Hindawi
Journal of Control Science and Engineering
Volume 2018, Article ID 5603639, 5 pages
https://doi.org/10.1155/2018/5603639

Research Article

Hybrid Synchronization of Uncertain Generalized Lorenz System by Adaptive Control

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Received 28 October 2017; Revised 18 December 2017; Accepted 19 December 2017; Published 24 January 2018

Academic Editor: Sundarapandian Vaidyanathan

This paper investigates hybrid synchronization of the uncertain generalized Lorenz system. Several useful criteria are given for synchronization of two generalized Lorenz systems, and the adaptive control law and the parameter update law are used. In comparison with those of existing synchronization methods, hybrid synchronization includes full-order synchronization, reduced-order synchronization, and modified projective synchronization. What is more, control of the stability point, complete synchronization, and antisynchronization can coexist in the same system. Numerical simulations show the effectiveness of this method in a class of chaotic systems.

1. Introduction

In 1990, Pecora and Carroll made chaos synchronization come true [1]; chaotic synchronization, as a very important topic in the nonlinear science, has been extensively studied in a variety of fields including secure communications and physical and biological systems [2, 3]. So far, a lot of methods about chaotic synchronization have been presented to prove that the chaotic synchronization method is feasible, such as linear and nonlinear feedback synchronization [4, 5], impulsive synchronization [6], adaptive synchronization [7], and observer based control method [8]. Among these schemes, hybrid synchronization is one in which some of the chaotic systems are synchronized whereas others are antisynchronized [9]. Due to its importance, hybrid synchronization has been the subject of many research works [10, 11]. Moreover, uncertainties exist widely in engineering and they often bring adverse effects to the stability and performance of real systems. So there is an increasing demand on developing better control techniques [12–16].

In the above discussed literature, the given systems usually were typical benchmark chaotic systems, such as the Lorenz system, Chen system, and Lü system. In this paper, we consider the generalized Lorenz system. Based on the stability theory of systems, several useful criteria are given for discussing synchronization of two generalized Lorenz systems, and the adaptive control law and the parameter update law are also given. In comparison with those of existing synchronization methods, hybrid synchronization includes full-order synchronization, reduced-order synchronization, and the modified projective synchronization. What is more, control of the stability point, complete synchronization, and antisynchronization can coexist in the same system. The rest of this paper is organized as follows: Section 2 gives theoretical analyses. Section 3 handles full-order synchronization and reduced-order hybrid synchronization. Section 4 gives the conclusion of the paper.

2. Theoretical Analyses

Let us consider a class of chaotic systems described by

$$\dot{X} = f(X) + g(X)\Theta, \tag{1}$$

where $X \in R^n$ is the state vector of the system, $f(X) : R^n \to R^n$ is vector-valued functions, $g(X) : R^n \to R^{n\times m}$ is a matrix function, and $\Theta = (\Theta_1, \Theta_2, \ldots, \Theta_m)^T \in R^m$ is parameter vector.

Remark 1. A number of well-known chaotic systems have the form of (1), such as Chua's circuit, hyperchaotic Lü system, and generalized Lorenz system.

Definition 2. For the drive and response chaotic systems

$$\dot{X} = f(X) + g(X)\Theta, \tag{2}$$

$$\dot{Y} = f(Y) + g(Y)\Theta + U, \tag{3}$$

if

$$\lim_{t\to\infty} \|Y - \Phi X\| = 0, \tag{4}$$

then, systems (2) and (3) are said to achieve hybrid synchronization, where $\Phi \in R^{n\times n}$ is a constant matrix and $U = (u_1, u_2, \dots, u_n)^T$ is the controller.

Remark 3. If $\mathrm{Rank}(\Phi) = n$, hybrid synchronization is full-order hybrid synchronization; if $0 < \mathrm{Rank}(\Phi) < n$, hybrid synchronization is reduced-order hybrid synchronization or coexistence of control problem and reduced-order hybrid synchronization; if $\mathrm{Rank}(\Phi) = 0$, hybrid synchronization problem will be turned into a chaos control problem.

Remark 4. If $\Phi = \mu I$, $\mu \in R$, hybrid synchronization problem will be reduced to projective synchronization, where I is an $n\times n$ identity matrix. In particular, if $\Phi = I$ and $-I$, hybrid synchronization problem is further simplified to complete synchronization and antiphase synchronization, respectively. And if $\Phi = \mathrm{diag}\{\eta_1, \eta_2, \dots, \eta_n\}$, $\eta_i \in R$, modified projective synchronization will appear.

Our objective is to design the controller U to achieve hybrid synchronization.

Theorem 5. *For the drive system (2) and the response system (3), let $e(t) = Y - \Phi X$; if the controller U is given by*

$$U = -f(Y) - g(Y)\widehat{\Theta} + \Phi f(X) + \Phi g(X)\Theta + Ke, \tag{5}$$

where $e(t) = (e_1, e_2, \dots, e_n)^T$ and $\widehat{\Theta}$ represent the estimate vectors of uncertain parameter vector Θ, then, if the coupling strength $K = \mathrm{diag}(k_1, k_2, \dots, k_n)$ is updated according to the following laws:

$$\dot{k}_i = -\lambda_i e_i^2, \tag{6}$$

and parameter update laws of the drive system are given as follows:

$$\dot{\widehat{\Theta}}\left(g(Y)\right)^T e, \tag{7}$$

thus, hybrid synchronization between the drive system (2) and the response system (3) can be achieved globally asymptotically. Furthermore, the unknown parameter $\widehat{\Theta}$ can be identified in the process of hybrid synchronization.

Proof. From (2)-(3), we get the following error dynamical system:

$$\dot{e} = f(Y) + g(Y)\Theta - \Phi f(X) - \Phi g(X)\Theta + U. \tag{8}$$

Choose a candidate Lyapunov function as follows:

$$V = \frac{1}{2}e^T e + \frac{1}{2}e_{\widehat{\Theta}}^T e_{\widehat{\Theta}} + \frac{1}{2}\sum_{i=1}^{n}\frac{1}{\lambda_i}\left(k_i + L\right)^2, \tag{9}$$

where $e = (e_1, e_2, \dots, e_n)^T$, $e_{\widehat{\Theta}} = (e_{\widehat{\Theta}_1}, e_{\widehat{\Theta}_2}, \dots, e_{\widehat{\Theta}_m})^T$, $e_{\widehat{\Theta}} = \Theta - \widehat{\Theta}$, $L > 0$.

Then the differentiation of V along the trajectories of (9) is

$$\begin{aligned}\dot{V} &= e^T\dot{e} + \dot{e}_{\widehat{\Theta}}^T e_{\widehat{\Theta}} - \sum_{i=1}^{n}\left(k_i + L\right)e_i^2 \\ &= e^T\left(g(Y)\Theta - g(Y)\widehat{\Theta} + Ke\right) - \left(\left(g(Y)\right)^T e\right)^T e_{\widehat{\Theta}} \\ &\quad - e^T Ke - Le^T e = -Le^T e < 0.\end{aligned} \tag{10}$$

Then, according to the Lyapunov stability theorem, the error system is asymptotically stable at the origin. Hence, hybrid synchronization between the drive system (2) and the response system (3) can be achieved under the controller (5) and parameter update laws (7). This completes the proof. □

3. Illustrative Example

In this section, we give some examples to show the effectiveness of this method.

In 1963, Lorenz found the first classical chaotic attractor [17]. In 1999, Chen found the Chen attractor which is similar but not topologically equivalent to Lorenz chaotic attractor [18]. In 2002, Lü found another new critical chaotic system [19]. These systems can be included in the following generalized Lorenz system [20] which is described by

$$\begin{aligned}\dot{x}_1 &= a_{11}x_1 + a_{12}x_2, \\ \dot{x}_2 &= a_{21}x_1 + a_{22}x_2 - x_1x_3, \\ \dot{x}_3 &= a_{33}x_3 + x_1x_2.\end{aligned} \tag{11}$$

Throughout the paper, the generalized Lorenz system (11) is chosen as the drive system, and the response system is

$$\begin{aligned}\dot{y}_1 &= a_{11}y_1 + a_{12}y_2 + u_1. \\ \dot{y}_2 &= a_{21}y_1 + a_{22}y_2 - y_1y_3 + u_2, \\ \dot{y}_3 &= a_{33}y_3 + y_1y_2 + u_3,\end{aligned} \tag{12}$$

We have the following corollaries for the generalized Lorenz system.

3.1. The Full-Order Hybrid Synchronization

Corollary 6. *If* Rank(Φ) = 3, *for example,* $\Phi = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 1 & 0 & 2 \end{pmatrix}$, *adaptive controller and parameter estimation adaptive laws are as follows:*

$$\begin{aligned}
u_1 &= -y_1 a_{11}^* - y_2 a_{12}^* + x_1 a_{11} + x_2 a_{12} + k_1 e_1, \\
u_2 &= y_1 y_3 - y_1 a_{21}^* - y_2 a_{22}^* + x_1 x_3 - x_1 a_{21} - x_2 a_{22} \\
&\quad + k_2 e_2, \\
u_3 &= -y_1 y_2 - y_3 a_{33}^* + 2 x_1 x_2 + x_1 a_{11} + x_2 a_{12} \\
&\quad + 2 x_3 a_{33} + k_3 e_3, \\
\dot{k}_1 &= -\lambda_1 e_1^2, \\
\dot{k}_2 &= -\lambda_2 e_2^2, \\
\dot{k}_3 &= -\lambda_3 e_3^2, \\
\dot{a}_{11}^* &= y_1 e_1, \\
\dot{a}_{12}^* &= y_2 e_1, \\
\dot{a}_{21}^* &= y_1 e_2, \\
\dot{a}_{22}^* &= y_2 e_2, \\
\dot{a}_{33}^* &= y_3 e_3.
\end{aligned} \tag{13}$$

Systems (11) and (12) can realize full-order hybrid synchronization.

The proof of Corollary 6 follows directly from Theorem 5; thus we leave out its proof here.

In the simulations, suppose that the "unknown" parameters of the drive and response Lü chaotic systems are chosen as $(a_{11}, a_{12}, a_{21}, a_{22}, a_{33}) = (-36, 36, 0, 20, -3)$. The initial values of the drive and response systems are taken as $(1, 2, 3)$ and $(2, 1, 3)$, respectively. The initial values estimated for "unknown" parameters are taken as $(2, 3, 1, 2, 3)$. Let $k_1 = 15$, $k_{2,3} = 1$, $\lambda_i = 1$, $i = 1, 2, 3$, and the simulated results are shown in Figures 1 and 2. In Figure 1, three state errors versus time are shown and the state errors tend to zero asymptotically as time evolves. Figure 2 shows that estimated values of parameters finally evolve to the true values.

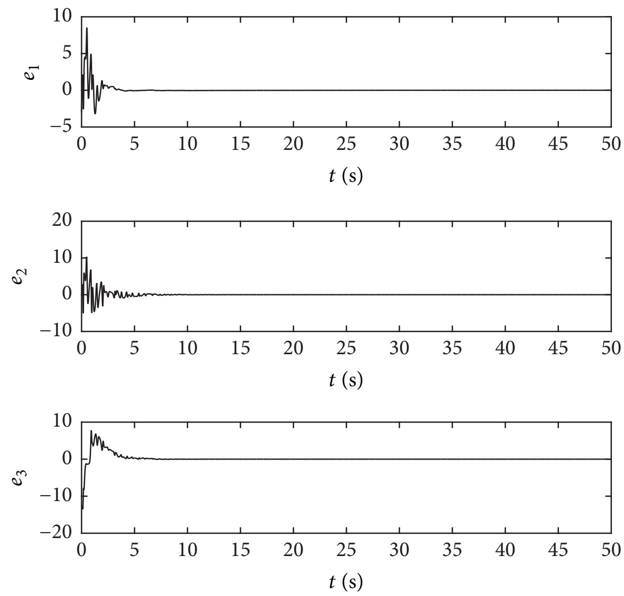

Figure 1: Synchronization errors between two Lü chaotic systems; the full-order hybrid synchronization is realized.

3.2. Coexistence of Control Problem and Reduced-Order Hybrid Synchronization

Corollary 7. *If* $0 <$ Rank(Φ) < 3, *for example,* $\Phi = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, *the adaptive controller and parameter estimation adaptive laws are as follows:*

$$\begin{aligned}
u_1 &= -y_1 a_{11}^* - y_2 a_{12}^* + x_1 a_{11} + x_2 a_{12} + k_1 e_1, \\
u_2 &= y_1 y_3 - y_1 a_{21}^* - y_2 a_{22}^* + x_1 x_3 - x_1 a_{21} - x_2 a_{22} \\
&\quad + k_2 e_2, \\
u_3 &= -y_1 y_2 - y_3 a_{33} + k_3 y_3, \\
\dot{k}_1 &= -\lambda_1 e_1^2, \\
\dot{k}_2 &= -\lambda_2 e_2^2, \\
\dot{k}_3 &= -\lambda_3 y_3^2, \\
\dot{a}_{11}^* &= y_1 e_1, \\
\dot{a}_{12}^* &= y_2 e_1, \\
\dot{a}_{21}^* &= y_1 e_2, \\
\dot{a}_{22}^* &= y_2 e_2.
\end{aligned} \tag{14}$$

Systems (11) and (12) can realize coexistence of the control problem, complete synchronization, and antisynchronization.

Proof. From $\Phi = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, we have $e_1 = y_1 - x_1$, $e_2 = y_2 + x_2$, $e_3 = y_3$; thus, we get the following dynamical system by condition of Corollary 7:

$$\begin{aligned}
\dot{e}_1 &= y_1 (a_{11} - a_{11}^*) + y_2 (a_{12} - a_{12}^*) + k_1 e_1, \\
\dot{e}_2 &= y_1 (a_{21} - a_{21}^*) + y_2 (a_{22} - a_{22}^*) + k_2 e_2, \\
\dot{e}_3 &= \dot{y}_3 = k_3 y_3.
\end{aligned} \tag{15}$$

Similarly to proofs of Theorem 5, we choose a candidate Lyapunov function as follows:

$$V = \frac{1}{2} e^T e + \frac{1}{2} e_{\widehat{\Theta}}^T e_{\widehat{\Theta}} + \frac{1}{2} \sum_{i=1}^{3} \frac{1}{\lambda_i} (k_i + L)^2, \tag{16}$$

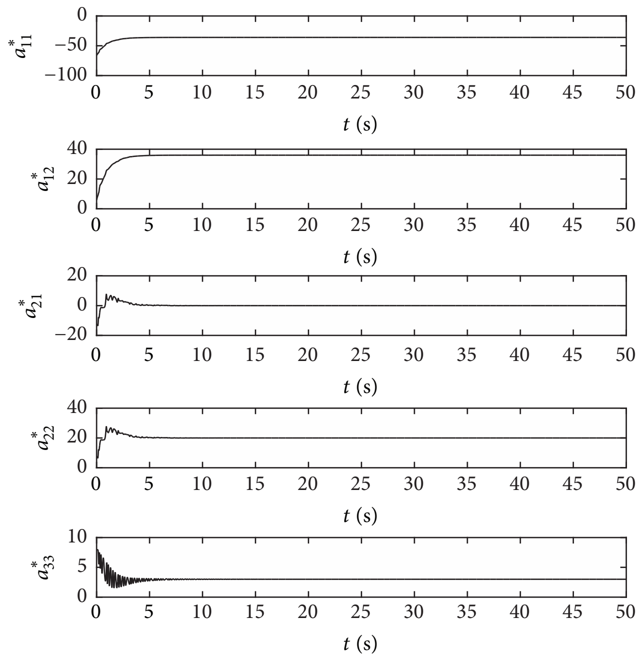

Figure 2: Estimated parameters of the Lü system finally evolve to the true values $(a_{11}, a_{12}, a_{21}, a_{22}, a_{33}) = (-36, 36, 0, 20, -3)$.

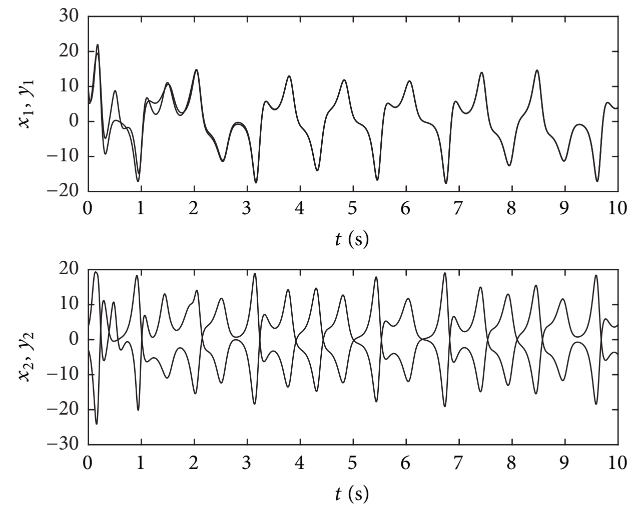

Figure 3: Complete synchronization and antisynchronization are realized for the Chen system.

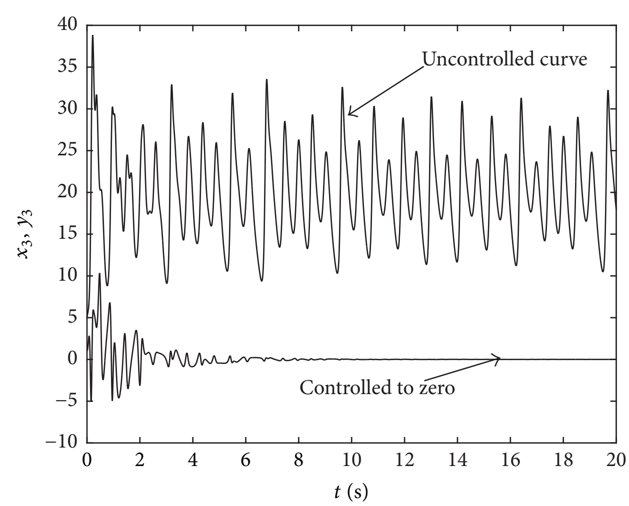

Figure 4: The third state vector (y_3) of the Chen system is controlled to the zero.

where $e = (e_1, e_2, y_3)^T$, $e_{\widehat{\Theta}} = (e_{\widehat{\Theta}_1}, e_{\widehat{\Theta}_2})^T$, and $e_{\widehat{\Theta}} = \Theta - \widehat{\Theta}$, $L > 0$.

Then the differentiation of V along the trajectories of (16) is

$$\begin{aligned}\dot{V} &= e^T \dot{e} + \dot{e}_{\widehat{\Theta}}^T e_{\widehat{\Theta}} - \sum_{i=1}^{n} (k_i + L) e_i^2 = -L \left(e_1^2 + e_2^2 + y_3^2\right) \\ &= -Le^T e < 0.\end{aligned} \tag{17}$$

Then, according to the Lyapunov stability theorem, system (15) is asymptotically stable at the origin. Hence, systems (11) and (12) can realize coexistence of the control problem, complete synchronization, and antisynchronization. This completes the proof. □

Remark 8. Obviously, the controllers u_1, u_2 realize reduced-order hybrid synchronization and the controller u_3 makes the third state vector (y_3) of the system controlled to the zero. Of course, Theorem 5 is still valid on carrying out complete reduced-order hybrid synchronization; for example, $\Phi = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$; hybrid synchronization problem will be turned into a complete reduced-order hybrid synchronization problem.

In the simulations, suppose that the "unknown" parameters of the drive and response Chen chaotic systems are chosen as $(a_{11}, a_{12}, a_{21}, a_{22}, a_{33}) = (-35, 35, -7, 28, -3)$. The initial values of the drive and response systems are taken as $(1, 2, 3)$ and $(2, 1, 3)$, respectively. The initial values estimated for "unknown" parameters are taken as $(2, 3, 1, 2)$. Let $k_i = 1$, $\lambda_i = 20$, $i = 1, 2, 3$; simulated results are shown in Figures 3–5. In Figure 3, complete synchronization and antisynchronization are realized for the Chen system. Figure 4 shows that the third state vector of the system is controlled to the zero. Figure 5 shows that estimated values of parameters finally evolve to the true values.

4. Conclusion

This paper has discussed hybrid synchronization of the uncertain generalized Lorenz system. Based on the stability theory systems, several useful criteria have been given for synchronization of two generalized Lorenz system systems, and the adaptive control law and the parameter update law were also given. In comparison with those of existing synchronization methods, hybrid synchronization includes full-order synchronization, reduced-order synchronization, and modified projective synchronization. Simulation results were presented to demonstrate the application of theoretical

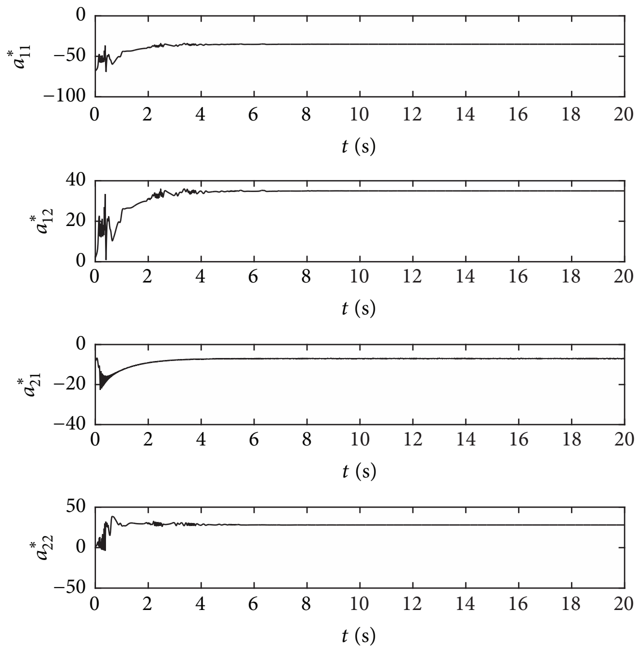

Figure 5: Estimated parameters of the Chen system finally evolve to the true values $(a_{11}, a_{12}, a_{21}, a_{22}) = (-35, 35, -7, 28)$.

results. Our future work is to study hybrid synchronization of Markovian jump complex networks with time-varying delay.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

This work is supported by the National Natural Science Foundation of China (61673221), the Youth Fund Project of the Humanities and Social Science Research for the Ministry of Education of China (14YJCZH173), and the Science and Technology Research Key Program for the Education Department of Hubei Province of China (D20156001).

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