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Research Article

Demand Management Based on Model Predictive Control Techniques

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Demand management (DM) is the process that helps companies to sell the right product to the right customer, at the right time, and for the right price. Therefore the challenge for any company is to determine how much to sell, at what price, and to which market segment while maximizing its profits. DM also helps managers efficiently allocate undifferentiated units of capacity to the available demand with the goal of maximizing revenue. This paper introduces control system approach to demand management with dynamic pricing (DP) using the model predictive control (MPC) technique. In addition, we present a proper dynamical system analogy based on active suspension and a stability analysis is provided via the Lyapunov direct method.

1. Introduction

A supply chain is a complex system that involves suppliers, producers, distribution centers, retailers, and finally the customers. The main purpose of the supply chain management (SCM) is to optimize the entire chain. In other words, SCM manages efficiently the flow of products and information between elements of the chain in order to attain goals that cannot be reached in an isolated manner. The application of DM to SCM allows us to efficiently balance the customer's requirements and the capabilities of the supply chain (e.g., see [1]).

Nowadays, in such dynamic business world, the rapid learning about the behaviour of demand and its yield is required. The adequate management of these two variables is an important source of competitive advantage for any company in the supply chain. Moreover, traditional yield management is switching into the more complex activity of DM. Therefore our research focuses on the integration of yield or revenue management into the framework of DM. It is well known that hoteliers, airlines, and car rentals companies have implemented yield management activities successfully. According to [2], to include yield management activities into DM enhances profitability and long run sustainability. Furthermore, DM allows managers to sell the right product or service to the right customer, at the right time, and for the right price with the main objective of maximizing the profits margins. This action can be made through the assignation of units of capacity to the available demand. For example, when dealing with perishable products, DM assumes a fixed capacity. However, in the case of wafer fabrication facilities, if product mix is highly predictable, or if all products use each piece of production equipment equally, all overall production forecast is needed to determine equipment requirements. However, demand has multiple dimensions such as the different products sold, the types of customers served (preferences and purchase behavior), and time. In this context, decisions made about the price or quantity of a product may affect the demand for related products and/or may also affect the future demand for the same product. This scenario represents an opportunity to develop a model that determines how much to sell at what price and to which market segment. Thus we propose the application of DM to SCM as a helpful tool which integrates capacity, inventory level, and demand in DP framework. Dynamic pricing (DP) is the core element of DM and refers to fluid pricing between the buyer and the seller, rather than the more traditional fixed pricing [3]. Based on this, the company's main goal is to maximize its total expected revenues by making the correct decision. From here our premise is to show that DP can be modeled applying control systems and specifically addressed by the techniques of optimal control in real time, as the well-known model predictive control (MPC). This research work introduces a second order dynamical system which integrates capacity, inventory level, and biding price in the framework of DM via DP. We utilize a dynamical 1/4 active suspension system to model and prove the applicability of DP to SCM. Moreover, by using the Lyapunov direct method we concluded that a sufficient and necessary condition for stability is the presence of inventory level along the supply chain.

The remainder of this paper's structure is as follows. Section 2 describes DM in semiconductor manufacturing; Section 3 presents sensitivity and stability analysis for DM; Section 4 considers model predictive control formulation for DM with respective simulations, while conclusions are provided in Section 5.

2. Demand Management Sector Applications

2.1. Demand Management Sector Applications: A Review of the Current Literature. The techniques of DM are relatively new and the first research that dealt directly with these issues appeared less than 20 years ago. The major sector of application of DM has been in the airline industry and different approaches have been presented in the literature for airlines and hotels [4–12] (see Table 1), manufacturing, services, and transportation [13–22] (see Table 2). So much research has been done in the DM field. Please refer to Tables 1 and 2 for an extended list of authors and a brief description of their work. There are many different areas in which DM has been applied successfully, as shown in Figure 1. To this day, most of the research has been performed in the airline sector (which is justified because DM was developed to solve problems in this area) and in the area of hotel management, where, by applying DM techniques, it has been possible to mitigate some of the booking limit problems based on reservations. We present a novel approach for dynamic pricing-inventory level based on DM, assuming that we have a pair of products subject to the same demand profile.

2.2. From Supply to Demand Management: The Damping Effect. Synchronization between supply and demand is the optimal scenario in any complex supply network. In dynamical systems, dissipation is considered the loss of energy over time; this is because of damping. It is common to take into account the damping effect in different situations like economics, finance, and electromechanical systems and much more in biological systems. In DM, the notion of damping is related to inventory level, basically to achieve stability conditions over the supply chain.

2.2.1. Finite Dimensional Formulation for Demand Management. The ordinary differential equation (ODE) for the dynamic system (DS) with damping is

$$M\frac{d^2x}{dt^2} + \beta\frac{dx}{dt} + Kx = f(t). \tag{1}$$

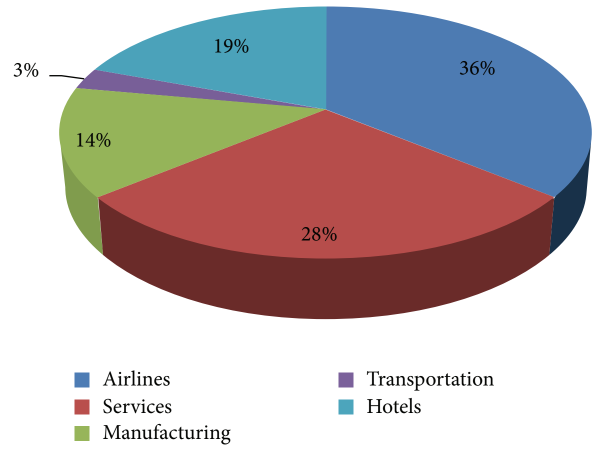

FIGURE 1: Demand management sector applications.

The present model in (1) describes the dynamics of mass-spring-damper second order dynamical system. Also, published by us [23] this model proposes a dynamic pricing for DM applying fast model predictive control approach. The present work is an extension of our previously published work.

For purposes of demand management we present the following model in the damping part of (1):

$$C\frac{d^2p}{dt^2} + \left[\int_0^\infty \beta(t)\,dt\right]\frac{dp}{dt} + Kp = d(t). \tag{2}$$

Equation (2) presents p—price level, C—capacity, $d(t)$—demand, K—biding price restitution coefficient, and $\beta(t)$—damping coefficient-inventory level.

The damping factor in (2) has the characteristic to be a function over time and then Lipschitz property can be applied and requires convergence in a scalar value (see Appendix A).

The demand management problem for optimal control has been addressed previously by different research groups [24]; dynamic pricing for optimum inventory level has been previously integrated as a first order ODE as shown in [25]. Moreover, [26] has extended this work by integrating dynamic pricing and demand level into a second order ODE. The interaction of dynamic pricing and demand level [27] presents a dynamic pricing model on web service, in which case the demand function is nonlinear. Optimization techniques have been shown [28] and have been used to quantify demand response in energy industry applying bid-pricing approaches; also, integer linear programming has been applied to model inventory level, production level, and capacity for dynamic pricing in [29].

2.2.2. Infinite Dimensional Formulation for Demand Management. An extended version of ODE in (2) is expressed as a generalized wave equation first proposed in [30] for flexible systems such as

$$C(p)\frac{\partial^2\varphi}{\partial t^2} + \beta\frac{\partial\varphi}{\partial t} + K\varphi = D(p,t). \tag{3}$$

Table 1: Demand management for airlines and hotels: literature review.

Author	Sector application	Methodology	Contribution
Abdel Aziz et al. [4]	Hotels	Research article (case studies)	Application to address the problem of room pricing in hotels. Optimization techniques are present. Multiclass scheme similar to the one implemented in airlines is studied
de Boer [5]	Airlines	Research article (theoretical)	Determination of a revenue management policy in response to the realization of the demand. This is done by matching supply and demand for seats
Hung and Chen [6]	Airlines	Research article (case studies)	This study develops and tests two heuristics approaches: the dynamic seat rationing and the expected revenue gap to help the airline make a fulfillment or rejection decision when a customer arrives
Gosavi et al. [7]	Airlines	Research article (simulation based optimization)	Application of simulation based optimization for seat allocation in airlines. The model considers real life assumption such as cancelations and overbooking
Graf and Kimms [8]	Airlines	Research article (case studies)	The optimal transfer prices are determined by a negotiation process. Option-based capacity control process results combined with transfer price optimization are compared with a first come first served approach
Kimms and Müller-Bungart [9]	Airlines	Research article (simulation based case study)	Application of heuristics and optimal networks in revenue management. Assuming stochastic demand
Netessine and Shumsky [10]	Airlines	Survey	Analysis of fundamental concepts and trade-offs of yield management to describe the parallels between yield management and inventory management
Rannou and Melli [11]	Hotels	Research article (case study)	Analysis and application of revenue management in hospitality industry. The idea is to achieve a performance measure for the return of investment
Walczak and Brumelle [12]	Airlines	Research article (theoretical)	Problem formulation of semi-Markov model for dynamic pricing and revenue management in airline industry, assuming continuous demand

Equation (3) is called master damping equation for demand management, which can be written as partial differential equation (PDE) as follows:

$$\begin{aligned} v_p \frac{\partial^2 \varphi(p,t)}{\partial p \partial t} + \left(\beta + a_p\right) \frac{\partial \varphi(p,t)}{\partial p} \\ + \alpha\beta \frac{\partial \varphi(p,t)}{\partial t} + K\varphi(p,t) = D(p,t), \end{aligned} \tag{4}$$

where first derivative of price in time is v_p and second derivative in time is a_p. See Appendix B for a detailed proof of (3) and (4).

3. Robust and Stability Analysis for Demand Management

DM can be stated as follows: how to select the product's mix amount and price in order to satisfy a fluctuating demand with the main goal of maximizing the profit. For this reason, in this document we propose the use of MPC by comparing DM and DP using an active suspension with damping.

3.1. Physical Modeling in DM-DP Problem. The use of a physical model represents an approach to obtain a suitable model to be controlled. According to [31], DM is the creation across the supply chain and its markets of a coordinated flow of demand. Also, the role of DM is to decrease demand. The application of the active suspension analogy to track both problems is suitable, because the price level (which represents outputs) and demand (which represents input) converge to a stability condition that requires inventory level in the dissipation of the dynamical system. DM assumes fixed capacity (which in terms of active suspension is the mass), inventory level (damping of the system), and biding price (restitution coefficient). In this case the idea is to present a model in which, by using DM, we can assess how demand does affect prices. We have chosen an active suspension system to approach the system model; see Figure 2.

The dynamic equations for the system are as follows.

For the capacity fixed C_f:

$$\begin{aligned} C_f \ddot{p_f} = k_f\left(w - p_f\right) - k_u\left(p_f - p_u\right) \\ - \beta\left(\dot{p_f} - \dot{p_u}\right) - \alpha \cdot d. \end{aligned} \tag{5}$$

Table 2: Demand management for manufacturing and services: literature review.

Author	Sector application	Methodology	Contribution
Bertsimas and Shioda [13]	Services	Research article (case studies)	Application of revenue management in restaurants. Using stochastic optimization via dynamic programming. Computational experiments are presented for reservation and without reservation case studies
Boyd and Bilegan [14]	Services	Research article (case study in e-commerce)	Analysis of revenue management applied to e-commerce in the airline industry. Optimization techniques are shown in inventory control mechanism
Defregger and Kuhn [15]	Manufacturing	Research article (case studies)	Application of revenue management to a make-to-order manufacturing company with limited inventory capacity. Classifying the underlying Markov decision process, by introducing a heuristic procedure
Kimes [16]	Services	Survey	Revenue management analysis via the application of optimization, forecasting, and overbooking
Kumar and Frederik [17]	Services	Survey	Application of revenue management in construction industry. The benefits of revenue management can be realized at manufacturing companies. Uncertain demand and pricing of available capacity are analyzed via this approach
Lee et al. [18]	Transportation	Research article (case studies)	Analysis and application of heuristic to solve a single revenue management problem with postponement, arising from the sea cargo industry. Optimization based case studies are addressed
Nair and Bapna [19]	Services	Research article	This paper studies optimal policies for allocating modems capacity among segments of customers using a continuous time Markov decision
Spengler et al. [20]	Manufacturing	Research article (case studies)	Revenue management approach for companies in the iron and steel industry is developed. The aim is to improve short-term order selection
Shin and Park [21]	Manufacturing	Research article (theoretical)	Yield improvement is analyzed in semiconductor manufacturing via machine learning
Steinhardt and Gönsch [22]	Services	Research article (case studies)	This paper addresses the problem of integrating revenue management capacity control with upgrade decision making. A new structural property for an integrated dynamic programming formulation is present

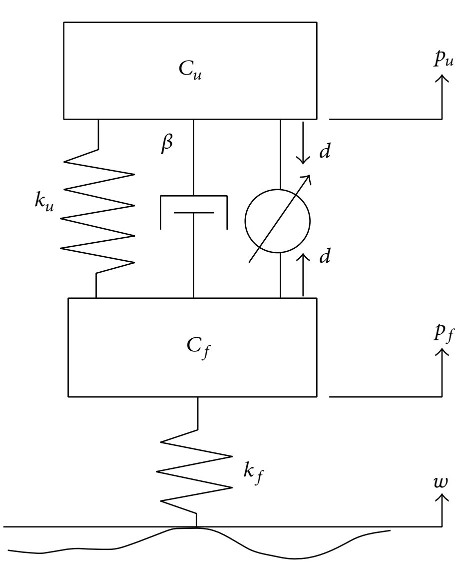

Figure 2: Active suspension analogy for DM-DP model.

For the capacity unfixed subject to active suspension performance C_u:

$$C_u \ddot{p}_u = k_u \left(p_f - p_u\right) + \beta \left(\dot{p}_f - \dot{p}_u\right) + (1 - \alpha)\, d. \quad (6)$$

Variables definition is as follows:

C_f: capacity fixed,

C_u: capacity fixed subject to active suspension,

p_f: price level for the C_f capacity fixed,

p_u: price level for the C_u subject to active suspension,

w: price level disturbance in the market,

d: demand,

k_f: biding price restitution coefficient based on price for C_f,

k_u: biding price restitution coefficient based on price for C_u,

β : damping-inventory level in the system,

α : embedding parameter $0 \leq \alpha \leq 1$.

As it is stated in DM, C_f and C_u are assumed fixed; the characteristic of this system is such that the damping coefficient is β. The nature of the input signal is the demand and the output is the price level for the capacity fixed and for the price level subject to active suspension. There exists a relation between biding prices and restitution coefficients k_f and k_u; this means the amount of product per unit of price. The model presented in the active suspension for demand and price modeling, from Figure 2, is based on the assumption that the analysis is for two products in the system. We plan to extend our model to a half-car active suspension with balanced capacity and unbalanced capacity for four products. Finally, in future work, we intend to extend it to a full-car model for a multiproduct system and analyze if the system is balanced (with the same amount of capacity in each active suspension).

3.2. Robust Policies in DM: A Sensitivity Analysis. The following approach can be used for the design of robust controllers, as suggested in [32], which analyzed the robustness for mass-spring system without damping. Consider a representation of an uncertain linear dynamical system in state-space form:

$$\dot{z} = A(q)z + B(q)u, \tag{7}$$

where $z \in \mathfrak{R}^n$, $u \in \mathfrak{R}^m$, and q is a vector of uncertain parameters. The sensitivity of the states with respect to parameter q is

$$\frac{d\dot{z}}{dq_i} = \frac{\partial A}{\partial q_i}z + A\frac{\partial z}{\partial q_i} + \frac{\partial B}{\partial q_i}u. \tag{8}$$

Consider the quarter active suspension system; from Figure 2, the equations of motion, assuming that the uncertain parameter is the damping coefficient (β), and the partial derivative of (5) and (6) with respect to β are

$$\frac{\partial \ddot{p}_f}{\partial \beta} = \frac{1}{C_f}\left[\left(\dot{p}_u - \dot{p}_f\right) + \beta\left(\frac{\partial \dot{p}_u}{\partial \beta} - \frac{\partial \dot{p}_f}{\partial \beta}\right)\right], \tag{9}$$

$$\frac{\partial \ddot{p}_u}{\partial \beta} = -\frac{1}{C_u}\left[\left(\dot{p}_u - \dot{p}_f\right) - \beta\left(\frac{\partial \dot{p}_f}{\partial \beta} - \frac{\partial \dot{p}_u}{\partial \beta}\right)\right]. \tag{10}$$

Both equations are not independent of one another; the sensitivity states are expressed as

$$\frac{\partial \ddot{p}_f}{\partial \beta} = -\frac{C_u}{C_f}\left(\frac{\partial \ddot{p}_u}{\partial \beta}\right). \tag{11}$$

The following boundary conditions increase the robustness:

$$\frac{\partial p_f}{\partial \beta} = \frac{\partial p_u}{\partial \beta} = \frac{\partial \dot{p}_f}{\partial \beta} = \frac{\partial \dot{p}_u}{\partial \beta} = 0. \tag{12}$$

Integrating (11) in time and subject to boundary conditions (12), finally the relationship is such that

$$\frac{\partial \dot{p}_f}{\partial \beta} = -\frac{C_u}{C_f}\left(\frac{\partial \dot{p}_u}{\partial \beta}\right). \tag{13}$$

Substituting (13) into (10) permits achieving the general state sensitivity equation of the form:

$$\frac{\partial \ddot{p}_u}{\partial \beta} = -\frac{1}{C_u}\left(\dot{p}_u - \dot{p}_f\right) + \left(\frac{\beta}{C_f} + \frac{\beta}{C_u}\right)\left(\frac{\partial \dot{p}_u}{\partial \beta}\right). \tag{14}$$

The sensitivity analysis presented in Section 3.2 considers variations in the inventory level, which gives a capacity equivalent, $C_{\text{eq}} = C_u C_f/(C_u + C_f)$, to be used in the dynamical system for modeling consideration and it refers to robust policies. We conclude that a robust inventory policy is represented by the following equation: $\beta/C_{\text{eq}} = \beta((1/C_f) + (1/C_u))$.

3.3. Stability Analysis for DM. For the behavior of the damping effect in DM, this section proposes the stability criteria via Lyapunov direct method.

Definition 1. A linear system is stabilizable if all unstable modes are controllable.

DM-ODE equation (2) is necessary to analyze the stability performance. Assume the DM-ODE equation has the structure $\dot{x} = Ax + Bu$.

Theorem 2. *For the dynamical system* (2), *and considering that (A, B) are controllable, a necessary and sufficient condition for stability in DM is* $\beta > 0$.

Proof. The following condition in the demand of the system is assumed:

$$d(t) = 0. \tag{15}$$

Equation (2) can be formulated as

$$C_{\text{eq}}\ddot{p} + \beta\dot{p} + kp = 0. \tag{16}$$

From (16) we use energy as the Lyapunov function:

$$V(p, \dot{p}) = \frac{1}{2}C_{\text{eq}}\dot{p}^2 + \frac{1}{2}kp^2, \tag{17}$$

$$\frac{dV}{dt} = \frac{\partial V}{\partial p}\frac{dp}{dt} + \frac{\partial V}{\partial \dot{p}}\frac{d\dot{p}}{dt}, \tag{18}$$

$$\frac{dV}{dt} = kp \cdot \dot{p} + C_{\text{eq}}\dot{p} \cdot \ddot{p}, \tag{19}$$

$$\frac{dV}{dt} = kp \cdot \dot{p} + C_{\text{eq}} \cdot \dot{p}\left(\frac{-\beta}{C_{\text{eq}}}\dot{p} - \frac{k}{C_{\text{eq}}}p\right). \tag{20}$$

Eliminating terms from (20), the following condition is achieved: $-\beta \cdot \dot{p}^2 \leq 0$. Finally, the presence of inventory level in the system $\beta > 0$ is a necessary and sufficient condition for stability purposes. □

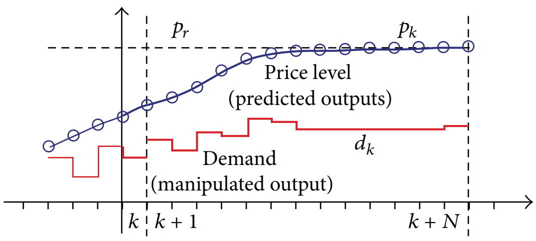

Figure 3: Schematic receding horizon control philosophy.

4. Model Predictive Control Analysis for Demand Management

4.1. Model Predictive Control: Towards Optimal Policies in DM. Model predictive control (MPC) is a real time optimal control strategy that has been applied in process control, aerospace, automotive, management science, and robotic applications. Besides these applications, MPC has other advantages such as good tracking performance, physical constraints handling, and extension to nonlinear systems [33]. MPC has been used to address production-inventory systems. For example, in adaptive MPC, the adapted model along with a smoothed estimation of the future customer demand is used to predict inventory levels over the optimization horizon [34]. MPC policy shows improved performance, greater flexibility, and higher functionality relative to an advanced order-up-to policy based on control engineering principles found in the literature [35]. The philosophy of MPC, also known as receding horizon control (RHC), has the advantage of handling control and state constraints [36]. The RHC strategy takes into account an objective function and many constraints; see Figure 3. In RHC to define an optimal strategy, the first step requires calculating a control law and repeating this process indefinitely, each sampling time.

With MPC, an optimization problem is solved at each time step to determine a plan of action over a fixed time horizon [37]. MPC is a nonlinear control policy that handles input and output constraints, as well as various control objectives. Using MPC, a system can be controlled near its physical limits, often outperforming linear control. It is well known that the computation of predictive control laws is a crucial task in every MPC application to systems with fast dynamics due to the fact that an optimization problem has to be solved online [38]. Other approaches, applying MPC techniques, have been developed such as simplified MPC algorithm for Markov jump systems [39] and constrained robust MPC effective for uncertain Markov jump systems, as previously shown [40].

For the active suspension analogy to DM-DP, we will use a linear MPC for purposes of analysis and control. Based on this, we are interested in optimal control problems of the form

$$\min_{d_k,\dots,d_{k+N-1}} \quad J = \sum_{k=0}^{N-1} \left\| p_k - p_r \right\|^2 + \left\| d_k - d_r \right\|^2, \tag{21a}$$

Table 3: MPC design parameters for optimal and robust policies.

Policy case	T_s	N_p	N_u
Optimal policies	0.05	10	2
Robust policies	0.03	10	2

$$\begin{aligned} \text{s.t.} \quad & x_{k+1} = A \cdot x_k + B \cdot d_k, \\ & p_k = C \cdot x_k + D \cdot d_k, \\ & d_{\min} \le d_k \le d_{\max}, \\ & p_{\min} \le p_k \le p_{\max}, \\ & x_k = x(k), \quad k = 0, \dots, N-1. \end{aligned} \tag{21b}$$

Here $x : \mathfrak{R} \rightarrow \mathfrak{R}^n$ denotes the state, $d : \mathfrak{R} \rightarrow \mathfrak{R}^n$ the control input, and $p : \mathfrak{R} \rightarrow \mathfrak{R}^n$ the output of the system.

The conventional linear MPC law is based on the following algorithm.

Algorithm (linear MPC)

(1) Achieve the new state p_k.

(2) Solve the optimization problem (21a) and (21b).

(3) Calculate the law $d(k) = d(k + 0 \mid k)$.

(4) $k \leftarrow k + 1$. Go to (1).

Computational Complexity Analysis. The complexity of the solver for the optimization problem (21a) and (21b) depends on the choice of the performance index, the model, and constraints. For our algorithm the optimization problem is a quadratic program (QP).

Based on the nature of the proposed algorithm, which is a QP related to an active set method, each iteration has cost $O(n^2)$ floating point operations (flops), where n is the number of decision variables and n is proportional to horizon N_p. It is important to notice that active set methods are exponential in the worst case but show good practical performance. Also, active set methods work best for small and medium size problems.

4.2. Simulations Results. After the presented analysis two scenario simulations are shown for optimal policies in Figures 4 and 5. The robust policies for DM are presented in Figures 6 and 7. The main goal is to contribute in the performance of MPC for optimal policies and robust policies in DM and to compare it with linear quadratic regulator (LQR). The application of tracking MPC in DM approaches achieves a suitable response for optimal policies. In Figure 6 price level is shown taking into account capacity fixed and unfixed responses. It is important to notice that the demand response, in Figure 7, is constrained.

MPC design parameters for optimal and robust policies are present in Table 3. It is important to note that T_s is the sampling time, N_p is the prediction horizon, and N_u is the control horizon, where $N_u < N_p$.

In Figure 4, the MPC simulation reflects under optimal policies a performance with small overshoot in prices and

Table 4: Design parameter for DM-active suspension dynamical system.

Policy case	C_u	C_f	k_u	k_f	β
Optimal policies	300	50	2000	2000	500
Robust policies	500	300	10000	10000	1000

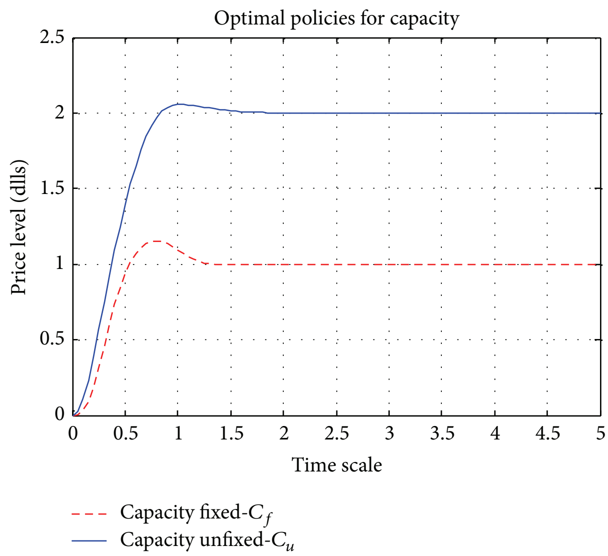

Figure 4: Simulation response for optimal policy price level-tracking MPC.

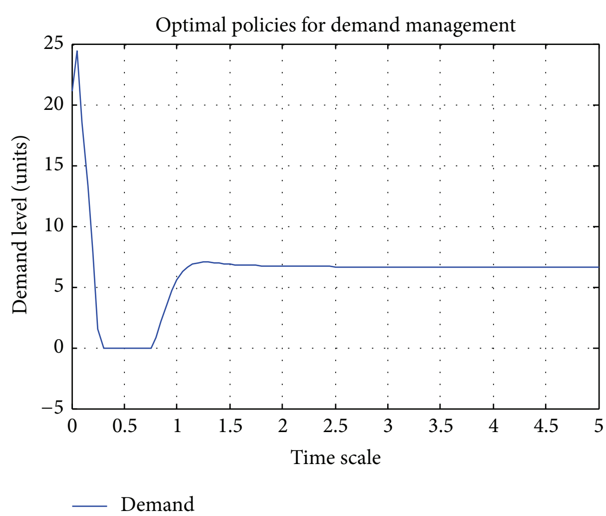

Figure 5: Optimal policies for demand management.

considerable fluctuations in demand. It is important to remember that we have the price level as output for the dynamical system and demand as input.

The MPC simulation shows in Figure 6 the performance in the application of the robust policies obtained from the analysis showed in Section 3.2. It is important to note that the price level in both responses is reduced by half compared with scenario 1 from Figure 10. This reduction is based on the notion that there is a relation between inventory level and

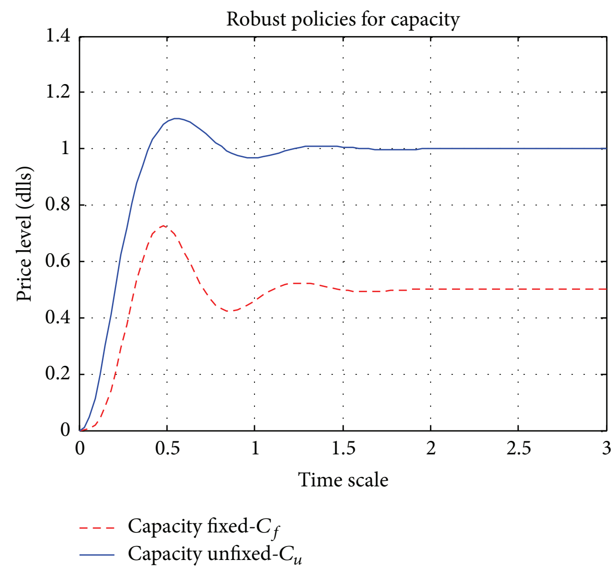

Figure 6: Simulation response for robust policy price level-tracking MPC.

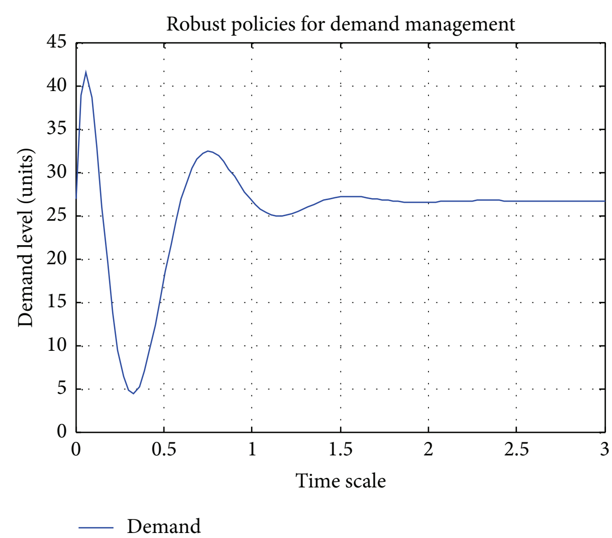

Figure 7: Robust policies for demand management.

capacity equivalent for the robust policy. Under variations in this relation, the price level is reduced and the demand presents small variations.

Design parameters for DM-active suspension are shown in Table 4, for optimal and robust policies. It is important to note that these parameters are the same over both policies for purposes of control via MPC and LQR.

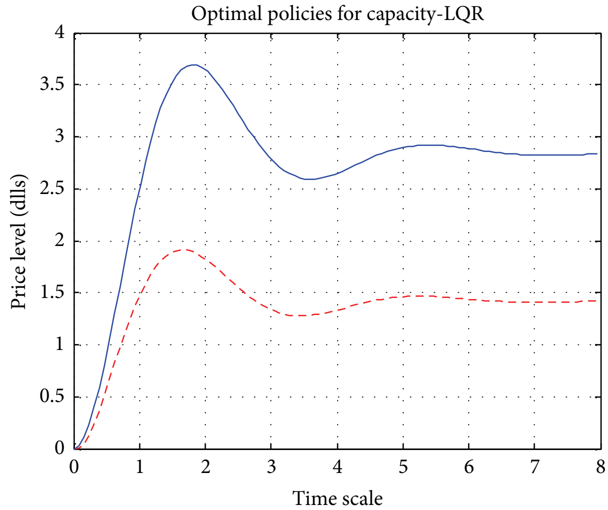

FIGURE 8: Optimal policies for capacity via LQR.

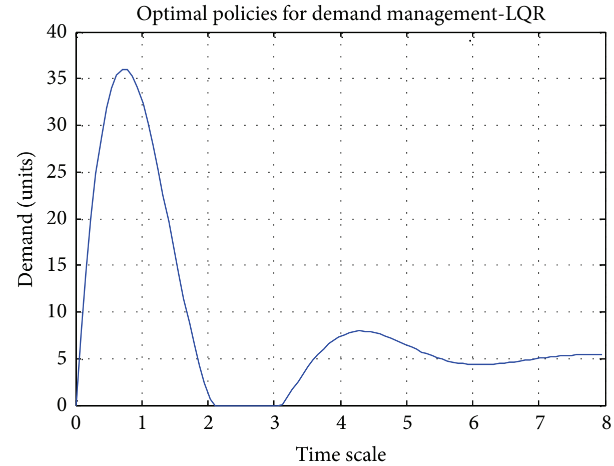

FIGURE 9: Optimal policies for demand management via LQR.

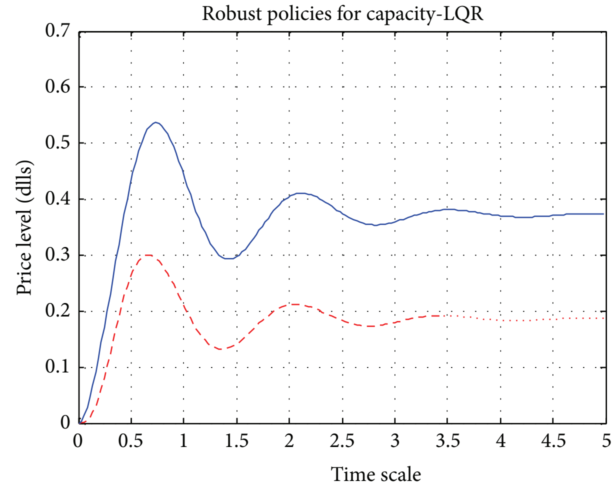

FIGURE 10: Robust policies for capacity via LQR.

In summary, from demand curve in economics, which establish that if price is lower consumers are ready to buy more, based on this context, with an optimal policy, the prices are higher and demand is low. However for robust policies the prices are lower than in optimal policies and the demand is higher. In this case the relation between the price and the demand is satisfied from economic theory.

Linear Quadratic Regulator. Consider that MPC is an online solution of a LQR optimization problem. The approach here is to introduce the LQR methodology and compare it with the MPC results. In the state-feedback version of the LQR problem we assume that the whole state x can be measured and therefore it is available to control [41]. The state-feedback control law $u(t) = -Kx(t)$ minimizes the cost function:

$$J_{\mathrm{LQR}} = \int_0^{\infty} \left(x^T Q x + u^T R u\right) dt, \tag{22}$$

where $K = R^{-1}B^T P$, and P is solved by an algebraic Riccati equation. The results achieved by the application of LQR in the optimal and robust policies are presented in Figures 8 and 9.

Once LQR is applied for optimal and robust policies, simulations present more oscillations when compared to MPC, using the same parameters from Table 4. In Figure 8, for optimal policies the price level achieves more overshoot and a larger setting time. Also in Figure 9 the demand level presents higher variation than in MPC optimal policies. The demand level is reduced at the end of time horizon, as it has been proposed in DM theory.

Robust policies achieve an oscillatory performance in the price level as it is noted in Figure 10; however, the price level is reduced considerably when compared to optimal policies via LQR. In Figure 11, demand level achieves a low value in units at the end of time horizon, which is expected, but the oscillation is persistent.

In summary, MPC simulations from Figures 4 to 7 present better performance than LQR which are shown in Figures 8 to 11, for optimal and robust policies.

Based on this context and these results, we aim to explore other control oriented approaches for DM-active suspension analogy via dynamic pricing such as robust control techniques, as developed in [42, 43] with the goal to compare its performance with MPC techniques. Also we plan to explore adaptive control techniques [44], output feedback control [45], and sample data control present [46] and to develop a trade-off with optimal control techniques.

5. Conclusions

This research work presents a novel DM-DP approach based on MPC for second order systems with damping, which

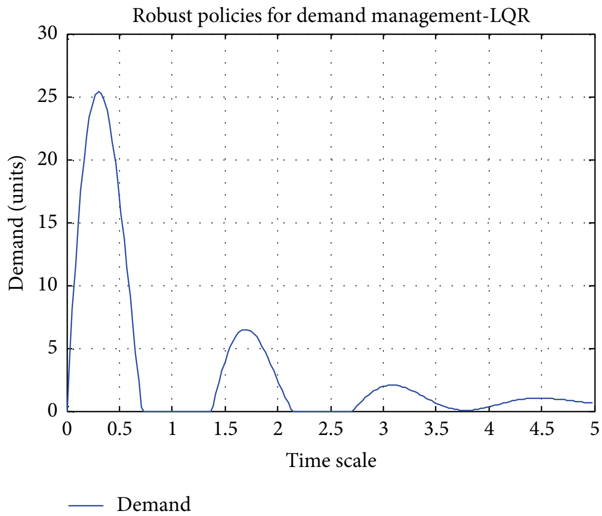

FIGURE 11: Robust policies for demand management via LQR.

in terms of DM tell us that dissipation process is present with inventory storage level in the system. Our results of the stability analysis from Lyapunov direct method show that an inventory level is needed in the system. The one-quarter active suspension model presents the analogy for a DM-DP process system behavior. Besides the sensitivity analysis shows us that an equivalent capacity level will provide reduced price level. Simulations results show that under robust policies the price level is reduced by half; this is because of the ratio between inventory level and capacity in the active suspension dynamical system. For future work, the intention is to develop a full active suspension model for multiple product system as well as to achieve inventory level dynamics via first order ordinary differential equations. Another future research area will focus on the exploration of a full active suspension model for one or multiple products in a multistage supply chain.

Appendices

A. Inventory Level Derivation

The derivation of the value of β requires the following analysis:

$$\text{Let } \beta(t) = \int_0^\infty e^{-t/\beta} dt, \quad \text{Finally } \beta(t) = \beta. \tag{A.1}$$

B. Infinite Dimensional Analysis for Demand Management

We propose the following generalized wave equation which describes the behavior of flexible systems, presented first in [30]:

$$m(x)\frac{\partial^2 u(x,t)}{\partial t^2} + \beta\frac{\partial u(x,t)}{\partial t} + Ku(x,t) = F(x,t). \tag{B.1}$$

The purpose is to obtain a PDE description for demand management via dynamic pricing. Based on this

$$C(p)\frac{\partial^2\varphi(p,t)}{\partial t^2} + \beta\left[\frac{\partial\varphi(p,t)}{\partial p} + \alpha\frac{\partial\varphi(p,t)}{\partial t}\right] + k\varphi(p,t) = D(p,t), \tag{B.2}$$

where

$$\frac{\partial\varphi}{\partial t} = \frac{\partial\varphi}{\partial p}\frac{\partial p}{\partial t} = v_p\frac{\partial\varphi}{\partial p}, \tag{B.3}$$

substituting (B.3) in (B.2) which gives

$$C(p)\left[\frac{\partial}{\partial t}\left(v_p\frac{\partial\varphi(p,t)}{\partial p}\right)\right] + \beta\frac{\partial\varphi(p,t)}{\partial p} + \alpha\beta\frac{\partial\varphi(p,t)}{\partial t} + K\varphi(p,t) = D(p,t). \tag{B.4}$$

Developing partial derivatives:

$$C(p)\left(v_p\frac{\partial^2\varphi(p,t)}{\partial p\partial t} + a_p\frac{\partial\varphi(p,t)}{\partial p}\right) + \beta\frac{\partial\varphi(p,t)}{\partial p} + \alpha\beta\frac{\partial\varphi(p,t)}{\partial t} + K\varphi(p,t) = D(p,t). \tag{B.5}$$

Define $C(p) \equiv 1$ and $0 \le \alpha \le 1$.

The PDE description for demand management is

$$v_p\frac{\partial^2\varphi(p,t)}{\partial p\partial t} + \left(\beta + a_p\right)\frac{\partial\varphi(p,t)}{\partial p} + \alpha\beta\frac{\partial\varphi(p,t)}{\partial t} + K\varphi(p,t) = D(p,t), \tag{B.6}$$

where $\beta + a_p > 0$.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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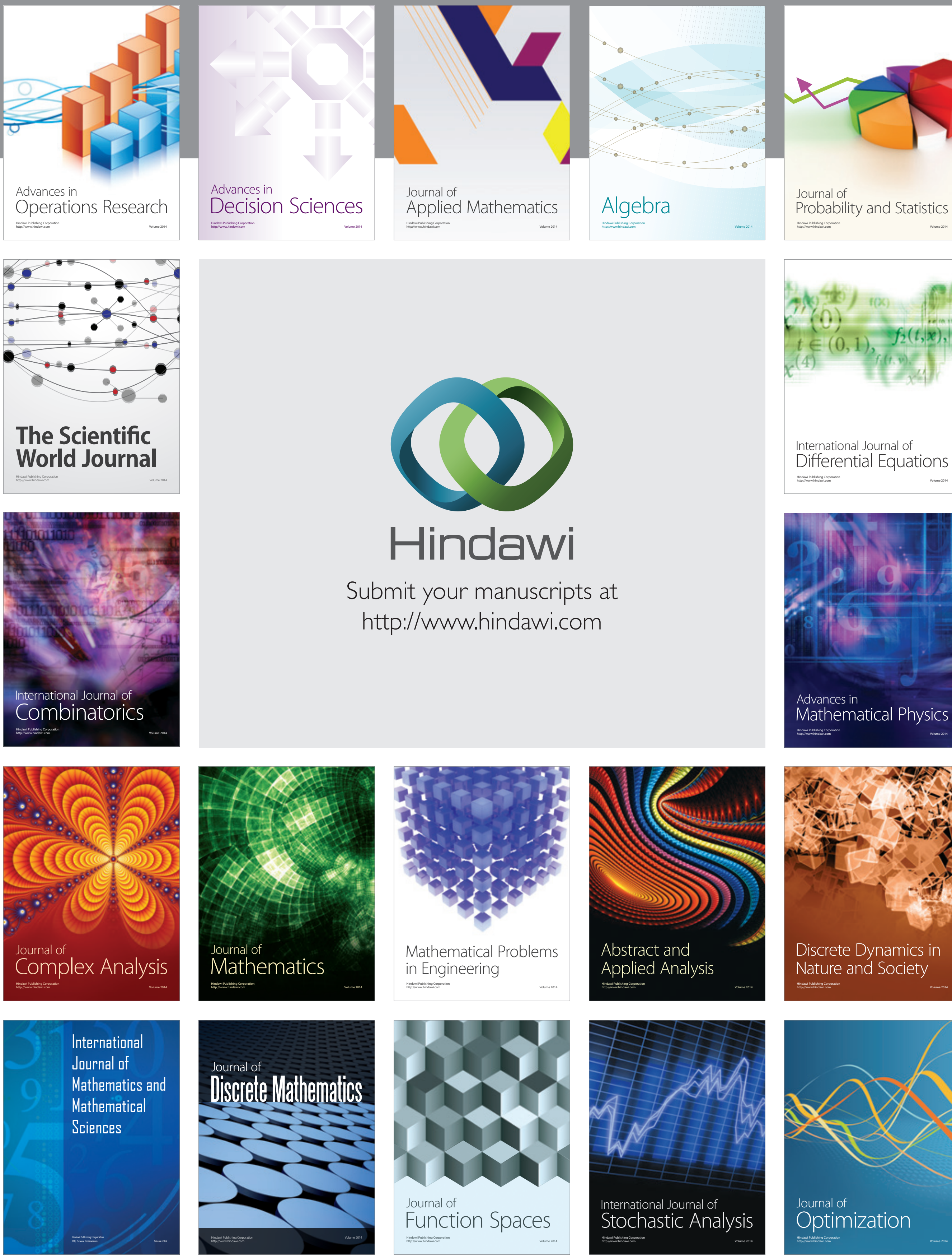
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