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Research Article

Delay-Dependent Finite-Time H_∞ Controller Design for a Kind of Nonlinear Descriptor Systems via a T-S Fuzzy Model

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Delay-dependent finite-time H_∞ controller design problems are investigated for a kind of nonlinear descriptor system via a T-S fuzzy model in this paper. The solvable conditions of finite-time H_∞ controller are given to guarantee that the loop-closed system is impulse-free and finite-time bounded and holds the H_∞ performance to a prescribed disturbance attenuation level γ. The method given is the ability to eliminate the impulsive behavior caused by descriptor systems in a finite-time interval, which confirms the existence and uniqueness of solutions in the interval. By constructing a nonsingular matrix, we overcome the difficulty that results in an infeasible linear matrix inequality (LMI). Using the FEASP solver and GEVP solver of the LMI toolbox, we perform simulations to validate the proposed methods for a nonlinear descriptor system via the T-S fuzzy model, which shows the application of the T-S fuzzy method in studying the finite-time control problem of a nonlinear system. Meanwhile the method was also applied to the biological economy system to eliminate impulsive behavior at the bifurcation value, stabilize the loop-closed system in a finite-time interval, and achieve a H_∞ performance level.

1. Introduction

The Lyapunov stability theory can be regarded as a relatively mature field, which focuses on the asymptotic stability of a system state on an infinite interval. However, it is noteworthy that the behavior of the system over a fixed finite-time interval needs to be considered in many practical problems, and this just leads to the finite-time or short time stability issues. In short, a system is said to be finite-time stable, if its state keeps within the prescribed bounds on a fixed time interval for given some initial conditions [1, 2]. It has been shown that the finite-time stability issues play an important role in the theory and practical application; for instance, the trajectory control of space vehicles, the disease control, the population quantity control, and so on will lead to a discussion on the finite-time stability or boundedness. This fact also motivates many scholars to study the finite-time stability or boundedness and robust control problems [1–6].

As is known to all, time-delays exist inevitably in many practical engineering systems, such as nuclear reactors, electronics, chemical processes, and hydraulic and biological systems. They can frequently lead to instability and poor performance. So there has been an increasing interest in the stability analysis and stabilization for time-delay systems in the last decades [4, 5, 7–15]. The results and discussions may be classified as delay-independent or -dependent. Delay-dependent discussions which take into account the size of delays are of less conservative, especially in the case of small time-delays [7]. The main issue of delay-dependent is that the presence of an integral term normally does not found in LMIs. There may be several ways to deal with the integral term, such as Jesson's inequality [10, 11], Moon et al.'s inequality [12], Park's inequality [13], Wirtinger's inequality [14], and the free-weighting-matrix approach [15]. However, each method has its own benefits and shortcomings in the conservative viewpoint. According to the characteristics of our research issue, we will investigate the delay-dependent finite-time H_∞ control problems of T-S descriptor system in this paper by using Jesson's inequality approach to overcoming the conservativeness.

The stability or robust stabilization of a nonlinear descriptor system has been an acknowledged difficult problem [16], especially with regard to the finite-time robust stabilization issues. Fortunately, Taniguchi et al. established a T-S fuzzy descriptor system model and proposed an innovative and simple method to solve control problems of a kind of nonlinear descriptor systems [17]. Many nonlinear dynamic systems are represented as T-S fuzzy systems, which is a universal fuzzy approximator [18–26]. The descriptor systems have been one of the major research fields of control theory, which is due to the fact that a descriptor system may exhibit impulsive or noncausal behavior along with the derivatives of these impulses. It just results in the fact that the discussion of descriptor systems is much more complex and more challenging than ordinary systems [7]. However, it is noteworthy that singular systems not only describe actual physical systems better but also are also more widely utilized than ordinary ones. So in the past decades, descriptor systems have attracted much attention [7, 17, 19, 25–30].

On the other hand, the problem of H_∞ control has been a topic of growing interestover the past decades. A great number of results on H_∞ control for the state-space and descriptor systems have been reported in literatures [29, 31–35]. However, the related research on finite-time H_∞ control of fuzzy descriptor systems is relatively less.

In view of the above-mentioned facts, this paper investigates the delay-dependent finite-time H_∞ controller design problems for a kind of nonlinear descriptor system via a T-S fuzzy model. The solvable conditions of finite-time H_∞ controller are given to guarantee that the loop-closed system is impulse-free and finite-time bounded and holds the H_∞ performance to a prescribed disturbance attenuation level γ. The method given is the ability to eliminate the impulsive behavior caused by descriptor systems in a finite-time interval, which confirms the existence and uniqueness of solutions in the interval. By constructing a nonsingular matrix, we overcome the difficulty that results in an infeasible linear matrix inequality (LMI). By using the FEASP solver and GEVP solver of the LMI toolbox together with SIMULINK simulation technology, we perform simulations to validate the proposed methods for a nonlinear descriptor system via the T-S fuzzy model, which shows the application of the T-S fuzzy method in studying the finite-time control problem for a nonlinear system. Meanwhile the method was also applied to the biological economy system to eliminate impulsive behavior at the bifurcation value, stabilize the loop-closed system in a finite-time interval, and achieve a H_∞ performance level.

This paper is organized as follows. Section 2 provides preliminaries and the formulation. The finite-time H_∞ control design scheme is proposed in Section 3. In Section 4, two design examples are given to show the advantage of developed results. Finally, concluding remark is made in Section 5.

2. System Formulation and Preliminaries

In the following, we consider the delay-depended finite-time H_∞ control problems for a kind of nonlinear descriptor systems via T-S fuzzy models. The nonlinear dynamical descriptor system can be described by the following fuzzy IF-THEN rules:

$$\begin{aligned}
\text{Model rule } i\text{: IF}\quad & \xi_1(t) \text{ is } M_{i1} \text{ and } \xi_2(t) \text{ is } M_{i2} \cdots \text{ and } \xi_p(t) \text{ is } M_{ip},\\
\text{THEN}\quad & E\dot{x}(t) = A_i x(t) + A_{id} x(t-d) + B_i u(t) + D_{1i} w(t),\\
& z(t) = C_i x(t) + D_{2i} w(t), \quad i = 1, 2, \ldots, r,\\
& x(t) = \varphi(t), \quad t \in [-d, 0],
\end{aligned} \tag{1}$$

where r is the number of IF-THEN rules and M_{ij} ($j = 1, 2, \ldots, p$) are fuzzy sets; $x(t) \in R^n$ is the state; $u(t) \in R^m$ is control input; $z(t) \in R^q$ is the controlled output; $w(t) \in R^p$ is the exogenous disturbance and satisfies $w^T(t)w(t) \le c_w$, where c_w is a positive number; the matrix $E \in R^{n\times n}$ may be singular; and we let $\operatorname{rank} E = r \le n$. The matrices A_i, A_{id}, B_i, C_i, D_{1i}, and D_{2i} are known real constant matrices with appropriate dimensions. The delay d is a positive constant. $\varphi(t)$ is a continuous initial function on $[-d, 0]$. $\xi_1(t), \xi_2(t), \ldots, \xi_p(t)$ are premise variables and write $\xi(t) = [\xi_1(t), \xi_2(t), \ldots, \xi_p(t)]^T$.

By using the fuzzy inference method of singleton fuzzifier and weighted average defuzzifier, the overall fuzzy model can be inferred as follows:

$$\begin{aligned}
E\dot{x}(t) &= \widetilde{A}_h x(t) + \widetilde{A}_{hd} x(t-d) + \widetilde{B}_h u(t) + \widetilde{D}_{1h} w(t),\\
z(t) &= \widetilde{C}_h x(t) + \widetilde{D}_{2h} w(t),\\
x(t) &= \varphi(t), \quad t \in [-d, 0],
\end{aligned} \tag{2}$$

where

$$\begin{aligned}
\widetilde{A}_h &= \sum_{i=1}^{r} h_i(\xi(t)) A_i,\\
\widetilde{A}_{hd} &= \sum_{i=1}^{r} h_i(\xi(t)) A_{id},\\
\widetilde{B}_h &= \sum_{i=1}^{r} h_i(\xi(t)) B_i,\\
\widetilde{D}_{1h} &= \sum_{i=1}^{r} h_i(\xi(t)) D_{1i},\\
\widetilde{C}_h &= \sum_{i=1}^{r} h_i(\xi(t)) C_i,\\
\widetilde{D}_{2h} &= \sum_{i=1}^{r} h_i(\xi(t)) D_{2i},
\end{aligned} \tag{3}$$

as follows: $M_{ij}(\xi_j(t))$ is the grade of membership of $\xi_j(t)$ in M_{ij}. It can be seen that $\beta_i(\xi(t)) = \prod_{j=1}^{p} M_{ij}(\xi_j(t)) \geq 0$, $i = 1, 2, \ldots, r$. Thus, $h_i(\xi(t)) = \beta_i(\xi(t)) / \sum_{i=1}^{r} \beta_i(\xi(t)) \geq 0$, $\sum_{i=1}^{r} h_i(\xi(t)) = 1, t \in [0, T]$. In this paper, let $\sum_{i=1}^{r} \beta_i(\xi(t)) > 0$, $t \in [0, T]$.

Note that $\operatorname{rank}(E) = r < n$. Without loss of generality, decompose matrices in (2) as follows:

$$E = \begin{pmatrix} I_r & 0 \\ 0 & 0 \end{pmatrix},$$

$$\widetilde{A}_h = \begin{pmatrix} \widetilde{A}_{h11} & \widetilde{A}_{h12} \\ \widetilde{A}_{h21} & \widetilde{A}_{h22} \end{pmatrix}, \tag{4}$$

$$\widetilde{A}_{hd} = \begin{pmatrix} \widetilde{A}_{hd11} & \widetilde{A}_{hd12} \\ \widetilde{A}_{hd21} & \widetilde{A}_{hd22} \end{pmatrix},$$

where $\widetilde{A}_{h11}, \widetilde{A}_{hd11} \in R^{r \times r}$, $\widetilde{A}_{h22} \in R^{(n-r)\times(n-r)}$, and $\widetilde{A}_{hd22} \in R^{(n-r)\times(n-r)}$.

Now we consider the systems

$$E\dot{x}(t) = \widetilde{A}_h x(t), \tag{5}$$

$$\begin{aligned} E\dot{x}(t) &= \widetilde{A}_h x(t) + \widetilde{A}_{hd} x(t-d), \\ x(t) &= \varphi(t), \\ & \qquad t \in [-d, 0]. \end{aligned} \tag{6}$$

Definition 1 (see [7]). System (5) is said to be regular if $\det(sE - \widetilde{A}_h)$ is not identically 0 for $t \in [0, T]$. The system is said to be impulse-free if $\deg_s \det(sE - \widetilde{A}_h) = \operatorname{rank} E$, for $t \in [0, T]$.

Lemma 2 (see [7]). *System (6) is regular and impulse-free, if system (5) is regular and impulse-free. The descriptor system (5) is impulse-free if and only if $\widetilde{A}_{h22}$ for $t \in [0, T]$ is nonsingular.*

Lemma 3 (Jensen's inequality [10]). *For any positive symmetric constant matrix $Z \in R^{n\times n}$, scalars a, b satisfying $a < b$, a vector function x in $[a, b] \to R^n$ such that the integrations concerned, are well defined, and then*

$$\int_a^b x^T(t) Z x(t)\, dt \geq \frac{1}{b-a} \int_a^b x^T(t)\, dt Z \int_a^b x(t)\, dt. \tag{7}$$

Lemma 4 (see [35]). *Given any real matrices X, Y, and W with appropriate dimensions such that $Y > 0$ and is symmetric, then one has*

$$X^T Y X + X^T W + W^T X + W^T Y^{-1} W \geq 0. \tag{8}$$

Definition 5 (see [4, 5] (finite-time stability)). The fuzzy descriptor time-delay system (6) without impulse is said to be finite-time stable with respect to (c_1, c_2, T), $0 < c_1 < c_2$, if $\sup_{t\in[-d,0]} \varphi^T(t)\varphi(t) \leq c_1$ implies $x^T(t)E^T E x(t) < c_2$, $\forall t \in [0, T]$.

Remark 6. In Definition 5, the finite-time stability of dynamic state is defined for the time-delay descriptor system. In some actual systems, we should focus on the dynamic state and the static state stability or focus only on the dynamic state stability and so forth. For example, in power systems and economic systems, there exists the question on the dynamic state stability analysis to be considered. When we need to consider the finite-time stability of dynamic and static state, we should redefine it (see the following Definition 7). In this paper, we discuss only the finite-time stability analysis and H_∞ control problems for the dynamic state of a fuzzy descriptor system with time-delay.

Definition 7 (finite-time stability). The fuzzy descriptor system (6) without impulse is said to be finite-time bounded with respect to (c_1, c_2, T, R), $0 < c_1 < c_2$, $R > 0$, if $\sup_{t\in[-d,0]} \varphi^T(t)R\varphi(t) \leq c_1$ implies $x^T(t)Rx(t) < c_2$, $\forall t \in [0, T]$.

Amato et al. pointed out in [1] that the Lyapunov asymptotic stability and the finite-time stability are independent concepts: a system which is finite-time stable may be not Lyapunov asymptotic stable, while a Lyapunov asymptotic stable system may be not finite-time stable if its state exceeds the prescribed bounds during the transients.

3. Finite-Time H_∞ Control

For the fuzzy model (2), we consider the following fuzzy controller via the PDC:

$$u(t) = \sum_{i=1}^{r} h_i(\xi(t)) K_i x(t). \tag{9}$$

We apply this controller $u(t)$ to system (2) which will result in the following closed-loop systems:

$$\begin{aligned} E\dot{x}(t) &= \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\xi(t)) h_j(\xi(t)) \\ &\quad \cdot \left\{ \left(A_i + B_i K_j\right) x(t) + A_{id} x(t-d) + D_{1i} w(t) \right\} \\ &\quad := \widetilde{A}_{hk} x(t) + \widetilde{A}_{hd} x(t-d) + \widetilde{D}_{1h} w(t), \\ x(t) &= \varphi(t), \quad t \in [-d, 0], \\ z(t) &= \sum_{i=1}^{r} h_i(\xi(t)) \left\{ C_i x(t) + D_{2i} w(t) \right\} := \widetilde{C}_h x(t) \\ &\quad + \widetilde{D}_{2h} w(t), \quad t \in [0, T]. \end{aligned} \tag{10}$$

The so-called finite-time H_∞ fuzzy control is to design a state feedback controller (9) such that the closed-loop system (10) is impulse-free and finite-time bounded and holds the H_∞ performance to the prescribed disturbance attenuation level γ. And the controller is called a finite-time H_∞ fuzzy controller. We also say that a finite-time H_∞ fuzzy control problem is solvable.

Definition 8 (finite-time boundedness). The time-delay descriptor system without impulse (10) is said to be finite-time boundedness with respect to (c_1, c_2, T, c_w) where $0 \le c_1 < c_2$ and $c_w \ge 0$, if $\sup_{t\in[-d,0]}\varphi^T(t)\varphi(t) \le c_1$ implies $x^T(t)E^T Ex(t) < c_2$, $\forall t \in [0,T]$, $\forall w(t) : w^T(t)w(t) \le c_w$.

Theorem 9. *Given positive numbers d_0 and γ, then for any delay $0 < d \le d_0$, system (10) is impulse-free and finite-time bounded with respect to (c_1, c_2, T, c_w) and holds the H_∞ performance level to the prescribed disturbance attenuation level γ, if there exist matrices $P = \begin{pmatrix} P_1 & 0 \\ P_2 & P_3 \end{pmatrix}$ $(P_1 \in R^{r\times r}, P_3 \in R^{(n-r)\times(n-r)}, P_1 > 0, |P_3| \ne 0)$, $\widehat{P} = \begin{pmatrix} P_1 & 0 \\ 0 & \widehat{P}_3 \end{pmatrix}$ $(\widehat{P}_3 \in R^{(n-r)\times(n-r)}, |\widehat{P}_3| \ne 0)$, K_i, positive matrices $Q = \begin{pmatrix} Q_1 & Q_2 \\ Q_2^T & Q_3 \end{pmatrix}$, $Z = \begin{pmatrix} Z_1 & Z_2 \\ Z_2^T & Z_3 \end{pmatrix}$, and a positive scalar ρ such that the following set of matrix inequalities hold:*

$$\Psi_{ii} < 0, \quad i = 1, 2, \ldots, r, \tag{11}$$

$$\Psi_{ij} + \Psi_{ji} < 0, \quad j < i = 1, 2, \ldots, r, \tag{12}$$

$$\begin{aligned} &\lambda_{\max}(P_1)\, c_1 + d\lambda_{\max}(Q)\, c_1 + d^2\lambda_{\max}(Z)\, c_0 \\ &\quad + \gamma^2 e^{-\rho T} c_w T < c_2 e^{-\rho T}\lambda_{\min}(\widehat{P}), \end{aligned} \tag{13}$$

where

$$\Psi_{ij} = \begin{pmatrix} \begin{pmatrix} (A_i + B_iK_j)^T P \\ +P^T(A_i + B_iK_j) + Q \\ -\frac{1}{d}E^T ZE - \rho E^T P \end{pmatrix} & P^T A_{id} + \frac{1}{d}E^T ZE & P^T D_{1i} & (A_i + B_iK_j)^T & C_i^T \\ A_{id}^T P + \frac{1}{d}E^T ZE & -Q - \frac{1}{d}E^T ZE & 0 & A_{id}^T & 0 \\ D_{1i}^T P & 0 & -\gamma^2 e^{-\rho T} I & D_{1i}^T & D_{2i}^T \\ A_i + B_iK_j & A_{id} & D_{1i} & -\frac{1}{d}Z^{-1} & 0 \\ C_i & 0 & D_{2i} & 0 & -I \end{pmatrix} < 0, \quad \forall t \in [0, T], \tag{14}$$

$$c_0 = \sup_{-d\le t\le 0} \dot{x}^T(t)\, E^T E\dot{x}(t).$$

Proof. We prove first that the system $E\dot{x}(t) = \widetilde{A}_{hk}x(t)$ is impulse-free. Thus the system $E\dot{x}(t) = \widetilde{A}_{hk}x(t) + \widetilde{A}_{hd}x(t-d)$ is impulse-free by Lemma 2. This also implies that it is regular [7]. To achieve this goal, we set $\Psi_{hl} = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\xi(t))h_j(\xi(t))\Psi_{ij}$. Then it can be derived by inequalities (11) and (12) along with $\Psi_{hl} = \sum_{i=1}^{r} h_i(\xi(t))h_i(\xi(t))\Psi_{ii} + \sum_{j<i}\sum h_i(\xi(t))h_j(\xi(t))(\Psi_{ij} + \Psi_{ji})$ that

$$\begin{pmatrix} \begin{pmatrix} \widetilde{A}_{hk}^T P + P^T\widetilde{A}_{hk} + Q \\ -\frac{1}{d}E^T ZE - \rho E^T P \end{pmatrix} & P^T\widetilde{A}_{hd} + \frac{1}{d}E^T ZE & P^T\widetilde{D}_{1h} & \widetilde{A}_{hk}^T & \widetilde{C}_h^T \\ \widetilde{A}_{hd}^T P + \frac{1}{d}E^T ZE & -Q - \frac{1}{d}E^T ZE & 0 & \widetilde{A}_{hd}^T & 0 \\ \widetilde{D}_{1h}^T P & 0 & -\gamma^2 e^{-\rho T} I & \widetilde{D}_{1h}^T & \widetilde{D}_{2h}^T \\ \widetilde{A}_{hk} & \widetilde{A}_{hd} & \widetilde{D}_{1h} & -\frac{1}{d}Z^{-1} & 0 \\ \widetilde{C}_h & 0 & \widetilde{D}_{2h} & 0 & -I \end{pmatrix} < 0, \quad \forall t \in [0, T]. \tag{15}$$

By applying the properties of Schur complement and noting inequality (15), then it is easy to see that

$$\begin{pmatrix} \begin{pmatrix} \widetilde{A}_{hk}^T P + P^T \widetilde{A}_{hk} + Q \\ -\frac{1}{d} E^T Z E - \rho E^T P \end{pmatrix} & P^T \widetilde{A}_{hd} + \frac{1}{d} E^T Z E & P^T \widetilde{D}_{1h} \\ \widetilde{A}_{hd}^T P + \frac{1}{d} E^T Z E & -Q - \frac{1}{d} E^T Z E & 0 \\ \widetilde{D}_{1h}^T P & 0 & -\gamma^2 e^{-\rho T} I \end{pmatrix} + d \begin{pmatrix} \widetilde{A}_{hk}^T \\ \widetilde{A}_{hd}^T \\ \widetilde{D}_{1h}^T \end{pmatrix} Z \begin{pmatrix} \widetilde{A}_{hk} & \widetilde{A}_{hd} & \widetilde{D}_{1h} \end{pmatrix} < 0, \quad \forall t \in [0, T]. \tag{16}$$

This implies $\widetilde{A}_{hk}^T P + P^T \widetilde{A}_{hk} + Q - (1/d) E^T Z E - \rho E^T P < 0, \forall t \in [0, T]$. We decompose the matrix $\widetilde{A}_{hk}$ into $\begin{pmatrix} \widetilde{A}_{hk11} & \widetilde{A}_{hk12} \\ \widetilde{A}_{hk21} & \widetilde{A}_{hk22} \end{pmatrix}$ which is compatible with that of $\widetilde{A}_h$ in (4). Then it turns out with some classical manipulations that $\begin{pmatrix} \otimes & \otimes \\ \otimes & \widetilde{A}_{hk22}^T P_3 + P_3^T \widetilde{A}_{hk22} + Q_3 \end{pmatrix} < 0$, $\forall t \in [0, T]$, in which $\otimes$ stands for the matrix that is not relevant in the discussion. It follows that $\widetilde{A}_{hk22}^T P_3 + P_3^T \widetilde{A}_{hk22} < 0$, $\forall t \in [0, T]$, which implies that $\widetilde{A}_{hk22}$ is nonsingular for $\forall t \in [0, T]$. So the system $E\dot{x}(t) = \widetilde{A}_{hk} x(t)$ is impulse-free from Lemma 2.

Then we proof that system (10) is finite-time bounded. To this end, we consider the following Lyapunov-Krasovskii functional candidate:

$$V(x(t)) = x^T(t) E^T P x(t) + \int_{t-d}^{t} x^T(s) Q x(s) \, ds + \int_{-d}^{0} \int_{t+\theta}^{t} \dot{x}^T(s) E^T Z E \dot{x}(s) \, ds \, d\theta. \tag{17}$$

It can be obtained that $E^T P = P^T E \geq 0$ from the structures of the matrices E and P. Then it is shown by classical computations that the derivative of the functional (17) along the trajectories of system (10) satisfies

$$\begin{aligned} \dot{V}(x(t)) &= \dot{x}^T(t) E^T P x(t) + x^T(t) P^T E \dot{x}(t) + x^T(t) \\ &\quad \cdot Q x(t) - x^T(t-d) Q x(t-d) + d \dot{x}^T(t) \\ &\quad \cdot E^T Z E \dot{x}(t) - \int_{-d}^{0} \dot{x}^T(t+\theta) E^T Z E \dot{x}(t+\theta) \, d\theta \\ &= \left(\widetilde{A}_{hk} x(t) + \widetilde{A}_{hd} x(t-d) + \widetilde{D}_{1h} w(t) \right)^T P x(t) \\ &\quad + x^T(t) P^T \left(\widetilde{A}_{hk} x(t) + \widetilde{A}_{hd} x(t-d) + \widetilde{D}_{1h} w(t) \right) \\ &\quad + x^T(t) Q x(t) - x^T(t-d) Q x(t-d) \\ &\quad + d \left(\widetilde{A}_{hk} x(t) + \widetilde{A}_{hd} x(t-d) + \widetilde{D}_{1h} w(t) \right)^T \\ &\quad \cdot Z \left(\widetilde{A}_{hk} x(t) + \widetilde{A}_{hd} x(t-d) + \widetilde{D}_{1h} w(t) \right) \\ &\quad - \int_{-d}^{0} \dot{x}^T(t+\theta) E^T Z E \dot{x}(t+\theta) \, d\theta. \end{aligned} \tag{18}$$

By Jensen's inequality in Lemma 3 and considering (18) together with inequality (16), then it can be reduced that

$$\begin{aligned} \dot{V}(x(t)) &\leq \left(x^T(t), x^T(t-d), w^T(t) \right) \begin{pmatrix} \begin{pmatrix} \widetilde{A}_{hk}^T P + P^T \widetilde{A}_{hk} + Q \\ -\frac{1}{d} E^T Z E - \rho E^T P \end{pmatrix} & P^T \widetilde{A}_{hd} + \frac{1}{d} E^T Z E & P^T \widetilde{D}_{1h} \\ \widetilde{A}_{hd}^T P + \frac{1}{d} E^T Z E & -Q - \frac{1}{d} E^T Z E & 0 \\ \widetilde{D}_{1h}^T P & 0 & -\gamma^2 e^{-\rho T} I \end{pmatrix} \begin{pmatrix} x(t) \\ x(t-d) \\ w(t) \end{pmatrix} \\ &\quad + d \left(x^T(t), x^T(t-d), w^T(t) \right) \begin{pmatrix} \widetilde{A}_{hk}^T \\ \widetilde{A}_{hd}^T \\ \widetilde{D}_{1h}^T \end{pmatrix} Z \begin{pmatrix} \widetilde{A}_{hk} & \widetilde{A}_{hd} & \widetilde{D}_{1h} \end{pmatrix} \begin{pmatrix} x(t) \\ x(t-d) \\ w(t) \end{pmatrix} \\ &\quad + \left(x^T(t), x^T(t-d), w^T(t) \right) \begin{pmatrix} \rho E^T P & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \gamma^2 e^{-\rho T} I \end{pmatrix} \begin{pmatrix} x(t) \\ x(t-d) \\ w(t) \end{pmatrix} \\ &< \left(x^T(t), x^T(t-d), w^T(t) \right) \begin{pmatrix} \rho E^T P & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \gamma^2 e^{-\rho T} I \end{pmatrix} \begin{pmatrix} x(t) \\ x(t-d) \\ w(t) \end{pmatrix}. \end{aligned} \tag{19}$$

Therefore, the following inequalities can be derived:

$$\dot{V}(x(t)) - \rho V(x(t)) < \gamma^2 e^{-\rho T} w^T(t) w(t). \tag{20}$$

Before and after multiplying (20) by $e^{-\rho t}$, then integrating the inequality form 0 to t leads to

$$V(x(t)) < e^{\rho t}\left(V(x(0)) + \gamma^2 e^{-\rho T} c_w T\right), \quad \forall t \in [0, T]. \tag{21}$$

It is easy to see that $E^T P = E^T \widehat{P} E = \begin{pmatrix} P_1 & 0 \\ 0 & 0 \end{pmatrix}$ from the structures of the matrices E, P, and $\widehat{P}$. Then we can derive that

$$\begin{aligned} V(x(0)) &= x^T(0) E^T P x(0) + \int_{-d}^{0} x^T(s) Q x(s)\,ds \\ &\quad + \int_{-d}^{0}\int_{\theta}^{0} \dot{x}^T(s) E^T Z E \dot{x}(s)\,ds\,d\theta \\ &= x^T(0) E^T \widehat{P} E x(0) + \int_{-d}^{0} x^T(s) Q x(s)\,ds \\ &\quad + \int_{-d}^{0}\int_{\theta}^{0} \dot{x}^T(s) E^T Z E \dot{x}(s)\,ds\,d\theta \\ &\le \lambda_{\max}(P_1) c_1 + d\lambda_{\max}(Q) c_1 + d^2 \lambda_{\max}(Z) c_0, \\ \lambda_{\min}(\widehat{P}) x^T(t) E^T E x(t) &\le x^T(t) E^T \widehat{P} E x(t) \\ &= x^T(t) E^T P x(t) < V(x(t)). \end{aligned} \tag{22}$$

Thus from inequalities (22) and (13), we can obtain that

$$\begin{aligned} x^T(t) E^T E x(t) &< e^{\rho T} \frac{1}{\lambda_{\min}(\widehat{P})} \left(\lambda_{\max}(P_1) c_1 \right. \\ &\quad \left. + d\lambda_{\max}(Q) c_1 + d^2 \lambda_{\max}(Z) c_0 + \gamma^2 e^{-\rho T} c_w T\right) < c_2, \\ &\quad \forall t \in [0, T]. \end{aligned} \tag{23}$$

By Definition 8, it is easy to see that system (10) is finite-time bounded.

Finally we prove that system (10) also achieves H_∞ performance to the prescribed disturbance attenuation level γ.

Noting inequality (19) and by some algebraic manipulations, then it is derived that

$$\begin{aligned} &\dot{V}(x(t)) - \rho V(x(t)) + z^T(t) z(t) - \gamma^2 e^{-\rho T} w^T(t) w(t) \\ &\le \left(x^T(t), x^T(t-d), w^T(t)\right) \begin{pmatrix} \begin{pmatrix} \widetilde{A}_{hk}^T P + P^T \widetilde{A}_{hk} + Q \\ -\frac{1}{d} E^T Z E - \rho E^T P \end{pmatrix} & P^T \widetilde{A}_{hd} + \frac{1}{d} E^T Z E & P^T \widetilde{D}_{1h} \\ \widetilde{A}_{hd}^T P + \frac{1}{d} E^T Z E & -Q - \frac{1}{d} E^T Z E & 0 \\ \widetilde{D}_{1h}^T P & 0 & 0 \end{pmatrix} \begin{pmatrix} x(t) \\ x(t-d) \\ w(t) \end{pmatrix} \\ &\quad + d\left(x^T(t), x^T(t-d), w^T(t)\right) \begin{pmatrix} \widetilde{A}_{hk}^T \\ \widetilde{A}_{hd}^T \\ \widetilde{D}_{1h}^T \end{pmatrix} Z \left(\widetilde{A}_{hk} \;\; \widetilde{A}_{hd} \;\; \widetilde{D}_{1h}\right) \begin{pmatrix} x(t) \\ x(t-d) \\ w(t) \end{pmatrix} \\ &\quad + \left(x^T(t), x^T(t-d), w^T(t)\right) \begin{pmatrix} \widetilde{C}_h^T \\ 0 \\ \widetilde{D}_{2h}^T \end{pmatrix} \left(\widetilde{C}_h \;\; 0 \;\; \widetilde{D}_{2h}\right) \begin{pmatrix} x(t) \\ x(t-d) \\ w(t) \end{pmatrix} - \gamma^2 e^{-\rho T} w^T(t) w(t). \end{aligned} \tag{24}$$

By inequalities (15) and (24), it is obtained that

$$\dot{V}(x(t)) - \rho V(x(t)) < -z^T(t) z(t) + \gamma^2 e^{-\rho T} w^T(t) w(t). \tag{25}$$

Before and after multiplying (25) by $e^{-\rho t}$, then integrating the inequality from 0 to T leads to

$$\begin{aligned} 0 \le e^{-\rho T} V(x(T)) &= e^{-\rho T} V(x(T)) - V(x(0)) \\ &< \int_0^T \left(-e^{-\rho t} z^T(t) z(t) + \gamma^2 e^{-\rho T} e^{-\rho t} w^T(t) w(t)\right) dt. \end{aligned} \tag{26}$$

Thus, we can obtain that

$$\begin{aligned} e^{-\rho T} \int_0^T z^T(t)\, z(t)\, \mathrm{d}t &\leq \int_0^T e^{-\rho t} z^T(t)\, z(t)\, \mathrm{d}t \\ &\leq e^{-\rho T} \int_0^T e^{-\rho t} \gamma^2 w^T(t)\, w(t)\, \mathrm{d}t \\ &\leq e^{-\rho T} \gamma^2 \int_0^T w^T(t)\, w(t)\, \mathrm{d}t. \end{aligned} \tag{27}$$

That is, $\int_0^T z^T(t) z(t) \mathrm{d}t \leq \gamma^2 \int_0^T w^T(t) w(t) \mathrm{d}t$. Thus system (10) holds the H_∞ performance for the prescribed disturbance attenuation level γ. This completes the proof. □

Remark 10. In the proceeding of the finite-time H_∞ controller design, one of the difficulties is a nonpositive matrix produced from a descriptor system, which will result in an infeasible linear matrix inequality. For overcoming the difficulty, we construct a nonsingular matrix $\widehat{P} = \begin{pmatrix} P_1 & 0 \\ 0 & \widehat{P}_3 \end{pmatrix}$ satisfying $E^T P = E^T \widehat{P} E$, which can provide the feasibility conditions to solve the finite-time boundedness problem of a descriptor system via LMIs. And the proof is simpler and easier to understand than one of the existing literatures on the descriptor systems.

Remark 11. In the following, we give the design method of the finite-time H_∞ fuzzy controller design via LMIs. In order to simplify the process of controller design, we select the positive matrix Z of Theorem 9 as $\begin{pmatrix} Z_1 & 0 \\ 0 & Z_3 \end{pmatrix}$. The main issue of the delay-dependent robust control is that the presence of inverse matrices of Q, Z normally does not found LMIs. This problem was successfully solved By Lemma 4 and its deformation.

Theorem 12. *Given positive numbers d_0, ρ, and γ, then for any delay $0 < d \leq d_0$, system (10) is impulse-free and finite-time bounded with respect to (c_1, c_2, T, c_w) and holds H_∞ performance for the prescribed disturbance attenuation level γ, if there exist a common matrix $X = P^{-T} = \begin{pmatrix} P_1^{-1} & -P_1^{-1} P_2^T P_3^{-T} \\ 0 & P_3^{-T} \end{pmatrix} := \begin{pmatrix} X_1 & X_2 \\ 0 & X_3 \end{pmatrix}$, matrices $N_i = K_i P^{-1}$, nonsingular matrix $\widehat{X} = \widehat{P}^{-1} = \begin{pmatrix} P_1^{-1} & 0 \\ 0 & \widehat{P}_3^{-1} \end{pmatrix} = \begin{pmatrix} P_1^{-1} & 0 \\ 0 & \widehat{P}_3^{-1} \end{pmatrix} := \begin{pmatrix} X_1 & 0 \\ 0 & \widehat{X}_3 \end{pmatrix}$, and positive matrices $Q = \begin{pmatrix} Q_1 & Q_2 \\ Q_2^T & Q_3 \end{pmatrix}$, $\overline{Z} = Z^{-1} = \begin{pmatrix} Z_1^{-1} & 0 \\ 0 & Z_3^{-1} \end{pmatrix} := \begin{pmatrix} \overline{Z}_1 & 0 \\ 0 & \overline{Z}_3 \end{pmatrix}$, such that the following set of LMIs hold:*

$$\begin{gathered} \overline{\Psi}_{ii} < 0, \\ i = 1, 2, \ldots, r, \end{gathered} \tag{28}$$

$$\begin{gathered} \overline{\Psi}_{ij} + \overline{\Psi}_{ji} < 0, \\ j < i = 1, 2, \ldots, r, \end{gathered} \tag{29}$$

$$\begin{pmatrix} X_1 & I \\ I & \lambda_1 I \end{pmatrix} > 0, \tag{30}$$

$$Q < \lambda_2 I_n, \tag{31}$$

$$\begin{pmatrix} \lambda_3 I & I \\ I & \overline{Z} \end{pmatrix} > 0 \tag{32}$$

$$\begin{pmatrix} X_1 & 0 \\ 0 & \widehat{X}_3 \end{pmatrix} < I, \tag{33}$$

$$\left(\lambda_1 c_1 + d\lambda_2 c_1 + d^2 \lambda_3 c_0 + \gamma^2 e^{-\rho T} c_w T\right) e^{\rho T} < c_2, \tag{34}$$

where

$$\overline{\Psi}_{ij} = \begin{pmatrix} \begin{pmatrix} X A_i^T + N_j^T B_i^T \\ + A_i X^T + B_i N_j \\ \frac{1}{d} E^T \overline{Z} E + \frac{1}{d} E^T E X^T \\ + \frac{1}{d} X E^T E - \rho X E^T \end{pmatrix} & A_{id} & D_{1i} & \begin{pmatrix} X A_i^T \\ + N_j^T B_i^T \end{pmatrix} & X C_i^T & X & X E^T \\ A_{id}^T & -Q & 0 & A_{id}^T & 0 & 0 & 0 \\ D_{1i}^T & 0 & -\gamma^2 e^{-\rho T} I & D_{1i}^T & D_{2i}^T & 0 & 0 \\ A_i X^T + B_i N_j & A_{id} & D_{1i} & -\frac{1}{d} \overline{Z} & 0 & 0 & 0 \\ C_i X^T & 0 & D_{2i} & 0 & -I & 0 & 0 \\ X^T & 0 & 0 & 0 & 0 & Q - 2I & 0 \\ E X^T & 0 & 0 & 0 & 0 & 0 & -d\overline{Z} \end{pmatrix} < 0. \tag{35}$$

Proof. Before and after multiplying inequality (11) by $\operatorname{diag}\{P^{-T}, I, I, I, I\}$ and its transpose respectively, then it is obtained that

$$\begin{pmatrix} \begin{pmatrix} P^{-T}A_i^T + P^{-T}K_i^T B_i^T \\ +A_i P^{-1} + B_i K_i P^{-1} \\ +P^{-T}QP^{-1} - \rho P^{-T}E^T \\ -\frac{1}{d}P^{-T}E^T ZEP^{-1} \end{pmatrix} & A_{id} + \frac{1}{d}P^{-T}E^T ZE & D_{1i} & \begin{pmatrix} P^{-T}A_i^T \\ +P^{-T}K_i^T B_i^T \end{pmatrix} & P^{-T}C_i^T \\ A_{id}^T + \frac{1}{d}E^T ZEP^{-1} & -Q - \frac{1}{d}E^T ZE & 0 & A_{id}^T & 0 \\ D_{1i}^T & 0 & -\gamma^2 e^{-\rho T} I & D_{1i}^T & D_{2i}^T \\ A_i P^{-1} + B_i K_i P^{-1} & A_{id} & D_{1i} & -\frac{1}{d}Z^{-1} & 0 \\ C_i P^{-1} & 0 & D_{2i} & 0 & -I \end{pmatrix} < 0. \quad (36)$$

By Lemma 4, it can be seen that $-(1/d)P^{-T}E^T ZEP^{-1} \le (1/d)E^T Z^{-1}E + (1/d)E^T EP^{-1} + (1/d)P^{-T}E^T E$. Therefore, inequality (36) gives

$$\begin{pmatrix} \begin{pmatrix} P^{-T}A_i^T + P^{-T}K_i^T B_i^T \\ +A_i P^{-1} + B_i K_i P^{-1} \\ +P^{-T}QP^{-1} - \rho P^{-T}E^T \\ +\frac{1}{d}E^T Z^{-1}E \\ +\frac{1}{d}E^T EP^{-1} + \frac{1}{d}P^{-T}E^T E \end{pmatrix} & A_{id} + \frac{1}{d}P^{-T}E^T ZE & D_{1i} & \begin{pmatrix} P^{-T}A_i^T \\ +P^{-T}K_i^T B_i^T \end{pmatrix} & P^{-T}C_i^T \\ A_{id}^T + \frac{1}{d}E^T ZEP^{-1} & -Q - \frac{1}{d}E^T ZE & 0 & A_{id}^T & 0 \\ D_{1i}^T & 0 & -\gamma^2 e^{-\rho T} I & D_{1i}^T & D_{2i}^T \\ A_i P^{-1} + B_i K_i P^{-1} & A_{id} & D_{1i} & -\frac{1}{d}Z^{-1} & 0 \\ C_i P^{-1} & 0 & D_{2i} & 0 & -I \end{pmatrix} < 0. \quad (37)$$

Now, set $X = P^{-T}$, $N_i = K_i P^{-1}$. Then, by inequality (37), it can be obtained that

$$\begin{pmatrix} \begin{pmatrix} XA_i^T + N_i^T B_i^T + A_i X^T + B_i N_i \\ +XQX^T - \rho XE^T + \frac{1}{d}E^T Z^{-1}E \\ +\frac{1}{d}E^T EX^T + \frac{1}{d}XE^T E \end{pmatrix} & A_{id} + \frac{1}{d}XE^T ZE & D_{1i} & \begin{pmatrix} XA_i^T \\ +N_i^T B_i^T \end{pmatrix} & XC_i^T \\ A_{id}^T + \frac{1}{d}E^T ZEX^T & -Q - \frac{1}{d}E^T ZE & 0 & A_{id}^T & 0 \\ D_{1i}^T & 0 & -\gamma^2 e^{-\rho T} I & D_{1i}^T & D_{2i}^T \\ A_i X^T + B_i N_i & A_{id} & D_{1i} & -\frac{1}{d}Z^{-1} & 0 \\ C_i X^T & 0 & D_{2i} & 0 & -I \end{pmatrix} < 0. \quad (38)$$

From Lemma 4, it can be seen that

$$\frac{1}{d}\begin{pmatrix} XE^T \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} Z \begin{pmatrix} 0 & E & 0 & 0 & 0 \end{pmatrix} + \frac{1}{d}\begin{pmatrix} 0 \\ E^T \\ 0 \\ 0 \\ 0 \end{pmatrix} Z \begin{pmatrix} EX^T & 0 & 0 & 0 & 0 \end{pmatrix}$$
$$\leq \frac{1}{d}\begin{pmatrix} XE^T \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} Z \cdot Z^{-1} Z \begin{pmatrix} EX^T & 0 & 0 & 0 & 0 \end{pmatrix} + \frac{1}{d}\begin{pmatrix} 0 \\ E^T \\ 0 \\ 0 \\ 0 \end{pmatrix} Z \begin{pmatrix} 0 & E & 0 & 0 & 0 \end{pmatrix}. \quad (39)$$

Thus, inequalities (38) and (39) give that

$$\begin{pmatrix} \begin{pmatrix} XA_i^T + N_i^T B_i^T \\ +A_i X^T + B_i N_i \\ +XQX^T - \rho XE^T \\ +\frac{1}{d}E^T Z^{-1} E \\ +\frac{1}{d}E^T EX^T + \frac{1}{d}XE^T E \\ +\frac{1}{d}XE^T ZEX^T \end{pmatrix} & A_{id} & D_{1i} & \left(XA_i^T + N_i^T B_i^T\right) & XC_i^T \\ A_{id}^T & -Q & 0 & A_{id}^T & 0 \\ D_{1i}^T & 0 & -\gamma^2 e^{-\rho T} I & D_{1i}^T & D_{2i}^T \\ A_i X^T + B_i N_i & A_{id} & D_{1i} & -\frac{1}{d}Z^{-1} & 0 \\ C_i X^T & 0 & D_{2i} & 0 & -I \end{pmatrix} < 0. \quad (40)$$

Then, by Schur complement, inequality (40) is equivalent to

$$\begin{pmatrix} \begin{pmatrix} XA_i^T + N_i^T B_i^T \\ +A_i X^T + B_i N_i \\ \frac{1}{d}E^T Z^{-1} E \\ +\frac{1}{d}E^T EX^T \\ +\frac{1}{d}XE^T E \\ -\rho XE^T \end{pmatrix} & A_{id} & D_{1i} & \begin{pmatrix} XA_i^T \\ +N_i^T B_i^T \end{pmatrix} & XC_i^T & X & XE^T \\ A_{id}^T & -Q & 0 & A_{id}^T & 0 & 0 & 0 \\ D_{1i}^T & 0 & -\gamma^2 e^{-\rho T} I & D_{1i}^T & D_{2i}^T & 0 & 0 \\ A_i X^T + B_i N_i & A_{id} & D_{1i} & -\frac{1}{d}Z^{-1} & 0 & 0 & 0 \\ C_i X^T & 0 & D_{2i} & 0 & -I & 0 & 0 \\ X^T & 0 & 0 & 0 & 0 & -Q^{-1} & 0 \\ EX^T & 0 & 0 & 0 & 0 & 0 & -dZ^{-1} \end{pmatrix} < 0. \quad (41)$$

By Lemma 4, we have $-Q^{-1} \leq Q - 2I$. Let $\overline{Z} = Z^{-1}$. Then, inequality (41) implies $\overline{\Psi}_{ii} < 0$, $i = 1, 2, \ldots, r$.

Similarly, we can prove that $\Psi_{ij}+\Psi_{ji} < 0$ only if $\overline{\Psi}_{ij}+\overline{\Psi}_{ji} < 0$. The proof is omitted to save space.

Inequality (30) is equivalent to $X_1 > (1/\lambda_1)I$, which implies $\lambda_{\max}(X_1^{-1}) = \lambda_{\max}(P_1) < \lambda_1$. Inequality (31) gives $\lambda_{\max}(Q) < \lambda_2$. Inequality (32) is equivalent to $0 < \lambda_3 I - \overline{Z}^{-1}$, which results in $\lambda_{\max}(Z) < \lambda_3$. Inequality (33) is equivalent to $\widehat{P}^{-1} < I$, which can derive $\lambda_{\min}(\widehat{P}) > 1$. Thus, from the above inequalities and (34), it can be derived that

$$\begin{aligned}&\left(\lambda_{\max}\left(P_1\right)c_1 + d\lambda_{\max}\left(Q\right)c_1 + d^2\lambda_{\max}\left(Z\right)c_0\right.\\&\quad\left.+\gamma^2 e^{-\rho T}c_w T\right)e^{\rho T} \leq \left(\lambda_1 c_1 + d\lambda_2 c_1 + d^2\lambda_3 c_0\right.\\&\quad\left.+\gamma^2 e^{-\rho T}c_w T\right)e^{\rho T} < c_2 < c_2\lambda_{\min}\left(\widehat{P}\right).\end{aligned} \tag{42}$$

That is, condition (13) of Theorem 9 holds only if all conditions of inequalities (30), (31), (32), (33), and (34) in Theorem 12 are met. This completes the proof. □

By using Theorem 12, we can also obtain the following finite-time H_∞ controller design method for system (10) without time-delay.

Corollary 13. *Given positive numbers ρ and γ, then system (10) without time-delay is impulse-free and finite-time bounded with respect to (c_1, c_2, T, c_w) and holds H_∞ performance for the prescribed disturbance attenuation level γ, if there exist a common matrix $X = P^{-T} = \begin{pmatrix} P_1^{-1} & -P_1^{-1}P_2^TP_3^{-T} \\ 0 & P_3^{-T}\end{pmatrix} := \begin{pmatrix} X_1 & X_2 \\ 0 & X_3\end{pmatrix}$, matrices $N_i = K_i P^{-1}$, and nonsingular matrices $\widehat{X} = \widehat{P}^{-1} = \begin{pmatrix} P_1^{-1} & 0 \\ 0 & \widehat{P}_3^{-1}\end{pmatrix} = \begin{pmatrix} P_1^{-1} & 0 \\ 0 & \widehat{P}_3^{-1}\end{pmatrix} := \begin{pmatrix} X_1 & 0 \\ 0 & \widehat{X}_3\end{pmatrix}$, such that the following set of LMIs hold:*

$$\begin{gathered}\widehat{\Psi}_{ii} < 0, \quad i = 1, 2, \ldots, r,\\ \widehat{\Psi}_{ij} + \widehat{\Psi}_{ji} < 0, \quad j < i = 1, 2, \ldots, r,\\ \begin{pmatrix} X_1 & I \\ I & \lambda_1 I\end{pmatrix} > 0,\\ \begin{pmatrix} X_1 & 0 \\ 0 & \widehat{X}_3\end{pmatrix} < I,\\ \left(\lambda_1 c_1 + \gamma^2 e^{-\rho T}c_w T\right)e^{\rho T} < c_2,\end{gathered} \tag{43}$$

where

$$\widehat{\Psi}_{ij} = \begin{pmatrix} \left(XA_i^T + N_j^TB_i^T + A_iX^T + B_iN_j - \rho XE^T\right) & D_{1i} & XC_i^T \\ D_{1i}^T & -\gamma^2 e^{-\rho T}I & D_{2i}^T \\ C_iX^T & D_{2i} & -I\end{pmatrix}. \tag{44}$$

4. Design Examples

Example 1. We know that it is more convenient to express practical models as the descriptor systems than ordinary ones such as a biological economic model with differential and algebraic equation established in [27]. The following economic model is expressed by a descriptor system instead of the ordinary one as a result of an algebraic equation:

$$\begin{aligned}\dot{\varsigma}_1(t) &= \left(-\frac{\alpha\beta}{r_2} - \frac{\eta c}{p}\right)\varsigma_1(t) + \alpha\varsigma_2(t) - \frac{c}{p}\varsigma_3(t)\\&\quad - \eta\varsigma_1^2(t) - \varsigma_1(t)\varsigma_3(t) + b_{11}w(t),\\ \dot{\varsigma}_2(t) &= \beta\varsigma_1(t) - r_2\varsigma_2(t),\\ 0 &= p\left(\frac{\alpha\beta}{r_2} - r_1 - \beta - \frac{\eta c}{p}\right)\varsigma_1(t) + p\varsigma_1(t)\varsigma_3(t)\\&\quad - m,\end{aligned} \tag{45}$$

where $m = 0$ is a bifurcation value. Equation (45) describes a biological economic model obtained by the transformation. It is a differential algebraic equation, where the meanings of parameters can be found in [27]. When m transitions 0 from negative to positive, an impulse phenomenon is formed. Carry outing a control for system (45), then the following system is given:

$$\begin{aligned}\dot{\varsigma}_1(t) &= \left(-\frac{\alpha\beta}{r_2} - \frac{\eta c}{p}\right)\varsigma_1(t) + \alpha\varsigma_2(t) - \frac{c}{p}\varsigma_3(t)\\&\quad - \eta\varsigma_1^2(t) - \varsigma_1(t)\varsigma_3(t) + b_{11}w(t),\\ \dot{\varsigma}_2(t) &= \beta\varsigma_1(t) - r_2\varsigma_2(t),\\ 0 &= p\left(\frac{\alpha\beta}{r_2} - r_1 - \beta - \frac{\eta c}{p}\right)\varsigma_1(t) + p\varsigma_1(t)\varsigma_3(t)\\&\quad + u(t),\\ z(t) &= \varsigma_1(t).\end{aligned} \tag{46}$$

Equation (46) can be represented by the following fuzzy descriptor system:

$$\begin{aligned}E\dot{x}(t) &= \sum_{i=1}^{2}\lambda_i\left(\varsigma_1(t)\right)\left(A_ix(t) + D_{1i}w(t) + B_iu(t)\right),\\ z(t) &= \sum_{i=1}^{2}\lambda_i\left(\varsigma_1(t)\right)C_ix(t),\end{aligned} \tag{47}$$

where

$$E = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$A_1 = \begin{pmatrix} -\frac{\alpha\beta}{r_2} - \frac{\eta c}{p} + \eta l & \alpha & -\frac{c}{p} + l \\ \beta & -r_2 & 0 \\ p\left(\frac{\alpha\beta}{r_2} - r_1 - \beta - \frac{\eta c}{p}\right) & 0 & -pl \end{pmatrix},$$

$$A_2 = \begin{pmatrix} -\frac{\alpha\beta}{r_2} - \frac{\eta c}{p} - \eta l & \alpha & -\frac{c}{p} - l \\ \beta & -r_2 & 0 \\ p\left(\frac{\alpha\beta}{r_2} - r_1 - \beta - \frac{\eta c}{p}\right) & 0 & pl \end{pmatrix},$$

$$D_{11} = D_{12} = \begin{pmatrix} b_{11} \\ 0 \\ 0 \end{pmatrix}, \tag{48}$$

$$B_1 = B_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix},$$

$$D_{21} = D_{22} = 0,$$

$$C_1 = C_2 = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix},$$

$$\lambda_1(\varsigma_1(t)) = \frac{1}{2}\left(1 - \frac{\varsigma_1(t)}{l}\right),$$

$$\lambda_2(\varsigma_1(t)) = \frac{1}{2}\left(1 + \frac{\varsigma_1(t)}{l}\right),$$

$$|\varsigma_1(t)| < l, \quad l > 0,$$

$$x(t) = [\varsigma_1(t), \varsigma_2(t), \varsigma_3(t)]^T.$$

Select the coefficients of the matrices $\alpha = 0.15$, $\beta = 0.5$, $r_1 = 0.2$, $r_2 = 0.1$, $\eta = 0.001$, $p = 1$, $c = 40$, $b_{11} = 0.1$, and $l = 10$ in [27]. Let $T = 2$, $c_1 = 0.2$, $c_w = 1$, $\rho = 0.02$, and $\lambda_1 = 0.01$. By using Corollary 13 and the FEASP solver and GEVP solver of the LMI toolbox, we can obtain $\min c_2 = 0.2100$. For $c_2 = 22$, we can obtain the feasible solutions of a finite-time fuzzy controller with the norm H_∞ bound $\gamma = 0.01$ as follows:

$$X = 10^3 \cdot \begin{bmatrix} 0.0006 & -0.0002 & 0.0143 \\ -0.0002 & 0.0007 & 0.0000 \\ 0 & 0 & -1.6642 \end{bmatrix},$$

$$\widehat{X}_3 = -1.6617 \cdot 10^3,$$

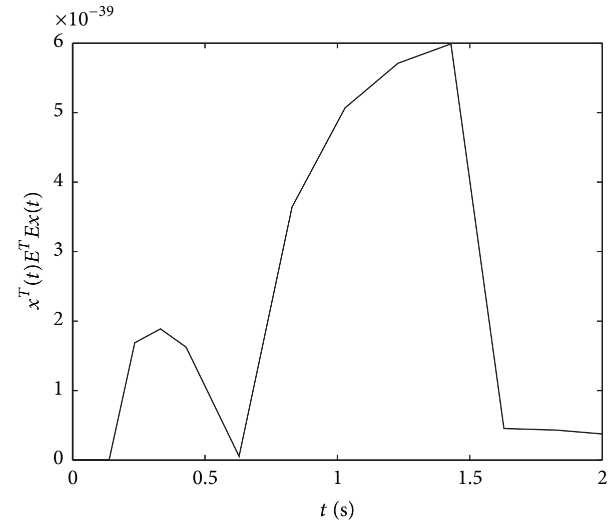

Figure 1: The trajectory of $x^T(t)E^T Ex(t)$ for the nonlinear system approximated by the fuzzy system.

$$N_1 = 10^4 \cdot [-4.9784 \;\; 0.0000 \;\; -1.7224],$$

$$N_2 = 10^4 \cdot [-8.3353 \;\; -0.0000 \;\; 1.6061],$$

$$\begin{aligned} u(t) = {} & \frac{1}{2}\left(1 - \frac{\varsigma_1(t)}{10}\right) \cdot 10^4 \\ & \cdot [-8.7569 \;\; -2.1106 \;\; 0.0010]\, x(t) \\ & + \frac{1}{2}\left(1 + \frac{\varsigma_1(t)}{10}\right) \cdot 10^4 \\ & \cdot [-1.4594 \;\; -0.3518 \;\; -0.0001]\, x(t). \end{aligned} \tag{49}$$

Let $w(t) = e^{-0.1t} \sin t$. Then the trajectory of $x^T(t)E^T Ex(t)$ for the nonlinear system approximated by a fuzzy system is shown in Figure 1.

Example 2. Consider the following nonlinear descriptor system with time-delay:

$$\begin{aligned} \dot{\varsigma}_1(t) = {} & \left(-\frac{\alpha\beta}{r_2} - \frac{\eta c}{p}\right)\varsigma_1(t) + \alpha\varsigma_2(t) - \frac{c}{p}\varsigma_3(t) \\ & - \eta\varsigma_1^2(t) - \varsigma_1(t)\varsigma_3(t) + d_{11}w(t) \\ & + \tau_{11}\varsigma_1(t-d), \\ \dot{\varsigma}_2(t) = {} & \beta\varsigma_1(t) - r_2\varsigma_2(t), \\ 0 = {} & p\left(\frac{\alpha\beta}{r_2} - r_1 - \beta - \frac{\eta c}{p}\right)\varsigma_1(t) + p\varsigma_1(t)\varsigma_3(t) \\ & + b_{11}u(t), \\ z(t) = {} & c_{11}\varsigma_1(t). \end{aligned} \tag{50}$$

Defining $x(t) = [\varsigma_1(t), \varsigma_2(t), \varsigma_3(t)]^T$, $\varsigma_i(t) \in R^1$ $(i = 1, 2, 3)$. The above equation can be rewritten as

$$\begin{aligned}
&\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \dot{x}(t) \\
&= \begin{pmatrix} -\dfrac{\alpha\beta}{r_2} - \dfrac{\eta c}{p} - \eta\varsigma_1(t) & \alpha & -\dfrac{c}{p} - \varsigma_1(t) \\ \beta & -r_2 & 0 \\ p\left(\dfrac{\alpha\beta}{r_2} - r_1 - \beta - \dfrac{\eta c}{p}\right) & 0 & p\varsigma_1(t) \end{pmatrix} x(t) \\
&\quad + \begin{pmatrix} \tau_{11} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} x(t-d) + \begin{pmatrix} 0 \\ 0 \\ b_{11} \end{pmatrix} u(t) \\
&\quad + \begin{pmatrix} d_{11} \\ 0 \\ 0 \end{pmatrix} w(t), \\
&z(t) = \begin{pmatrix} c_{11} & 0 & 0 \end{pmatrix} x(t).
\end{aligned} \tag{51}$$

Choose, respectively, the membership functions of the fuzzy sets M_1, M_2 as $\lambda_1(\varsigma_1(t)) = (1/2)(1 - \varsigma_1(t)/l)$, $\lambda_2(\varsigma_1(t)) = (1/2)(1 + \varsigma_1(t)/l)$, and $|\varsigma_1(t)| < l$, $l > 0$. The following T-S fuzzy model exactly represents the dynamics of nonlinear descriptor system (50) or (51):

$$\begin{aligned}
\text{Model Rule 1: If}\quad & \varsigma_1(t) \text{ is } M_1, \\
\text{Then}\quad & E\dot{x}(t) \\
& = A_1 x(t) + A_{1d} x(t-d) \\
& \quad + B_1 u(t) + D_{11} w(t), \\
& z(t) = C_1 x(t) + D_{21} w(t), \\
& x(t) = \varphi(t), \quad t \in [-d, 0], \\
\text{Model Rule 2: If}\quad & \varsigma_1(t) \text{ is } M_2, \\
\text{Then}\quad & E\dot{x}(t) \\
& = A_2 x(t) + A_{2d} x(t-d) \\
& \quad + B_2 u(t) + D_{12} w(t), \\
& z(t) = C_2 x(t) + D_{22} w(t), \\
& x(t) = \varphi(t), \quad t \in [-d, 0],
\end{aligned} \tag{52}$$

where

$$\begin{aligned}
E &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \\
A_1 &= \begin{pmatrix} -\dfrac{\alpha\beta}{r_2} - \dfrac{\eta c}{p} + \eta l & \alpha & -\dfrac{c}{p} + l \\ \beta & -r_2 & 0 \\ p\left(\dfrac{\alpha\beta}{r_2} - r_1 - \beta - \dfrac{\eta c}{p}\right) & 0 & -pl \end{pmatrix}, \\
A_2 &= \begin{pmatrix} -\dfrac{\alpha\beta}{r_2} - \dfrac{\eta c}{p} - \eta l & \alpha & -\dfrac{c}{p} - l \\ \beta & -r_2 & 0 \\ p\left(\dfrac{\alpha\beta}{r_2} - r_1 - \beta - \dfrac{\eta c}{p}\right) & 0 & pl \end{pmatrix}, \\
A_{1d} &= A_{2d} = \begin{pmatrix} \tau_{11} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \\
D_{11} &= D_{12} = \begin{pmatrix} d_{11} \\ 0 \\ 0 \end{pmatrix}, \\
B_1 &= B_2 = \begin{pmatrix} 0 \\ 0 \\ b_{11} \end{pmatrix}, \\
D_{21} &= D_{22} = 0, \\
C_1 &= C_2 = \begin{pmatrix} c_{11} & 0 & 0 \end{pmatrix}.
\end{aligned} \tag{53}$$

Equation (50) can be represented by the following fuzzy descriptor system:

$$\begin{aligned}
E\dot{x}(t) &= \sum_{i=1}^{2} \lambda_i(\varsigma_1(t)) \\
&\quad \cdot \left(A_i x(t) + A_{id} x(t-d) + B_i u(t) + D_{1i} w(t)\right), \\
z(t) &= \sum_{i=1}^{2} \lambda_i(\varsigma_1(t)) \left(C_i x(t) + D_{2i} u(t)\right).
\end{aligned} \tag{54}$$

Select the coefficients of the matrices $\alpha = 0.15$, $\beta = 0.5$, $r_1 = 0.2$, $r_2 = 0.1$, $\eta = 0.001$, $p = 1$, $c = 220$, $\tau_{11} = 2.0034 \cdot 10^{-5}$, $d_{11} = -4.2344 \cdot 10^{-5}$, $b_{11} = -0.0725$, $c_{11} = 0.0317$, and $l = 220$. Let $d = 0.0001$, $T = 2$, $c_0 = 0$ $c_1 = 0.02$, $c_w = 2$, and $\rho = 0.00002$. By using Theorem 12 and the FEASP solver and GEVP solver of the LMI toolbox, we can obtain $\min c_2 = 20$.

For $c_2 = 32$, we can obtain the feasible solutions of a finite-time fuzzy controller with the H_∞ norm bound $\gamma = 0.0001$ as follows:

$$\begin{aligned} X_1 &= 10^{-7} \cdot \begin{pmatrix} 0.2887 & -0.0000 \\ -0.0000 & 0.2900 \end{pmatrix}, \\ X_3 &= \widehat{X}_3 = -0.0122, \\ N_1 &= 10^3 \cdot [0.0000 \;\; 0.0000 \;\; 4.3297], \\ N_2 &= 10^3 \cdot [-8.3353 \;\; -0.0000 \;\; 4.3320], \\ Q &= \begin{pmatrix} 0.9973 & -0.0000 & 0 \\ -0.0000 & 0.9973 & 0 \\ 0 & 0 & 0.9973 \end{pmatrix}, \\ Z &= 10^7 \cdot \begin{pmatrix} 0.0000 & -0.0000 & 0 \\ -0.0000 & 0.0000 & 0 \\ 0 & 0 & 2.6151 \end{pmatrix}. \end{aligned} \tag{55}$$

The finite-time H_∞ fuzzy controller is as follows:

$$\begin{aligned} u(t) = {} & \frac{1}{2}\left(1 - \frac{\varsigma_1(t)}{220}\right) 10^5 \\ & \cdot [0.0245 \;\; 0.0000 \;\; -3.5489]\, x(t) \\ & + \frac{1}{2}\left(1 + \frac{\varsigma_1(t)}{220}\right) 10^5 \\ & \cdot [0.2828 \;\; 0.0000 \;\; -3.5508]\, x(t). \end{aligned} \tag{56}$$

Let $w(t) = e^{-t} \sin t$. Then the trajectories of $x^T(t)E^T Ex(t)$ for the approximated nonlinear system are shown in Figure 2.

5. Conclusions

This paper investigates the delay-dependent finite-time H_∞ controller design problems for a kind of nonlinear descriptor system via a T-S fuzzy model. The solvable conditions of finite-time H_∞ controller are given to guarantee that the loop-closed system is impulse-free and finite-time bounded and holds the H_∞ performance to a prescribed disturbance attenuation level γ. The method given is the ability to eliminate the impulsive behavior caused by descriptor systems in a finite-time interval, which confirms the existence and uniqueness of solutions in the interval. We perform simulations to validate the proposed methods for a nonlinear descriptor system via the T-S fuzzy model, which shows the application of the T-S fuzzy method in studying the finite-time control problem of a nonlinear system. Meanwhile the method was also applied to the biological economy system to eliminate impulsive behavior at the bifurcation value, stabilize the loop-closed system in a finite-time interval, and achieve a H_∞ performance level.

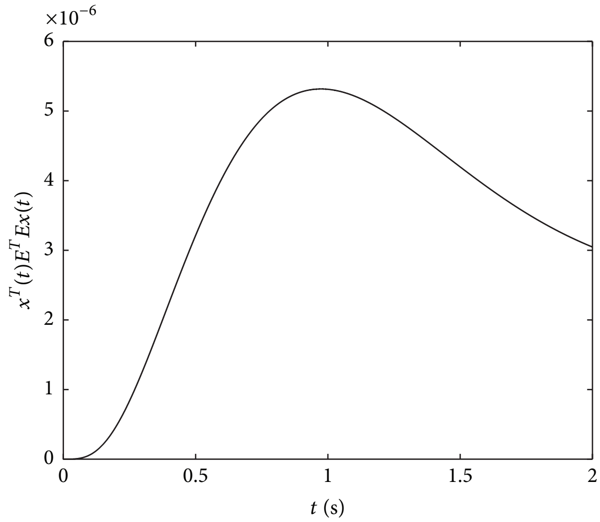

FIGURE 2: The trajectory of $x^T(t)E^T Ex(t)$ for the nonlinear system approximated by the fuzzy system.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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