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Research Article

New Results on Passivity for Discrete-Time Stochastic Neural Networks with Time-Varying Delays

Wei Kang,[1,2] Jun Cheng,[3] and Xiangyang Cheng[4]

[1]*School of Mathematical Sciences, University of Electronic Science and Technology of China, Chengdu 611731, China*
[2]*School of Information Engineering, Fuyang Teachers College, Fuyang 236041, China*
[3]*School of Electrical and Information Technology, Yunnan Minzu University, Kunming 650500, China*
[4]*School of Business, Fuyang Teachers College, Fuyang 236041, China*

Correspondence should be addressed to Wei Kang; kangwei0830@163.com

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The problem of passivity analysis for discrete-time stochastic neural networks with time-varying delays is investigated in this paper. New delay-dependent passivity conditions are obtained in terms of linear matrix inequalities. Less conservative conditions are obtained by using integral inequalities to aid in the achievement of criteria ensuring the positiveness of the Lyapunov-Krasovskii functional. At last, numerical examples are given to show the effectiveness of the proposed method.

1. Introduction

Neural networks have been greatly applied in many areas in the past few decades, such as static processing, pattern recognition, and combinatorial optimization [1–3]. In practice, time-delays are frequently encountered in neural networks. As the finite signal propagation time and the finite speed of information processing, the existence of the delays may cause oscillation, instability, and divergence in neural networks. Moreover, stochastic perturbations and parameter uncertainties are two main resources which could reduce the performances of delayed neural networks. Due to the importance in both theory and practice, the problem of stability for stochastic delayed neural networks with parameter uncertainties is one of hot issues. Therefore, there have been lots of important and interesting results in this field [3–17].

It should be noticed that most neural networks are focused on continuous-time case [3, 7–12]. However, discrete-time systems play crucial roles in today's information society. Particulary, when implementing the delayed continuous-time neural networks for computer simulation, it needs to formulate discrete-time system. Thus, it is necessary to research the dynamics of discrete-time neural networks. In recent years, a lot of important results have been published in the literatures [13–17]. Kwon et al. [14] discussed the stability criteria for the discrete-time system with time varying delays. Wang et al. [16] researched the exponential stability of discrete-time neural networks with distributed delays by means of Lyapunov-Krasovskill functional theory and linear matrix inequalities technology. In [17], the authors are concerned with the robust state estimation for discrete neural networks with successive packet dropouts, linear fractional uncertainties, and mixed time-delays.

On the other hand, passivity is a significant concept that represents input-output feature of dynamic systems, which can offer a powerful tool for analyzing mechanical systems, nonlinear systems, and electrical circuits [18]. The passivity theory was firstly presented in the circuit analysis [19]. During the past several decades, the passivity theory has found successful applications in various areas such as complexity, signal processing, stability, chaos control, and fuzzy control. Thus, the problem of passivity for time-delay neural networks has received much attention and lots of effective approaches have been proposed in this research area [20–27]. The authors [21, 22] discussed the problem of passivity for neural networks with time-delays. Recently, Lee et al. [23] further studied the problem of dissipative analysis for neural networks with times-delays by using reciprocally convex approach and

linear matrix inequality technology. Very recently, in [24], the problem of passivity criterion of discrete-time stochastic bidirectional associative memory neural networks with time-varying delays has been developed. In [25], some delay-dependent sufficient passivity conditions have been obtained for stochastic discrete-time neural networks with time-varying delays in terms of linear matrix inequalities technology and free-weighting matrices approach. A less conservative passivity criterion for discrete-time stochastic neural networks with time-varying delays was derived in [26]. However, there is still a room for decreasing the conservatism.

Motivated by the above discussion, the problem of passivity for discrete-time stochastic neural networks with time-varying delays is studied. The major contribution of this paper lies in that, first of all, different from the traditional way, a new inequality is introduced to deal with terms $\sum_{i=k-\tau_m}^{k-1} \eta^T(i)T_1\eta(i)$ and $\sum_{i=k-\tau_M}^{k-\tau_m-1} \eta^T(i)T_1\eta(i)$. This method can effectively reduce the conservatism. Secondly, we do not need all the symmetric matrices in the Lyapunov functional to be positive definite and take advantage of the relationships of $\tau(k)-\tau_m$ and $\tau_M-\tau(k)$. New passivity conditions are presented in terms of matrix inequalities. Finally, numerical examples are given to indicate the effectiveness of the proposed method.

Notations. Throughout this paper, the superscripts "-1" and "T" stand for the inverse and transpose of a matrix, respectively; $P>0$ ($P\geqslant 0$, $P<0$, $P\leqslant 0$) means that the matrix P is symmetric positive definite (positive semidefinite, negative definite, and negative semidefinite); $E\{\cdot\}$ stands for the mathematical expectation operator with respect to the given probability measure; $\|\cdot\|$ refers to the Euclidean vector norm; $(\Omega, \mathscr{F}, \mathscr{P})$ denotes a complete probability space with a filtration containing all p-null sets and is right conditions; $N[a,b]$ denotes the discrete interval given $N[a,b]=\{a,a+1,\ldots,b-1,b\}$; R^n denotes n-dimensional Euclidean space; $R^{m\times n}$ is the set of $m\times n$ real matrices; $*$ denotes the symmetric block in symmetric matrix; $\lambda_{\max}(Q)$ and $\lambda_{\min}(Q)$ denote, respectively, the maximal and minimal eigenvalue of matrix Q.

2. Problem Statement and Preliminaries

Consider the following DSNN with time-varying delays:

$$\begin{aligned} x(k+1) &= Ax(k)+B_1g(x(k))+B_2g(x(k-\tau(k))) \\ &\quad +u(k)+\delta(x(k),x(k-\tau(k)))\omega(k), \\ y(k) &= g(x(k)), \\ x(j) &= \psi(j), \quad j=-\tau_M,-\tau_M+1,\ldots,0, \end{aligned} \tag{1}$$

where $x(k)=[x_1(k),x_2(k),\ldots,x_n(k)]^T\in R^n$ is the neuron state vector of the system; $y(k)$ is the output of the neural networks; $u(k)=[u_1(k),u_2(k),\ldots,u_n(k)]^T$ is the input vector; $\psi(j)$ is the initial condition; $A=\mathrm{diag}(a_1,a_2,\ldots,a_n)$ is the state feedback coefficient matrix; B_1 and B_2 are the connection weight matrices and the delayed connection weight matrices, respectively; $g(x(k))=[g_1(x_1(k)), g_2(x_2(k)),\ldots,g_n(x_n(k))]^T\in R^n$ represents the neuron activation functions; $\tau(k)$ denotes the known time-varying delay and satisfies $0<\tau_m\leq\tau(k)\leq\tau_M$; δ is diffusion coefficient vector and $\omega(k)$ is a scalar Brownian motion defined on the probability space $(\Omega,\mathscr{F},\mathscr{P})$ with

$$\begin{aligned} E\{\omega(k)\}&=0, \\ E\{\omega(k)\omega(k)\}&=1, \\ E\{\omega(i)\omega(j)\}&=0, \quad (i\neq j). \end{aligned} \tag{2}$$

Assumption 1. The neuron activation function $g(\cdot)$ satisfies

$$l_s^-\leqslant\frac{g_s(a)-g_s(b)}{a-b}\leqslant l_s^+, \quad g_s(0)=0,\ s=1,2,\ldots,n, \tag{3}$$

for all $a,b\in R$, $a\neq b$, where l_s^- and l_s^+ are known real constants.

Remark 2. In Assumption 1, l_s^- and l_s^+ can be positive, negative, or zero. Moreover, when $l_s^-=0$, then $l_s^+>0$.

Assumption 3. $\delta(x(k),x(k-\tau(k)))$ is the continuous function satisfying

$$\begin{aligned} &\delta^T(x(k),x(k-\tau(k)))\,\delta(x(k),x(k-\tau(k))) \\ &\quad\leq\rho_1x^T(k)x(k)+\rho_2x^T(k-\tau(k))x(k-\tau(k)), \end{aligned} \tag{4}$$

where ρ_1, ρ_2 are known constant scalars.

The following lemmas and definition will be used in proof of main results.

Lemma 4. *For integers $\tau(k)$ and vector function $x(k+\cdot): N[-\tau_M,-\tau_m]\mapsto R^n$, $\eta(k)=x(k+1)-x(k)$, for any positive semidefinite matrix*

$$X=\begin{bmatrix} X_{11} & X_{12} & X_{13} \\ X_{12}^T & X_{22} & X_{23} \\ X_{13}^T & X_{23}^T & X_{33} \end{bmatrix}\geq 0, \tag{5}$$

the following inequality holds:

$$\begin{aligned} &-\sum_{i=k-\tau(k)}^{k-1}\eta^T(i)X_{33}\eta(i) \\ &\leq\sum_{i=k-\tau(k)}^{k-1}\begin{bmatrix} x^T(k) & x^T(k-\tau(k)) & \eta^T(i) \end{bmatrix} \\ &\quad\cdot\begin{bmatrix} X_{11} & X_{12} & X_{13} \\ X_{12}^T & X_{22} & X_{23} \\ X_{13}^T & X_{23}^T & 0 \end{bmatrix}\begin{bmatrix} x(k) \\ x(k-\tau(k)) \\ \eta(i) \end{bmatrix}. \end{aligned} \tag{6}$$

Proof. In fact, we have

$$\begin{aligned}0 \le & \sum_{i=k-\tau(k)}^{k-1} \left[x^T(k) \;\; x^T(k-\tau(k)) \;\; \eta^T(i)\right] \\ & \cdot \begin{bmatrix} X_{11} & X_{12} & X_{13} \\ X_{12}^T & X_{22} & X_{23} \\ X_{13}^T & X_{23}^T & X_{33} \end{bmatrix} \begin{bmatrix} x(k) \\ x(k-\tau(k)) \\ \eta(i) \end{bmatrix} \\ = & \sum_{i=k-\tau(k)}^{k-1} \Bigg(\eta^T(i) X_{33} \eta(i) \\ & + \left[x^T(k) \;\; x^T(k-\tau(k)) \;\; \eta^T(i)\right] \\ & \cdot \begin{bmatrix} X_{11} & X_{12} & X_{13} \\ X_{12}^T & X_{22} & X_{23} \\ X_{13}^T & X_{23}^T & 0 \end{bmatrix} \begin{bmatrix} x(k) \\ x(k-\tau(k)) \\ \eta(i) \end{bmatrix} \Bigg). \end{aligned} \tag{7}$$

Thus, one can easily obtain

$$\begin{aligned} & -\sum_{i=k-\tau(k)}^{k-1} \eta^T(i) X_{33} \eta(i) \\ & \le \sum_{i=k-\tau(k)}^{k-1} \left[x^T(k) \;\; x^T(k-\tau(k)) \;\; \eta^T(i)\right] \\ & \cdot \begin{bmatrix} X_{11} & X_{12} & X_{13} \\ X_{12}^T & X_{22} & X_{23} \\ X_{13}^T & X_{23}^T & 0 \end{bmatrix} \begin{bmatrix} x(k) \\ x(k-\tau(k)) \\ \eta(i) \end{bmatrix}. \end{aligned} \tag{8}$$

The proof is completed. □

Remark 5. The new inequality was proposed in [5, 6] for continuous-time systems; it is worth noting that we firstly extend this method to study discrete-time neural networks in this paper.

Lemma 6 (see [4, 13]). *Let $M \in R^{n\times n}$ be a positive-definite matrix, $X_i \in R^n$; then*

$$\begin{aligned} & -(m-n)\sum_{i=k-m}^{k-n-1} X_i^T M X_i \\ & \leqslant -\left(\sum_{i=k-m}^{k-n-1} X_i\right)^T M \left(\sum_{i=k-m}^{k-n-1} X_i\right). \end{aligned} \tag{9}$$

Lemma 7 (see [24]). *Let $A = A^T, D, E$ be real matrices with appropriate dimensions, with matrix $F(k)$ satisfying $F^T(k)F(k) \le I$; there exists a scalar $\varepsilon > 0$ such that*

$$A + DF(t)E + E^T F^T(t) D^T \le A + \varepsilon DD^T + \varepsilon^{-1} E^T E. \tag{10}$$

Lemma 8 (see [3]). *For any constant matrices Ω, Υ_1, Υ_2 with appropriate dimensions, and a function $\tau(k)$ satisfying $\tau_m \le \tau(k) \le \tau_M$, then*

$$\Omega + (\tau(k) - \tau_m)\Upsilon_1 + (\tau_M - \tau(k))\Upsilon_2 < 0, \tag{11}$$

if and only if

$$\begin{aligned} \Omega + (\tau_M - \tau_m)\Upsilon_1 < 0, \\ \Omega + (\tau_M - \tau_m)\Upsilon_2 < 0. \end{aligned} \tag{12}$$

Definition 9 (see [25]). The system (1) is said to be passive if there exists a scalar $\gamma > 0$ satisfying

$$2\sum_{i=0}^{k_0} E\left\{y^T(i) u(i)\right\} \ge -\gamma \sum_{i=0}^{k_0} E\left\{u^T(i) u(i)\right\}, \tag{13}$$

for all $k_0 \in N$ and for all solution of (1) with $\psi(0) = 0$.

3. Main Results

In this section, the passivity of discrete-time stochastic neural networks with time-varying delays will be investigated by use of the new integral inequality and Lyapunov method. In the paper, some of symmetric matrices in Lyapunov-Krasovskii functional are not necessarily required to be positive definite.

Denote

$$\begin{aligned} L_1 &= \operatorname{diag}\{l_1^- l_1^+, \ldots, l_n^- l_n^+\}, \\ L_2 &= \operatorname{diag}\left\{\frac{l_1^- + l_1^+}{2}, \ldots, \frac{l_n^- + l_n^+}{2}\right\}, \\ \tau_{Mm} &= \tau_M - \tau_m, \\ \eta(k) &= x(k+1) - x(k). \end{aligned} \tag{14}$$

Main results are given in the following theorems.

Theorem 10. *Under Assumptions 1 and 3, the discrete-time stochastic neural network (1) is passive, if there exist matrices P, $R_1, R_2, Q = \begin{bmatrix} Q_{11} & Q_{12} \\ * & Q_{22} \end{bmatrix} > 0$, $T_1 > 0$, $T_2 > 0$, $X = \begin{bmatrix} X_{11} & X_{12} & X_{13} \\ * & X_{22} & X_{23} \\ * & * & X_{33} \end{bmatrix} \ge 0$, $Y = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ * & Y_{22} & Y_{23} \\ * & * & Y_{33} \end{bmatrix} \ge 0$, $Z = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} \\ * & Z_{22} & Z_{23} \\ * & * & Z_{33} \end{bmatrix} \ge 0$, the positive diagonal matrices $S_k = \operatorname{diag}\{s_{1k}, s_{2k}, \ldots, s_{nk}\}$ $(k = 1, 2)$, and scalars $\lambda > 0$, $\gamma > 0$, such that the following matrix inequalities hold:*

$$\begin{aligned} \begin{bmatrix} T_1 + \dfrac{\tau_m}{\tau_M} P & -T_1 \\ * & T_1 + \tau_m R_1 \end{bmatrix} > 0, \\ \begin{bmatrix} T_2 + P & -T_2 \\ * & T_2 + \tau_M R_2 \end{bmatrix} > 0, \end{aligned} \tag{15}$$

$$\begin{aligned} T_1 - X_{33} \ge 0, \\ T_2 - Y_{33} \ge 0, \\ T_2 - Z_{33} \ge 0, \end{aligned} \tag{16}$$

$$\begin{aligned} \Theta \le \lambda I, \\ \Omega + \Upsilon_1 < 0, \\ \Omega + \Upsilon_2 < 0, \end{aligned} \tag{17}$$

where

$$\Theta = P + \tau_m T_1 + \tau_{Mm} T_2,$$

$$\Omega = \begin{bmatrix} \Omega_{11} & \cdots & \Omega_{17} \\ * & \ddots & \vdots \\ * & * & \Omega_{77} \end{bmatrix},$$

$$\Upsilon_1 = \begin{bmatrix} d_{11} & \cdots & d_{17} \\ * & \ddots & \vdots \\ * & * & d_{77} \end{bmatrix},$$

$$\Upsilon_2 = \begin{bmatrix} e_{11} & \cdots & e_{17} \\ * & \ddots & \vdots \\ * & * & e_{77} \end{bmatrix},$$

$$\begin{aligned}
\Omega_{11} &= \left(\tau_{Mm} + 1\right) Q_{11} + R_1 + \tau_m X_{11} + X_{13} + X_{13}^T \\
&\quad + \tau_m (A - I)\, T_1 (A - I) + \tau_{Mm} (A - I)\, T_2 (A - I) \\
&\quad + \rho_1 \lambda I + APA - P - S_1 L_1, \\
\Omega_{12} &= \tau_m X_{12} - X_{13} + X_{23}^T, \\
\Omega_{15} &= \left(\tau_{Mm} + 1\right) Q_{12} + L_2 S_1 + \tau_m (A - I)\, T_1 B_1 \\
&\quad + \tau_{Mm} (A - I)\, T_2 B_1 + A^T P B_1, \\
\Omega_{16} &= \tau_m (A - I)\, T_1 B_1 + \tau_{Mm} (A - I)\, T_2 B_1 + APB_1, \\
\Omega_{17} &= \tau_m (A - I)\, T_1 + \tau_{Mm} (A - I)\, T_2 + AP, \\
\Omega_{22} &= -R_1 + R_2 + \tau_m X_{22} - X_{23} - X_{23}^T + Y_{13} + Y_{13}^T, \\
\Omega_{23} &= -Y_{13} + Y_{23}^T, \\
\Omega_{33} &= -Q_{11} - S_2 L_1 + \rho_2 \lambda I - Y_{23} - Y_{23}^T + Z_{13} + Z_{13}^T, \\
\Omega_{34} &= -Z_{13} + Z_{23}^T, \\
\Omega_{36} &= -Q_{12} + L_2 S_2, \\
\Omega_{44} &= -R_2 - Z_{23} - Z_{23}^T, \\
\Omega_{55} &= B_1^T P B_1 + \tau_m B_1^T T_1 B_1 + \tau_{Mm} B_1^T T_2 B_1 \\
&\quad + \left(\tau_{Mm} + 1\right) Q_{22} - S_1, \\
\Omega_{56} &= B_1^T P B_2 + \tau_m B_1^T T_1 B_2 + \tau_{Mm} B_1^T T_2 B_2, \\
\Omega_{57} &= B_1^T P + \tau_m B_1^T T_1 + \tau_{Mm} B_1^T T_2 - I, \\
\Omega_{66} &= B_2^T P B_2 + \tau_m B_2^T T_1 B_2 + \tau_{Mm} B_2^T T_2 B_2 \\
&\quad - Q_{22} - S_2, \\
\Omega_{67} &= B_2^T P + \tau_m B_2^T T_1 + \tau_{Mm} B_2^T T_2, \\
\Omega_{77} &= P + \tau_m T_1 + \tau_{Mm} T_2 - \gamma I, \\
d_{22} &= Y_{11}, \\
d_{23} &= Y_{12}, \\
d_{33} &= Y_{22}, \\
e_{33} &= Z_{11}, \\
e_{34} &= Y_{12}, \\
d_{44} &= Z_{22}, \quad \textit{elsewhere}, \\
\Omega_{ij} &= 0, \\
d_{ij} &= 0, \\
e_{ij} &= 0, \\
i, j &= 1, 2, \ldots, 8.
\end{aligned} \tag{18}$$

Proof. Define a new augmented of Lyapunov-Krasovskii functional as follows:

$$V(k) = V_1(k) + V_2(k) + V_3(k) + V_4(k), \tag{19}$$

where

$$\begin{aligned}
V_1(k) &= x^T(k) P x(k), \\
V_2(k) &= \sum_{i=k-\tau_m}^{k-1} x^T(i) R_1 x(i) + \sum_{i=k-\tau_M}^{k-\tau_m-1} x^T(i) R_2 x(i), \\
V_3(k) &= \sum_{i=k-\tau(k)}^{k-1} \begin{bmatrix} x(i) \\ g(x(i)) \end{bmatrix}^T \begin{bmatrix} Q_{11} & Q_{12} \\ * & Q_{22} \end{bmatrix} \begin{bmatrix} x(i) \\ g(x(i)) \end{bmatrix} \\
&\quad + \sum_{j=-\tau_M}^{-\tau_m-1} \sum_{i=k+j}^{k-1} \begin{bmatrix} x(i) \\ g(x(i)) \end{bmatrix}^T \begin{bmatrix} Q_{11} & Q_{12} \\ * & Q_{22} \end{bmatrix} \begin{bmatrix} x(i) \\ g(x(i)) \end{bmatrix}, \\
V_4(k) &= \sum_{j=-\tau_m}^{-1} \sum_{i=k+j}^{k-1} \eta^T(i) T_1 \eta(i) \\
&\quad + \sum_{j=-\tau_M}^{-\tau_m-1} \sum_{i=k+j}^{k-1} \eta^T(i) T_2 \eta(i).
\end{aligned} \tag{20}$$

Firstly, we show that the Lyapunov-Krasovskii functional $V(k)$ is positive definite. By using Lemma 6, one can obtain

$$\begin{aligned}
&\sum_{j=-\tau_m}^{-1} \sum_{i=k+j}^{k-1} \eta^T(i) T_1 \eta(i) \\
&\quad \geq \sum_{j=-\tau_m}^{-1} \frac{-1}{j} \left(\sum_{i=k+j}^{k-1} \eta(i) \right)^T T_1 \left(\sum_{i=k+j}^{k-1} \eta(i) \right) \\
&\quad = \sum_{j=-\tau_m}^{-1} \frac{-1}{j} \begin{bmatrix} x(k) \\ x(k+j) \end{bmatrix}^T \begin{bmatrix} T_1 & -T_1 \\ * & T_1 \end{bmatrix} \begin{bmatrix} x(k) \\ x(k+j) \end{bmatrix} \\
&\quad \geq \frac{1}{\tau_m} \sum_{i=k-\tau_m}^{k-1} \begin{bmatrix} x(k) \\ x(i) \end{bmatrix}^T \begin{bmatrix} T_1 & -T_1 \\ * & T_1 \end{bmatrix} \begin{bmatrix} x(k) \\ x(i) \end{bmatrix},
\end{aligned}$$

$$\begin{aligned}
&\sum_{j=-\tau_M}^{-\tau_m-1}\sum_{i=k+j}^{k-1}\eta^T(i)\,T_2\eta(i)\\
&\quad\geq \sum_{j=-\tau_M}^{-\tau_m-1}\frac{-1}{j}\left(\sum_{i=k+j}^{k-1}\eta(i)\right)^T Z_2\left(\sum_{i=k+j}^{k-1}\eta(i)\right)\\
&\quad= \sum_{j=-\tau_M}^{-\tau_m-1}\frac{-1}{j}\begin{bmatrix}x(k)\\ x(k+j)\end{bmatrix}^T\begin{bmatrix}T_1 & -T_1\\ * & T_1\end{bmatrix}\begin{bmatrix}x(k)\\ x(k+j)\end{bmatrix}\\
&\quad\geq \frac{1}{\tau_M}\sum_{i=k-\tau_M}^{k-\tau_m-1}\begin{bmatrix}x(k)\\ x(i)\end{bmatrix}^T\begin{bmatrix}T_2 & -T_2\\ * & T_2\end{bmatrix}\begin{bmatrix}x(k)\\ x(i)\end{bmatrix},
\end{aligned}\tag{21}$$

$$\begin{aligned}
&x^T(k)\,Px(k)\\
&\quad=\sum_{i=k-\tau_m}^{k-1}\frac{1}{\tau_M}x^T(k)\,Px(k)\\
&\qquad+\sum_{i=k-\tau_M}^{k-\tau_m-1}\frac{1}{\tau_M}x^T(k)\,Px(k).
\end{aligned}\tag{22}$$

Then, it follows from (19)–(21) that

$$\begin{aligned}
V(k)\geq{}&\sum_{i=k-\tau_m}^{k-1}\frac{1}{\tau_M}x^T(k)\,Px(k)+\sum_{i=k-\tau_M}^{k-\tau_m-1}\frac{1}{\tau_M}x^T(k)\,Px(k)\\
&+\sum_{i=k-\tau_m}^{k-1}x^T(i)\,R_1x(i)+\sum_{i=k-\tau_M}^{k-\tau_m-1}x^T(i)\,R_2x(i)\\
&+\frac{1}{\tau_m}\sum_{i=k-\tau_m}^{k-1}\begin{bmatrix}x(k)\\ x(i)\end{bmatrix}^T\begin{bmatrix}T_1 & -T_1\\ * & T_1\end{bmatrix}\begin{bmatrix}x(k)\\ x(i)\end{bmatrix}\\
&+\frac{1}{\tau_M}\sum_{i=k-\tau_M}^{k-\tau_m-1}\begin{bmatrix}x(k)\\ x(i)\end{bmatrix}^T\begin{bmatrix}T_2 & -T_2\\ * & T_2\end{bmatrix}\begin{bmatrix}x(k)\\ x(i)\end{bmatrix}\\
\geq{}&\frac{1}{\tau_m}\sum_{i=k-\tau_m}^{k-1}\begin{bmatrix}x(k)\\ x(i)\end{bmatrix}^T\begin{bmatrix}T_1+\dfrac{\tau_m}{\tau_M}P & -T_1\\ -T_1 & T_1+\tau_mR_1\end{bmatrix}\begin{bmatrix}x(k)\\ x(i)\end{bmatrix}\\
&+\frac{1}{\tau_M}\sum_{i=k-\tau_M}^{k-\tau_m-1}\begin{bmatrix}x(k)\\ x(i)\end{bmatrix}^T\begin{bmatrix}T_2+P & -T_2\\ -T_2 & T_2+\tau_mR_2\end{bmatrix}\begin{bmatrix}x(k)\\ x(i)\end{bmatrix}.
\end{aligned}\tag{23}$$

From condition (15), there exists a scalar $\delta_1 > 0$, for any $x(k)\neq 0$, such that

$$V(k)\geq\delta_1\|x(k)\|^2>0.\tag{24}$$

Now, taking the forward difference of $V(k)$ along the trajectories of system (1), it yields that

$$\begin{aligned}
&E\{\Delta V_1(k)\}\\
&\quad=E\left\{(\eta(k)+x(k))^T P(\eta(k)+x(k))-x^T(k)\,Px(k)\right\}\\
&\quad=E\left\{\eta^T(k)\,P\eta(k)+2x^T(k)\,P\eta(k)\right\},\\
&E\{\Delta V_2(k)\}\\
&\quad=E\left\{x^T(k)\,R_1x(k)-x^T(k-\tau_m)(R_1-R_2)x(k-\tau_m)\right.\\
&\qquad\left.-x^T(k-\tau_M)\,R_2x(k-\tau_M)\right\},\\
&E\{\Delta V_3(k)\}\\
&\quad\leq E\left\{(\tau_{Mm}+1)\begin{bmatrix}x(k)\\ g(x(k))\end{bmatrix}^T\begin{bmatrix}Q_{11} & Q_{12}\\ * & Q_{22}\end{bmatrix}\begin{bmatrix}x(k)\\ g(x(k))\end{bmatrix}\right.\\
&\qquad-\begin{bmatrix}x(k-\tau(k))\\ g(x(k-\tau(k)))\end{bmatrix}^T\begin{bmatrix}Q_{11} & Q_{12}\\ * & Q_{22}\end{bmatrix}\\
&\qquad\left.\cdot\begin{bmatrix}x(k-\tau(k))\\ g(x(k-\tau(k)))\end{bmatrix}\right\}\\
&\quad=E\left\{(\tau_{Mm}+1)\,x^T(k)\,Q_{11}x(k)+2x^T(k)\,Q_{12}g(x(k))\right.\\
&\qquad+g^T(x(k))\,Q_{22}g(x(k))\\
&\qquad-x^T(k-\tau(k))\,Q_{11}x(k-\tau(k))\\
&\qquad-2x^T(k-\tau(k))\,Q_{12}g(x(k-\tau(k)))\\
&\qquad\left.-g^T(x(k-\tau(k)))\,Q_{22}g(x(k-\tau(k)))\right\},\\
&E\{\Delta V_4(k)\}\\
&\quad=E\left\{\eta^T(k)(\tau_mT_1+\tau_{Mm}T_2)\eta(k)-\sum_{i=k-\tau_m}^{k-1}\eta^T(i)\,T_1\eta(i)\right.\\
&\qquad\left.-\sum_{i=k-\tau(k)}^{k-\tau_m-1}\eta^T(i)\,T_2\eta(i)-\sum_{i=k-\tau_M}^{k-\tau(k)-1}\eta^T(i)\,T_2\eta(i)\right\},\\
&E\left\{-\sum_{i=k-\tau_m}^{k-1}\eta^T(i)\,T_1\eta(i)-\sum_{i=k-\tau(k)}^{k-\tau_m-1}\eta^T(i)\,T_2\eta(i)\right.\\
&\qquad\left.-\sum_{i=k-\tau_M}^{k-\tau(k)-1}\eta^T(i)\,T_2\eta(i)\right\}\\
&\quad=E\left\{-\sum_{i=k-\tau_m}^{k-1}\eta^T(i)(T_1-X_{33})\eta(i)\right.\\
&\qquad\left.-\sum_{i=k-\tau(k)}^{k-\tau_m-1}\eta^T(i)(T_2-Y_{33})\eta(i)\right.
\end{aligned}$$

$$
\begin{aligned}
&- \sum_{i=k-\tau_M}^{k-\tau(k)-1} \eta^T(i)\left(T_2 - Z_{33}\right)\eta(i) - \sum_{i=k-\tau_m}^{k-1} \eta^T(i) X_{33}\eta(i) \\
&- \sum_{i=k-\tau(k)}^{k-\tau_m-1} \eta^T(i) Y_{33}\eta(i) - \sum_{i=k-\tau_M}^{k-\tau(k)-1} \eta^T(i) Z_{33}\eta(i)\Bigg\}.
\end{aligned} \tag{25}
$$

Form the new inequality of Lemma 4, one can get

$$
\begin{aligned}
&- \sum_{i=k-\tau_m}^{k-1} \eta^T(i) X_{33}\eta(i) \\
&\le \sum_{i=k-\tau_m}^{k-1} \begin{bmatrix} x^T(k) & x^T(k-\tau_m) & \eta^T(i) \end{bmatrix} \\
&\qquad \cdot \begin{bmatrix} X_{11} & X_{12} & X_{13} \\ X_{12}^T & X_{22} & X_{23} \\ X_{13}^T & X_{23}^T & 0 \end{bmatrix} \begin{bmatrix} x(k) \\ x(k-\tau_m) \\ \eta(i) \end{bmatrix} \\
&= x^T(k)\left(\tau_m X_{11} + X_{13}^T + X_{13}\right) x(k) \\
&\quad + x^T(k)\left(\tau_m X_{12} - X_{13} + X_{23}^T\right) x(k-\tau_m) \\
&\quad + x^T(k-\tau_m)\left(\tau_m X_{12}^T - X_{13}^T + X_{23}\right) x(k) \\
&\quad + x^T(k-\tau_m)\left(\tau_m X_{22} - X_{23}^T - X_{23}\right) x(k-\tau_m), \\
&- \sum_{i=k-\tau(k)}^{k-\tau_m-1} \eta^T(i) Y_{33}\eta(i) \\
&\le \sum_{i=k-\tau(k)}^{k-\tau_m-1} \begin{bmatrix} x^T(k-\tau_m) & x^T(k-\tau(k)) & \eta^T(i) \end{bmatrix} \\
&\qquad \cdot \begin{bmatrix} X_{11} & X_{12} & X_{13} \\ X_{12}^T & X_{22} & X_{23} \\ X_{13}^T & X_{23}^T & 0 \end{bmatrix} \begin{bmatrix} x(k-\tau_m) \\ x(k-\tau(k)) \\ \eta(i) \end{bmatrix} \\
&= x^T(k-\tau_m)\left(\left(\tau(k)-\tau_m\right) Y_{11} + Y_{13}^T + Y_{13}\right) x(k-\tau_m) \\
&\quad + x^T(k-\tau_m)\left(\left(\tau(k)-\tau_m\right) Y_{12} - Y_{13} + Y_{23}^T\right) x(k-\tau(k)) \\
&\quad + x^T(k-\tau(k))\left(\left(\tau(k)-\tau_m\right) Y_{12}^T - Y_{13}^T + Y_{23}\right) x(k-\tau_m) \\
&\quad + x^T(k-\tau(k))\left(\left(\tau(k)-\tau_m\right) Y_{22} - Y_{23}^T - Y_{23}\right) x(k-\tau(k)), \\
&- \sum_{i=k-\tau_M}^{k-\tau(k)-1} \eta^T(i) Z_{33}\eta(i) \\
&\le \sum_{i=k-\tau_M}^{k-\tau(k)-1} \begin{bmatrix} x^T(k-\tau(k)) & x^T(k-\tau_M) & \eta^T(i) \end{bmatrix} \\
&\qquad \cdot \begin{bmatrix} X_{11} & X_{12} & X_{13} \\ X_{12}^T & X_{22} & X_{23} \\ X_{13}^T & X_{23}^T & 0 \end{bmatrix} \begin{bmatrix} x(k-\tau(k)) \\ x(k-\tau_M) \\ \eta(i) \end{bmatrix} \\
&= x^T(k-\tau(k))\left(\left(\tau_M-\tau(k)\right) Z_{11} + Z_{13}^T + Z_{13}\right) x(k-\tau(k)) \\
&\quad + x^T(k-\tau(k))\left(\left(\tau_M-\tau(k)\right) Z_{12} - Z_{13} + Z_{23}^T\right) x(k-\tau_M) \\
&\quad + x^T(k-\tau_M)\left(\left(\tau_M-\tau(k)\right) Z_{12}^T - Z_{13}^T + Z_{23}\right) x(k-\tau(k)) \\
&\quad + x^T(k-\tau_M)\left(\left(\tau_M-\tau(k)\right) Z_{22} - Z_{23}^T - Z_{23}\right) x(k-\tau_M).
\end{aligned} \tag{26}
$$

It is easy to get

$$
\begin{aligned}
&E\left\{\eta^T(k)\,\Theta\,\eta(k)\right\} \\
&\quad = E\Big\{\Big[(A-I)x(k) + B_1 g(x(k)) + B_2 g(x(k-\tau(k))) \\
&\qquad + u(k) + \delta(x(k), x(k-\tau(k)))\,\omega(k)\Big]^T \\
&\qquad \cdot \Theta\Big[(A-I)x(k) + B_1 g(x(k)) \\
&\qquad\quad + B_2 g(x(k-\tau(k))) \\
&\qquad\quad + u(k) + \delta(x(k), x(k-\tau(k)))\,\omega(k)\Big]\Big\}.
\end{aligned} \tag{27}
$$

From Assumption 3 and inequality (16), we have

$$
\begin{aligned}
&\delta(x(k), x(k-\tau(k)))\,\Theta\,\delta(x(k), x(k-\tau(k))) \\
&\quad \le \lambda\left[\rho_1 x^T(k)x(k) + \rho_2 x^T(k-\tau(k))x(k-\tau(k))\right].
\end{aligned} \tag{28}
$$

From Assumption 1, it follows that

$$
\begin{aligned}
&\left(g_i(x_i(k)) - l_i^+ x_i(k)\right)\left(g_i(x_i(k)) - l_i^- x_i(k)\right) \le 0, \\
&\left(g_i(x_i(k-\tau(k))) - l_i^+ x_i(k-\tau(k))\right) \\
&\qquad \cdot \left(g_i(x_i(k-\tau(k))) - l_i^- x_i(k-\tau(k))\right) \le 0.
\end{aligned} \tag{29}
$$

Thus, for the diagonal matrices $S_k = \mathrm{diag}\{s_{1k}, s_{2k}, \ldots, s_{nk}\}$ $(k = 1, 2)$, one can receive the following inequalities:

$$
\begin{aligned}
&- \begin{bmatrix} x(k) \\ g(x(k)) \end{bmatrix}^T \begin{bmatrix} S_1 L_1 & -S_1 L_2 \\ * & S_1 \end{bmatrix} \begin{bmatrix} x(k) \\ g(x(k)) \end{bmatrix} \ge 0, \\
&- \begin{bmatrix} x(k-\tau(k)) \\ g(x(k-\tau(k))) \end{bmatrix}^T \\
&\qquad \cdot \begin{bmatrix} S_2 L_1 & -S_2 L_2 \\ * & S_2 \end{bmatrix} \begin{bmatrix} x(k-\tau(k)) \\ g(x(k-\tau(k))) \end{bmatrix} \ge 0.
\end{aligned} \tag{30}
$$

Combining (25)–(30), it yields

$$\begin{aligned} & E\left\{\Delta V(k) - 2y^T(k)u(k) - \gamma u^T(k)u(k)\right\} \\ & \le E\left\{\alpha^T(k)\left(\Omega + \left(\tau(k) - \tau_m\right)\Upsilon_1 + \left(\tau_M - \tau(k)\right)\Upsilon_2\right)\alpha(k)\right. \\ & \quad - \sum_{i=k-\tau_m}^{k-1}\eta^T(i)\left(T_1 - X_{33}\right)\eta(i) \\ & \quad - \sum_{i=k-\tau(k)}^{k-\tau_m-1}\eta^T(i)\left(T_2 - Y_{33}\right)\eta(i) \\ & \quad \left. - \sum_{i=k-\tau_M}^{k-\tau(k)-1}\eta^T(i)\left(T_2 - Z_{33}\right)\eta(i)\right\}, \end{aligned} \tag{31}$$

where

$$\begin{aligned} \alpha^T(k) = & \left[x^T(k), x^T\left(k-\tau_m\right), x^T\left(k-\tau(k)\right), x^T\left(k-\tau_M\right),\right. \\ & \left. g^T(x(k)), g^T\left(x\left(k-\tau(k)\right)\right), u^T(k)\right]. \end{aligned} \tag{32}$$

From (15)–(17), observing that $T_1 - X_{33} \ge 0$, $T_2 - Y_{33} \ge 0$, and $T_2 - Z_{33} \ge 0$, one can conclude that

$$E\left\{\Delta V(k) - 2y^T(k)u(k) - \gamma u^T(k)u(k)\right\} \le 0. \tag{33}$$

Then,

$$\begin{aligned} 2\sum_{i=0}^{k_0}E\left\{y^T(i)u(i)\right\} \ge & \sum_{i=0}^{k_0}E\left\{\Delta V(i)\right\} \\ & - \gamma\sum_{i=0}^{k_0}E\left\{u^T(i)u(i)\right\}. \end{aligned} \tag{34}$$

By the definition of $V(k)$ and inequality (23), one can find that

$$\sum_{i=0}^{k_0}E\left\{\Delta V(i)\right\} = E\left\{V\left(k_0+1\right) - V(0)\right\} \ge 0. \tag{35}$$

So, one can have

$$2\sum_{i=0}^{k_0}E\left\{y^T(i)u(i)\right\} \ge -\gamma\sum_{i=0}^{k_0}E\left\{u^T(i)u(i)\right\}, \tag{36}$$

for all $k_0 \in N$. This completes the proof. □

Remark 11. It should be pointed out that the new inequality is introduced to deal with $\sum_{i=k-\tau_m}^{k-1}\eta^T(i)T_1\eta(i)$ and $\sum_{i=k-\tau_M}^{k-\tau_m-1}\eta^T(i)T_1\eta(i)$, which is immensely different from traditional ways. This method can effectively reduce the conservatism of the results.

Remark 12. In this paper, not all the matrices in the Lyapunov functional need to be positive definite. In fact, the conditions in (15) assure the positive definiteness of the Lyapunov functional; this is greatly different from traditional ways for passivity researches of discrete-time neural network, because the traditional methods always need Lyapunov matrices to be positive definite.

Remark 13. It can be seen that the term $\sum_{i=k-\tau_M}^{k-\tau_m-1}\eta^T(i)T_1\eta(i)$ is divided into two parts; the aim is to make full use of the relationship of $\tau_M - \tau(k)$ and $\tau(k) - \tau_m$, and then, taking advantage of the integral inequality and Lemma 8, new passivity conditions are obtained in terms of LMIs.

Corollary 14. *Under Assumptions 1 and 3, the discrete-time stochastic neural network (1) is passive, if there exist scalars $\lambda > 0$, $\gamma > 0$, matrices $P > 0$, $R_1 > 0$, $R_2 > 0$, $Q = \begin{bmatrix} Q_{11} & Q_{12} \\ * & Q_{22} \end{bmatrix} > 0$, $T_1 > 0$, $T_2 > 0$, $X = \begin{bmatrix} X_{11} & X_{12} & X_{13} \\ * & X_{22} & X_{23} \\ * & * & X_{33} \end{bmatrix} \ge 0$, $Y = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ * & Y_{22} & Y_{23} \\ * & * & Y_{33} \end{bmatrix} \ge 0$, $Z = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} \\ * & Z_{22} & Z_{23} \\ * & * & Z_{33} \end{bmatrix} \ge 0$, and the positive diagonal matrices $S_k = \operatorname{diag}\{s_{1k}, s_{2k}, \ldots, s_{nk}\}$ $(k = 1, 2)$, such that linear matrix inequalities (16) and (17) hold.*

Now, we will consider the stochastic discrete-time neural networks with time-varying delay and parameter uncertainties as follows:

$$\begin{aligned} x(k+1) = & \left(A + \Delta A(k)\right)x(k) \\ & + \left(B_1 + \Delta B_1(k)\right)g(x(k)) \\ & + \left(B_2 + \Delta B_2(k)\right)g\left(x\left(k-\tau(k)\right)\right) + u(k) \\ & + \delta\left(x(k), x\left(k-\tau(k)\right)\right)\omega(k), \end{aligned} \tag{37}$$

where $\Delta A(k)$, $\Delta B_1(k)$, and $\Delta B_2(k)$ denote the parameter uncertainties that are assumed to be of the form

$$\left[\Delta A(k) \;\; \Delta B_1(k) \;\; \Delta B_2(k)\right] = MF(k)\left[E_1 \;\; E_2 \;\; E_3\right], \tag{38}$$

where M, E_i $(i = 1, 2, 3)$ are known constant matrices, and $F(k)$ is the unknown matrix valued function subject to

$$F(k)^T F(k) \le I. \tag{39}$$

Theorem 15. *Under Assumptions 1 and 3, the discrete-time stochastic uncertain neural network (37) is robustly passive, if there exist scalars $\lambda > 0$, $\gamma > 0$, $\varepsilon > 0$, matrices P, R_1, R_2, $Q = \begin{bmatrix} Q_{11} & Q_{12} \\ * & Q_{22} \end{bmatrix} > 0$, $T_1 > 0$, $T_2 > 0$, $X = \begin{bmatrix} X_{11} & X_{12} & X_{13} \\ * & X_{22} & X_{23} \\ * & * & X_{33} \end{bmatrix} \ge 0$, $Y = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ * & Y_{22} & Y_{23} \\ * & * & Y_{33} \end{bmatrix} \ge 0$, $Z = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} \\ * & Z_{22} & Z_{23} \\ * & * & Z_{33} \end{bmatrix} \ge 0$, and the positive*

diagonal matrices $S_k = \mathrm{diag}\{s_{1k}, s_{2k}, \ldots, s_{nk}\}$ $(k = 1, 2)$, *such that the following matrix inequalities hold:*

$$\begin{bmatrix} T_1 + \dfrac{\tau_m}{\tau_M} P & -T_1 \\ * & T_1 + \tau_m R_1 \end{bmatrix} > 0,$$

$$\begin{bmatrix} T_2 + P & -T_2 \\ * & T_2 + \tau_M R_2 \end{bmatrix} > 0,$$

$$T_1 - X_{33} \geq 0,$$

$$T_2 - Y_{33} \geq 0,$$

$$T_2 - Z_{33} \geq 0,$$

$$\Theta \leq \lambda I,$$

$$\begin{bmatrix} \Sigma + \Upsilon_i & \Xi_1 & \sqrt{\tau_m}\Xi_2 & \sqrt{\tau_{Mm}}\Xi_3 & 0 & \varepsilon\Xi_4 \\ * & -P & 0 & 0 & PM & 0 \\ * & * & -T_1 & 0 & \sqrt{\tau_m}T_1 M & 0 \\ * & * & * & -T_2 & \sqrt{\tau_{Mm}}T_2 M & 0 \\ * & * & * & * & -\varepsilon I & 0 \\ * & * & * & * & * & -\varepsilon I \end{bmatrix} < 0, \quad (i = 1, 2), \tag{40}$$

where

$$\Xi_1 = \begin{bmatrix} PA & 0 & 0 & 0 & PB_1 & PB_2 & P \end{bmatrix}^T,$$

$$\Xi_2 = \begin{bmatrix} T_1(A - I) & 0 & 0 & 0 & T_1 B_1 & T_1 B_2 & T_1 \end{bmatrix}^T,$$

$$\Xi_3 = \begin{bmatrix} T_2(A - I) & 0 & 0 & 0 & T_2 B_1 & T_2 B_2 & T_2 \end{bmatrix}^T,$$

$$\Xi_4 = \begin{bmatrix} E_1 & 0 & 0 & 0 & E_2 & E_3 & 0 \end{bmatrix}^T,$$

$$\Theta = P + \tau_m T_1 + \tau_{Mm} T_2,$$

$$\Sigma = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} & 0 & 0 & (\tau_{Mm} + 1)Q_{12} + L_2 S_1 & 0 & 0 \\ * & \Sigma_{22} & -Y_{13} + Y_{23}^T & 0 & 0 & 0 & 0 \\ * & * & \Sigma_{33} & -Z_{13} + Z_{23}^T & 0 & -Q_{12} + L_2 S_2 & 0 \\ * & * & * & -Z_{23} - Z_{23}^T - R_2 & 0 & 0 & 0 \\ * & * & * & * & (\tau_{Mm} + 1)Q_{22} - S_1 & 0 & -I \\ * & * & * & * & * & -Q_{22} - S_2 & 0 \\ * & * & * & * & * & * & \Sigma_{77} \end{bmatrix}, \tag{41}$$

$$\Sigma_{11} = (\tau_{Mm} + 1)Q_{11} + R_1 + \tau_m X_{11} + X_{13} + X_{13}^T + \rho_1 \lambda I - P - S_1 L_1,$$

$$\Sigma_{12} = \tau_m X_{12} - X_{13} + X_{23}^T,$$

$$\Sigma_{22} = -R_1 + R_2 + \tau_m X_{22} - X_{23} - X_{23}^T + Y_{13} + Y_{13}^T,$$

$$\Sigma_{33} = -Q_{11} - S_2 L_1 + \rho_2 \lambda I - Y_{23} - Y_{23}^T + Z_{13} + Z_{13}^T,$$

$$\Sigma_{77} = P + \tau_m T_1 + \tau_{Mm} T_2 - \gamma I.$$

Υ_1, Υ_2 *are the same as defined in Theorem 10.*

Proof. By replacing A, B_1, B_2 in (17) with $A + \Delta A(k)$, $B_1 + \Delta B_1(k)$, $B_2 + \Delta B_2(k)$, respectively, then using Lemma 7, the desired results can be obtained immediately. The proof is completed. □

4. Numerical Examples

In this section, some numerical examples are proposed to show the effectiveness of the results obtained in this paper.

Example 1. Consider the system (1) with the following parameters:

$$A = \begin{bmatrix} 0.8 & 0 \\ 0 & 0.9 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0.001 & 0 \\ 0 & 0.005 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} -0.1 & 0.01 \\ -0.2 & -0.1 \end{bmatrix},$$

$$\rho_1 = 0.003,$$
$$\rho_2 = 0.002. \tag{42}$$

The activation functions are taken as

$$f_1(x) = f_2(x) = \tanh(0.5x). \tag{43}$$

It can be verified that

$$L_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad L_2 = \begin{bmatrix} 0.25 & 0 \\ 0 & 0.25 \end{bmatrix}. \tag{44}$$

In this example, if $\tau_m = 3$ and $\tau_M = 12$, the optimal passivity performance obtained is $\gamma = 12.2439$ by the method in [25] and $\gamma = 5.4007$ by the method in [26], while by Theorem 10 in this paper, the optimal passivity performance $\gamma = 3.9546$. The comparisons of γ are listed in Table 1, when $\tau_m = 3$, $\tau_M = 8, 9, 10, 11, 12$. Then, when we assume $\tau_M = 13$, the optimal passivity performance γ obtained by Theorem 10 for different τ_m can be found in Table 2. It can be seen that our results are less conservative than the ones in [25, 26].

Example 2. Consider the system (1) with the following parameters:

$$A = \begin{bmatrix} 0.4 & 0 \\ 0 & 0.5 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0.001 & 0 \\ 0 & 0.005 \end{bmatrix}, \quad B_2 = \begin{bmatrix} -0.1 & 0.01 \\ -0.2 & -0.1 \end{bmatrix}, \quad \rho_1 = \rho_2 = 0.003. \tag{45}$$

The activation functions are taken as

$$f_1(x) = f_2(x) = \tanh(x). \tag{46}$$

It can be verified that

$$L_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad L_2 = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}. \tag{47}$$

For this example, when $\gamma = 3$ and $\tau_m = 1$, by Theorem 10, we can get that the upper bound of the time-varying delay is $\tau_M = 8$. When $\gamma = 4$ and $\tau_m = 1$, by Theorem 10, we can get $\tau_M = 10$; we can obtain upper bound of τ_M for different γ and τ_m, which are summarized in Table 3. It can be found from Table 3 that, for the same τ_m, a larger passivity performance γ corresponds to a larger τ_M; with the same τ_M, a smaller passivity performance γ corresponds to a larger τ_m.

Table 1: Optimal passivity performance γ for different τ_M.

Methods	8	9	10	11	12
Theorem 1 [25]	3.9708	4.8691	6.2501	8.3991	12.2439
Corollary 2 [26]	2.9660	3.3483	3.8428	4.4968	5.4007
Theorem 10	2.8481	3.2154	3.5172	3.7438	3.9546

Table 2: Optimal passivity performance γ for different τ_m.

Methods	4	5	6	7	8
Theorem 1 [25]	16.1853	13.2125	11.0130	9.2978	7.9134
Corollary 2 [26]	6.1649	5.6743	5.2451	4.8664	4.5300
Theorem 10	4.4126	4.3752	4.2563	4.1871	4.0802

Table 3: Allowable upper bounds of τ_M for different γ and τ_m.

τ_m	1	3	5	7	9
$\gamma = 3$	8	10	12	14	16
$\gamma = 4$	10	12	14	16	18
$\gamma = 6$	12	13	15	17	19

5. Conclusions

In this paper, the problem of passivity analysis for discrete-time stochastic neural networks with time-varying delays has been investigated. The presented sufficient conditions are based on the Lyapunov-Krasovskii functional, a new inequality and linear matrix inequality approach. Numerical examples are given to demonstrate the usefulness and effectiveness of the proposed results. Finally, it should be worth noticing that the proposed method in this paper may be extensively applicable in many other areas, such as Markov jump neural networks, Markov jump neural networks with incomplete transition descriptions, and switched neural networks, which deserves further investigation.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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References

[1] M. M. Gupta, L. Jin, and N. Homma, *Static and Dynamic Neural Networks: From Fundamentals to Advanced Theory*, John Wiley & Sons, New York, NY, USA, 2003.

[2] J. J. Hopfield, "Neurons with graded response have collective computational properties like those of two-state neurons," *Proceedings of the National Academy of Sciences of the United States of America*, vol. 81, no. 10 I, pp. 3088–3092, 1984.

[3] D. Yue, Y. Zhang, and E. Tian, "Improved global robust delay-dependent stability criteria for delayed cellular neural networks," *International Journal of Computer Mathematics*, vol. 85, no. 8, pp. 1265–1277, 2008.

[4] S. Xu, J. Lam, B. Zhang, and Y. Zou, "A new result on the delay-dependent stability of discrete systems with time-varying delays," *International Journal of Robust and Nonlinear Control*, vol. 24, no. 16, pp. 2512–2521, 2014.

[5] P.-L. Liu, "Delay-dependent global exponential robust stability for delayed cellular neural networks with time-varying delay," *ISA Transactions*, vol. 52, no. 6, pp. 711–716, 2013.

[6] P. L. Liu, "A delay decomposition approach to robust stability analysis of uncertain systems with time-varying delay," *ISA Transactions*, vol. 51, no. 6, pp. 694–701, 2012.

[7] J. Sun and J. Chen, "Stability analysis of static recurrent neural networks with interval time-varying delay," *Applied Mathematics and Computation*, vol. 221, pp. 111–120, 2013.

[8] D. Zhang and L. Yu, "Exponential state estimation for Markovian jumping neural networks with time-varying discrete and distributed delays," *Neural Networks*, vol. 35, pp. 103–111, 2012.

[9] P. G. Park, J. W. Ko, and C. Jeong, "Reciprocally convex approach to stability of systems with time-varying delays," *Automatica*, vol. 47, no. 1, pp. 235–238, 2011.

[10] C. Lin, Q.-G. Wang, and T. H. Lee, "A less conservative robust stability test for linear uncertain time-delay systems," *IEEE Transactions on Automatic Control*, vol. 51, no. 1, pp. 87–91, 2006.

[11] R. Lu, H. Wu, and J. Bai, "New delay-dependent robust stability criteria for uncertain neutral systems with mixed delays," *Journal of the Franklin Institute. Engineering and Applied Mathematics*, vol. 351, no. 3, pp. 1386–1399, 2014.

[12] J. Cheng, H. Zhu, S. Zhong, F. Zheng, and Y. Zeng, "Finite-time filtering for switched linear systems with a mode-dependent average dwell time," *Nonlinear Analysis: Hybrid Systems*, vol. 15, pp. 145–156, 2015.

[13] Z. Wang, G. Wei, and G. Feng, "Reliable H_∞ control for discrete-time piecewise linear systems with infinite distributed delays," *Automatica*, vol. 45, no. 12, pp. 2991–2994, 2009.

[14] O. M. Kwon, M. J. Park, J. H. Park, S. M. Lee, and E. J. Cha, "New criteria on delay-dependent stability for discrete-time neural networks with time-varying delays," *Neurocomputing*, vol. 121, pp. 185–194, 2013.

[15] J. Cheng, H. Zhu, S. Zhong, Q. Zhong, and Y. Zeng, "Finite-time H_∞ estimation for discrete-time Markov jump systems with time-varying transition probabilities subject to average dwell time switching," *Communications in Nonlinear Science and Numerical Simulation*, vol. 20, no. 2, pp. 571–582, 2015.

[16] T. Wang, C. Zhang, S. Fei, and T. Li, "Further stability criteria on discrete-time delayed neural networks with distributeddelay," *Neurocomputing*, vol. 111, pp. 195–203, 2013.

[17] X. Kan, H. Shu, and Z. Li, "Robust state estimation for discrete-time neural networks with mixed time-delays, linear fractional uncertainties and successive packet dropouts," *Neurocomputing*, vol. 135, pp. 130–138, 2014.

[18] R. Lozano, B. Brogliato, O. Egeland, and B. Maschke, *Dissipative Systems Analysis and Control: Theory and Applications*, Springer, London, UK, 2nd edition, 2007.

[19] V. Bevelevich, *Classical Network Synthesis*, Van Nostrand, New York, NY, USA, 1968.

[20] B. Zhang, S. Xu, and J. Lam, "Relaxed passivity conditions for neural networks with time-varying delays," *Neurocomputing*, vol. 142, pp. 299–306, 2014.

[21] D. H. Ji, J. H. Koo, S. C. Won, S. M. Lee, and J. H. Park, "Passivity-based control for Hopfield neural networks using convex representation," *Applied Mathematics and Computation*, vol. 217, no. 13, pp. 6168–6175, 2011.

[22] Z. G. Wu, J. H. Park, H. Su, and J. Chu, "New results on exponential passivity of neural networks with time-varying delays," *Nonlinear Analysis: Real World Applications*, vol. 13, no. 4, pp. 1593–1599, 2012.

[23] T. H. Lee, M.-J. Park, J. H. Park, O.-M. Kwon, and S.-M. Lee, "Extended dissipative analysis for neural networks with time-varying delays," *IEEE Transactions on Neural Networks and Learning Systems*, vol. 25, no. 10, pp. 1936–1941, 2014.

[24] R. Raja, U. K. Raja, R. Samidurai, and A. Leelamani, "Passivity analysis for uncertain discrete-time stochastic BAM neural networks with time-varying delays," *Neural Computing and Applications*, vol. 25, no. 3-4, pp. 751–766, 2014.

[25] Q. Song, J. Liang, and Z. Wang, "Passivity analysis of discrete-time stochastic neural networks with time-varying delays," *Neurocomputing*, vol. 72, no. 7–9, pp. 1782–1788, 2009.

[26] Z.-G. Wu, P. Shi, H. Su, and J. Chu, "Dissipativity analysis for discrete-time stochastic neural networks with time-varying delays," *IEEE Transactions on Neural Networks and Learning Systems*, vol. 24, no. 3, pp. 345–355, 2013.

[27] Z. Wu, P. Shi, H. Su, and J. Chu, "Stability and disspativity analysis of static neural networks with time delay," *IEEE Transactions on Neural Networks and Learning Systems*, vol. 23, no. 2, pp. 199–210, 2012.