Research Article

Switching Fuzzy Guaranteed Cost Control for Nonlinear Networked Control Systems

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This paper deals with the problem of guaranteed cost control for a class of nonlinear networked control systems (NCSs) with time-varying delay. A guaranteed cost controller design method is proposed to achieve the desired control performance based on the switched T-S fuzzy model. The switching mechanism is introduced to handle the uncertainties of NCSs. Based on Lyapunov functional approach, some sufficient conditions for the existence of state feedback robust guaranteed cost controller are presented. Simulation results show that the proposed method is effective to guarantee system's global asymptotic stability and quality of service (QoS).

1. Introduction

As network technology advanced in the last decade, networked control system (NCS) has increasingly become a research focus. Considerable attention on the modeling and controller design of NCSs has been paid in [1–5]. There are many advantages to NCSs, such as reduced system wiring, facilitated system maintenance, and increased systems flexibility.

However, due to the insertion of communication channels, this brings many challenging problems such as network-induced delay and data packet dropout. Regardless of the type of network used, these special issues degrade the system dynamic performance and are a source of potential instability. There are a number of design methods that have been proposed to deal with these problems. One of the most general methods is to model the NCS as a system with time-varying delays. So the stability of an NCS is equivalent to the stability of a system with time-varying delays [2]. Moreover, the sampling behavior has also an important impact on the design of the NCS controller because the states of the feedback controller are not continuous as a result of the existence of zeroth-order hold (ZOH). In [1], a model of NCS was provided considering network-induced delay and packet dropout in the transmission. In [3], an observer-based stabilizing controller was designed for the NCSs involving both random measurement and actuation delays. Robust controllers for uncertain NCSs were also obtained in [4, 5]. However, how to analyze the stability of nonlinear NCSs has increasingly become a challenging topic. Some results about the stability of nonlinear NCSs were obtained in [6, 7]. In [8], a stochastic optimal controller design for nonlinear NCSs with uncertain dynamics via neurodynamic programming was proposed. The closed-loop stability of the nonlinear NCSs was demonstrated by selecting neural network (NN) update laws. However, these methods often require some strict assumptions for a system model, so it is difficult for practical applications.

In the last few years, the fuzzy control is a useful approach to solve the control problems of nonlinear systems. The Takagi-Sugeno (T-S) fuzzy system proposed in [9] is widely applied to industrial control fields because of its simple structure with local dynamics. In the T-S fuzzy model, local dynamics in different state-space regions are represented by many linear models so that linear system theory can conveniently be employed to analyze the stability of overall closed-loop system and to design the feedback controller. The typical design approaches are carried out based on fuzzy model

via the so-called parallel distributed compensation (PDC) method [10]. Considering the time-delay characteristic of nonlinear systems, many results about the T-S fuzzy model with a time-delay term are obtained to deal with stability and stabilization problems of nonlinear systems with time delays [2, 11–13]. The guaranteed cost control for a T-S fuzzy system with time delays was presented in [2, 11, 12]. In [13], robust control problem was studied for a class of large-scale NCSs, and the decentralized design was presented using T-S fuzzy approach.

Nevertheless, an inherent drawback remains since the number of fuzzy rules of a T-S model increases exponentially with the number of nonlinearities constituting the matched nonlinear system [14]. This makes fuzzy controller design and implementation difficult as the complexity of the nonlinear system to be controlled increases [15]. To outline the problem of rules explosion in fuzzy T-S modeling, some authors have proposed to combine the merit of switched systems with T-S ones to deal with nonlinear control problems [16–18]. To do so, partitioning the state space of a nonlinear system allows defining a switched nonlinear system. Then, inside each partition, a T-S model can be obtained. As stated in [18], the resulting switched T-S system inherits some essential features of hybrid systems and maintains all the information and knowledge representation capacity of fuzzy systems. However, few papers have studied stabilization issues of switched fuzzy systems based on switching Lyapunov functions [16] or quadratic approaches [17]. In [19], based on barrier Lyapunov functions (BLFs), a new control design for constrained nonlinear switched systems was investigated to achieve output tracking. By ensuring the boundedness of the BLFs in the closed loop, the proposed approaches can guarantee that all states in the switched systems do not violate the desired constraints and that all closed-loop signals are bounded. For many nonlinear systems, some of the premise variables of the corresponding T-S fuzzy models are measurable when they are modeled as T-S fuzzy models, while the parts with unmeasurable premise variables can be modeled as uncertainties. Thus, the overall systems can be described by T-S fuzzy parts with measurable premise variables and uncertainty parts. In [20], a switching stabilizing controller is designed for a nonlinear system with unknown parameters or unmeasurable premise variables. However, the uncertainties cannot be well handled. In [21], a switching fuzzy dynamic output feedback control scheme is proposed, where the switching mechanism is introduced to handle the unknown parameters.

In designing a controller for a real plant, it is invariably necessary to design a control system which not only is stable but also possesses a strong robust performance. One way to deal with this is the so-called guaranteed cost control approach [2]. This addresses the robust performance problem and has the advantage of providing an upper bound on a given performance index guaranteeing that system performance degradation incurred by uncertainty is less than this bound. For the NCSs, the quality of service (QoS) is one of the most important performance indexes. Therefore, it is vital to design a guaranteed cost controller such that the NCSs are stable as well as satisfactory with the required QoS.
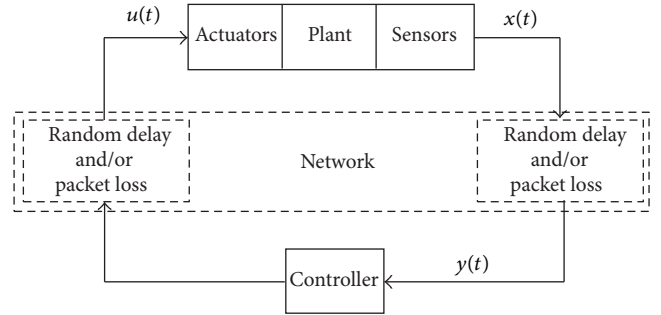

FIGURE 1: A general NCS.

In this paper, we aim at the problem of guaranteed cost control for a class of uncertain nonlinear NCSs with time delays. Considering the QoS of NCSs, we propose a guaranteed cost control scheme to achieve the desired control performance based on switched T-S fuzzy control method, where the switching mechanism is introduced to handle the uncertainties. Moreover, the sufficient condition for the existence of the robust guaranteed cost controller and the design method of the corresponding switching control law are obtained via Lyapunov functions. Comparing with [2, 11, 12], the proposed switching fuzzy approach inherits some essential features of hybrid systems to deal with the uncertain nonlinear NCSs and avoids the inherent drawback of rules explosion in modeling a fuzzy T-S model. In addition, we consider the stabilization problem of the switched fuzzy T-S system with time delays in NCSs.

The innovations of this paper are as follows: (1) the guaranteed cost controller is proposed for nonlinear NCSs with time-varying delay to achieve the desired control performance based on the switched T-S fuzzy model with uncertain parameters, and (2) the sufficient condition for the robust guaranteed cost control law is presented to uncertain nonlinear NCSs.

The paper is organized as follows. The basic problem formulation of the nonlinear networked control system is given in Section 2. The switching fuzzy controller via state feedback is analyzed in Section 3. In Section 4, the sufficient stability conditions and guaranteed cost control law are discussed by Lyapunov functions. Section 5 provides simulation results to demonstrate the effectiveness of the proposed method. Finally, concluding remarks are given in Section 6.

2. Problem Formulation

A general NCS configuration is illustrated in Figure 1, which is composed of a controller and a remote system containing a physical plant, sensors, and actuators. The controller and the plant are physically located at different locations and are directly linked by a data network in order to perform remote closed-loop control.

In Figure 1, the nonlinear networked control system can be described as follows:

$$\dot{x}(t) = f(x(t), u(t)),$$
$$y(t) = g(x(t), u(t)),$$
(1)

where $x(t) \in R^n$ is the state variation, $u(t) \in R^m$ is the control input, $y(t) \in R^s$ is the regulated output, $f(x, u)$ and $g(x, u)$ are the nonlinear functions. Currently, it has been proved that the T-S fuzzy models are universal approximations of many nonlinear dynamic systems. So we introduce a T-S fuzzy system to model a class of nonlinear NCSs. Without considering the disturbance input, we use the following T-S fuzzy model to approximate $f(x(t), u(t))$ and $g(x(t), u(t))$:

$$R_i: \text{IF } \theta_1(t) \text{ is } N_{i1}, \dots, \text{and } \theta_g(t) \text{ is } N_{ig},$$
$$\text{THEN } \begin{cases} \dot{x}(t) = A_i x(t) + B_i u(t), \\ y(t) = C_i x(t), \end{cases} \tag{2}$$

where $i = 1, 2, \dots, r$ is the index number of fuzzy rules, $\theta_1(t), \theta_2(t), \dots, \theta_g(t)$ are the known premise variables, N_{ik} is the fuzzy sets $(k = 1, 2, \dots, g)$, $x(t) \in R^n$ is the state vector, $y(t) \in R^s$ is the output vector, $u(t) \in R^m$ is the control input, $A_i \in R^{n \times n}$ is known system matrices, $B_i \in R^{n \times m}$ is the input matrix, and $C_i \in R^{n \times s}$ is the output matrix.

Before designing the controller, we make the following reasonable assumptions.

Assumption 1. The sensor is clock-driven. The controller and actuator are event-driven. The clocks among them are synchronized.

Assumption 2. Time-varying network-induced delay is less than one sampling period.

Assumption 3. The computational delay is negligible.

Assumption 4. The signal is single-packet transmission without packet drop.

Assuming the node sampling period is T, so we can obtain the discrete T-S fuzzy model of nonlinear networked control systems with time-varying delay:

$$R_i: \text{IF } \theta_1(k) \text{ is } N_{i1}, \dots, \text{and } \theta_g(k) \text{ is } N_{ig},$$
$$\text{THEN } \begin{cases} x(k+1) \\ \quad = \Phi_i x(k) + \Gamma_{i0}(\tau_k) u(k) \\ \quad + \Gamma_{i1}(\tau_k) u(k-1), \\ y(k) = C_i x(k), \end{cases} \tag{3}$$

where $\Phi_i = e^{A_i T}$, $\Gamma_{i0}(\tau_k) = \int_0^{T-\tau_k} e^{A_i T} dt \cdot B_i$, $\Gamma_{i1}(\tau_k) = \int_{T-\tau_k}^{T} e^{A_i T} dt \cdot B_i$, and τ_k is the network-induced delay, $i = 0, 1, 2, \dots, r$.

Furthermore, (3) can be transformed into the following form with uncertain parameters:

$$x(k+1) = \Phi_i x(k) + (\Gamma_{i0} + D_i F_i(\tau_k) E_i) u(k)$$
$$+ (\Gamma_{i1} - D_i F_i(\tau_k) E_i) u(k-1), \tag{4}$$
$$y(k) = C_i x(k),$$

where D_i, E_i are known constant matrices of appropriate dimensions and F_i is an unknown matrix function satisfying

$$F^T F \le I,$$
$$\Gamma_{i0}(\tau_k) = \Gamma_{i0} + D_i F_i(\tau_k) E_i, \tag{5}$$
$$\Gamma_{i1}(\tau_k) = \Gamma_{i1} - D_i F_i(\tau_k) E_i.$$

For any given $x(k)$ and $u(k)$, by using a weighted-average defuzzifier, product inference, and singleton fuzzifier, the local models can be integrated into a global nonlinear model:

$$x(k+1) = \sum_{i=1}^{r} \mu_i(\theta(k)) [\Phi_i x(k) + \Gamma_{i0}(\tau_k) u(k)$$
$$+ \Gamma_{i1}(\tau_k) u(k-1)], \tag{6}$$
$$y(k) = C_i x(k),$$

where

$$\theta(k) = [\theta_1^T(k), \theta_2^T(k), \dots, \theta_g^T(k)]^T,$$
$$\mu_i(\theta(k)) = \frac{\prod_{l=1}^{g} N_{il}(q_l(k))}{\sum_{i=1}^{r} \prod_{l=1}^{g} N_{il}(\theta_l(k))}. \tag{7}$$

$N_{il}(\theta_l(k))$ is the grade of membership of $\theta_l(k)$ in the fuzzy set N_{il}. Notice the following facts:

$$\mu_i(\theta(k)) \ge 0, \qquad \sum_{i=1}^{r} \mu_i(\theta(k)) = 1. \tag{8}$$

The guaranteed cost function associated with system (3) is given by

$$J = \sum_{k=0}^{\infty} \left[x^T(k) Q x(k) + u_\sigma^T(k) R u_\sigma(k) \right]. \tag{9}$$

Definition 5. Consider the uncertain system (3) and cost function (9). If there exists a control law $u_\sigma^*(k)$ and a positive scalar J^* such that for all admissible uncertainties, the closed-loop system is asymptotically stable and the value of the cost function (9) satisfies $J \le J^*$, then J^* is said to be a guaranteed cost and $u_\sigma^*(k)$ is said to be a guaranteed cost control law.

3. Controller Design

We assume that switched fuzzy controller is constituted with N switching rules. The σth subfuzzy controller is

$$R^\sigma_i: \text{IF } \theta_1(t) \text{ is } N_{i1}, \dots, \text{and } \theta_g(t) \text{ is } N_{ig},$$
$$\text{THEN } u_\sigma(k) = K_{\sigma i} x(k), \tag{10}$$

where $\sigma = 1, 2, \dots, N$ is a piecewise constant function representing the switching signal and R_i^σ represents ith fuzzy

rule of the σth subfuzzy controller. The switching control law is constituted by the following fuzzy controller:

$$
\begin{aligned}
u_1(k) &= \sum_{i=1}^{r} \mu_i(\theta(k)) K_{1i} x(k), \\
u_2(k) &= \sum_{i=1}^{r} \mu_i(\theta(k)) K_{2i} x(k), \\
&\vdots \\
u_N(k) &= \sum_{i=1}^{r} \mu_i(\theta(k)) K_{Ni} x(k).
\end{aligned}
\tag{11}
$$

When the controlled system is in σth switching subsystem, the global fuzzy equation is as follows

$$
\begin{aligned}
&x(k+1) \\
&= \sum_{i=1}^{r} \mu_i \left[\Phi_i x(k) + \Gamma_{i0}(\tau_k) u(k) + \Gamma_{i1}(\tau_k) u(k-1) \right] \\
&= \sum_{i=1}^{r} \mu_i \Phi_i x(k) \\
&\quad + \sum_{i=1}^{r} \sum_{j=1}^{r} \mu_i \mu_j \left[\Gamma_{i0}(\tau_k) K_{\sigma j} x(k) + \Gamma_{i1}(\tau_k) K_{\sigma j} x(k-1) \right] \\
&= \sum_{i=1}^{r} \sum_{j=1}^{r} \mu_i \mu_j \\
&\quad \times \left[\Phi_i x(k) + \Gamma_{i0}(\tau_k) K_{\sigma j} x(k) \right. \\
&\quad \left. + \Gamma_{i1}(\tau_k) K_{\sigma j} x(k-1) \right] \\
&= \sum_{i=1}^{r} \sum_{j=1}^{r} \mu_i \mu_j \\
&\quad \times \left\{ \left[\Phi_i + \Gamma_{i0}(\tau_k) K_{\sigma j} \right] x(k) \right. \\
&\quad \left. + \Gamma_{i1}(\tau_k) K_{\sigma j} x(k-1) \right\} \\
&= \sum_{i=1}^{r} \sum_{j=1}^{r} \mu_i \mu_j \left\{ \left[\Phi_i + \left(\Gamma_{i1} - D_i F_i(\tau_k) E_i \right) K_{\sigma j} \right] x(k) \right. \\
&\quad \left. + \left(\Gamma_{i1} - D_i F_i(\tau_k) E_i \right) K_{\sigma j} x(k-1) \right\}.
\end{aligned}
\tag{12}
$$

Let $\Omega_1, \Omega_2, \ldots, \Omega_N$ be a partition of the set R^n; then $\bigcup_{i=1}^{N} \Omega_i = R^n \setminus \{0\}$, $\Omega_i \bigcap \Omega_j = \Phi$, $i = j$. The switching law that is determined by $\Omega_1, \Omega_2, \ldots,$ and Ω_N is $\sigma = \sigma(x(t)) = i$, when

$x(t) \in \Omega_i$. This switching law can be completely described by the following function:

$$
v_\sigma(x(t)) = \begin{cases} 1, & x(t) \in \Omega_\sigma, \\ 0, & x(t) \notin \Omega_\sigma, \end{cases} \quad \sigma = 1, 2, \ldots, N.
\tag{13}
$$

Thus, we have

$$
\begin{aligned}
x(k+1) &= \sum_{\sigma=1}^{N} \sum_{i=1}^{r} \sum_{j=1}^{r} v_\sigma \mu_i \mu_j \\
&\quad \times \left\{ \left[\Phi_i + \left(\Gamma_{i1} - D_i F_i(\tau_k) E_i \right) K_{\sigma j} \right] x(k) \right. \\
&\quad \left. + \left(\Gamma_{i1} - D_i F_i(\tau_k) E_i \right) K_{\sigma j} x(k-1) \right\},
\end{aligned}
\tag{14}
$$

where τ_k is the network-induced delay.

4. Sufficient Condition for Guaranteed Cost Control

Lemma 6 (see [22] (Schur complement)). *For a given symmetric matrix,*

$$
S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}.
\tag{15}
$$

Then, the following three conditions are mutually equivalent:

(1) $S < 0$,

(2) $S_{11} < 0$, $S_{22} - S_{12}^T S_{11}^{-1} S_{12} < 0$,

(3) $S_{22} < 0$, $S_{11} - S_{12} S_{22}^{-1} S_{12}^T < 0$.

Lemma 7 (see [23]). *Given matrices Y, D, E, and I of appropriate dimensions and with Y and I symmetrical and $I > 0$, then*

$$
Y + DFE + E^T F^T D^T < 0
\tag{16}
$$

for all F satisfying $F^T F \leq I$, if and only if there exists some $\varepsilon > 0$ such that

$$
Y + \varepsilon^2 DD^T + \varepsilon^{-2} E^T E < 0.
\tag{17}
$$

Theorem 8. *Consider the uncertain nonlinear networked control systems (3) and the cost function (9). If there exist some constants $\lambda_\sigma \in [0,1]$, $\sum_{\sigma=1}^{l} \lambda_\sigma = 1$, a group of positive constants $\varepsilon_\sigma > 0$, and positive definite matrices X, Z, $Y_{\sigma i}$, $\sigma = 1,\ldots,l$, $i = 1,\ldots,r$, such that the following matrix inequalities (18) hold:*

$$
\sum_{\sigma=1}^{l} \lambda_\sigma \begin{bmatrix} L & U & V & 0 & 0 & 0 & 0 \\ * & -2X+2Z & 0 & 2X & \left(Y_{\sigma i}+Y_{\sigma j}\right)^T & \left(E_i Y_{\sigma j}\right)^T & \left(E_j Y_{\sigma i}\right)^T \\ * & * & -2Z & 0 & 0 & -\left(E_i Y_{\sigma j}\right)^T & -\left(E_j Y_{\sigma i}\right)^T \\ * & * & * & -2Q^{-1} & 0 & 0 & 0 \\ * & * & * & * & -2R^{-1} & 0 & 0 \\ * & * & * & * & * & -\varepsilon_\sigma I & 0 \\ * & * & * & * & * & * & -\varepsilon_\sigma I \end{bmatrix} < 0,
\tag{18}
$$

$$
1 \leq i \leq j \leq r,
$$

where

$$X = P^{-1}, \qquad Y_{\sigma i} = k_{\sigma i} X,$$

$$Z = XSX,$$

$$L = -2X + \varepsilon_\sigma \left(D_i D_i^T + D_j D_j^T \right), \tag{19}$$

$$U = \Phi_i + \Phi_j + \Gamma_{i0} Y_{\sigma j} + \Gamma_{j0} Y_{\sigma i},$$

$$V = \Gamma_{i1} Y_{\sigma j} + \Gamma_{j1} Y_{\sigma i},$$

then close-loop system (14) *with the guaranteed cost controller* (11) *and the switching law* $\sigma = \sigma(x(k))$ *is globally asymptotically stable. The guaranteed cost function* (9) *satisfies the following bound:*

$$J \le x^T(0) Px(0) + x^T(-1) Sx(-1). \tag{20}$$

Proof. For the networked control system (3), we lead the performance index as follows:

$$J = \sum_{k=0}^{\infty} \left[x^T(k) Qx(k) + u_\sigma^T(k) Ru_\sigma(k) \right]$$

$$= \sum_{k=0}^{\infty} \left[\sum_{i=1}^{r} \sum_{j=1}^{r} \mu_i \mu_j x^T(k) \left(Q + K_{\sigma i}^T R K_{\sigma j} \right) x(k) \right]$$

$$= \sum_{k=0}^{\infty} \left[\sum_{i=1}^{r} \mu_i^2 x^T(k) \left(Q + K_{\sigma i}^T R K_{\sigma i} \right) x(k) \right.$$

$$\left. + 2\sum_{i=1}^{r} \sum_{j>i}^{r} \mu_i \mu_j x^T(k) \left(Q + K_{\sigma i}^T R K_{\sigma j} \right) x(k) \right]$$

$$= \sum_{k=0}^{\infty} \left[\sum_{i=1}^{r} \mu_i^2 x^T(k) \left(Q + K_{\sigma i}^T R K_{\sigma i} \right) x(k) \right.$$

$$+ 2\sum_{i=1}^{r} \sum_{j>i}^{r} \mu_i \mu_j x^T(k)$$

$$\left. \times \left(Q + \frac{\left(K_{\sigma i} + K_{\sigma j} \right)^T}{2} R \frac{\left(K_{\sigma i} + K_{\sigma j} \right)}{2} \right) x(k) \right]. \tag{21}$$

Consider the Lyapunov function as

$$V(x(k)) = x^T(k) Px(k)$$

$$+ x^T(k-1) Sx(k-1). \tag{22}$$

Let

$$\Theta = \begin{bmatrix} G_{\sigma ij}^T P G_{\sigma nm} - P + S & G_{\sigma ij}^T P H_{\sigma nm} \\ H_{\sigma ij}^T P G_{\sigma nm} & H_{\sigma ij}^T P H_{\sigma nm} - S \end{bmatrix},$$

$$G_{\sigma ij} = \Phi_i + \Gamma_{i0} K_{\sigma j} + D_i F_i(\tau_k) E_i K_{\sigma j},$$

$$\alpha_{\sigma ij} = G_{\sigma ij} + G_{\sigma ji}, \tag{23}$$

$$H_{\sigma ij} = \Gamma_{i1} K_{\sigma j} - D_i F_i(\tau_k) E_i K_{\sigma j},$$

$$\beta_{\sigma ij} = H_{\sigma ij} + H_{\sigma ji}.$$

Along any trajectory of the closed-loop system (14), the forward difference of $V(k)$ is

$$\Delta V(x(k)) = V(x(k+1)) - V(x(k))$$

$$= x^T(k+1) Px(k+1) + x^T(k) Sx(k) - x^T(k) Px(k) - x^T(k-1) Sx(k-1)$$

$$= \sum_{\sigma=1}^{N} \sum_{i=1}^{r} \sum_{j=1}^{r} \sum_{n=1}^{r} \sum_{m=1}^{r} \nu_\sigma \mu_i \mu_j \mu_n \mu_m \begin{bmatrix} x(k) \\ x(k-1) \end{bmatrix}^T \Theta \begin{bmatrix} x(k) \\ x(k-1) \end{bmatrix}$$

$$\le \sum_{\sigma=1}^{N} \nu_\sigma \left\{ \frac{1}{4} \sum_{i=1}^{r} \sum_{j=1}^{r} \mu_i \mu_j \begin{bmatrix} x(k) \\ x(k-1) \end{bmatrix}^T \begin{bmatrix} \alpha_{\sigma ij}^T P \alpha_{\sigma ij} - 4P + 4S & \alpha_{\sigma ij}^T P \beta_{\sigma ij} \\ \beta_{\sigma ij}^T P \alpha_{\sigma ij} & \beta_{\sigma ij}^T P \beta_{\sigma ij} - 4S \end{bmatrix} \begin{bmatrix} x(k) \\ x(k-1) \end{bmatrix} \right\} \tag{24}$$

$$= \sum_{\sigma=1}^{N} \nu_\sigma \left\{ \sum_{i=1}^{r} \mu_i^2 \begin{bmatrix} x(k) \\ x(k-1) \end{bmatrix}^T \begin{bmatrix} G_{\sigma ii}^T P G_{\sigma ii} - P + S & G_{\sigma ii}^T P H_{\sigma ii} \\ H_{\sigma ii}^T P G_{\sigma ii} & H_{\sigma ii}^T P H_{\sigma ii} - S \end{bmatrix} \begin{bmatrix} x(k) \\ x(k-1) \end{bmatrix} \right.$$

$$\left. + 2\sum_{i=1}^{r} \sum_{j>i}^{r} \mu_i \mu_j \begin{bmatrix} x(k) \\ x(k-1) \end{bmatrix}^T \begin{bmatrix} \left[\dfrac{\alpha_{\sigma ij}}{2}\right]^T P \dfrac{\alpha_{\sigma ij}}{2} - P + S & \left[\dfrac{\alpha_{\sigma ij}}{2}\right]^T P \dfrac{\beta_{\sigma ij}}{2} \\ \left[\dfrac{\beta_{\sigma ij}}{2}\right]^T P \dfrac{\alpha_{\sigma ij}}{2} & \left[\dfrac{\beta_{\sigma ij}}{2}\right]^T P \dfrac{\beta_{\sigma ij}}{2} - S \end{bmatrix} \begin{bmatrix} x(k) \\ x(k-1) \end{bmatrix} \right\}.$$

Thus

$$
\Delta V\left(x\left(k\right)\right) + x^T\left(k\right) Q x\left(k\right) + u_\sigma^T\left(k\right) R u_\sigma\left(k\right)
$$

$$
\leq \sum_{\sigma=1}^{N} \nu_\sigma \left\{ \sum_{i=1}^{r} \mu_i^2 \begin{bmatrix} x\left(k\right) \\ x\left(k-1\right) \end{bmatrix}^T \right.
$$

$$
\times \begin{bmatrix} G_{\sigma ii}^T P G_{\sigma ii} - P + S & G_{\sigma ii}^T P H_{\sigma ii} \\ H_{\sigma ii}^T P G_{\sigma ii} & H_{\sigma ii}^T P H_{\sigma ii} - S \end{bmatrix} \begin{bmatrix} x\left(k\right) \\ x\left(k-1\right) \end{bmatrix}
$$

$$
+ 2\sum_{i=1}^{r}\sum_{j>i}^{r} \mu_i \mu_j \begin{bmatrix} x\left(k\right) \\ x\left(k-1\right) \end{bmatrix}^T \begin{bmatrix} \left[\dfrac{\alpha_{\sigma ij}}{2}\right]^T P \dfrac{\alpha_{\sigma ij}}{2} - P + S & \left[\dfrac{\alpha_{\sigma ij}}{2}\right]^T P \dfrac{\beta_{\sigma ij}}{2} \\ \left[\dfrac{\beta_{\sigma ij}}{2}\right]^T P \dfrac{\alpha_{\sigma ij}}{2} & \left[\dfrac{\beta_{\sigma ij}}{2}\right]^T P \dfrac{\beta_{\sigma ij}}{2} - S \end{bmatrix} \begin{bmatrix} x\left(k\right) \\ x\left(k-1\right) \end{bmatrix}
$$

$$
+ \sum_{i=1}^{r} \mu_i^2 x^T\left(k\right) \left(Q + K_{\sigma i}^T R K_{\sigma i}\right) x\left(k\right)
$$

$$
+ 2\sum_{i=1}^{r}\sum_{j>i}^{r} \mu_i \mu_j x^T\left(k\right) \left(Q + \frac{\left(K_{\sigma i} + K_{\sigma j}\right)^T}{2} R \frac{\left(K_{\sigma i} + K_{\sigma j}\right)}{2}\right) x\left(k\right) \right\}
\tag{25}
$$

$$
= \sum_{\sigma=1}^{N} \nu_\sigma \left\{ \sum_{i=1}^{r} \mu_i^2 \begin{bmatrix} x\left(k\right) \\ x\left(k-1\right) \end{bmatrix}^T \right.
$$

$$
\times \begin{bmatrix} G_{\sigma ii}^T P G_{\sigma ii} - P + S + Q + K_{\sigma i}^T R K_{\sigma i} & G_{\sigma ii}^T P H_{\sigma ii} \\ H_{\sigma ii}^T P G_{\sigma ii} & H_{\sigma ii}^T P H_{\sigma ii} - S \end{bmatrix} \begin{bmatrix} x\left(k\right) \\ x\left(k-1\right) \end{bmatrix}
$$

$$
+ 2\sum_{i=1}^{r}\sum_{j>i}^{r} \mu_i \mu_j \begin{bmatrix} x\left(k\right) \\ x\left(k-1\right) \end{bmatrix}^T
$$

$$
\times \begin{bmatrix} \left[\dfrac{\alpha_{\sigma ij}}{2}\right]^T P \dfrac{\alpha_{\sigma ij}}{2} - P + S + Q + \dfrac{\left(K_{\sigma i} + K_{\sigma j}\right)^T}{2} R \dfrac{\left(K_{\sigma i} + K_{\sigma j}\right)}{2} & \left[\dfrac{\alpha_{\sigma ij}}{2}\right]^T P \dfrac{\beta_{\sigma ij}}{2} \\ \left[\dfrac{\beta_{\sigma ij}}{2}\right]^T P \dfrac{\alpha_{\sigma ij}}{2} & \left[\dfrac{\beta_{\sigma ij}}{2}\right]^T P \dfrac{\beta_{\sigma ij}}{2} - S \end{bmatrix}
$$

$$
\times \left. \begin{bmatrix} x\left(k\right) \\ x\left(k-1\right) \end{bmatrix} \right\}.
$$

Thus, if the matrix inequalities (26) and (27) hold

$$
\begin{bmatrix} G_{\sigma ii}^T P G_{\sigma ii} - P + S + Q + K_{\sigma i}^T R K_{\sigma i} & G_{\sigma ii}^T P H_{\sigma ii} \\ H_{\sigma ii}^T P G_{\sigma ii} & H_{\sigma ii}^T P H_{\sigma ii} - S \end{bmatrix} < 0, \quad 1 \leq i \leq r,
\tag{26}
$$

$$
\begin{bmatrix} \left[\dfrac{\alpha_{\sigma ij}}{2}\right]^T P \dfrac{\alpha_{\sigma ij}}{2} - P + S + Q + \dfrac{\left(K_{\sigma i} + K_{\sigma j}\right)^T}{2} R \dfrac{\left(K_{\sigma i} + K_{\sigma j}\right)}{2} & \left[\dfrac{\alpha_{\sigma ij}}{2}\right]^T P \dfrac{\beta_{\sigma ij}}{2} \\ \left[\dfrac{\beta_{\sigma ij}}{2}\right]^T P \dfrac{\alpha_{\sigma ij}}{2} & \left[\dfrac{\beta_{\sigma ij}}{2}\right]^T P \dfrac{\beta_{\sigma ij}}{2} - S \end{bmatrix} < 0, \quad 1 \leq i < j \leq r,
\tag{27}
$$

then, the following inequality can hold:

$$\Delta V(x(k)) + x^T(k)Qx(k) + u_\sigma^T(k)Ru_\sigma(k) < 0, \quad (28)$$

and (26) is the special case of (27) when $i = j$. Thus, when (27) holds, the inequality (28) can hold. Consider

$$\theta_{\sigma ij} = \begin{bmatrix} \left[\dfrac{\alpha_{\sigma ij}}{2}\right]^T P \dfrac{\alpha_{\sigma ij}}{2} - P + S + Q + \dfrac{(K_{\sigma i} + K_{\sigma j})^T}{2} R \dfrac{(K_{\sigma i} + K_{\sigma j})}{2} & \left[\dfrac{\alpha_{\sigma ij}}{2}\right]^T P \dfrac{\beta_{\sigma ij}}{2} \\ \left[\dfrac{\beta_{\sigma ij}}{2}\right]^T P \dfrac{\alpha_{\sigma ij}}{2} & \left[\dfrac{\beta_{\sigma ij}}{2}\right]^T P \dfrac{\beta_{\sigma ij}}{2} - S \end{bmatrix} < 0, \quad (29)$$

$$1 \le i \le j \le r.$$

Let

$$\Omega_{\sigma ij} = -4P + 4S + 4Q + (K_{\sigma i} + K_{\sigma j})^T R (K_{\sigma i} + K_{\sigma j}),$$

$$\theta_{\sigma ij} = \begin{bmatrix} \Omega_{\sigma ij} + \alpha_{\sigma ii}^T P \alpha_{\sigma ij} & \alpha_{\sigma ii}^T P \beta_{\sigma ij} \\ \beta_{\sigma ii}^T P \alpha_{\sigma ij} & \beta_{\sigma ii}^T P \beta_{\sigma ij} - 4S \end{bmatrix} < 0,$$

$$1 \le i \le j \le r. \quad (30)$$

Therefore, according to the fact of $\mu_i(\theta(k)) \ge 0$ in (8), the inequality (30) and the equality (24), we can obtain $\Delta V < 0$.

Define sets $\Omega_\sigma = \{y \in R^{2n} : y^T \theta_{\sigma ij} y < 0\}$; thus $\cup \Omega_i = R^{2n} \setminus \{0\}$. Construct the sets $\widetilde{\Omega}_1 = \Omega_1, \ldots, \widetilde{\Omega}_\sigma = \Omega_\sigma - \bigcup_{i=1}^{\sigma-1} \widetilde{\Omega}_i, \ldots$.

Obviously, we have $\bigcup_{i=1}^l \widetilde{\Omega}_i = R^{2n} \setminus \{0\}, \widetilde{\Omega}_i \cap \widetilde{\Omega}_j = \Phi, i \ne j$. Construct a switching law as follows:

$$\sigma(x(k)) = i, \quad \text{when } x(k) \in \widetilde{\Omega}_i, \ \sigma \in M. \quad (31)$$

Thus, $\Delta V(x(k)) + x^T(k)Qx(k) + u_\sigma^T(k)Ru_\sigma(k) \le \xi^T(k)\theta_{\sigma ij}\xi(k) < 0, \forall \xi(k) \ne 0$, where $\xi(k) = [x^T(k) \ x^T(k-1)]^T$.

Notice that $F^T F \le I$. Applying Lemmas 6 and 7 to the inequality (29), we have

$$\begin{bmatrix} \Psi & \Lambda & \Pi & 0 & 0 & 0 & 0 \\ * & -2P+2S & 0 & 2I & (k_{\sigma i}+k_{\sigma j})^T & (E_i k_{\sigma j})^T & (E_j k_{\sigma i})^T \\ * & * & -2S & 0 & 0 & -(E_i k_{\sigma j})^T & (E_j k_{\sigma i})^T \\ * & * & * & -2Q^{-1} & 0 & 0 & 0 \\ * & * & * & * & -2R^{-1} & 0 & 0 \\ * & * & * & * & * & -\varepsilon_\sigma I & 0 \\ * & * & * & * & * & * & -\varepsilon_\sigma I \end{bmatrix} < 0, \quad (32)$$

where

$$\Psi = -2P^{-1} + \varepsilon_\sigma \left(D_i D_i^T + D_j D_j^T\right),$$

$$\Lambda = \Phi_i + \Phi_j + \Gamma_{i0} k_{\sigma j} + \Gamma_{j0} k_{\sigma i}, \quad (33)$$

$$\Pi = \Gamma_{i1} k_{\sigma j} + \Gamma_{j1} k_{\sigma i}.$$

By inequality (32) left-multiplied and right-multiplied by $\mathrm{diag}(I \ P^{-1} \ P^{-1} \ I \ I \ I \ I)$ and defining new variables $X = P^{-1}, Y_{\sigma i} = k_{\sigma i} X$, and $Z = XSX$, we have

$$\begin{bmatrix} L & U & V & 0 & 0 & 0 & 0 \\ * & -2X+2Z & 0 & 2X & (Y_{\sigma i}+Y_{\sigma j})^T & (E_i Y_{\sigma j})^T & (E_j Y_{\sigma i})^T \\ * & * & -2Z & 0 & 0 & -(E_i Y_{\sigma j})^T & (E_j Y_{\sigma i})^T \\ * & * & * & -2Q^{-1} & 0 & 0 & 0 \\ * & * & * & * & -2R^{-1} & 0 & 0 \\ * & * & * & * & * & -\varepsilon_\sigma I & 0 \\ * & * & * & * & * & * & -\varepsilon_\sigma I \end{bmatrix} < 0, \quad (34)$$

where

$$L = -2X + \varepsilon_\sigma \left(D_i D_i^T + D_j D_j^T \right),$$

$$U = \Phi_i + \Phi_j + \Gamma_{i0} Y_{\sigma j} + \Gamma_{j0} Y_{\sigma i},$$

$$V = \Gamma_{i1} Y_{\sigma j} + \Gamma_{j1} Y_{\sigma i}.$$

(35)

Taking (28) into account, for all admissible uncertainties, we can infer that as follows:

$$\Delta V (x (k)) \le -x^T (k) Q x (k) - u_\sigma^T (k) R u_\sigma (k).$$

(36)

According to (18), we can infer that there exists at least one σ such that (29) is established. Therefore, the closed-loop system (14) is asymptotically stable under the controller (11) and the switching law (31). Moreover, we have

$$\Delta V < \sum_{i=1}^{r} \mu_i^2 x^T (k) \left(Q + K_{\sigma i}^T R K_{\sigma i} \right) x (k)$$

$$+ 2 \sum_{i=1}^{r} \sum_{j>i}^{r} \mu_i \mu_j x^T (k)$$

$$\times \left(Q + \frac{(K_{\sigma i} + K_{\sigma j})^T}{2} R \frac{(K_{\sigma i} + K_{\sigma j})}{2} \right) x (k),$$

(37)

thus

$$\sum_{i=1}^{r} \mu_i^2 x^T (k) \left(Q + K_{\sigma i}^T R K_{\sigma i} \right) x (k)$$

$$+ 2 \sum_{i=1}^{r} \sum_{j>i}^{r} \mu_i \mu_j x^T (k)$$

$$\times \left(Q + \frac{(K_{\sigma i} + K_{\sigma j})^T}{2} R \frac{(K_{\sigma i} + K_{\sigma j})}{2} \right) x (k) < -\Delta V.$$

(38)

The inequalities (38) are added up together in the case that k is $0, 1, 2, \ldots, \infty$; we have

$$J = \sum_{k=0}^{\infty} \left[\sum_{i=1}^{r} \mu_i^2 x^T (k) \left(Q + K_{\sigma i}^T R K_{\sigma i} \right) x (k) \right.$$

$$+ 2 \sum_{i=1}^{r} \sum_{j>i}^{r} \mu_i \mu_j x^T (k)$$

$$\left. \times \left(Q + \frac{(K_{\sigma i} + K_{\sigma j})^T}{2} R \frac{(K_{\sigma i} + K_{\sigma j})}{2} \right) x (k) \right]$$

$$\le x^T (0) P x (0) + x^T (-1) S x (-1).$$

(39)

Therefore, Theorem 8 is proved.   □

Theorem 9. *Consider the uncertain nonlinear networked control systems (3) and the cost function (9). If there exist some nonpositive or nonnegative constants $\delta_{\sigma\lambda}$ ($\sigma, \lambda = 1, 2, \ldots, l$), a group of positive constants $\varepsilon_\sigma > 0$, and positive-definite matrices X_σ, Z_σ, matrix $Y_{\sigma i}$, $\sigma = 1, \ldots, l$, $i = 1, \ldots, r$, such that the following matrix inequalities:*

$$\begin{bmatrix} L & U & V & 0 & 0 & 0 & 0 \\ * & \Xi + \sum_{\lambda=1, \lambda \ne \sigma}^{l} \delta_{\sigma\lambda} (X_\lambda - X_\sigma) & 0 & 2X & (Y_{\sigma i} + Y_{\sigma j})^T & (E_i Y_{\sigma j})^T & (E_j Y_{\sigma i})^T \\ * & * & -2Z & 0 & 0 & -(E_i Y_{\sigma j})^T & -(E_j Y_{\sigma i})^T \\ * & * & * & -2Q^{-1} & 0 & 0 & 0 \\ * & * & * & * & -2R^{-1} & 0 & 0 \\ * & * & * & * & * & -\varepsilon_\sigma I & 0 \\ * & * & * & * & * & * & -\varepsilon_\sigma I \end{bmatrix} < 0, \quad 1 \le i \le j \le r,$$

(40)

hold, where

$$\Xi = -2X_\sigma + 2Z_\sigma, \qquad X_\sigma = P_\sigma^{-1},$$

$$Y_{\sigma i} = k_{\sigma i} X_\sigma, \qquad Z_\sigma = X_\sigma S_\sigma X_\sigma,$$

$$L = -2X_\sigma + \varepsilon_\sigma \left(D_i D_i^T + D_j D_j^T \right),$$

$$U = \Phi_i + \Phi_j + \Gamma_{i0} Y_{\sigma j} + \Gamma_{j0} Y_{\sigma i},$$

$$V = \Gamma_{i1} Y_{\sigma j} + \Gamma_{j1} Y_{\sigma i},$$

(41)

then system (14) with the guaranteed cost controller (11) and the switching law $\sigma = \sigma(x(k))$ is globally asymptotically stable. The guaranteed cost function (9) satisfies the following bound:

$$J \le x^T (0) P_\sigma x (0) + x^T (-1) S_\sigma x (-1).$$

(42)

Proof. By the Lemmas 6 and 7, and $X_\sigma = P_\sigma^{-1}$, $Y_{\sigma i} = k_{\sigma i} X_\sigma$, $Z_\sigma = X_\sigma S_\sigma X_\sigma$, from (40), we have

$$
\begin{bmatrix} \left[\dfrac{\alpha_{\sigma ij}}{2}\right]^T P_\sigma \dfrac{\alpha_{\sigma ij}}{2} - P_\sigma + S_\sigma + Q + \dfrac{\left(K_{\sigma i} + K_{\sigma j}\right)^T}{2} R \dfrac{\left(K_{\sigma i} + K_{\sigma j}\right)}{2} + \displaystyle\sum_{\lambda=1, \lambda \neq \sigma}^{l} \delta_{\sigma\lambda}\left(P_\lambda - P_\sigma\right) & \left[\dfrac{\alpha_{\sigma ij}}{2}\right]^T P_\sigma \dfrac{\beta_{\sigma ij}}{2} \\[2ex] \left[\dfrac{\beta_{\sigma ij}}{2}\right]^T P_\sigma \dfrac{\alpha_{\sigma ij}}{2} & \left[\dfrac{\beta_{\sigma ij}}{2}\right]^T P_\sigma \dfrac{\beta_{\sigma ij}}{2} - S_\sigma \end{bmatrix} < 0.
\tag{43}
$$

Without loss of generality, we assume that $\delta_{\sigma\lambda} \geq 0$.

Obviously, there exists at least one $\sigma \in M$ when $x(k) \in R^n \setminus \{0\}$, such that $x^T(k)(P_\lambda - P_\sigma)x(k) \geq 0, \forall \lambda \in M$.

Let $\Omega_\sigma = \{x(k) \in R^n \mid x^T(k)(P_\lambda - P_\sigma)x(k) \geq 0, \forall \lambda \in M, \lambda \neq \sigma, x(k) \neq 0\}$; thus $\cup \Omega_i = R^n \setminus \{0\}$.

Construct the sets $\widetilde{\Omega}_1 = \Omega_1, \ldots, \widetilde{\Omega}_\sigma = \Omega_\sigma - \bigcup_{i=1}^{\sigma-1} \widetilde{\Omega}_i, \ldots$.

Obviously, we have $\bigcup_{i=1}^{l} \widetilde{\Omega}_i = R^n \setminus \{0\}, \widetilde{\Omega}_i \cap \widetilde{\Omega}_j = \Phi, i \neq j$. Construct a switched law by

$$
\sigma(x(k)) = i, \quad \text{when } x(k) \in \widetilde{\Omega}_i, \ \sigma \in M. \tag{44}
$$

Thus, from (43), we have

$$
\begin{bmatrix} x(k) \\ x(k-1) \end{bmatrix}^T \begin{bmatrix} \left[\dfrac{\alpha_{\sigma ij}}{2}\right]^T P_\sigma \dfrac{\alpha_{\sigma ij}}{2} - P_\sigma + S_\sigma + Q + \dfrac{\left(K_{\sigma i} + K_{\sigma j}\right)^T}{2} R \dfrac{\left(K_{\sigma i} + K_{\sigma j}\right)}{2} & \left[\dfrac{\alpha_{\sigma ij}}{2}\right]^T P_\sigma \dfrac{\beta_{\sigma ij}}{2} \\[2ex] \left[\dfrac{\beta_{\sigma ij}}{2}\right]^T P_\sigma \dfrac{\alpha_{\sigma ij}}{2} & \left[\dfrac{\beta_{\sigma ij}}{2}\right]^T P_\sigma \dfrac{\beta_{\sigma ij}}{2} - S \end{bmatrix}
$$
$$
\begin{bmatrix} x(k) \\ x(k-1) \end{bmatrix} < 0, \quad 1 \leq i \leq j \leq r. \tag{45}
$$

Following the similar lines as in the proof of Theorem 8, we have

$$
\theta_{\sigma ij} = \begin{bmatrix} \left[\dfrac{\alpha_{\sigma ij}}{2}\right]^T P_\sigma \dfrac{\alpha_{\sigma ij}}{2} - P_\sigma + S_\sigma + Q + \dfrac{\left(K_{\sigma i} + K_{\sigma j}\right)^T}{2} R \dfrac{\left(K_{\sigma i} + K_{\sigma j}\right)}{2} & \left[\dfrac{\alpha_{\sigma ij}}{2}\right]^T P_\sigma \dfrac{\beta_{\sigma ij}}{2} \\[2ex] \left[\dfrac{\beta_{\sigma ij}}{2}\right]^T P_\sigma \dfrac{\alpha_{\sigma ij}}{2} & \left[\dfrac{\beta_{\sigma ij}}{2}\right]^T P_\sigma \dfrac{\beta_{\sigma ij}}{2} - S \end{bmatrix} < 0,
$$
$$
1 \leq i \leq j \leq r, \tag{46}
$$

Theorem 9 is proved. □

then, $\Delta V_\sigma(x(k)) + x^T(k)Qx(k) + u_\sigma^T(k)Ru_\sigma(k) < 0$. Thus,

$$
\sum_{i=1}^{r} \mu_i^2 x^T(k)\left(Q + K_{\sigma i}^T R K_{\sigma i}\right) x(k)
$$
$$
+ 2\sum_{i=1}^{r}\sum_{j>i}^{r} \mu_i \mu_j x^T(k)\left(Q + \dfrac{\left(K_{\sigma i} + K_{\sigma j}\right)^T}{2} R \dfrac{\left(K_{\sigma i} + K_{\sigma j}\right)}{2}\right) x(k)
$$
$$
< -\Delta V_\sigma. \tag{47}
$$

The above inequality (47) is added up together in the case that k is $0, 1, 2, \ldots, \infty$, according to the stability of the system, then

$$
J \leq x^T(0) P_\sigma x(0) + x^T(-1) S_\sigma x(-1). \tag{48}
$$

5. Simulation Example

Consider the nonlinear system with the following differential equation [24]:

$$
\ddot{s}(t) + f(s(t), \dot{s}(t)) - 0.1 s(t) = F(t), \tag{49}
$$

where $f(s(t), \dot{s}(t)) = 0.5 s(t) + 0.75 \sin(\dot{s}(t)/0.5)$.

Choose the state variable and the input variable as $x(t) = [s(t), \dot{s}(t)]^T$, $u(t) = F(t)$, respectively. It can be represented by the following fuzzy model consisting of two rules:

$$R^1: \text{ IF } x_2(t) \text{ is } M_1,$$

$$\text{THEN } \dot{x}(t) = A_1 x(t) + B_1 u(t),$$

$$R^2: \text{ IF } x_2(t) \text{ is } M_2,$$

$$\text{THEN } \dot{x}(t) = A_2 x(t) + B_2 u(t),$$

where $x(t) = [x_1(t), x_2(t)]^T$

$$A_1 = \begin{bmatrix} 0 & 1 \\ 0.1 & -2 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} 0 & 1 \\ 0.1 & -0.5 - 1.5\beta \end{bmatrix},$$

$$B_1 = B_2 = [-0.1 \ -0.2]^T, \qquad \beta = \frac{0.01}{\pi}, \tag{51}$$

and β is used to avoid system matrices being singular.

The sampling period $T = 0.3$ s, then the discrete model of the system is as follows:

$$R^1: \text{ IF } \frac{x_2(t)}{0.5} \text{ is about } 0,$$

$$\text{THEN } x(k+1) = \Phi_1 x(k) + (\Gamma_{10} + D_1 F_1(\tau_k) E_1) u(k)$$
$$+ (\Gamma_{11} - D_1 F_1(\tau_k) E_1) u(k-1),$$

$$R^2: \text{ IF } \frac{x_2(t)}{0.5} \text{ is about } \pi \text{ or } -\pi,$$

$$\text{THEN } x(k+1) = \Phi_2 x(k) + (\Gamma_{20} + D_2 F_2(\tau_k) E_2) u(k)$$
$$+ (\Gamma_{21} - D_2 F_2(\tau_k) E_2) u(k-1), \tag{52}$$

where

$$\Phi_1 = \begin{bmatrix} 1.0037 & 0.2259 \\ 0.0226 & 0.5519 \end{bmatrix}, \qquad \Phi_2 = \begin{bmatrix} 1.0043 & 0.2788 \\ 0.0279 & 0.8635 \end{bmatrix},$$

$$\Gamma_{10} = \begin{bmatrix} -10.0018 \\ -0.0001 \end{bmatrix}, \qquad \Gamma_{11} = \begin{bmatrix} -10.0390 \\ -0.2260 \end{bmatrix},$$

$$\Gamma_{20} = \begin{bmatrix} -10 \\ 0 \end{bmatrix}, \qquad \Gamma_{21} = \begin{bmatrix} -10.0432 \\ -0.2788 \end{bmatrix},$$

$$D_1 = \begin{bmatrix} 20.7693 & 0.2141 \\ 1.0137 & -0.4386 \end{bmatrix},$$

$$D_2 = \begin{bmatrix} 6.8691 & 0.6676 \\ 0.3353 & -1.3678 \end{bmatrix},$$

$$E_1 = \begin{bmatrix} 0.4773 \\ 1.0869 \end{bmatrix}, \qquad E_2 = \begin{bmatrix} 1.2500 \\ 1.4786 \end{bmatrix},$$

$$F_1(\tau_k) = \begin{bmatrix} e^{-0.0488\tau_k} & 0 \\ 0 & e^{-2.0488(0.3-\tau_k)} \end{bmatrix},$$

$$F_2(\tau_k) = \begin{bmatrix} e^{-0.1522\tau_k} & 0 \\ 0 & e^{-0.6570(0.3-\tau_k)} \end{bmatrix}. \tag{53}$$
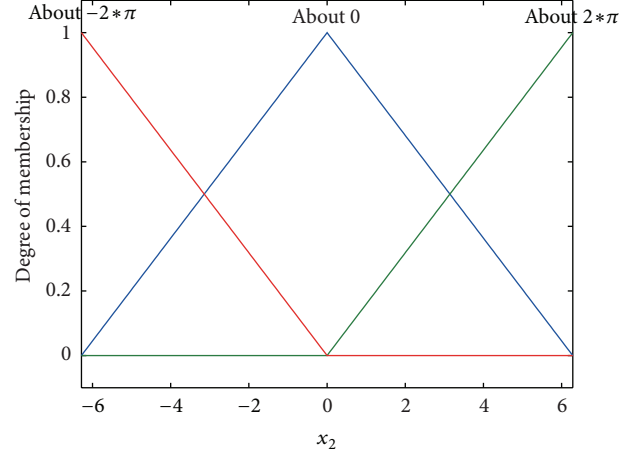

FIGURE 2: The membership function of input x_2.

Obviously, $F_1(\tau_k)$ and $F_2(\tau_k)$ satisfy the uncertain matching conditions:

$$F_1^T(\tau_k) F_1(\tau_k) \leq I, \qquad F_2^T(\tau_k) F_2(\tau_k) \leq I. \tag{54}$$

The membership functions of "about 0" and "about π or $-\pi$" are selected as in Figure 2.

Suppose the switched fuzzy feed-back controllers are the following fuzzy controllers:

$$u_1(k) = \sum_{i=1}^{2} \mu_i(x_2(k)) k_{1i} x_i(k),$$

$$u_2(k) = \sum_{i=1}^{2} \mu_i(x_2(k)) k_{2i} x_i(k). \tag{55}$$

Choose $Q = 0.1 I_{2 \times 2}$, $R = 1$. Carrying out computations for matrices inequality (40), we obtain

$$P_1 = \begin{bmatrix} 8.0177 & 5.2362 \\ 5.2362 & 8.5369 \end{bmatrix},$$

$$P_2 = \begin{bmatrix} 7.8926 & 5.4026 \\ 5.4026 & 9.1487 \end{bmatrix}. \tag{56}$$

The controller gain:

$$k_{11} = [-0.6138 \ -0.4204],$$

$$k_{12} = [-0.2936, \ -0.3036],$$

$$k_{21} = [-0.4156 \ -0.3852],$$

$$k_{22} = [-0.0893, \ -0.1025]. \tag{57}$$

Let

$$\Omega_1 = \left\{ x(k) \in R^2 \mid x^T(k)(P_2 - P_1) x(k) \geq 0, x(k) \neq 0 \right\},$$

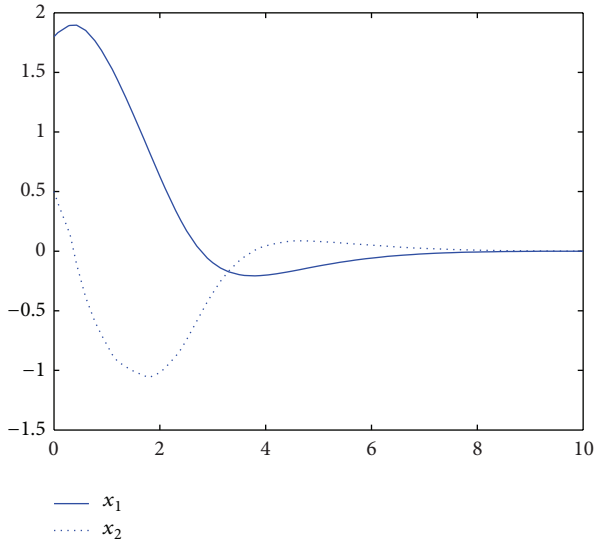$$\Omega_2 = \left\{ x(k) \in R^2 \mid x^T(k)(P_1 - P_2) x(k) \geq 0, x(k) \neq 0 \right\}. \tag{58}$$

FIGURE 3: The state trajectory using fuzzy controller 1.
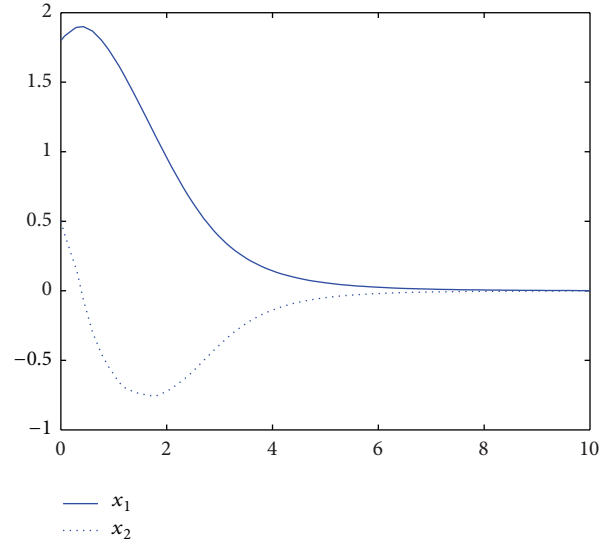

FIGURE 4: The state trajectory using fuzzy controller 2.

Then $\Omega_1 \cup \Omega_2 = R^2 \setminus \{0\}$. We design a switched law as follows:

$$\sigma(x(k)) = \begin{cases} 1, & x(k) \in \Omega_1, \\ 2, & x(k) \in \Omega_2 \setminus \Omega_1. \end{cases} \quad (59)$$

The initial condition is $[1.8 \ 0.5]^T$, guaranteed cost bound $J^* = 36.53$.

Figures 3 and 4 show the system state trajectories that use fuzzy controller 1 and fuzzy controller 2, respectively. Figure 5 shows the simulating results for the proposed switched fuzzy controller method. In Figure 5, the system state trajectories indicate that nonlinear networked control system is asymptotically stable and satisfies the performance index via the designed guaranteed cost controller and the switching law. From the simulating results, we can confirm that the guaranteed cost controller in the switched fuzzy model is able to stabilize the nonlinear delay system via switching T-S fuzzy method. In addition, the performance of switching fuzzy controller is better than that of the fuzzy controllers.

6. Conclusions

In this paper, we have presented a novel controller design methodology for a class of nonlinear NCSs based on switched T-S fuzzy model. By introducing the switching mechanism into the fuzzy T-S systems, the proposed methods can deal with the uncertainties of nonlinear NCSs with time delays and furthermore avoid the inherent drawback of a fuzzy T-S model in controller design and implementation of nonlinear systems. In addition, considering QoS of nonlinear NCSs, some sufficient conditions for the existence of the robust guaranteed cost control law have been built via Lyapunov functional approach. Simulation results have verified and confirmed the effectiveness of the guaranteed cost controller based on the switched T-S fuzzy model for nonlinear NCSs.

At present, this paper only presents a numerical example to show the validity of our control scheme on the nonlinear
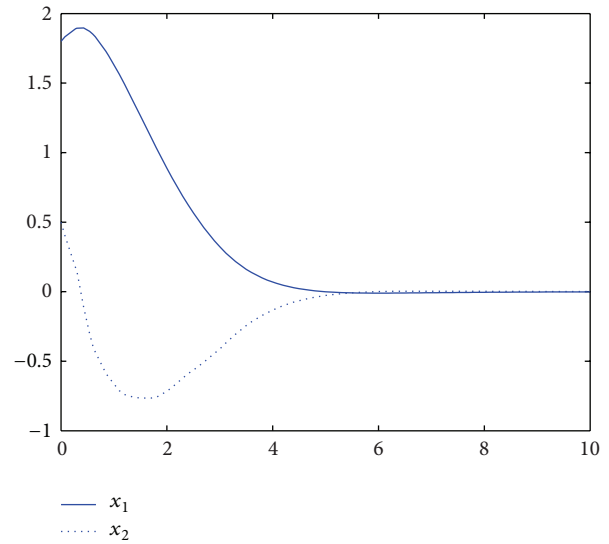

FIGURE 5: The state trajectory using switched fuzzy controller.

NCS with time delays. In next step, we plan to further verify this control scheme via practical NCSs and investigate the stability analysis and controller design with multiple-packet transmission in nonlinear NCSs. Moreover, the boundedness of the parameter constraints for NCSs will be studied. The switched dynamics of nonlinear NCSs will also be considered in future investigation.
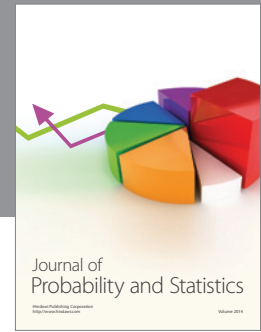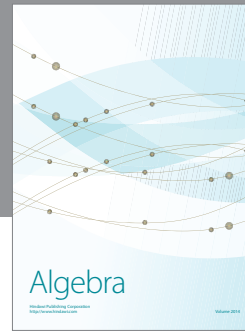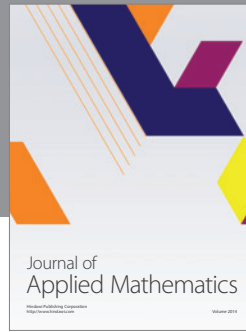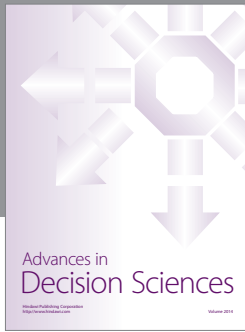
Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgments

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