration (3).

Theorem 2. *Let C be a nonempty convex subset of a normed linear space E. Let T be a contraction mapping of C into itself and $F(T) \neq \phi$. If*

$$\overline{\alpha} + \overline{\beta}\mu - 1 < \underline{k} \le k_i \le \overline{k} \le \underline{\alpha}, \tag{5}$$

where $\overline{\alpha} = \max\{\alpha_i\}$, $\underline{\alpha} = \min\{\alpha_i\}$, $\overline{\beta} = \max\{\beta_i\}$, $\overline{k} = \max\{k_i\}$, *and* $\underline{k} = \min\{k_i\}$, $i = 1, 2, \ldots, n$, *then* $\lim_{n\to\infty}\|x_{n+1} - y_{n+1}\| = 0$, *which also implies that scheme (3) and scheme (2) converge to the same fixed point of* T.

Proof. From (5), we have

$$\begin{aligned} & 0 < \alpha_i - k_i, \\ & 0 < \alpha_i - k_i + \beta_i \mu \end{aligned} \tag{6}$$

for $i = 1, 2, \ldots, n$. Then,

$$\begin{aligned} \|e_{n+1}\| &= \|(\alpha_n - k_n) e_n + \beta_n (T x_n - T y_n)\| \\ &\le (\alpha_n - k_n)\|e_n\| + \beta_n \|T x_n - T y_n\| \\ &\le (\alpha_n - k_n)\|e_n\| + \beta_n \mu \|x_n - y_n\| \\ &\le (\alpha_n - k_n + \beta_n \mu)\|e_n\| \\ &\ \ \vdots \\ &\le \prod_{i=1}^{n} (\alpha_i - k_i + \beta_i \mu)\|e_i\|. \end{aligned} \tag{7}$$

It follows from (5) that $0 < \overline{\alpha} + \overline{\beta}\mu - \underline{k} < 1$, where $\overline{\alpha} = \max\{\alpha_i\}$, $\overline{\beta} = \max\{\beta_i\}$, and $\underline{k} = \min\{k_i\}$, $i = 1, 2, \ldots, n$. Thus,

$$\|e_{n+1}\| \le \prod_{i=1}^{n} (\alpha_i - k_i + \beta_i \mu)\|e_i\| \le (\overline{\alpha} + \overline{\beta}\mu - \underline{k})^n \|e_1\|. \tag{8}$$

It is easy to see that $\|e_{n+1}\| \to 0$, as $n \to \infty$; that is, $\lim_{n\to\infty}\|x_{n+1} - y_{n+1}\| = 0$. This completes the proof. □

Remark 3. It follows from Theorem 2 that $|k_i| < 1$ for $i = 1, 2, \ldots, n$.

Theorem 2 can be applied to approximating the fixed point of a nonexpansive mapping where the contraction constant $\mu = 1$. If $\mu = 1$, Theorem 2 reduces to the following result.

Corollary 4. *Let C be a nonempty convex subset of a normed linear space E. Let T be a nonexpansive mapping of C into itself and $F(T) \neq \phi$. If*

$$\overline{\alpha} + \overline{\beta} - 1 < \underline{k} \le k_i \le \overline{k} \le \underline{\alpha}, \tag{9}$$

where $\overline{\alpha} = \max\{\alpha_i\}$, $\underline{\alpha} = \min\{\alpha_i\}$, $\overline{\beta} = \max\{\beta_i\}$, $\overline{k} = \max\{k_i\}$, *and* $\underline{k} = \min\{k_i\}$, $i = 1, 2, \ldots, n$, *then* $\lim_{n\to\infty}\|x_{n+1} - y_{n+1}\| = 0$.

3.2. Three Special Cases. Now, we use Theorem 2 to construct the associated schemes for Picard iteration scheme, Mann iteration scheme, and Ishikawa iteration scheme for contraction mappings and derive the convergence theorems for those schemes, respectively. First, we consider the Picard iteration scheme. The Picard iteration scheme is defined by

$$\begin{aligned} & \widehat{x}_1 \in C, \\ & \widehat{x}_{n+1} = T \widehat{x}_n, \quad n \in \mathbb{N}. \end{aligned} \tag{10}$$

We define the iteration scheme associated with Picard iteration scheme (10):

$$\begin{aligned} & \widehat{y}_1 \in C, \\ & \widehat{y}_{n+1} = T \widehat{y}_n + k_n (\widehat{x}_n - \widehat{y}_n), \quad n \in \mathbb{N}. \end{aligned} \tag{11}$$

Let $\hat{e}_n = \hat{x}_n - \hat{y}_n$ for $n \in \mathbb{N}$. Schemes (2) and (3) give the following scheme:

$$\begin{aligned}&\hat{e}_1 = \hat{x}_1 - \hat{y}_1 \in C,\\&\hat{e}_{n+1} = -k_n\hat{e}_n + \left(T\hat{x}_n - T\hat{y}_n\right), \quad n \in \mathbb{N}.\end{aligned} \tag{12}$$

Then, by the similar proof of Theorem 2, we have the following convergence theorem.

Theorem 5. *Let C be a nonempty convex subset of a normed linear space E. Let T be a contraction mapping of C into itself and $F(T) \neq \phi$. If $\max\{|k_i|\} + \mu < 1$, $i = 1, 2, \ldots, n$, then* $\lim_{n\to\infty}\|\hat{x}_{n+1} - \hat{y}_{n+1}\| = 0$.

Second, we consider the Mann iteration scheme. The Mann iteration scheme is defined by

$$\begin{aligned}&\check{x}_1 \in C,\\&\check{x}_{n+1} = \left(1-\beta_n\right)\check{x}_n + \beta_n T\check{x}_n, \quad n \in \mathbb{N}.\end{aligned} \tag{13}$$

We construct the following iteration scheme associated with Mann iteration scheme (13):

$$\begin{aligned}&\check{y}_1 \in C,\\&\check{y}_{n+1} = \left(1-\beta_n\right)\check{y}_n + \beta_n T\check{y}_n + k_n\left(\check{x}_n - \check{y}_n\right), \quad n \in \mathbb{N}.\end{aligned} \tag{14}$$

By defining an error variable $\check{e}_i = \check{x}_i - \check{y}_i$ for $i = 1, 2, \ldots, n$, we obtain the following iteration scheme:

$$\begin{aligned}&\check{e}_1 = \check{x}_1 - \check{y}_1 \in C,\\&\check{e}_{n+1} = \left(1-\beta_n-k_n\right)\check{e}_n + \beta_n\left(T\check{x}_n - T\check{y}_n\right), \quad n \in \mathbb{N}.\end{aligned} \tag{15}$$

Then, from the similar proof for Theorem 2, we derive the following convergence result.

Theorem 6. *Let C be a nonempty convex subset of a normed linear space E. Let T be a contraction mapping of C into itself and $F(T) \neq \phi$. If $\overline{\beta}\mu - \underline{\beta} < \underline{k}_i \leq k_i \leq \overline{k} < 1 - \overline{\beta}$, where $\underline{\beta} = \min\{\beta_i\}$, $\overline{\beta} = \max\{\beta_i\}$, $\overline{k} = \max\{k_i\}$, and $\underline{k} = \min\{k_i\}$, $i = 1, 2, \ldots, n$, then* $\lim_{n\to\infty}\|\check{x}_{n+1} - \check{y}_{n+1}\| = 0$.

Third, we consider the Ishikawa iteration scheme. The Ishikawa iteration scheme is defined by

$$\begin{aligned}&\tilde{x}_1 \in C,\\&\tilde{x}_{n+1} = \left(1-\beta_n\right)\tilde{x}_n + \beta_n\widetilde{T}\left(\left(1-\gamma_n\right)\tilde{x}_n + \gamma_n\widetilde{T}\tilde{x}_n\right),\\&\qquad n \in \mathbb{N},\end{aligned} \tag{16}$$

where $\widetilde{T}$ is a contraction mapping of C into itself with the contraction constant μ. We generate the following iteration scheme associated with Ishikawa iteration scheme (16):

$$\begin{aligned}&\tilde{y}_1 \in C,\\&\tilde{y}_{n+1} = \left(1-\beta_n\right)\tilde{y}_n + \beta_n\widetilde{T}\left(\left(1-\gamma_n\right)\tilde{y}_n + \gamma_n\widetilde{T}\tilde{y}_n\right)\\&\qquad + k_n\left(\tilde{x}_n - \tilde{y}_n\right), \quad n \in \mathbb{N}.\end{aligned} \tag{17}$$

After defining an error variable $\tilde{e}_i = \tilde{x}_i - \tilde{y}_i$ for $i = 1, 2, \ldots, n$, we obtain the error scheme:

$$\begin{aligned}&\tilde{e}_1 = \tilde{x}_1 - \tilde{y}_1 \in C,\\&\tilde{e}_{n+1} = \left(1-\beta_n-k_n\right)\tilde{e}_n + \beta_n\left(\widetilde{T}\left(\left(1-\gamma_n\right)\tilde{x}_n + \gamma_n\widetilde{T}\tilde{x}_n\right)\right.\\&\qquad\left. - \widetilde{T}\left(\left(1-\gamma_n\right)\tilde{y}_n + \gamma_n\widetilde{T}\tilde{y}_n\right)\right), \quad n \in \mathbb{N}.\end{aligned} \tag{18}$$

Then, from the similar proof for Theorem 2, we achieve the following convergence theorem.

Theorem 7. *Let C be a nonempty convex subset of a normed linear space E. Let T be a contraction mapping of C into itself and $F(T) \neq \phi$. If $\overline{\beta}\mu(1-\underline{\gamma}+\overline{\gamma}\mu) - \underline{\beta} < \underline{k} \leq k_i \leq \overline{k} < 1 - \overline{\beta}$, where $\underline{\beta} = \min\{\beta_i\}$, $\overline{\beta} = \max\{\beta_i\}$, $\underline{\gamma} = \min\{\gamma_i\}$, $\overline{\gamma} = \max\{\gamma_i\}$, $\overline{k} = \max\{k_i\}$, and $\underline{k} = \min\{k_i\}$, $i = 1, 2, \ldots, n$, then* $\lim_{n\to\infty}\|\tilde{x}_{n+1} - \tilde{y}_{n+1}\| = 0$.

3.3. Impact of k_i to the Convergent Rate. Next, we analyze the influence of size k_i to the convergence rate of (3). We first give another iteration scheme associated with iteration scheme (2):

$$\begin{aligned}&z_1 \in C,\\&z_{n+1} = \alpha_n z_n + \beta_n T z_n + \check{k}_n\left(x_n - z_n\right), \quad n \in \mathbb{N},\end{aligned} \tag{19}$$

where $\check{k}_n > k_n$. In order to compare the convergence rate of (3) with that of (19), we give the following definitions for the convergent rates of two different iteration schemes.

Definition 8 (see [4]). Let $\{a_n\}$ and $\{b_n\}$ be two sequences of real numbers which converges to a and b, respectively. We say that the sequence $\{a_n\}$ converges faster than $\{b_n\}$ if $\lim_{n\to\infty}(|a_n - a|/|b_n - b|) = 0$.

Definition 9. Let $\{x_n\}$, $\{y_n\}$, $\{z_n\}$ be three iterative schemes which satisfy $z_n \to x_n, y_n \to x_n$ as $n \to \infty$. Let $\{a_n\}$ and $\{b_n\}$ be two sequences of real numbers which converge to 0. We say that the scheme $\{z_n\}$ converges faster than $\{y_n\}$ to $\{x_n\}$, if

$$\begin{aligned}&\left\|z_n - x_n\right\| \leq a_n, \quad \text{for all } n \in \mathbb{N},\\&\left\|y_n - x_n\right\| \leq b_n, \quad \text{for all } n \in \mathbb{N},\end{aligned} \tag{20}$$

and $\{a_n\}$ converges faster than $\{b_n\}$.

By the similar method for Theorem 2, we have

$$\left\|z_{n+1} - x_{n+1}\right\| \leq \left(\overline{\alpha} + \overline{\beta}\mu - \underline{\check{k}}\right)^n \left\|z_1 - x_1\right\|, \tag{21}$$

where $\underline{\check{k}} = \min\{\check{k}_i\}$, $i = 1, 2, \ldots, n$, and $0 < \overline{\alpha} + \overline{\beta}\mu - \underline{\check{k}} < 1$. It follows from $\underline{k} < \underline{\check{k}}$ that $0 < \overline{\alpha} + \overline{\beta}\mu - \underline{\check{k}} < \overline{\alpha} + \overline{\beta}\mu - \underline{k} < 1$, which implies

$$\begin{aligned}\lim_{n\to\infty}\frac{\left\|z_{n+1} - x_{n+1}\right\|}{\left\|y_{n+1} - x_{n+1}\right\|} &= \lim_{n\to\infty}\left(\frac{\overline{\alpha} + \overline{\beta}\mu - \underline{\check{k}}}{\overline{\alpha} + \overline{\beta}\mu - \underline{k}}\right)^n \frac{\left\|z_1 - x_1\right\|}{\left\|y_1 - x_1\right\|}\\&= 0.\end{aligned} \tag{22}$$

Hence, from the above mentions, we have the following comparison result for the convergence rate according to the size k_i.

Theorem 10. *The iteration scheme defined by (19) converges faster than the iteration scheme defined by (3).*

Remark 11. The convergence rate of iteration scheme defined by (3) increases as k_i increases which means that the convergence rate of iteration scheme defined by (3) can be controlled by the adjustment of size k_i.

Remark 12. If T is a nonexpansive mapping, i.e., $\mu = 1$, limitation (22) reduces to

$$\lim_{n\to\infty}\frac{\|z_{n+1}-x_{n+1}\|}{\|y_{n+1}-x_{n+1}\|}=\lim_{n\to\infty}\left(\frac{\overline{\alpha}+\overline{\beta}-\check{k}}{\overline{\alpha}+\overline{\beta}-\underline{k}}\right)^n\frac{\|z_1-x_1\|}{\|y_1-x_1\|} = 0, \tag{23}$$

which means that Theorem 10 is still valid for the nonexpansive mapping.

4. An Application to Synchronization of Logistic Maps

Logistic maps are classical discrete systems which can generate bifurcation and chaos. Synchronization of two logistic maps, which means the state variable of one logistic map is eventually equal to the counterpart of another logistic map, has been widely used in secure communication, image encryption, and signal transmission [22, 31]. Our results can be applied to studying the synchronization of logistic maps.

If $Tx_n = -\beta_n x_n^2$ and $\alpha_n = \beta_n = r$, then scheme (2) reduces to the following logistic map

$$x_{n+1} = rx_n - rx_n^2, \quad n \in \mathbb{N}, \tag{24}$$

where $x_1 \in C$. If $0 < r < 0.5$ and $0 < x_1 < 1$, then we can derive $0 < x_n < 1$, which implies that

$$\left|Tx_n - Ty_n\right| = r\left|x_n^2 - y_n^2\right| = r\left|x_n + y_n\right|\left|x_n - y_n\right| < \left|x_n - y_n\right| \tag{25}$$

for any $0 < x_n, y_n < 1$.

Here, we consider another logistic map

$$y_{n+1} = ry_n - ry_n^2, \quad n \in \mathbb{N}, \tag{26}$$

where $y_1 \in C$. Defining $e_n = x_n - y_n$, we can have the following scheme:

$$e_{n+1} = r\left(1-\left(x_n+y_n\right)\right)e_n, \quad n \in \mathbb{N}, \tag{27}$$

where $e_1 = x_1 - y_1 \in C$.

Definition 13. If $\lim_{n\to\infty}|x_n - y_n| = 0$, the logistic map described by (24) is said to achieve the global synchronization with the logistic map described by (26).

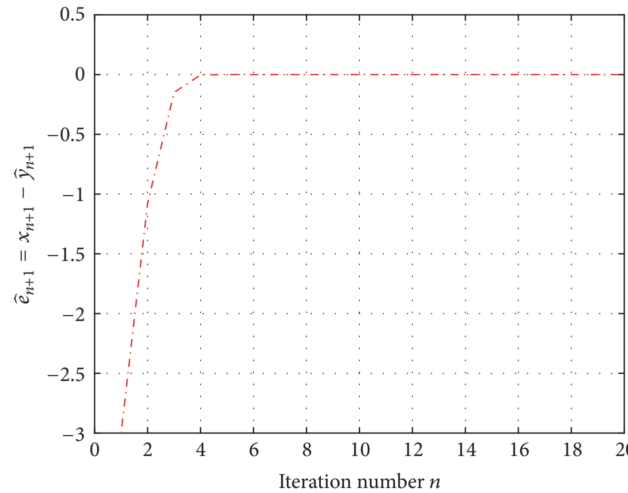

Figure 1: The trajectory of $\hat{e}_{n+1}$ with $\mu = 0.78$ and $k_i = 0.21$.

By using the similar proof method of Theorem 2 with $k_i = 0$ and $|r(1-(x_n+y_n))| < 0.5$, we can derive the following result.

Theorem 14. *If $0 < r < 0.5$ and $0 < x_1, y_1 < 1$, the logistic map described by (24) achieves the global synchronization with the logistic map described by (26).*

5. Two Illustrated Examples

Example 15. Now we give an example for the main theorems with numerical analysis.

Consider $Tx = \sqrt{x^2 - 8x + 40}$ on $[4.1, 10]$ with the contraction constant $\mu = 0.78$. The fixed point of T is $x = 5$. We first construct the iteration scheme (11) associated with Picard iteration (10). From Theorem 5, we know that $\max\{|k_i|\} + \mu < 1$, $i = 1, 2, \ldots, n$, which implies $|k_i| < 0.22$ for $i = 1, 2, \ldots, n$. We choose $k_i = 0.21$, for $i = 1, 2, \ldots, n$, and $x_1 = 6$ and $\hat{y}_1 = 8$. Figure 1 gives a demonstration of trajectory of $\hat{e}_{n+1}$ of (11). From Figure 1, we can see that $\lim_{n\to\infty}\|\hat{e}_{n+1}\| = 0$; that is, $\lim_{n\to\infty}\|x_{n+1} - \hat{y}_{n+1}\| = 0$, which indicates the effectiveness of Theorem 5.

Then, we construct the iteration scheme (14) associated with Mann iteration (13) with $\beta_i = 0.75$ for $i = 1, 2, \ldots, n$. From Theorem 6, we can get $\overline{\beta}\mu - \underline{\beta} < \underline{k} \le k_i \le \overline{k} < 1 - \overline{\beta}$, which indicates $-0.165 < k_i < 0.25$ for $i = 1, 2, \ldots, n$. We choose $k_i = 0.24$, for $i = 1, 2, \ldots, n$, and $x_1 = 6$ and $\check{y}_1 = 8$. Figure 2 provides the trajectory of $\check{e}_{n+1}$ of (14). From Figure 2, we can observe that $\lim_{n\to\infty}\|\check{e}_{n+1}\| = 0$; that is, $\lim_{n\to\infty}\|x_{n+1} - \check{y}_{n+1}\| = 0$, which indicates the effectiveness of Theorem 6.

Now, we construct the iteration scheme (17) associated with Ishikawa iteration (16) with $\beta_i = 0.5$ and $\gamma_i = 0.75$ for $i = 1, 2, \ldots, n$. From Theorem 7, we can have $\overline{\beta}\mu(1-\underline{\gamma}+\overline{\gamma}\mu) - \underline{\beta} < \underline{k} \le k_i \le \overline{k} < 1 - \overline{\beta}$, which implies $-0.1744 < k_i < 0.5$ for $i = 1, 2, \ldots, n$. We choose $k_i = 0.49$, for $i = 1, 2, \ldots, n$, and $x_1 = 6$ and $\tilde{y}_1 = 8$. Figure 3 reveals the trajectory of $\tilde{e}_{n+1}$ of (17). It follows from Figure 3 that $\lim_{n\to\infty}\|\tilde{e}_{n+1}\| = 0$; that is,

Table 1: Comparison for convergent rates of iteration scheme (3) with different k_i.

	$n = 1$	$n = 2$	$n = 3$	$\cdots$	$n = 17$	$n = 18$	$n = 19$	$n = 20$
y_n as $k_i = 0.39$	9.00	6.63	5.58	$\cdots$	5.00	5.00	5.00	5.00
y_n as $k_i = 0.01$	9.00	7.77	6.80	$\cdots$	5.02	5.01	5.01	5.00
y_n as $k_i = -0.1$	9.00	8.10	7.30	$\cdots$	5.05	5.03	5.02	5.01

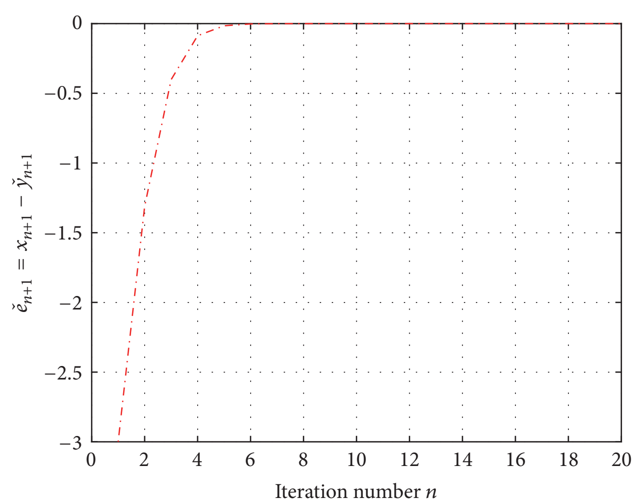

Figure 2: The trajectory of $\check{e}_{n+1}$ with $\mu = 0.78$ and $k_i = 0.24$.

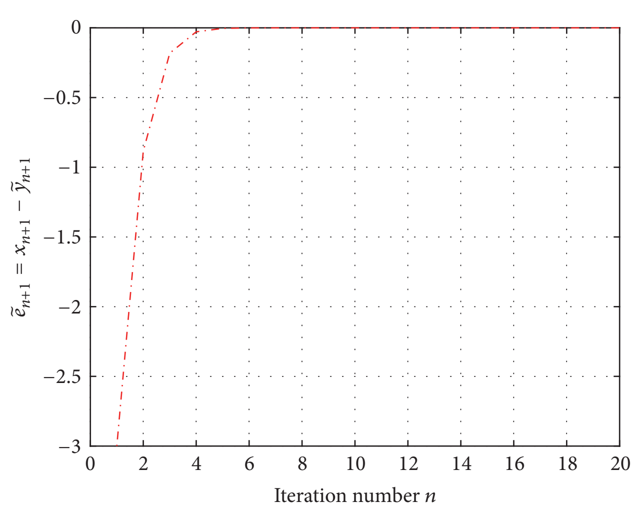

Figure 3: The trajectory of $\tilde{e}_{n+1}$ with $\mu = 0.78$ and $k_i = 0.49$.

$\lim_{n\to\infty}\|x_{n+1} - \tilde{y}_{n+1}\| = 0$, which indicates the effectiveness of Theorem 7.

Finally, we compare the convergence rates of (3) with different k_i. We choose scheme (14) with $\beta_i = 0.6$. From Theorem 6, we can have $-0.132 < k_i < 0.4$ for $i = 1, 2, \ldots, n$. Let $x_1 = 6$ and $y_1 = z_1 = 9$. We choose k_i as 0.39, 0.01, and −0.1 to approximate the fixed point, respectively. Table 1 shows that the convergence rate of (3) increases as k_i increases, which also illustrates the effectiveness of Theorem 10.

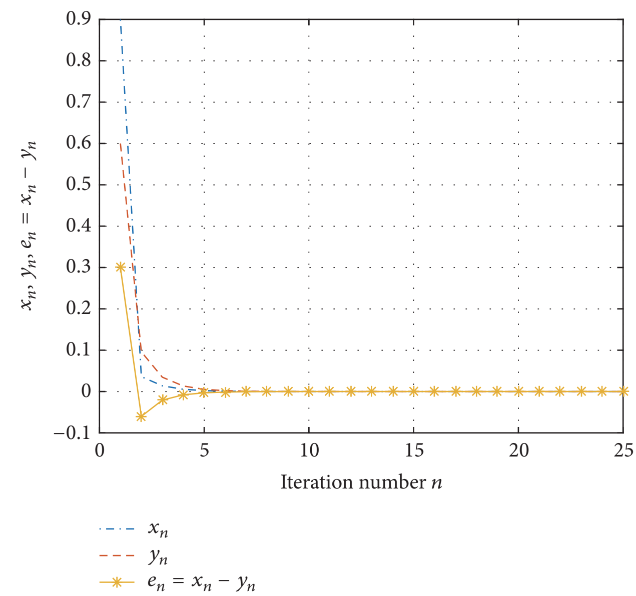

Figure 4: The trajectories of x_n, y_n, e_n.

Example 16. Consider the logistic maps described by (24) and (26) with $r = 0.4$ and $x_1 = 0.9$ and $y_1 = 0.6$. Figure 4 reveals the global synchronization of the logistic maps described by (24) and (26) which illustrates the effectiveness of Theorem 14.

6. Conclusions and Future Works

For a given convergent scheme to approximate the fixed point of a contraction mapping, we have provided an associated scheme which had a similar structure to that of the given scheme. We have derived conditions to ensure this new scheme and the given scheme to converge to the same fixed point. We have used our derived results to construct the associated schemes for Picard, Mann, and Ishikawa iterative schemes for contraction mappings and derived the convergence theorems for those schemes, respectively. Moreover, we can accelerate the convergence rate of this new scheme by controlling the feedback parameter. We have extended those convergence and comparison results to nonexpansive mappings. In addition, we have utilized those derived results to investigate the synchronization of logistic maps. We have used two examples to illustrate the effectiveness of our derived results. In this paper, we only consider the linear feedback in the scheme. Our future research focus is to design a faster scheme by using the nonlinear feedback.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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