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Research Article

Robust Fuzzy Control for Fractional-Order Uncertain Hydroturbine Regulating System with Random Disturbances

Fengjiao Wu,[1] Guitao Zhang,[2] and Zhengzhong Wang[1]

[1] *Department of Electrical Engineering, Northwest A&F University, Shaanxi, Yangling 712100, China*
[2] *School of Electrical Engineering, Xi'an Jiaotong University, Shaanxi, Xi'an 710049, China*

Correspondence should be addressed to Zhengzhong Wang; wangzz0910@163.com

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The robust fuzzy control for fractional-order hydroturbine regulating system is studied in this paper. First, the more practical fractional-order hydroturbine regulating system with uncertain parameters and random disturbances is presented. Then, on the basis of interval matrix theory and fractional-order stability theorem, a fuzzy control method is proposed for fractional-order hydroturbine regulating system, and the stability condition is expressed as a group of linear matrix inequalities. Furthermore, the proposed method has good robustness which can process external random disturbances and uncertain parameters. Finally, the validity and superiority are proved by the numerical simulations.

1. Introduction

Due to the advantages of describing actual projects, especially for the description of memory and hereditary properties of numerous materials and processes [1, 2], fractional calculus has attracted more and more people's attention. It has been verified that many practical systems could be elegantly expressed with fractional calculus, such as power system [3], permanent magnet synchronous motor system [4], mechanical system [5], and chemical processing system [6].

The hydroturbine regulating system is a core component for safe and stable operation of hydropower station system. For a long time up to now, the integer-order model is usually adopted [7–9]. However, as we all know, the hydroturbine regulating system is integration of hydraulic, mechanical, and electrical parts. This complex composition makes it a strong coupling, nonlinear, and nonminimum phase system [10–12]. So the integer-order model may not be suitable for perfectly describing the hydroturbine regulating system. Besides, due to the memory character and history-dependence of hydraulic servo system, the more practical fractional-order model is considered accordingly in this study. Many studies have shown that the hydroturbine regulating system may exhibit irregular nonlinear vibrations when the system is under parameter variations and external random disturbances [13–15]. Therefore, it is very important to design nonlinear controller for the stable operation of fractional-order hydroturbine regulating system. Until now, some nonlinear control schemes have been proposed for fractional-order systems, such as sliding mode control [16], predictive control [17], adaptive control [18], and pinning control [19]. However, few of the above-mentioned methods consider the uncertainty and random disturbances.

Fuzzy control is a robust method, which can deal with external disturbances [20–22]. Besides, with the help of fuzzy linearization and linear matrix theory, the uncertain parameters can be well processed [23–25]. There have been many results applying fuzzy control to integer-order nonlinear systems [26–29]. However, there is little literature about fractional-order nonlinear fuzzy control and there is almost no relevant result for fractional-order hydroturbine regulating system.

According to the above analysis, some advantages are shown in this study. Firstly, the fractional-order hydroturbine regulating system with uncertain parameters and random disturbances is presented, which is more in accordance with practical engineering. Secondly, a fuzzy control method is proposed for fractional-order hydroturbine regulating system, and the stability condition is expressed as a group of linear matrix inequalities (LMIs). Furthermore, the proposed method has good robustness which can process external random disturbances and uncertain parameters. Even if the system is with six uncertain parameters, the controller designed for fractional-order hydroturbine regulating system is still valid. Lastly, numerical simulations have demonstrated the effectiveness and superiority when compared with traditional PID control method.

The rest of this paper is organized as follows. In Section 2, the fractional-order hydroturbine regulating system is introduced. Section 3 presents the robust fuzzy controller design for fractional-order hydroturbine regulating system. Simulations are shown in Section 4. And the paper is ended in Section 5.

2. The Fractional-Order Hydroturbine Regulating System

The integer-order hydroturbine regulation system is given as [30]

$$\begin{aligned}
\frac{d\delta}{dt} &= \omega_0 \omega, \\
\frac{d\omega}{dt} &= \frac{1}{T_{ab}} \left(m_t - D\omega - \frac{E'_q V_s}{x'_{d\Sigma}} \sin\delta \right. \\
&\quad \left. - \frac{V_s^2}{2} \frac{x'_{d\Sigma} - x_{q\Sigma}}{x'_{d\Sigma} x_{q\Sigma}} \sin 2\delta \right), \\
\frac{dm_t}{dt} &= \frac{1}{e_{qh} T_w} \left(-m_t + e_y y + \frac{e e_y T_w}{T_y} y \right), \\
\frac{dy}{dt} &= -\frac{1}{T_y} y,
\end{aligned} \tag{1}$$

where δ, ω, m_t, and y are the rotor angle deviation of the generator, the relative deviation of the rotational speed of the generator, the hydroturbine output incremental torque deviation, and the incremental deviation of the guide vane opening, respectively.

The hydraulic servo system has significant historical reliance. Since it is a powerful advantage for fractional calculus to describe the function which has significant historical reliance, the fractional-order hydraulic servo system is adopted.

According to fractional calculus, the fractional-order hydraulic servo system is obtained as

$$D^{\alpha} y = \frac{1}{T_y} (u - y), \tag{2}$$

where T_y is the major relay connector response time.

According to (1) and (2) and for convenience, we use x_1, x_2, x_3, x_4 to replace δ, ω, m_t, y. Then, the fractional-order mathematical model of hydroturbine regulation system could be represented as

$$\begin{aligned}
\frac{dx_1}{dt} &= \omega_0 x_2, \\
\frac{dx_2}{dt} &= \frac{1}{T_{ab}} \left(x_3 - D x_2 - \frac{E'_q V_s}{x'_{d\Sigma}} \sin x_1 \right. \\
&\quad \left. - \frac{V_s^2}{2} \frac{x'_{d\Sigma} - x_{q\Sigma}}{x'_{d\Sigma} x_{q\Sigma}} \sin 2x_1 \right), \\
\frac{dx_3}{dt} &= \frac{1}{e_{qh} T_w} \left(-x_3 + e_y x_4 + \frac{e e_y T_w}{T_y} x_4 \right), \\
\frac{d^{\alpha} x_4}{dt^{\alpha}} &= -\frac{1}{T_y} x_4,
\end{aligned} \tag{3}$$

where $\omega_0 = 314$, $T_{ab} = 9.0$ s, $D = 2.0$, $E'_q = 1.35$, $x'_{d\Sigma} = 1.15$, $x_{q\Sigma} = 1.474$, $T_w = 0.8$ s, $T_y = 0.1$ s, $V_s = 1.0$, $e_{qh} = 0.5$, $e_y = 1.0$, $e = 0.7$, and $\alpha = 0.9$.

Considering the universality of disturbances, the fractional-order hydroturbine regulating system could be represented as

$$\begin{aligned}
\frac{dx_1}{dt} &= \omega_0 x_2 + \text{rand}(1) \times x_1, \\
\frac{dx_2}{dt} &= \frac{1}{T_{ab}} \left(x_3 - D x_2 - \frac{E'_q V_s}{x'_{d\Sigma}} \sin x_1 \right. \\
&\quad \left. - \frac{V_s^2}{2} \frac{x'_{d\Sigma} - x_{q\Sigma}}{x'_{d\Sigma} x_{q\Sigma}} \sin 2x_1 \right) + \text{rand}(1) \times x_2, \\
\frac{dx_3}{dt} &= \frac{1}{e_{qh} T_w} \left(-x_3 + e_y x_4 + \frac{e e_y T_w}{T_y} x_4 \right) + \text{rand}(1) \\
&\quad \times x_3, \\
\frac{d^{\alpha} x_4}{dt^{\alpha}} &= -\frac{1}{T_y} x_4 + \text{rand}(1) \times x_4.
\end{aligned} \tag{4}$$

Figure 1 shows the state trajectories of fractional-order hydroturbine regulating system (4) with initial value $[x_1 \ \ x_2 \ \ x_3 \ \ x_4]^T = [0.01 \ \ 0.01 \ \ 0.01 \ \ 0.01]^T$. It clearly shows that the system states are in irregular and unstable nonlinear vibrations. So the effective controller should be designed for the vibration inhibition and stable operation of the fractional-order hydroturbine regulating system.

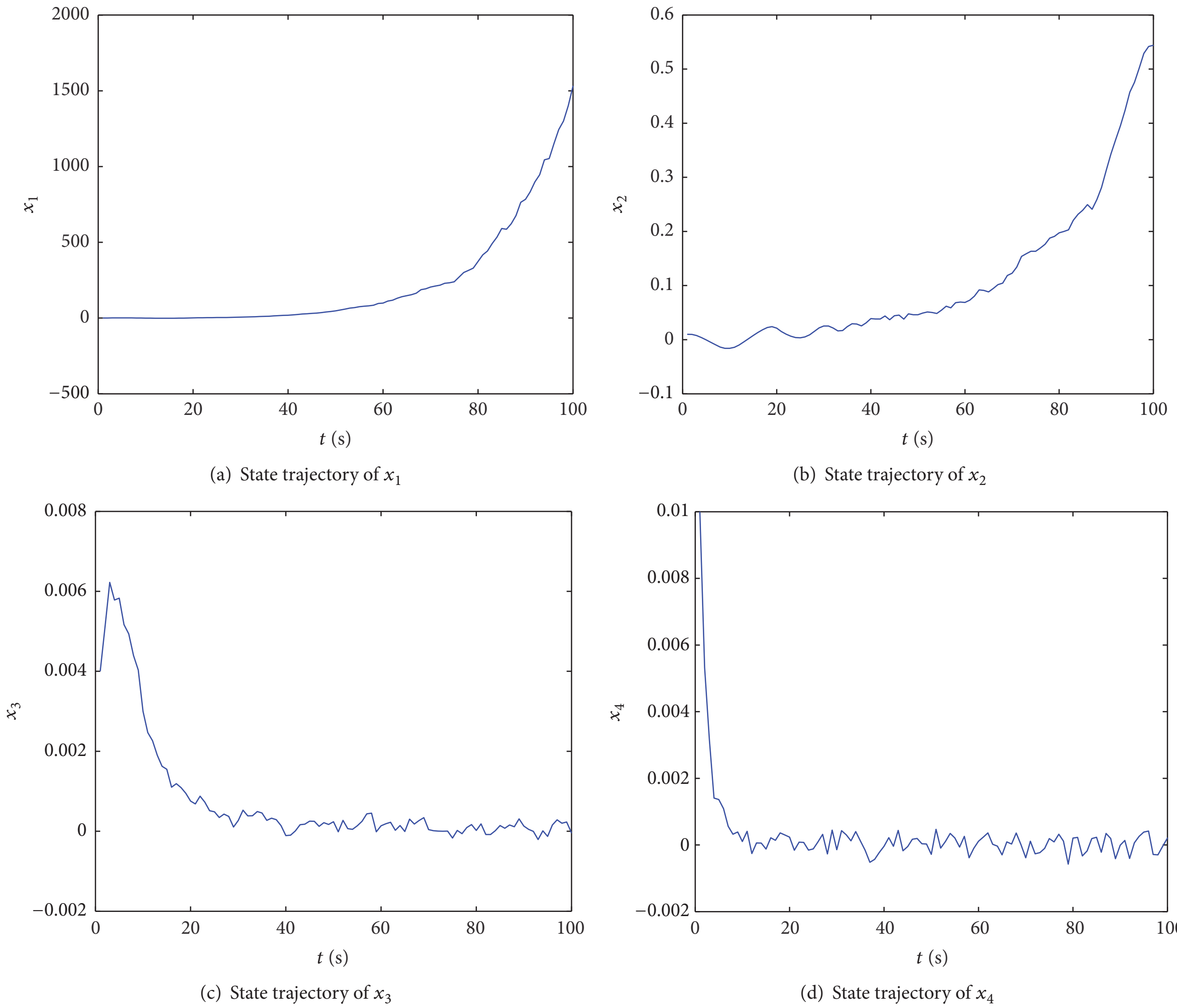


Figure 1: State trajectories of fractional-order hydroturbine regulating system (4).

3. Robust Fuzzy Controller Design

3.1. Interval Matrix Theory. When the uncertain parameters are considered, the fractional-order hydroturbine regulating system could be rewritten as

$$\begin{aligned}\frac{dx_1}{dt} &= \tilde{a}x_2,\\ \frac{dx_2}{dt} &= \tilde{b}x_3 - \tilde{c}x_2 - \frac{1}{T_{ab}} \times \frac{E'_q V_s}{x'_{d\Sigma}} \sin x_1 - \frac{1}{T_{ab}}\\ &\quad \times \frac{V_s^2}{2} \frac{x'_{d\Sigma} - x_{q\Sigma}}{x'_{d\Sigma} x_{q\Sigma}} \sin 2x_1,\\ \frac{dx_3}{dt} &= -\tilde{d}x_3 + \tilde{e}x_4,\\ \frac{d^\alpha x_4}{dt^\alpha} &= \tilde{f}x_4,\end{aligned} \tag{5}$$

where $\tilde{a} \in [a_1, a_2]$, $\tilde{b} \in [b_1, b_2]$, $\tilde{c} \in [c_1, c_2]$, $\tilde{d} \in [d_1, d_2]$, $\tilde{e} \in [e_1, e_2]$, and $\tilde{f} \in [f_1, f_2]$.

To discuss the parameter uncertainties of the coefficient matrix of the fractional-order hydroturbine regulating system (5), the following interval matrix theory is introduced.

The linear fractional-order interval system is given as [31]

$$\frac{d^\alpha x}{dt^\alpha} = \tilde{A}x(t), \tag{6}$$

where the interval uncertain matrix $\tilde{A}$ satisfies

$\tilde{A} \in N[A^l, A^u] = \{\tilde{A} \in R^{n\times n} \mid a_{ij}^l \le a_{ij} \le a_{ij}^u,\ i, j = 1, \dots, n\}$, where A^l and A^u are the lower and upper bounds of matrix $\tilde{A}$, respectively.

The matrix $\tilde{A}$ could be equivalent presented as

$$\tilde{A} = A_{0i} + E\Sigma F, \tag{7}$$

where

$$A_{0i}=\frac{1}{2}\left(A_i^l+A_i^u\right),$$
$$\Sigma\in\Sigma^*=\left\{\Sigma_i\in R^{n\times n}\mid\Sigma_i=\operatorname{diag}\left(\varepsilon_{11},\ldots,\varepsilon_{1n},\ldots,\varepsilon_{n1},\ldots,\varepsilon_{nn}\right),\ \left|\varepsilon_{ij}\right|\le 1,\ i,j=1,\ldots,n\right\},$$
$$E=\left(\sqrt{h_{11}}e_1,\ldots,\sqrt{h_{1n}}e_1,\ldots,\sqrt{h_{n1}}e_n,\ldots,\sqrt{h_{nn}}e_n\right),$$
$$F=\left(\sqrt{h_{11}}e_1,\ldots,\sqrt{h_{1n}}e_n,\ldots,\sqrt{h_{n1}}e_1,\ldots,\sqrt{h_{nn}}e_n\right)^T, \quad (8)$$

where $H=(h_{ij})_{n\times n}=H_i=(1/2)(A_i^u-A_i^l)$, e_i $(i=1,\ldots,n)$ is the $n\times n$ identity matrix ith column.

Note that, for any i and $\Sigma\in\Sigma^*$, there is

$$\Sigma\Sigma^T=\Sigma^T\Sigma\le I,\quad (I \text{ is } n\times n \text{ unit matrix}). \quad (9)$$

3.2. Parallel Distributed Compensation (PDC) Controller. Rule R^i: IF $z_1(t)$ is M_{i1} and ... and $z_n(t)$ is M_{ip}

$$\text{THEN } \frac{d^\alpha x(t)}{dt^\alpha}=\left(\widetilde{A}_i+\Delta A_i\right)x(t)+\left(B_i+\Delta B_i\right)u(t),\quad r=(1,2,\ldots,r), \quad (10)$$

where the fuzzy set is M_{ij} $(j=1,2,\ldots,n)$ and r is the IF-THEN rules number, $x(t)\in R^n$ is the state vector, $A_i\in R^{n\times n}$, $z(t)=[z_1(t),z_2(t),\ldots,z_p(t)]$ are the premise variables, the fractional order is α $(0<\alpha<1)$, and the control input is $u(t)$. The Takagi-Sugeno fuzzy model total output is introduced as

$$\frac{d^\alpha x}{dt^\alpha}=\sum_{i=1}^r h_i(z(t))\left(A_i+\Delta A_i\right)x(t)+\sum_{i=1}^r h_i(z(t))\left(B_i+\Delta B_i\right)u(t), \quad (11)$$

where

$$h_i(z(t))=\frac{\prod_{j=1}^n M_{ij}\left(z_j(t)\right)}{\sum_{i=1}^r\prod_{j=1}^n M_{ij}\left(z_j(t)\right)}\ge 0,\qquad \sum_{i=1}^r h_i(z(t))=1. \quad (12)$$

The new fuzzy controller $u(t)$ is designed on the basis of parallel distributed compensation (PDC) and represented as follows.

Rule R^i: IF $z_1(t)$ is M_{i1} and ... and $z_n(t)$ is M_{ip}

$$\text{THEN } u(t)=K_i x(t)\quad (i=1,2,\ldots,r) \quad (13)$$

The parallel distributed compensation controller is shown as follows:

$$u(t)=\sum_{i=1}^r h_i(z(t))K_i x(t), \quad (14)$$

where K_i $(i=1,2,\ldots,r)$ represents the feedback gain.

By substituting (14) to (11), there follows

$$\frac{d^\alpha x}{dt^\alpha}=\sum_{i=1}^r h_i(z(t))\left(A_i+\Delta A_i\right)x(t)+\sum_{i=1}^r h_i(z(t))\left(B_i+\Delta B_i\right)\sum_{i=1}^r h_i(z(t))K_i x(t). \quad (15)$$

According to the term $\sum_{i=1}^r h_i(z(t))=1$, (15) can be equally written as

$$\frac{d^\alpha x}{dt^\alpha}=\sum_{i=1}^r\sum_{j=1}^r h_i(z(t))h_j(z(t))\cdot\left(A_i+\Delta A_i+\left(B_i+\Delta B_i\right)K_j\right)x(t). \quad (16)$$

3.3. Takagi-Sugeno Fuzzy Model of Fractional-Order Hydro-turbine Regulating System. For the convenience of the coefficient matrix, the Maclaurin series expansion is introduced:

$$\sin x=x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\cdots+(-1)^n\frac{x^{2n+1}}{(2n+1)!}+\cdots. \quad (17)$$

Assume $x_2\in[-d,d]$; here $d=2$. The Takagi-Sugeno fuzzy model is established, with the following two rules to describe the dynamic behavior of the system:

R^1: IF x_2 is $M_1(x_2(t))$, THEN $d^\alpha x(t)/dt^\alpha=(\widetilde{A}_1+\Delta A_1)x(t)+(B_1+\Delta B_1)u(t)$;

R^2: IF x_2 is $M_2(x_2(t))$, THEN $d^\alpha x(t)/dt^\alpha=(\widetilde{A}_2+\Delta A_2)x(t)+(B_2+\Delta B_2)u(t)$.

The membership functions are taken as follows:

$$M_1\left(x_2(t)\right)=\frac{1}{2}\left(1+\frac{x_2(t)}{d}\right),\qquad M_2\left(x_2(t)\right)=\frac{1}{2}\left(1-\frac{x_2(t)}{d}\right). \quad (18)$$

In view of the above description, there is

$$\widetilde{A}_1=\begin{bmatrix}0 & \widetilde{a} & 0 & 0\\ \frac{17231}{16951} & -\widetilde{b} & \widetilde{c} & 0\\ 0 & 0 & -\widetilde{d} & \widetilde{e}\\ 0 & 0 & 0 & -\widetilde{f}\end{bmatrix},$$

$$\widetilde{A}_2 = \begin{bmatrix} 0 & \widetilde{a} & 0 & 0 \\ \dfrac{1577}{16951} & -\widetilde{b} & \widetilde{c} & 0 \\ 0 & 0 & -\widetilde{d} & \widetilde{e} \\ 0 & 0 & 0 & -\widetilde{f} \end{bmatrix},$$

$$B_1 = B_2 = I_{4\times 4}. \quad (19)$$

So the fractional-order Takagi-Sugeno fuzzy model of the hydroturbine regulating system can be represented as

$$\frac{d^{\alpha}x}{dt^{\alpha}} = \sum_{i=1}^{2} h_i(z(t))\left(\widetilde{A}_i + \Delta A_i + (B_i + \Delta B_i)\, u(t)\right). \quad (20)$$

3.4. Controller Design. According to the PDC law, one has

$$u(t) = \sum_{j=1}^{2} h_j(z(t))\, K_j x(t). \quad (21)$$

Then, (20) could be rewritten as

$$\frac{d^{\alpha}x}{dt^{\alpha}} = \sum_{i=1}^{2}\sum_{j=1}^{2} h_i(x(t))\, h_j(x(t)) \cdot \left(\widetilde{A}_i + \Delta A_i + (B_i + \Delta B_i)\, K_j\right) x(t). \quad (22)$$

With the help of interval matrix theory in Section 3.1, (22) can be equivalently given as

$$\frac{d^{\alpha}x}{dt^{\alpha}} = \sum_{i=1}^{2}\sum_{j=1}^{2} h_i(x(t))\, h_j(x(t)) \cdot \left(A_{0i} + E\Sigma F + \Delta A_i + (B_i + \Delta B_i)\, K_j\right) x(t). \quad (23)$$

Assumption 1. The parameter uncertainties considered here are norm-bounded in the following form:

$$\Delta A_i = D_i F_i(t)\, E_{1i}, \qquad \Delta B_i = D_i F_i(t)\, E_{2i}, \quad (24)$$

where D_i, E_{1i}, E_{2i} are known real constant matrices of appropriate dimensions and F_i is a diagonal random matrix with Lebesgue-measurable elements and satisfies $F_i F_i^T \le I$; I is the identity matrix with an appropriate dimension.

According to Assumption 1, (23) can be equally written as

$$\frac{d^{\alpha}x}{dt^{\alpha}} = \sum_{i=1}^{2}\sum_{j=1}^{2} h_i(x(t))\, h_j(x(t)) \cdot \left(A_{0i} + E\Sigma F + D_i F_i E_{1i} + (B_i + D_i F_i E_{2i})\, K_j\right) x(t). \quad (25)$$

Lemma 2 (see [32]). *For the linear fractional-order system presented below,*

$$D^{\alpha}x = Ax, \qquad x(0) = x_0, \quad (26)$$

where $A \in R^{n\times n}$, $x \in R^n$, and $\alpha = [\alpha_1, \alpha_2, \ldots, \alpha_i, \ldots, \alpha_n]$ ($0 < \alpha_i \le 1$), for all eigenvalues λ_i of matrix A, when and only when $|\arg(\lambda_i)| > \alpha\pi/2$ is satisfied; system (26) is asymptotically stable.

Theorem 3. *Assume the system matrix A meets the Lyapunov function; that is, there is real positive definite symmetric matrix P as well as semipositive definite matrix Q meeting $A^TP + PA = -Q$. The fractional-order hydroturbine regulating system (25) then will converge to the equilibrium point asymptotically.*

Proof. Assume λ is an eigenvalue of the system matrix A and the eigenvector is ξ; one can easily get

$$A\xi = \lambda\xi. \quad (27)$$

For both sides of (27), making conjugation and transpose, one has

$$\overline{(A\xi)}^T = \bar{\lambda}\xi^T. \quad (28)$$

For (27) left side, multiplying $\xi^T P$, one can be obtain

$$\xi^T PA\xi = \lambda\xi^T P\xi. \quad (29)$$

For (28) right side, making the similar treatment and multiplying $P\xi$, one can also get

$$\overline{(A\xi)}^T P\xi = \bar{\lambda}\xi^T P\xi. \quad (30)$$

Combining (29) and (30), one can easily get

$$\xi^T\left(PA + A^TP\right)\xi = \left(\lambda + \bar{\lambda}\right)\xi^T P\xi. \quad (31)$$

According to Theorem 3, $A^TP + PA$ is a seminegative definite matrix. So, for any nonzero vector ξ, one has

$$\xi^T\left(PA + A^TP\right)\xi = \xi^T(-Q)\,\xi \le 0, \qquad \xi^T P\xi > 0. \quad (32)$$

Combining (31) and (32), one can easily get

$$\left(\lambda + \bar{\lambda}\right) = \xi^T(-Q)\,\xi\overline{\xi^T P\xi} \le 0. \quad (33)$$

It means, for all eigenvalues of matrix A, there is

$$\left|\arg(\lambda)\right| \ge \frac{\pi}{2} > \frac{\alpha\pi}{2}, \quad (\alpha < 1). \quad (34)$$

Referring to Lemma 2, the fractional-order hydroturbine regulating system (25) then will converge to the equilibrium point asymptotically. The proof is complete. □

The following more flexible theorem is presented on the basis of Theorem 3.

Theorem 4. *For any system variables $x = [x_1, x_2, \ldots, x_n]^T$, there exists a real positive definite symmetric matrix P meeting $J = x^T P(d^{\alpha}x/dt^{\alpha}) \le 0$, ($x^T P(d^{\alpha}x/dt^{\alpha})$ is called J function). The fractional-order hydroturbine regulating system (25) then* *will converge to the equilibrium point asymptotically. The condition $J = x^T P(d^\alpha x/dt^\alpha) \le 0$ could be equivalently given as*

$$J_0 = x^T P \frac{d^\alpha x}{dt^\alpha} + \left(\frac{d^\alpha x}{dt^\alpha}\right)^T Px \le 0. \quad (35)$$

Proof. From Theorem 3, there is

$$A^T P + PA = -Q. \quad (36)$$

According to (36), one can easily get

$$x^T \left(A^T P + PA\right) x = -x^T Qx, \quad (37)$$

where Q is a semipositive definite matrix. That is to say, for any system variable x, there is

$$x^T \left(A^T P + PA\right) x = -x^T Qx \le 0. \quad (38)$$

Substituting (26) to (38), one has

$$x^T P \frac{d^\alpha x}{dt^\alpha} + \left(\frac{d^\alpha x}{dt^\alpha}\right)^T Px \le 0. \quad (39)$$

The positive definite symmetric matrix P is supposed as

$$P = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}. \quad (40)$$

Introducing (40) to (39), one gets

$$\sum_{i,j=1}^{n} a_{ij} x_i \frac{d^\alpha x_j}{dt^\alpha} + \sum_{i,j=1}^{n} a_{ij} x_j \frac{d^\alpha x_i}{dt^\alpha} \le 0. \quad (41)$$

For $a_{ij} = a_{ji}$ $(\forall i, j)$, (41) could be rewritten as

$$\begin{aligned} \sum_{i,j=1}^{n} a_{ij} x_i \frac{d^\alpha x_j}{dt^\alpha} + \sum_{i,j=1}^{n} a_{ij} x_j \frac{d^\alpha x_i}{dt^\alpha} &= 2\sum_{i,j=1}^{n} a_{ij} x_j \frac{d^\alpha x_i}{dt^\alpha} \\ &= 2x^T P \frac{d^\alpha x}{dt^\alpha} = 2\left(\frac{d^\alpha x}{dt^\alpha}\right)^T Px \le 0. \end{aligned} \quad (42)$$

It is clear that Theorem 4 is equivalent to Theorem 3. The proof is finished. □

Based on the above theorems, the more practical stability conditions are proposed as follows.

Theorem 5. *The fractional-order hydroturbine regulating system (25) then will converge to the equilibrium point asymptotically, once there exist a plus constant η, a positive definite matrix P, and the controller gain matrices K_i $(i = 1, 2)$ which satisfy the following LMIs:*

$$\begin{gathered} \begin{bmatrix} \Phi_{ii} & QF^T \\ FQ & \eta^{-1} I \end{bmatrix} < 0, \\ \begin{bmatrix} \Phi_{ij} & QF^T \\ FQ & -\eta^{-1} I \end{bmatrix} < 0, \\ Q > 0, \end{gathered} \quad (43)$$

where

$$\begin{aligned} \Phi_{ii} &= Q\left(A_i + D_i F_i E_{1i}\right)^T + M_i^T \left(B_i + D_i F_i E_{2i}\right)^T \\ &\quad + \left(A_i + D_i F_i E_{1i}\right) Q + \left(B_i + D_i F_i E_{2i}\right) M_i \\ &\quad + \eta E E^T, \\ \Phi_{ij} &= Q\left(A_i + D_i F_i E_{1i}\right)^T + M_j^T \left(B_i + D_i F_i E_{2i}\right)^T \\ &\quad + Q\left(A_j + D_j F_j E_{1j}\right)^T \\ &\quad + M_i^T \left(B_j + D_j F_j E_{2j}\right)^T + \left(A_i + D_i F_i E_{1i}\right) Q \\ &\quad + \left(B_i + D_i F_i E_{2i}\right) M_j + \left(A_j + D_j F_j E_{1j}\right) Q \\ &\quad + \left(B_j + D_j F_j E_{2j}\right) M_i + \eta E E^T, \end{aligned} \quad (44)$$

$Q = P^{-1}$, $M_i = K_i P^{-1}$, $M_j = K_j P^{-1}$, and I is 4×4 unit matrix.

Proof. Based on Theorem 4, choose $J_0 = x^T P(d^\alpha x/dt^\alpha) + (d^\alpha x/dt^\alpha)^T Px$ as J function for system (25).

$$\begin{aligned} J_0 &= x^T P \frac{d^\alpha x}{dt^\alpha} + \left(\frac{d^\alpha x}{dt^\alpha}\right)^T Px = \sum_{i=1}^{4}\sum_{j=1}^{4} h_i h_j \left(A_{0i} + E\Sigma F + D_i F_i E_{1i} + \left(B_i + D_i F_i E_{2i}\right) K_j\right)^T x^T Px + x^T P \sum_{i=1}^{4}\sum_{j=1}^{4} h_i h_j \left(A_{0i}\right. \\ &\quad \left. + E\Sigma F + D_i F_i E_{1i} + \left(B_i + D_i F_i E_{2i}\right) K_j\right) = \sum_{i=1}^{4}\sum_{j=1}^{4} h_i h_j x^T \left\{\left[\left(A_{0i} + D_i F_i E_{1i} + \left(B_i + D_i F_i E_{2i}\right) K_j\right)^T P + F^T \Sigma E^T P\right]\right. \end{aligned}$$

$$+\left[P\left(A_{0i}+D_iF_iE_{1i}+\left(B_i+D_iF_iE_{2i}\right)K_j\right)+PE\Sigma F\right]x\Big\} = \sum_{i=1}^{4}h_i^2x^T\left[\left(A_{0i}+D_iF_iE_{1i}+\left(B_i+D_iF_iE_{2i}\right)K_i\right)^TP+F^T\Sigma E^TP\right.$$
$$+P\left(A_{0i}+D_iF_iE_{1i}+\left(B_i+D_iF_iE_{2i}\right)K_i\right)+PE\Sigma F\Big]x+\sum_{i<j}^{4}h_ih_jx^T\left[\left(A_{0i}+D_iF_iE_{1i}+\left(B_i+D_iF_iE_{2i}\right)K_j\right)^TP+F^T\Sigma E^TP\right.$$
$$+P\left(A_{0i}+D_iF_iE_{1i}+\left(B_i+D_iF_iE_{2i}\right)K_j\right)+PE\Sigma F\Big]x+\sum_{i>j}^{4}h_ih_jx^T\left[\left(A_{0j}+D_jF_jE_{1j}+\left(B_j+D_jF_jE_{2j}\right)K_i\right)^TP\right.$$
$$+F^T\Sigma E^TP+P\left(A_{0j}+D_jF_jE_{1j}+\left(B_j+D_jF_jE_{2j}\right)K_i\right)+PE\Sigma F\Big]x=\sum_{i=1}^{4}h_i^2x^T\left[\left(A_{0i}+D_iF_iE_{1i}+\left(B_i+D_iF_iE_{2i}\right)K_i\right)^TP\right.$$
$$+P\left(A_{0i}+D_iF_iE_{1i}+\left(B_i+D_iF_iE_{2i}\right)K_i\right)\Big]x+\sum_{i=1}^{4}h_i^2x^T\left(F^T\Sigma E^TP+PE\Sigma F\right)x+2\sum_{i<j}^{4}h_ih_jx^T\left(F^T\Sigma E^TP+PE\Sigma F\right)x$$
$$+\sum_{i<j}^{4}h_ih_jx^T\left[\left(A_{0i}+D_iF_iE_{1i}+\left(B_i+D_iF_iE_{2i}\right)K_j\right)^TP+P\left(A_{0i}+D_iF_iE_{1i}+\left(B_i+D_iF_iE_{2i}\right)K_j\right)\right]x+\sum_{i>j}^{4}h_ih_jx^T\left[\left(A_{0j}\right.\right.$$
$$\left.\left.+D_jF_jE_{1j}+\left(B_j+D_jF_jE_{2j}\right)K_i\right)^TP+P\left(A_{0j}+D_jF_jE_{1j}+\left(B_j+D_jF_jE_{2j}\right)K_i\right)\right]x. \tag{45}$$

Considering that $\sum_{i=1}^4 h_i^2 + 2\sum_{i<j}^4 h_ih_j = 1$ and

$$\begin{aligned}
&\sum_{i<j}^{4}h_ih_jx^T\left[\left(A_{0i}+D_iF_iE_{1i}+\left(B_i+D_iF_iE_{2i}\right)K_j\right)^TP+P\left(A_{0i}+D_iF_iE_{1i}+\left(B_i+D_iF_iE_{2i}\right)K_j\right)\right]x\\
&\quad+\sum_{i>j}^{4}h_ih_jx^T\left[\left(A_{0j}+D_jF_jE_{1j}+\left(B_j+D_jF_jE_{2j}\right)K_i\right)^TP+P\left(A_{0j}+D_jF_jE_{1j}+\left(B_j+D_jF_jE_{2j}\right)K_i\right)\right]x\\
&\quad=\sum_{i<j}^{4}h_ih_jx^T\left\{\left[\left(A_{0i}+D_iF_iE_{1i}+\left(B_i+D_iF_iE_{2i}\right)K_j\right)+\left(A_{0j}+D_jF_jE_{1j}+\left(B_j+D_jF_jE_{2j}\right)K_i\right)\right]^TP\right.\\
&\quad\left.+P\left[\left(A_{0i}+D_iF_iE_{1i}+\left(B_i+D_iF_iE_{2i}\right)K_j\right)+\left(A_{0j}+D_jF_jE_{1j}+\left(B_j+D_jF_jE_{2j}\right)K_i\right)\right]\right\}x\\
&\quad=2\sum_{i<j}^{4}h_ih_jx^T\left\{\left[\frac{\left(A_{0i}+D_iF_iE_{1i}+\left(B_i+D_iF_iE_{2i}\right)K_j\right)+\left(A_{0j}+D_jF_jE_{1j}+\left(B_j+D_jF_jE_{2j}\right)K_i\right)}{2}\right]^TP\right.\\
&\quad\left.+P\left[\frac{\left(A_{0i}+D_iF_iE_{1i}+\left(B_i+D_iF_iE_{2i}\right)K_j\right)+\left(A_{0j}+D_jF_jE_{1j}+\left(B_j+D_jF_jE_{2j}\right)K_i\right)}{2}\right]\right\}x,
\end{aligned} \tag{46}$$

select

$$\begin{aligned}
G_{ii}&=A_{0i}+D_iF_iE_{1i}+\left(B_i+D_iF_iE_{1i}\right)K_i,\\
G_{ij}&=\frac{\left(A_{0i}+D_iF_iE_{1i}+\left(B_i+D_iF_iE_{1i}\right)K_j\right)+\left(A_{0j}+D_jF_jE_{1j}+\left(B_j+D_jF_jE_{2j}\right)K_i\right)}{2}.
\end{aligned} \tag{47}$$

There follows

$$J_0 = \sum_{i=1}^{4} h_i^2 x^T \left(G_{ii}^T P + PG_{ii}\right) x + 2\sum_{i<j}^{4} h_i h_j x^T \left(G_{ij}^T P + PG_{ij}\right) x + x^T F^T \Sigma E^T Px + x^T PE\Sigma Fx. \quad (48)$$

Select $\xi^T = x^T PE$, $\theta = Fx$, and there is

$$2x^T PE\Sigma Fx \le \eta x^T PEE^T Px + \eta^{-1} x^T F^T Fx. \quad (49)$$

Taking the transpose of both sides in (49), one obtains

$$2x^T F^T \Sigma E^T Px \le \eta x^T PEE^T Px + \eta^{-1} x^T F^T Fx. \quad (50)$$

According to (50) and (49), one gets

$$x^T F^T \Sigma E^T Px + x^T PE\Sigma Fx \le \eta x^T PEE^T Px + \eta^{-1} x^T F^T Fx. \quad (51)$$

By substituting (51) into (48), one has

$$J_0 \le \sum_{i=1}^{4} h_i^2 x^T \left(G_{ii}^T P + PG_{ii}\right) x + 2\sum_{i<j}^{4} h_i h_j x^T \left(G_{ij}^T P + PG_{ij}\right) x + \eta x^T PEE^T Px + \eta^{-1} x^T F^T Fx. \quad (52)$$

Note that $\sum_{i=1}^{4} h_i^2 + 2\sum_{i<j}^{4} h_i h_j = 1$, so (52) can be equally represented as

$$J_0 \le \sum_{i=1}^{4} h_i^2 x^T \left(G_{ii}^T P + PG_{ii} + \eta PEE^T P + \eta^{-1} F^T F\right) x + 2\sum_{i<j}^{4} h_i h_j x^T \left(G_{ij}^T P + PG_{ij} + \eta PEE^T P + \eta^{-1} F^T F\right) \cdot x. \quad (53)$$

Accordingly, (25) can be assured as long as the following inequalities hold.

$$\begin{aligned} &G_{ii}^T P + PG_{ii} + \eta PEE^T P + \eta^{-1} F^T F < 0 \quad (i, j = 1, 2), \\ &G_{ij}^T P + PG_{ij} + \eta PEE^T P + \eta^{-1} F^T F < 0 \quad (i < j \le 2). \end{aligned} \quad (54)$$

It is clear that the fractional-order hydroturbine regulating system (25) then will converge to the equilibrium point asymptotically once (54) are satisfied.

With the help of Schur's theorem [33], one can easily transform (54) to the standard form of linear matrix inequalities, which are presented as (43). The proof is complete. □

4. Numerical Simulations

Considering the fractional-order hydroturbine regulating system (4) with uncertain parameters, $\widetilde{a} = 314 + 0.1\sin(t)$, $\widetilde{b} = 2/9 + 0.1\cos(t)$, $\widetilde{c} = 1/9 + 0.1\sin(t)$, $\widetilde{d} = 5/2 + 0.1\cos(t)$, $\widetilde{e} = 33/2 + 0.1\sin(t)$, and $\widetilde{f} = 10 + 0.1\cos(t)$.

Therefore, $\widetilde{a} \in [313.9, 314.1]$, $\widetilde{b} \in [11/90, 29/90]$, $\widetilde{c} \in [1/90, 19/90]$, $\widetilde{d} \in [2.4, 2.6]$, $\widetilde{e} \in [16.4, 16.6]$, and $\widetilde{f} \in [9.9, 10.1]$.

To take control of the fractional-order hydroturbine regulating system with uncertain parameters, we take $d = 2$, A_{0i} $(i = 1, 2)$, and E and F can be calculated. The value of Σ can also refer to Section 3.1.

$$\begin{aligned} &B_1 = B_2 = I_{4\times4}, \\ &D_1 = D_2 = E_{11} = E_{12} = E_{21} = E_{22} = I_{4\times4}, \\ &F_1 = F_2 = \operatorname{diag}\left(\operatorname{diag}\left(\operatorname{rand}(4,4)\right)\right). \end{aligned} \quad (55)$$

According to the above theorems and $\eta = 1000$, in the environment of Matlab 7.0 LMI toolbox we acquire

$$P = 10^{-4} \times \begin{bmatrix} 0.0000 & -0.0000 & 0.0000 & -0.0000 \\ -0.0000 & 0.4004 & 0.0013 & -0.0004 \\ 0.0000 & 0.0013 & 0.4955 & 0.0009 \\ -0.0000 & 0.0004 & 0.0093 & 0.4965 \end{bmatrix},$$

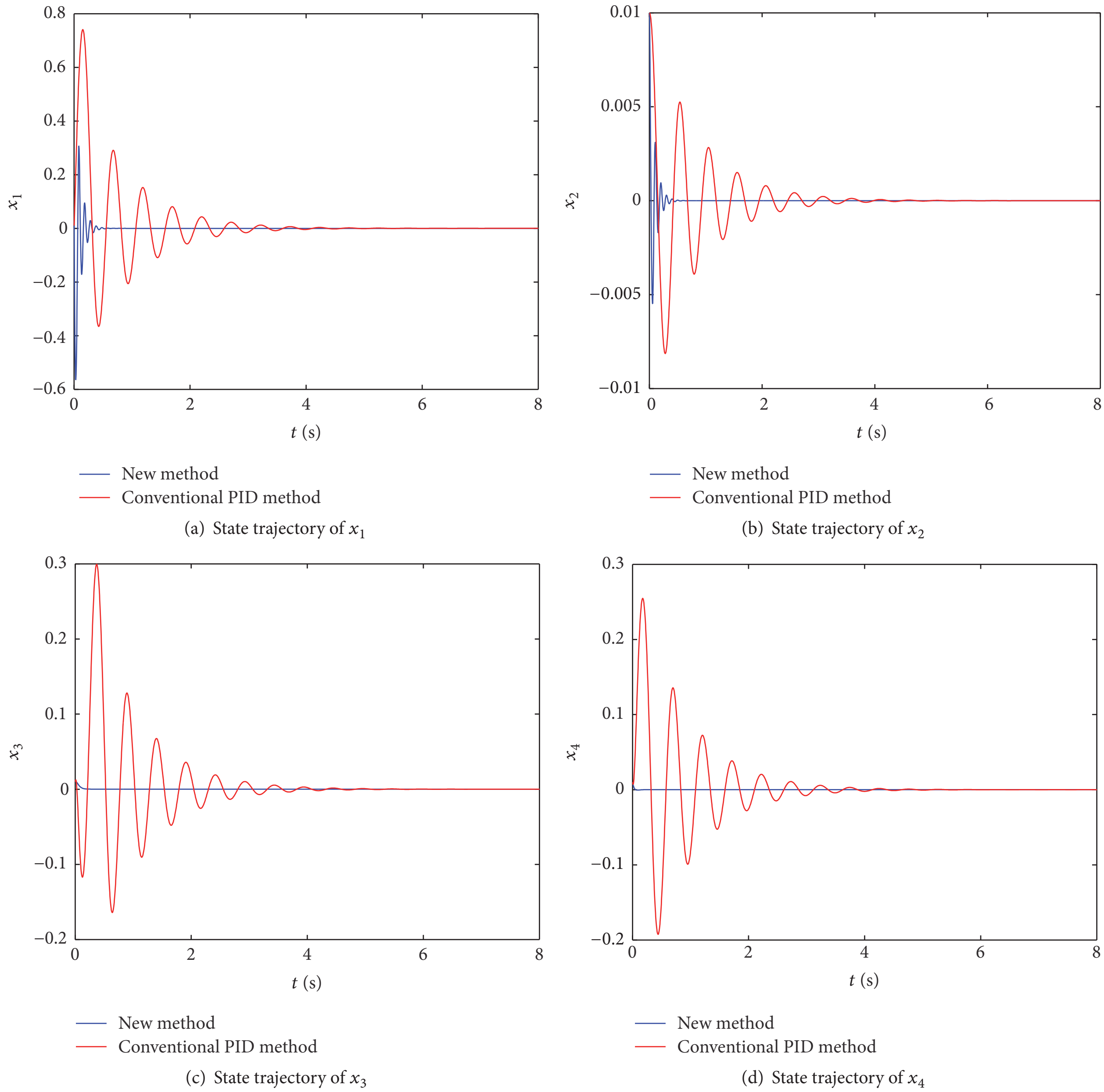
$$K_1 = 10^3 \times \begin{bmatrix} -0.0007 & -7.4759 & -0.0232 & 0.0079 \\ -0.0003 & -0.0038 & -0.0000 & 0.0000 \\ 0.0000 & 0.0000 & -0.0011 & -0.0072 \\ -0.0000 & -0.0000 & -0.0070 & 0.0052 \end{bmatrix}, \quad (56)$$

$$K_2 = \begin{bmatrix} -0.4730 & -797.0103 & -2.1527 & -0.7156 \\ -0.0270 & -4.2479 & -0.0467 & 0.0051 \\ 0.0000 & 0.0372 & -1.0621 & -7.2386 \\ -0.0000 & -0.0049 & -7.0022 & 5.2363 \end{bmatrix}.$$

Figure 2 shows the simulation results of the traditional PID control method as well as the proposed scheme in this paper. It is clear that when the controller is applied, the system state variables quickly converge to zero point, which verifies the effectiveness. Compared with the traditional PID control method, it needs shorter transition time and the overshoot is more smaller than the designed scheme, which shows the superiority and good robustness of the proposed method.

5. Conclusions

The robust fuzzy control for fractional-order hydroturbine regulating system was studied in this paper. First, mathematical model of the fractional-order hydroturbine regulating system with uncertain parameters and random disturbances



(a) State trajectory of x_1

(b) State trajectory of x_2

(c) State trajectory of x_3

(d) State trajectory of x_4

FIGURE 2: State trajectories of controlled fractional-order hydroturbine regulating system (4).

was introduced. Second, a fuzzy control method was proposed for fractional-order hydroturbine regulating system. Furthermore, the stability condition of the fractional-order hydroturbine regulating system was given as a group of linear matrix inequalities and the detailed mathematical proofs were presented. Besides, the method could handle the case of time-varying parameters and random disturbances, which has shown the good robustness. Finally, the validity and superiority were verified by the simulation results.

The scheme designed is simple and easy to implement and could be applied to similar fractional-order nonlinear systems. In the future, we will consider and extend the approach to other fractional-order hydroturbine governing systems, such as hydropower systems with time delay.

Competing Interests

The authors declare that there are no competing interests regarding the publication of this paper.

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