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Research Article

Throughput-Delay Trade-Off for Cognitive Radio Networks: A Convex Optimization Perspective

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The throughput-delay trade-off problem for cooperative spectrum sensing (CSS) is investigated. It is proved that the maximum achievable throughput and the minimum transmission delay cannot be obtained simultaneously. An efficient algorithm is proposed to optimize the sensing bandwidth and the final decision threshold jointly such that the throughput is maximized while the delay is constrained. It is demonstrated that convex optimization plays an essential role in solving the problem in an efficient way. Simulation results show that the proposed optimal scheme can significantly improve the throughput of the secondary users (SUs) under the constraint that the delay Quality of Service (QoS) requirements of the SUs are satisfied.

1. Introduction

Cognitive radio (CR) is a potential technology to improve the spectrum efficiency by allowing the secondary users to temporarily utilize the unused licensed spectrum of the primary users (PUs) [1, 2]. Efficient and effective spectrum sensing is required to guarantee the QoS of both PU and SU [3–5]. To mitigate the impact of fading and shadowing, a variety of cooperative spectrum sensing methods (e.g., [6–10]) have been proposed. Authors in [11] consider CSS when two SUs collaborate via relaying scheme. In [12], the SU throughput of a sensing-based CR system with Markovian traffic is analyzed and optimized.

In the frame structure of CSS [13–15], the SU must cease transmission during the sensing slot. Thus, the transmission delay will be long, and the QoS will not be guaranteed for delay sensitive applications. To achieve continuous spectrum sensing, authors in [16–18] split the PU spectrum band into K subbands and allocate one subband exclusively for detection purposes. In the new CSS frame structure shown in [19], the SUs conduct spectrum sensing and data transmission simultaneously over two different parts of the primary user spectrum band. In this way, the SUs do not need to cease transmission in the spectrum sensing stage, and the QoS can be guaranteed.

In this paper, we study the throughput-delay trade-off problem for CSS with the new frame structure. It is proved that the maximum throughput and the minimum transmission delay cannot be obtained simultaneously. Our object is to design the sensing parameters to maximize the throughput of the SUs under the constraint that the delay QoS requirements of the SUs are satisfied.

We focus on optimizing the sensing bandwidth and the final decision threshold jointly such that the throughput is maximized while the delay is constrained. An efficient algorithm is proposed to obtain the optimal scheme. It is demonstrated that convex optimization plays an essential role in solving the associated design problems. Simulation results show that different delay QoS requirements require different values of sensing bandwidth. The throughput of the SUs can be greatly improved by using the proposed optimal scheme.

The rest of this paper is organized as follows. In Section 2, the mathematical formulation of the throughput-delay trade-off problem with new CSS frame structure is presented. Section 3 is devoted to the analysis of throughput-delay trade-off from a convex optimization perspective. Simulation results are provided in Section 4. Finally, in Section 5, we present the conclusion.

2. Problem Formulation

In CSS, consider the case that the SUs know the PU transmission bandwidth W. It was shown that authors in [19] proposed

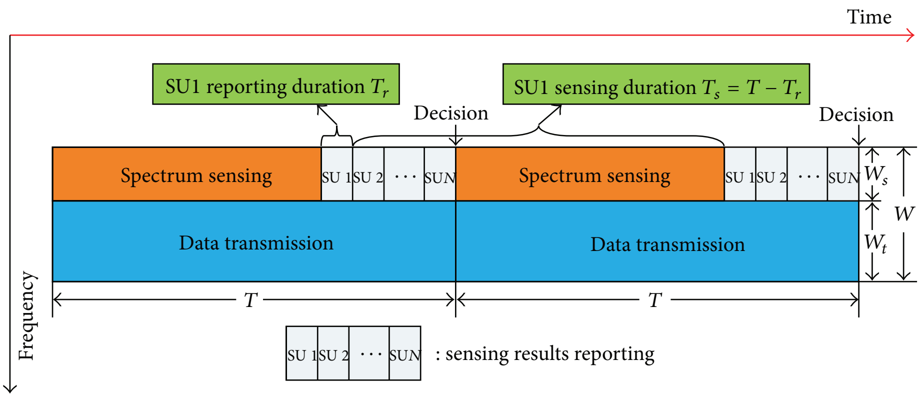

FIGURE 1: New frame structure for CSS.

a new CSS frame structure in which W is divided into two parts. As depicted in Figure 1, W_s is assigned exclusively for spectrum sensing and sensing results reporting, W_t is assigned exclusively for potential SU data transmission, and $W_s + W_t = W$. To implement the new CSS frame structure, the SU transmitter is equipped with two radios, one for spectrum sensing and the other for data transmission. When the sensing result in the previous frame indicated that the PU was absent, the SU is allowed to transmit data. Otherwise, the SU keeps silent to protect the PU. Suppose that the frame duration is T and the individual reporting duration is T_r. Since each SU continues spectrum sensing after sending its sensing result to the fusion center (FC), the sensing duration for each SU is $T_s = T - T_r$.

Considering that energy detection is employed to detect the status of PU in the local sensing step, the false alarm and detection probabilities at each SU are [18]

$$\mathscr{P}_{\mathrm{fa}} = \mathcal{Q}\left(\frac{\epsilon}{\sqrt{2T_sW_s}} - \sqrt{2T_sW_s}\right),$$
$$\mathscr{P}_{\mathrm{de}} = \mathcal{Q}\left(\frac{\epsilon}{(1+\gamma)\sqrt{2T_sW_s}} - \sqrt{2T_sW_s}\right), \quad (1)$$

where ϵ is the energy detection threshold and γ is the SNR of PU's signal at each SU. The false alarm probability is related to the detection probability as follows:

$$\mathscr{P}_{\mathrm{fa}} = \mathcal{Q}\left((1+\gamma)\,\mathcal{Q}^{-1}(\mathscr{P}_{\mathrm{de}}) + \gamma\sqrt{2T_sW_s}\right). \quad (2)$$

Suppose that the counting rule is employed in FC; then, the final false alarm and detection probabilities are [20]

$$\mathrm{Q}_{\mathrm{fa}} = \sum_{i=k}^{N}\binom{N}{i}(\mathscr{P}_{\mathrm{fa}})^i(1-\mathscr{P}_{\mathrm{fa}})^{N-i},$$
$$\mathrm{Q}_{\mathrm{de}} = \sum_{i=k}^{N}\binom{N}{i}(\mathscr{P}_{\mathrm{de}})^i(1-\mathscr{P}_{\mathrm{de}})^{N-i}, \quad (3)$$

where N is the number of SUs and k is the final decision threshold at FC.

There are two possible scenarios for which the SU can transmit its data. Scenario 1: the PU is correctly detected to be absent. The probability of scenario 1 happening is $(1 - \mathrm{Q}_{\mathrm{fa}})\mathscr{P}(H_0)$, where $\mathscr{P}(H_0)$ is the probability of absence of PU. In this scenario, the normalized achievable throughput of the SUs will be $((W - W_s)/W)C$, where $C = \log_2(1 + \gamma_s)$ and γ_s is the SNR for the secondary link [21]. Scenario 2: the PU is falsely detected to be absent; that is, missed detection occurs, and the probability of this scenario happening is $(1 - \mathrm{Q}_{\mathrm{de}})\mathscr{P}(H_1)$, where $\mathscr{P}(H_1)$ is the probability of presence of PU. In this scenario, the channel is occupied simultaneously by the PU and the SU to transmit data, and the normalized achievable throughput of the SUs will be $((W - W_s)/W)C_1$, where $C_1 = \log_2(1 + (\gamma_s/(1 + \gamma_p)))$ ($\gamma_p = P_p/N_0$, where P_p is the interference power of the PU at the secondary receiver and N_0 is the noise power) [21]. The normalized achievable throughput of the SUs can be computed by

$$\widehat{\mathscr{R}} = \mathscr{R} + \mathscr{R}_1, \quad (4)$$

where

$$\mathscr{R} = \frac{W - W_s}{W}C\left(1 - \mathrm{Q}_{\mathrm{fa}}\right)\mathscr{P}(H_0),$$
$$\mathscr{R}_1 = \frac{W - W_s}{W}C_1\left(1 - \mathrm{Q}_{\mathrm{de}}\right)\mathscr{P}(H_1). \quad (5)$$

In the new CSS frame structure, the SU transmission delay occurs in two cases: false alarm and correct detection of PU [18]. When false alarm happens, the transmission delay is

$$\mathscr{D}_1 = \mathscr{P}(H_0)\,\mathrm{Q}_{\mathrm{fa}}\cdot T. \quad (6)$$

In the case of correct detection of PU, the delay is

$$\mathscr{D}_2 = \mathscr{P}(H_1)\,\mathrm{Q}_{\mathrm{de}}\cdot T. \quad (7)$$

Thus, the normalized SU transmission delay is

$$\mathscr{D} = \frac{\mathscr{D}_1 + \mathscr{D}_2}{T} = \mathscr{P}(H_0)\,\mathrm{Q}_{\mathrm{fa}} + \mathscr{P}(H_1)\,\mathrm{Q}_{\mathrm{de}}. \quad (8)$$

To guarantee the QoS of the SUs, maximizing the throughput of the SUs $\widehat{\mathscr{R}}$ while minimizing the SU transmission delay $\mathscr{D}$ subject to $\mathrm{Q}_{\mathrm{de}} \geq \mathrm{Q}_{\mathrm{de}}^{\mathrm{th}}$, $0 < W_s < W$ and

$1 \leq k \leq N$, may be of practical interest. Mathematically, the problem is given as

$$\begin{aligned} &\max \quad \widehat{\mathscr{R}} \\ &\min \quad \mathscr{D} \end{aligned} \tag{9}$$

$$\text{s.t.} \quad Q_{\text{de}} \geq Q_{\text{de}}^{\text{th}} \tag{10}$$

$$0 < W_s < W, \tag{11}$$

$$1 \leq k \leq N. \tag{12}$$

For a given W_s and k, we take the first partial derivative of $\widehat{\mathscr{R}}$ with respect to Q_{de} and obtain

$$\begin{aligned} \frac{\partial \widehat{\mathscr{R}}}{\partial Q_{\text{de}}} &= -C\mathscr{P}(H_0)\frac{W-W_s}{W}\frac{dQ_{\text{fa}}}{dQ_{\text{de}}} - C_1\mathscr{P}(H_1)\frac{W-W_s}{W} \\ &= -C\mathscr{P}(H_0)\frac{W-W_s}{W}\frac{dQ_{\text{fa}}/d\mathscr{P}_{\text{de}}}{dQ_{\text{de}}/d\mathscr{P}_{\text{de}}} \\ &\quad - C_1\mathscr{P}(H_1)\frac{W-W_s}{W} \\ &= -C\mathscr{P}(H_0)\frac{W-W_s}{W} \\ &\quad \times \frac{N\binom{N-1}{k-1}\mathscr{P}_{\text{fa}}^{k-1}(1-\mathscr{P}_{\text{fa}})^{N-k}}{N\binom{N-1}{k-1}\mathscr{P}_{\text{de}}^{k-1}(1-\mathscr{P}_{\text{de}})^{N-k}}\frac{d\mathscr{P}_{\text{fa}}}{d\mathscr{P}_{\text{de}}} \\ &\quad - C_1\mathscr{P}(H_1)\frac{W-W_s}{W} \\ &= -C\mathscr{P}(H_0)\frac{W-W_s}{W}\left(\frac{\mathscr{P}_{\text{fa}}}{\mathscr{P}_{\text{de}}}\right)^{k-1}\left(\frac{1-\mathscr{P}_{\text{fa}}}{1-\mathscr{P}_{\text{de}}}\right)^{N-k} \\ &\quad \times \frac{d\mathscr{P}_{\text{fa}}/d\epsilon}{d\mathscr{P}_{\text{de}}/d\epsilon} - C_1\mathscr{P}(H_1)\frac{W-W_s}{W} \\ &= -C\mathscr{P}(H_0)(1+\gamma)\frac{W-W_s}{W}\left(\frac{\mathscr{P}_{\text{fa}}}{\mathscr{P}_{\text{de}}}\right)^{k-1} \\ &\quad \times \left(\frac{1-\mathscr{P}_{\text{fa}}}{1-\mathscr{P}_{\text{de}}}\right)^{N-k} e^{(\epsilon\gamma/(1+\gamma))-(\epsilon^2\gamma(\gamma+2)/4T_sW_s(1+\gamma)^2)} \\ &\quad - C_1\mathscr{P}(H_1)\frac{W-W_s}{W} < 0. \end{aligned} \tag{13}$$

Then, we take the first partial derivative of $\mathscr{D}$ with respect to Q_{de} and obtain

$$\begin{aligned} \frac{\partial \mathscr{D}}{\partial Q_{\text{de}}} &= \mathscr{P}(H_0)\frac{dQ_{\text{fa}}}{dQ_{\text{de}}} + \mathscr{P}(H_1) \\ &= \mathscr{P}(H_0)\frac{dQ_{\text{fa}}/d\mathscr{P}_{\text{de}}}{dQ_{\text{de}}/d\mathscr{P}_{\text{de}}} + \mathscr{P}(H_1) \\ &= \mathscr{P}(H_0)\frac{N\binom{N-1}{k-1}\mathscr{P}_{\text{fa}}^{k-1}(1-\mathscr{P}_{\text{fa}})^{N-k}}{N\binom{N-1}{k-1}\mathscr{P}_{\text{de}}^{k-1}(1-\mathscr{P}_{\text{de}})^{N-k}} \\ &\quad \times \frac{d\mathscr{P}_{\text{fa}}}{d\mathscr{P}_{\text{de}}} + \mathscr{P}(H_1) \\ &= \mathscr{P}(H_0)\left(\frac{\mathscr{P}_{\text{fa}}}{\mathscr{P}_{\text{de}}}\right)^{k-1} \\ &\quad \times \left(\frac{1-\mathscr{P}_{\text{fa}}}{1-\mathscr{P}_{\text{de}}}\right)^{N-k}\frac{d\mathscr{P}_{\text{fa}}/d\epsilon}{d\mathscr{P}_{\text{de}}/d\epsilon} + \mathscr{P}(H_1) \\ &= \mathscr{P}(H_0)(1+\gamma)\left(\frac{\mathscr{P}_{\text{fa}}}{\mathscr{P}_{\text{de}}}\right)^{k-1}\left(\frac{1-\mathscr{P}_{\text{fa}}}{1-\mathscr{P}_{\text{de}}}\right)^{N-k} \\ &\quad \times e^{(\epsilon\gamma/(1+\gamma))-(\epsilon^2\gamma(\gamma+2)/4T_sW_s(1+\gamma)^2)} + \mathscr{P}(H_1) \\ &> 0. \end{aligned} \tag{14}$$

Hence, $\widehat{\mathscr{R}}$ is a decreasing function of Q_{de}, and $\mathscr{D}$ is an increasing function of Q_{de}. This proves that the throughput of the SUs $\widehat{\mathscr{R}}$ is maximized and the SU transmission delay $\mathscr{D}$ is minimized only if the constraint (10) is at equality; that is, $Q_{\text{de}} = Q_{\text{de}}^{\text{th}}$. Therefore, the optimization problem can be reduced to

$$\begin{aligned} &\max \quad \widehat{\mathscr{R}} \\ &\min \quad \mathscr{D} \end{aligned} \tag{15}$$

$$\begin{aligned} \text{s.t.} \quad & Q_{\text{de}} = Q_{\text{de}}^{\text{th}}, \\ & 0 < W_s < W, \\ & 1 \leq k \leq N. \end{aligned} \tag{16}$$

In scenario 2, the CR network experiences interference from the PU; obviously, $C > C_1$. Also, it is assumed that the probability of presence of PU is small, say less than 0.3; hence, it is economically advisable to explore the secondary usage for the PU frequency band. Also, to sufficiently protect the PU, the target detection probability $Q_{\text{de}}^{\text{th}}$ is set not less than 0.9. Therefore, the optimization problem can be approximated by maximizing $\mathscr{R}$ while minimizing $\mathscr{D}$ subject to (16). Furthermore, in the next section, we will prove that the maximum throughput and the minimum delay cannot be obtained simultaneously. Therefore, the objective of throughput-delay trade-off is to design the sensing parameters such that the throughput $\mathscr{R}$ is maximized while the delay $\mathscr{D}$ is constrained. The problem formulation can be stated as follows:

$$\begin{aligned} &\max \quad \mathscr{R} \\ &\text{s.t.} \quad Q_{\text{de}} = Q_{\text{de}}^{\text{th}}, \\ &\qquad 0 < W_s < W, \\ &\qquad 1 \leq k \leq N, \\ &\qquad \mathscr{D} \leq \mathscr{D}^{\text{th}}, \end{aligned} \tag{17}$$

where $\mathscr{D}^{\text{th}}$ is the threshold of the delay constraint.

3. Throughput-Delay Trade-Off: A Convex Optimization Perspective

In this section, we will investigate the throughput-delay trade-off problem from a convex optimization perspective. Let $\theta = (1+\gamma)\mathcal{Q}^{-1}(\mathcal{P}_{\text{de}}) + \gamma\sqrt{2T_s W_s}$, and since

$$\frac{d\mathcal{P}_{\text{fa}}}{dW_s} = -\frac{\gamma}{2}\sqrt{\frac{T_s}{\pi W_s}}e^{-\theta^2/2} < 0, \tag{18}$$

we have

$$\begin{aligned}\frac{\partial \mathcal{D}}{\partial W_s} &= \mathcal{P}(H_0)\, N\binom{N-1}{k-1} \\ &\quad \times (\mathcal{P}_{\text{fa}})^{k-1}(1-\mathcal{P}_{\text{fa}})^{N-k}\frac{d\mathcal{P}_{\text{fa}}}{dW_s} \\ &< 0.\end{aligned} \tag{19}$$

Thus, $\mathcal{D}$ is a decreasing function of W_s. Then,

$$\frac{\partial \mathcal{R}}{\partial W_s} = -\frac{C}{W}(1-Q_{\text{fa}})\mathcal{P}(H_0) - \frac{W-W_s}{W}C\mathcal{P}(H_0)\frac{dQ_{\text{fa}}}{dW_s}, \tag{20}$$

where

$$\frac{dQ_{\text{fa}}}{dW_s} = N\binom{N-1}{k-1}(\mathcal{P}_{\text{fa}})^{k-1}(1-\mathcal{P}_{\text{fa}})^{N-k}\frac{d\mathcal{P}_{\text{fa}}}{dW_s}. \tag{21}$$

Obviously, $\lim_{W_s\to 0}(\partial\mathcal{R}/\partial W_s) = \infty$, $\lim_{W_s\to W}(\partial\mathcal{R}/\partial W_s) < 0$. Thus, there must exist an optimal value of W_s that maximizes $\mathcal{R}$ and the root of $\partial\mathcal{R}/\partial W_s = 0$ exists for $W_s \in (0, W)$. Next, we will prove that the maximum point of $\mathcal{R}$ is unique; namely, the root of $\partial\mathcal{R}/\partial W_s = 0$ is unique.

Substituting (18) and (21) into (20), we have

$$\begin{aligned}\frac{\partial \mathcal{R}}{\partial W_s} &= -\frac{C}{W}(1-Q_{\text{fa}})\mathcal{P}(H_0) + C\mathcal{P}(H_0)\, N\binom{N-1}{k-1} \\ &\quad \times (\mathcal{P}_{\text{fa}})^{k-1}(1-\mathcal{P}_{\text{fa}})^{N-k}\frac{\gamma}{2}\frac{W-W_s}{W}\sqrt{\frac{T_s}{\pi W_s}}e^{-\theta^2/2} \\ &= -\frac{C}{W}\mathcal{P}(H_0)\Bigg[\sum_{i=0}^{k-1}\binom{N}{i}(\mathcal{P}_{\text{fa}})^{i}(1-\mathcal{P}_{\text{fa}})^{N-i} \\ &\quad - N\binom{N-1}{k-1}(\mathcal{P}_{\text{fa}})^{k-1}(1-\mathcal{P}_{\text{fa}})^{N-k} \\ &\quad \times (W-W_s)\frac{\gamma}{2}\sqrt{\frac{T_s}{\pi W_s}}e^{-\theta^2/2}\Bigg].\end{aligned} \tag{22}$$

Setting $\partial\mathcal{R}/\partial W_s = 0$, we have

$$\begin{aligned}&\sum_{i=0}^{k-1}\binom{N}{i}(\mathcal{P}_{\text{fa}})^{i}(1-\mathcal{P}_{\text{fa}})^{N-i} \\ &\quad = N\binom{N-1}{k-1}(\mathcal{P}_{\text{fa}})^{k-1}(1-\mathcal{P}_{\text{fa}})^{N-k} \\ &\quad \times (W-W_s)\frac{\gamma}{2}\sqrt{\frac{T_s}{\pi W_s}}e^{-\theta^2/2}.\end{aligned} \tag{23}$$

Then,

$$\begin{aligned}&\frac{\sum_{i=0}^{k-1}\binom{N}{i}(\mathcal{P}_{\text{fa}})^{i}(1-\mathcal{P}_{\text{fa}})^{N-i}}{N\binom{N-1}{k-1}(\mathcal{P}_{\text{fa}})^{k-1}(1-\mathcal{P}_{\text{fa}})^{N-k}(W-W_s)(\gamma/2)\sqrt{T_s/\pi W_s}} \\ &\quad = e^{-\theta^2/2},\end{aligned} \tag{24}$$

$$\ln\left[\frac{\sum_{i=0}^{k-1}\binom{N}{i}(\mathcal{P}_{\text{fa}})^{i-k+1}(1-\mathcal{P}_{\text{fa}})^{k-i}}{N\binom{N-1}{k-1}(W-W_s)(\gamma/2)\sqrt{T_s/\pi W_s}}\right] = -\frac{1}{2}\theta^2. \tag{25}$$

According to (25), it can be derived that $u = v$, where

$$\begin{aligned}u &= \ln\Bigg[\frac{2(k-1)!(N-k)!}{\gamma(W-W_s)N!}\sqrt{\frac{\pi W_s}{T_s}} \\ &\quad \times\sum_{i=0}^{k-1}\binom{N}{i}(\mathcal{P}_{\text{fa}})^{i-k+1}(1-\mathcal{P}_{\text{fa}})^{k-i}\Bigg], \\ v &= -\frac{1}{2}\theta^2.\end{aligned} \tag{26}$$

Taking the first derivative of u with respect to W_s, it is derived that

$$\frac{du}{dW_s} = \phi_1 + \phi_2 + \phi_3, \tag{27}$$

where

$$\begin{aligned}\phi_1 &= \frac{W+W_s}{2W_s(W-W_s)}, \\ \phi_2 &= -\frac{d\mathcal{P}_{\text{fa}}}{dW_s}\frac{\sum_{i=0}^{k-1}\binom{N}{i}(k-1-i)(\mathcal{P}_{\text{fa}})^{i-k}(1-\mathcal{P}_{\text{fa}})^{k-i}}{\sum_{i=0}^{k-1}\binom{N}{i}(\mathcal{P}_{\text{fa}})^{i-k+1}(1-\mathcal{P}_{\text{fa}})^{k-i}}, \\ \phi_3 &= -\frac{d\mathcal{P}_{\text{fa}}}{dW_s}\frac{\sum_{i=0}^{k-1}\binom{N}{i}(k-i)(\mathcal{P}_{\text{fa}})^{i-k+1}(1-\mathcal{P}_{\text{fa}})^{k-i-1}}{\sum_{i=0}^{k-1}\binom{N}{i}(\mathcal{P}_{\text{fa}})^{i-k+1}(1-\mathcal{P}_{\text{fa}})^{k-i}}.\end{aligned} \tag{28}$$

Since $\phi_1 > 0$, $\phi_2 > 0$, and $\phi_3 > 0$, we have $du/dW_s > 0$, and u is an increasing function of W_s.

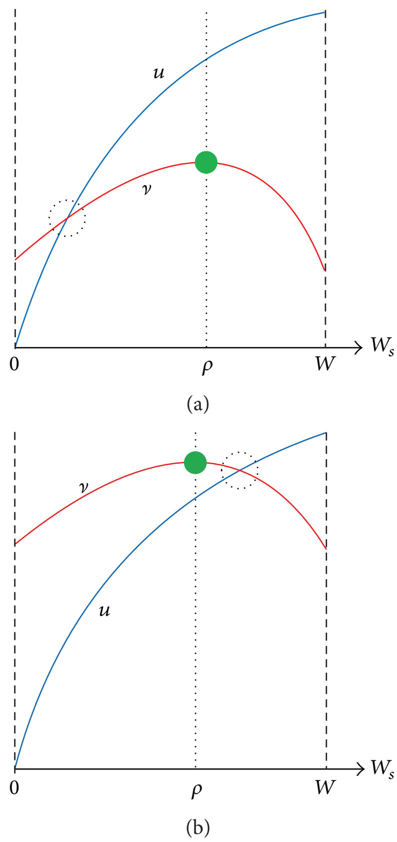

Figure 2: Illustration of the curves: u and v.

Taking the first derivative of v with respect to W_s, it is derived that

$$\begin{aligned} \frac{dv}{dW_s} &= -\theta \frac{d\theta}{dW_s} \\ &= -\left((1+\gamma)\mathscr{Q}^{-1}(\mathscr{P}_{\mathrm{de}}) + \gamma\sqrt{2T_s W_s}\right)\gamma\sqrt{\frac{T_s}{2W_s}}. \end{aligned} \tag{29}$$

Let $\rho = ((1+\gamma)\mathscr{Q}^{-1}(\mathscr{P}_{\mathrm{de}}))^2/2T_s\gamma^2$, for $W_s \in (0,\rho]$, $dv/dW_s \geq 0$; for $W_s \in (\rho, W)$, $dv/dW_s < 0$. Thus, v is an increasing function of W_s for $W_s \in (0,\rho]$ and a decreasing function of W_s for $W_s \in (\rho, W)$. We have proved that $du/dW_s > dv/dW_s$ for $W_s \in (0,\rho]$ in the Appendix.

We know that u and v must intersect each other since the root of $\partial\mathscr{R}/\partial W_s = 0$ exists for $W_s \in (0, W)$. If u and v have an intersection for $W_s \in (0,\rho]$, since u increases faster than v for $W_s \in (0,\rho]$ and $u > v$ for $W_s \in (\rho, W)$, there is only one intersection between u and v, which is illustrated in Figure 2(a). If u and v do not have an intersection for $W_s \in (0,\rho]$, they must have an intersection for $W_s \in (\rho, W)$. Since u is an increasing function of W_s and v is a decreasing function of W_s for $W_s \in (\rho, W)$, there is only one intersection between u and v, which is illustrated in Figure 2(b). Thus, the root of $\partial\mathscr{R}/\partial W_s = 0$ is unique.

From the above analysis, we can conclude that $\mathscr{R}$ is a unimodal function for $W_s \in (0, W)$. We define W_s^* as the root of $\partial\mathscr{R}/\partial W_s = 0$. Since $\mathscr{D}$ is a decreasing function of W_s, there

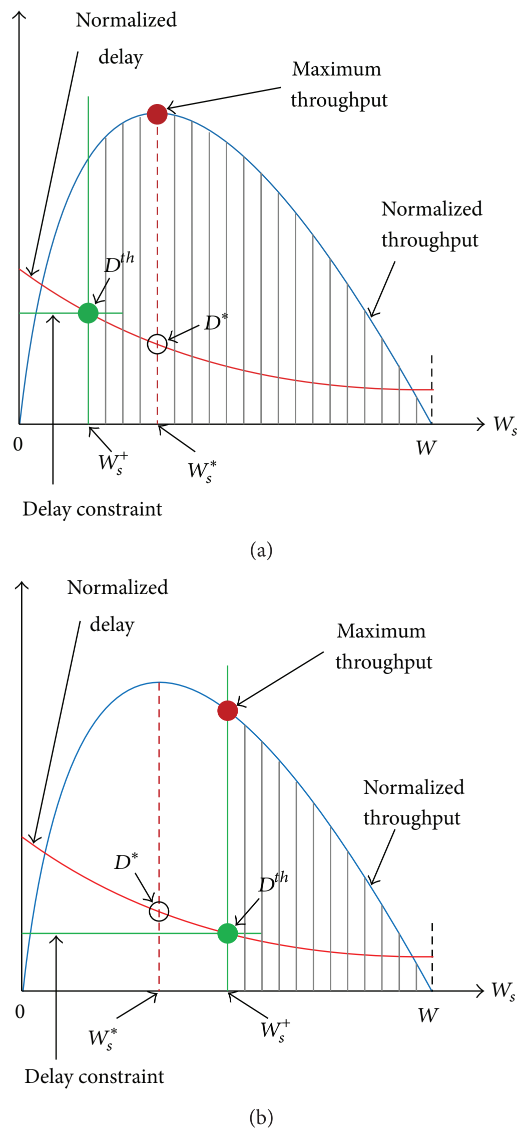

Figure 3: Illustration of the curves: normalized throughput $\mathscr{R}$ and normalized delay $\mathscr{D}$.

is no value of W_s that can maximize the throughput $\mathscr{R}$ and minimize the delay $\mathscr{D}$ simultaneously. We will optimize W_s and k jointly such that the throughput is maximized while the delay is constrained. To satisfy the delay QoS requirement, we set $\mathscr{D} \leq \mathscr{D}^{\mathrm{th}}$, where $\mathscr{D}^{\mathrm{th}}$ is the threshold of the delay constraint. Since $\mathscr{D}$ is a decreasing function of W_s, we should choose $W_s \geq W_s^+$, where W_s^+ is the minimum sensing bandwidth that can satisfy the delay QoS requirement and is determined by $\mathscr{D}^{\mathrm{th}}$.

Different SUs have different QoS requirements. Let $\mathscr{D}^*$ denote the transmission delay which is corresponding to W_s^*. For the SUs with relaxed delay QoS requirements, $\mathscr{D}^{\mathrm{th}}$ may be larger than $\mathscr{D}^*$ and $W_s^+ < W_s^*$. In this case, we should choose W_s^* to maximize the throughput $\mathscr{R}$, which is illustrated in Figure 3(a). For the SUs with stringent delay QoS requirements, $\mathscr{D}^{\mathrm{th}}$ may be smaller than or equal to

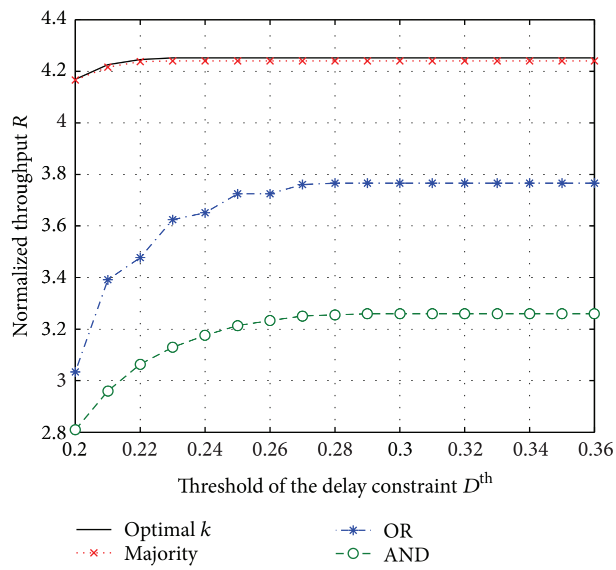

FIGURE 4: Normalized throughput $\mathscr{R}$ for various counting rules.

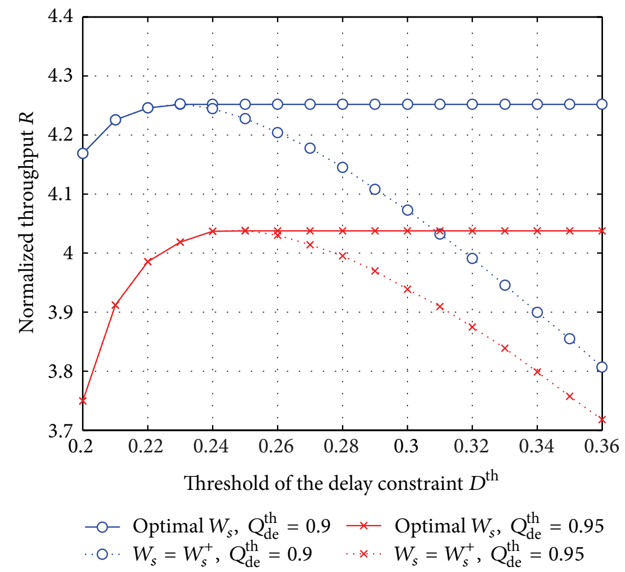

FIGURE 5: Normalized throughput $\mathscr{R}$ for various sensing schemes.

$\mathscr{D}^*$ and $W_s^+ \geq W_s^*$. In this case, we should choose W_s^+ to maximize the throughput $\mathscr{R}$, which is illustrated in Figure 3(b). For the optimal final decision threshold k_{opt}, no closed-form solution can be obtained. Hence, we will search through k from 1 to N to obtain k_{opt}. The optimal scheme that maximizes the throughput $\mathscr{R}$ can be divided into 3 steps as follows.

Step 1. For each k ($k = 1, 2, \ldots, N$), calculate the root $\mathscr{P}_{\text{de},k}$ of $\sum_{i=k}^{N} \binom{N}{i} (\mathscr{P}_{de})^i (1 - \mathscr{P}_{\text{de}})^{N-i} = Q_{\text{de}}^{\text{th}}$, the root $W_{s,k}^*$ of $\partial \mathscr{R} / \partial W_s = 0$, and the root $\mathscr{P}_{\text{fa},k}$ of $Q_{\text{fa}} = (\mathscr{D}^{\text{th}} - \mathscr{P}(H_1) Q_{\text{de}}^{\text{th}}) / \mathscr{P}(H_0)$ by using the Bisection method [22].

Step 2. According to $\mathscr{P}_{\text{de},k}$ and $\mathscr{P}_{\text{fa},k}$, calculate $W_{s,k}^+$. If $W_{s,k}^* > W_{s,k}^+$, choose optimal sensing bandwidth $W_{s,k}^{\text{opt}} = W_{s,k}^*$; otherwise, choose $W_{s,k}^{\text{opt}} = W_{s,k}^+$.

Step 3. Calculate and compare $\mathscr{R}_k$, and choose the maximum one.

4. Simulation Results

To evaluate the throughput-delay trade-off for various sensing schemes, simulation results have been conducted in this section. The frame duration is $T = 20$ ms; the individual reporting duration is $T_r = 1$ ms; the PU transmission bandwidth is $W = 2.5 \times 10^4$ Hz; the number of SUs is $N = 9$; the SNR of the PU's signal at the receiver of SU is $\gamma = -10$ dB unless otherwise stated; the SNR for the secondary link is $\gamma_s = 20$ dB; $\mathscr{P}(H_0) = 0.8$ unless otherwise stated.

Figure 4 illustrates the normalized throughput $\mathscr{R}$ versus the threshold of the delay constraint for various counting rules. Optimal W_s is employed and $Q_{\text{de}}^{\text{th}} = 0.9$. When $\mathscr{D}^{\text{th}}$ initially increases, the throughput increases. This means that relaxing the delay constraint will result in a higher throughput. However, when $\mathscr{D}^{\text{th}}$ is increased further, the throughput no longer increases and remains unchanged since the optimal sensing bandwidth is W_s^* in this case, and the throughput reaches the peak value. It is also seen that the optimal scheme can achieve a higher throughput than that using fixed thresholds. Majority rule is suboptimal and AND rule performs the worst. Thus, to enhance the throughput of the SUs, the final decision threshold k needs to be optimized.

In Figure 5, the optimal k values are used for each sensing scheme. For the SUs with stringent delay QoS requirements, $\mathscr{D}^{\text{th}} \leq \mathscr{D}^*$, it can be observed that the throughput with optimal W_s is the same as that using W_s^+ as the sensing bandwidth. This is because the optimal W_s is equal to W_s^+ in this case, which has been discussed in Section 3. For the SUs with relaxed delay QoS requirements, $\mathscr{D}^{\text{th}} > \mathscr{D}^*$, the throughput with optimal W_s reaches the peak value and remains unchanged. However, the throughput with $W_s = W_s^+$ decreases as the threshold of the delay constraint increases. This is because the optimal W_s is equal to W_s^* and $W_s^+ < W_s^*$ in this case, which has been discussed in Section 3. Larger $Q_{\text{de}}^{\text{th}}$ means better protection to PU. It is seen that relaxing the constraint on the protection of PU will result in a higher throughput.

In Figures 6 and 7, the sensing bandwidth W_s and the final decision threshold k are jointly optimized, $Q_{\text{de}}^{\text{th}} = 0.95$. Figure 6 is simulated to show the normalized throughput $\mathscr{R}$ versus the threshold of the delay constraint with different values of $\mathscr{P}(H_0)$. It is also observed that when $\mathscr{D}^{\text{th}}$ initially increases, the throughput $\mathscr{R}$ increases. However, when $\mathscr{D}^{\text{th}}$ is increased further ($\mathscr{D}^{\text{th}} > \mathscr{D}^*$), the throughput $\mathscr{R}$ reaches the peak value and remains unchanged. The larger the value of $\mathscr{P}(H_0)$, the higher the normalized throughput $\mathscr{R}$; this is because more spectrum opportunities can be reused by the SUs. In addition, $\mathscr{D}^*$ decreases as $\mathscr{P}(H_0)$ becomes larger. In Figure 7, it is shown that the larger the value of γ, the higher

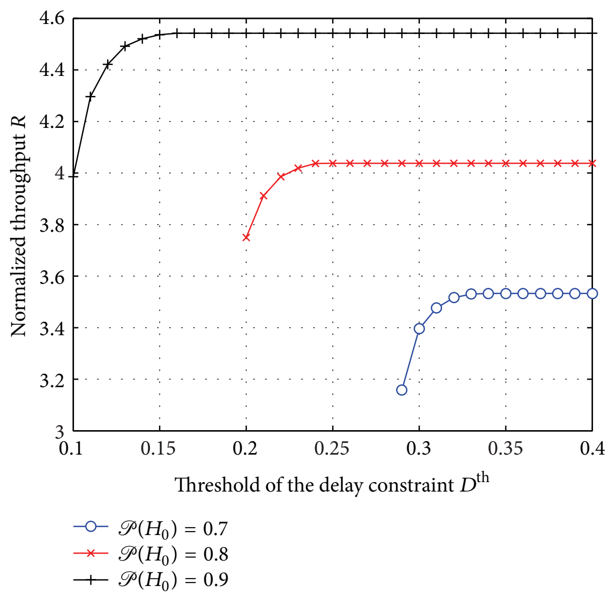

FIGURE 6: Normalized throughput $\mathcal{R}$ versus threshold of the delay constraint with different values of $\mathcal{P}(H_0)$.

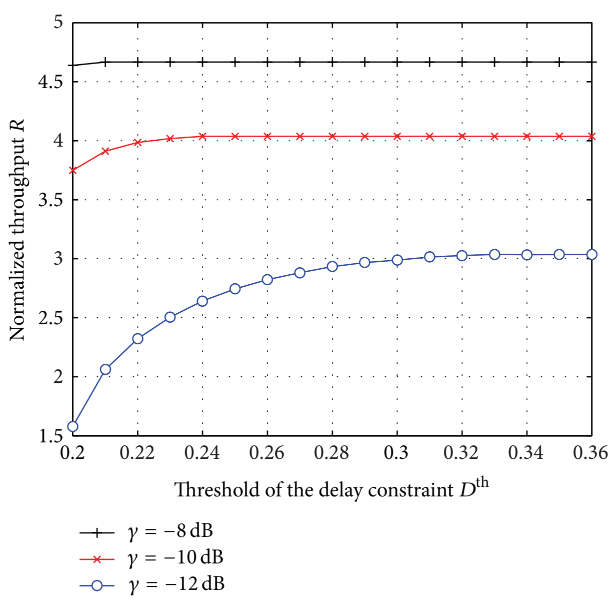

FIGURE 7: Normalized throughput $\mathcal{R}$ versus threshold of the delay constraint with different values of γ.

the normalized throughput $\mathcal{R}$. In addition, $\mathcal{D}^*$ decreases as the SNR of the PU's signal at the receiver of SU increases.

5. Conclusion

This paper studies the new CSS frame structure and formulates the throughput-delay trade-off problem. Particularly, we optimize the sensing bandwidth and the final decision threshold jointly to maximize the throughput of the SUs under the constraint that the delay QoS requirements of the SUs are satisfied. And we have demonstrated the key role of convex optimization in solving the associated design problems. Simulation results have shown that different delay QoS requirements require different optimal W_s values, and the optimal scheme can effectively improve the throughput of the SUs.

Appendix

Proof of $du/dW_s > dv/dW_s$ for $W_s \in (0,\rho]$

According to (27) and (29), we have $du/dW_s = \phi_1 + \phi_2 + \phi_3$, $dv/dW_s = -\gamma\sqrt{T_s/2W_s}\cdot\theta$, where ϕ_1, ϕ_2, ϕ_3, and θ are defined in Section 3. Obviously,

$$\frac{du}{dW_s} > \phi_3 > -\frac{d\mathcal{P}_{\text{fa}}}{dW_s}\frac{1}{1-\mathcal{P}_{fa}} = \frac{\gamma}{2}\sqrt{\frac{T_s}{\pi W_s}}\frac{e^{-\theta^2/2}}{\mathcal{Q}(-\theta)}. \quad \text{(A.1)}$$

According to [23], $e^{-x^2/2}/\mathcal{Q}(x) > \sqrt{2\pi}x$ for $x \geq 0$. For $W_s \in (0,\rho], \theta \leq 0$; then, $-\theta \geq 0$ and $e^{-\theta^2/2}/\mathcal{Q}(-\theta) > \sqrt{2\pi}(-\theta)$. Thus,

$$\frac{\gamma}{2}\sqrt{\frac{T_s}{\pi W_s}}\frac{e^{-\theta^2/2}}{\mathcal{Q}(-\theta)} > \gamma\sqrt{\frac{T_s}{2W_s}}(-\theta) = \frac{dv}{dW_s}. \quad \text{(A.2)}$$

Therefore, $du/dW_s > dv/dW_s$ for $W_s \in (0,\rho]$.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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