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Research Article

Compressive Sensing for High-Resolution Direction-of-Arrival Estimation via Iterative Optimization on Sensing Matrix

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A novel compressive sensing- (CS-) based direction-of-arrival (DOA) estimation algorithm is proposed to solve the performance degradation of the CS-based DOA estimation in the presence of sensing matrix mismatching. Firstly, a DOA sparse sensing model is set up in the presence of sensing matrix mismatching. Secondly, combining the Dantzig selector (DS) algorithm and least-absolute shrinkage and selection operator (LASSO) algorithm, a CS-based DOA estimation algorithm which performs iterative optimization alternatively on target angle information vector and sensing matrix mismatching error vector is proposed. The simulation result indicates that the proposed algorithm possesses higher angle resolution and estimation accuracy compared with conventional CS-based DOA estimation algorithms.

1. Introduction

The strong scatter centers of targets in area of interest only occupy finite angle resolution cells, and the echo signal of targets is sparse, so compressive sensing (CS) theory is widely studied in direction-of-arrival (DOA) estimation applications [1–5]. In [1], a CS-based DOA estimation algorithm of multiple input and multiple output (MIMO) radar is proposed, which makes use of the sparsity of radar echo signals to perform compressive sampling on array receipt signals in time-domain. In [2], an array with element randomly distributed is adopted to perform compressive sampling on space-domain signal, reducing the number of receiving front-end channels of the array. However, both [1, 2] treat the overcomplete based matrixes as the redundant dictionaries, obtained from the angle interval of uniform quantization area of interest, which cannot ensure that the corresponding sensing matrix meets the restricted isometry property (RIP) [3]. And then, [4] proves the RIP for MIMO radar application. Reference [5] uses random Gauss matrix to perform compressive sampling on space-domain signal and adopts regularized multivectors focal undetermined system solver (RMFOCUSS) algorithm to achieve high-resolution estimation. However, the computation complexity of RMFOCUSS algorithm increases dramatically with the increase of snapshots.

In addition, the estimation performance degrades seriously in the presence of sensing matrix mismatching in the above algorithms [6, 7]. The authors in [8] investigate the CS-based DOA estimation in the presence of sensing model mismatching errors, proving that the performance of CS-based DOA estimation algorithm degrades dramatically in that case. References [9–11] present a DOA estimation model under sensing model mismatching and then use Bayesian method to realize DOA estimation. Reference [12] proposes a joint least-absolute shrinkage and selection operator (LASSO) algorithm to achieve DOA estimation in the presence of mismatching.

In this paper, a new CS-based DOA estimation algorithm is proposed to decrease the effect of sensing matrix mismatching and achieve high resolution on DOA estimation. Firstly, a DOA sparse sensing model is set up in the presence of sensing matrix mismatching. Secondly, combining the Dantzig selector (DS) algorithm [13] and least-absolute shrinkage and selection operator (LASSO) algorithm [14], a CS-based DOA estimation algorithm which performs iterative optimization alternatively on target angle information vector and sensing matrix mismatching error vector is proposed to achieve high-resolution DOA estimates.

2. The Signal Model

Consider that K distant-field narrow-band signals enter the uniform linear array (ULA), made up by M array elements, and then the output signal $\mathbf{x}(t)$ of the array can be represented as

$$\mathbf{x}(t) = \sum_{k=1}^{K} s_k(t)\,\boldsymbol{\alpha}(\theta_k) + \mathbf{e}(t), \tag{1}$$

where $\boldsymbol{\alpha}(\theta_k) = [1, e^{j2\pi f_{\theta_k}}, \ldots, e^{j2\pi(M-1)f_{\theta_k}}]^T$ is steering vector of the kth receipt signal, $f_{\theta_k} = d\sin\theta_k/\lambda$, and d is the distance between the array elements. λ is the wavelength of carrier wave, $\mathbf{e}(t) = [e_1(t), e_2(t), \ldots, e_L(t)]^T$ is the array noise vector, and $s_k(t)$ is the signal plural envelope.

Assume that the angle resolution vector obtained from the angle interval of area of interest through uniform quantization is $\widetilde{\boldsymbol{\theta}} = [\widetilde{\theta}_1, \widetilde{\theta}_2, \ldots, \widetilde{\theta}_N]$, N is the number of angle resolution vectors, and $\varphi = \widetilde{\theta}_2 - \widetilde{\theta}_1$ is the angle resolution cell; then (1) can be rewritten as

$$\mathbf{x}(t) = \mathbf{A}(\widetilde{\boldsymbol{\theta}})\,\mathbf{s}(t) + \mathbf{e}(t), \tag{2}$$

where $\mathbf{s}(t) = [s_1(t), s_2(t), \ldots, s_N(t)]^T$ is the target angle information vector and $\mathbf{A}(\widetilde{\boldsymbol{\theta}}) = [\boldsymbol{\alpha}(\widetilde{\theta}_1), \boldsymbol{\alpha}(\widetilde{\theta}_2), \ldots, \boldsymbol{\alpha}(\widetilde{\theta}_N)]$ is the steering vector matrix of angle resolution cell of the array.

In practice, targets in area of interest only occupy finite angle resolution cells. So $\|\mathbf{s}(t)\|_0 = K \ll N$ and $\|\cdot\|_0$ denotes L_0 norm. Thus the output signal of the array $\mathbf{x}(t)$ is K sparse signal, $\mathbf{A}(\widetilde{\boldsymbol{\theta}})$ is the sparsity-based matrix, and K is the sparsity of target angle information vector.

3. The Proposed Algorithm

3.1. DOA Estimation Model under Sensing Model Mismatching. Assume that K targets' angle information vector of the array in area of interest $\boldsymbol{\theta} = [\theta_1, \theta_2, \ldots, \theta_k]$, and $\theta_k \notin \widetilde{\boldsymbol{\theta}}$. That is to say, the kth target's angle information mismatches the angle resolution vector defined before, which is called mismatching between sensing matrix and target angle information. According to CS theory, sensing model mismatching will lead to the angle information vector failing to represent target angle precisely, decreasing the estimation accuracy of target angles through conventional CS-based DOA estimation method [8].

Assume that $\widetilde{\theta}_{n_k} \in \widetilde{\boldsymbol{\theta}}$ ($n_k \in [1, 2, \ldots, N]$) is the angle resolution element nearest to target's angle θ_k in angle resolution vectors; then the steering vector of kth target can be denoted approximately as

$$\mathbf{a}(\theta_k) \approx \mathbf{a}(\widetilde{\theta}_{n_k}) + \mathbf{b}(\widetilde{\theta}_{n_k})(\theta_k - \widetilde{\theta}_{n_k}), \tag{3}$$

where $\mathbf{b}(\widetilde{\theta}_{n_k}) = d(\mathbf{a}(\widetilde{\theta}_{n_k}))/d\widetilde{\theta}_{n_k}$.

Thus the steering vector matrix when sensing matrix mismatches target angle information can be rewritten as

$$\boldsymbol{\Phi}(\widetilde{\boldsymbol{\theta}}) = \mathbf{A}(\widetilde{\boldsymbol{\theta}}) + \mathbf{B}(\widetilde{\boldsymbol{\theta}})\,\boldsymbol{\Lambda}, \tag{4}$$

where $\mathbf{B}(\widetilde{\boldsymbol{\theta}}) = [\mathbf{b}(\widetilde{\theta}_1), \mathbf{b}(\widetilde{\theta}_2), \ldots, \mathbf{b}(\widetilde{\theta}_N)]$, $\boldsymbol{\Lambda} = \mathrm{diag}(\boldsymbol{\beta})$, $\boldsymbol{\beta} = [\beta_1, \beta_2, \ldots, \beta_N]^T$, and

$$\beta_n = \begin{cases} \theta_k - \widetilde{\theta}_{n_k}, & n = n_k,\ (k \in [1, 2, \ldots, K]),\ \beta_n \in \left[-\frac{1}{2}\varphi, \frac{1}{2}\varphi\right], \\ 0, & \text{else.} \end{cases} \tag{5}$$

Therefore, taking no account of the approximation error of measurement noise, (2) can be rewritten as

$$\mathbf{x}(t) = \boldsymbol{\Phi}\mathbf{s}(t) + \mathbf{e}(t). \tag{6}$$

According to CS theory, we can recover the target angle information vector $\mathbf{s}(t)$ by sampling the receipt signal with only finite array elements. So we extract L elements from M elements of the array, and let $\boldsymbol{\Psi}$ be the corresponding line-extraction matrix; thus the output of the array after line extraction could be represented as

$$\mathbf{y}(t) = \boldsymbol{\Psi}\mathbf{x}(t) = \boldsymbol{\Psi}(\boldsymbol{\Phi}\mathbf{s}(t) + \mathbf{e}(t)) = \boldsymbol{\theta}\mathbf{s}(t) + \mathbf{n}(t). \tag{7}$$

By observing (7), we can conclude that sampling of space-domain signals can be regarded as measurement matrix $\boldsymbol{\Psi}$ performing random projection measurements on echo signal $\mathbf{x}(t)$. In addition, sensing matrix $\boldsymbol{\theta}$ is the product result of matrix $\boldsymbol{\Psi}$ whose elements are randomly distributed and sparsity-based matrix $\boldsymbol{\Phi}$ which can be treated as Fourier transform matrix of space-domain signal. Therefore, $\boldsymbol{\theta}$ meets the RIP condition with great probability, thus ensuring the effectiveness and robustness of using compressive sensing reconstruction algorithm to perform DOA estimation.

3.2. Derivation of the Proposed Algorithm. By comparing (2) and (6), we can find that the influences of measurement noise and sensing matrix mismatching error on DOA estimation can be summed up to "additive" disturbance and "productive" disturbance. So far, conventional CS-based DOA estimation algorithms only have constraints on "additive" disturbance but fail to take the influence of "productive" disturbance on the accuracy of target angle information estimation into consideration. Therefore, conventional CS-based DOA estimation algorithms cannot effectively reduce sensing matrix mismatching error when angle resolution vector, previously defined, fails to precisely represent target. That is to say, conventional CS-based DOA algorithms are unable to ensure DOA estimation's effectiveness and robustness when there are sensing matrix mismatching errors.

To solve these problems, a novel CS-based DOA estimation algorithm suitable for the situation when sensing matrix mismatches target angle information is proposed in this paper. The proposed algorithm combines DS algorithm and LASSO algorithm to achieve a high-resolution DOA estimation result by performing iterative optimization alternatively on target angle information vector and sensing matrix mismatching error vector.

First assume the vector of sensing matrix mismatching error $\boldsymbol{\beta} = \mathbf{0}$ in lack of prior information. According to CS theory, the optimization problem expressed in (7) can be solved by working out L_1 norm optimization under the circumstance of noise, consequently obtaining target angle information vector in space-domain:

$$\begin{aligned} \hat{\mathbf{s}}(t) &= \min \quad \|\mathbf{s}(t)\|_1 \\ \text{subject to} &\quad \left\|\boldsymbol{\theta}^H\left(\mathbf{y}(t)-\boldsymbol{\theta}\mathbf{s}(t)\right)\right\|_\infty < \mu, \end{aligned} \tag{8}$$

where constant μ is relevant to noise variance. This optimization problem can be perfectly solved by DS algorithm.

Take the estimation value of target angle information obtained by solving (8) to (7); we can get

$$\mathbf{y}(t) = \boldsymbol{\Psi}\left(\left(\mathbf{A}\left(\tilde{\boldsymbol{\theta}}\right)+\mathbf{B}\left(\tilde{\boldsymbol{\theta}}\right)\boldsymbol{\Lambda}\right)\cdot\hat{\mathbf{s}}(t)+\mathbf{e}(t)\right). \tag{9}$$

According to the property of vector, compiling (9), it can be achieved that

$$\begin{aligned} \mathbf{y}(t) &= \boldsymbol{\Psi}\left(\mathbf{A}\left(\tilde{\boldsymbol{\theta}}\right)\cdot\hat{\mathbf{s}}(t)+\mathbf{B}\left(\tilde{\boldsymbol{\theta}}\right)\boldsymbol{\Lambda}\hat{\mathbf{s}}(t)+\mathbf{e}(t)\right) \\ &= \boldsymbol{\Psi}\mathbf{A}\left(\tilde{\boldsymbol{\theta}}\right)\hat{\mathbf{s}}(t)+\boldsymbol{\Psi}\mathbf{B}\left(\tilde{\boldsymbol{\theta}}\right)\boldsymbol{\Omega}\boldsymbol{\beta}+\mathbf{n}(t), \end{aligned} \tag{10}$$

where $\boldsymbol{\Omega} = \operatorname{diag}(\hat{\mathbf{s}}(t))$.

Because of $\boldsymbol{\beta}$ being vector of sensing matrix mismatch error, from (5) we know that $\boldsymbol{\beta}$ shares the same sparsity with target angle information $\mathbf{s}(t)$. So (10) can be transformed to

$$\mathbf{y}'(t) = \nu\boldsymbol{\beta}+\mathbf{n}(t), \tag{11}$$

where $\mathbf{y}'(t) = \mathbf{y}(t) - \boldsymbol{\Psi}\mathbf{A}(\tilde{\boldsymbol{\theta}})\hat{\mathbf{s}}(t)$ and $\nu = \boldsymbol{\Psi}\mathbf{B}(\tilde{\boldsymbol{\theta}})\boldsymbol{\Omega}$.

Hence, (11) can be retreated as a CS optimization problem using sensing matrix mismatching error as the sparse signal, and this CS optimization problem can be denoted as

$$\begin{aligned} \boldsymbol{\beta} &= \arg\min_{\beta} \quad \left\|\mathbf{y}'-\nu\boldsymbol{\beta}\right\|_2^2 \\ \text{subject to} &\quad \left\|\boldsymbol{\beta}\right\|_1 \leq \frac{1}{2}K\varphi. \end{aligned} \tag{12}$$

The CS optimization problem in (12) can be perfectly solved by LASSO algorithm.

Take the vector of sensing matrix mismatching error $\boldsymbol{\beta}$ worked out from (12) to (8); estimation value of target angle information $\hat{\mathbf{s}}(t)$ can be resolved. Repeat the process mentioned above until the difference of two target angle information vector's norm is less than the certain predefined threshold. That is,

$$\frac{\left\|[\hat{\mathbf{s}}(t)]^{(p+1)} - [\hat{\mathbf{s}}(t)]^{(p)}\right\|_2^2}{\left\|[\hat{\mathbf{s}}(t)]^{(p)}\right\|_2^2} \leq \Delta. \tag{13}$$

Stop iteration at this moment; then $[\hat{\mathbf{s}}(t)]^{(p+1)}$ that we get is the target angle estimation information, where Δ is the certain predefined threshold.

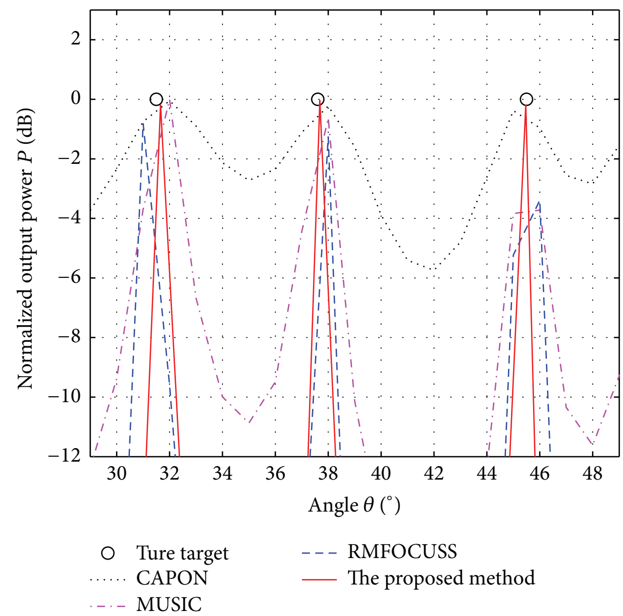

Figure 1: Angle resolution of estimated spatial spectrum.

4. Simulation

In this part, numerical simulations are presented to examine the performance of the proposed method. Consider $L = 10$ array elements spaced randomly in $[0, 30\lambda]$.

In the first examples, the angle resolution cell is set to be $\varphi = 1^\circ$, and the snapshots of the echo signal $P = 200$. The angles of input signals are assumed to be $[31.3^\circ, 37.6^\circ, 45.2^\circ]$. Figure 1 illustrates the spatial spectrum, when the signal-to-noise ratio (SNR) is set to 0 dB. It can be seen that both MUSIC algorithm and CAPON algorithm fail to achieve high-resolution estimation on target angle information. The CS-based DOA algorithm based on RMFOCUSS is endowed with better angle resolution compared to conventional DOA estimation algorithms while its estimation accuracy still suffers from the impacts of mismatching between sensing matrix and target angle information. In contrast, the proposed algorithm remarkably increases the DOA estimation accuracy by performing calibration on sensing matrix.

In the second examples, we consider the root-mean-square error (RMSE) of different DOA estimation algorithms versus different SNR. As shown in Figure 2, we can observe that the proposed algorithm possesses better estimation performance and achieves high resolution on DOA estimation, for the reason that it successfully calibrates sensing matrix mismatching error and impairs the effect from system sensing matrix mismatching in low SNR.

In the third experiment, the simulations on randomly generated DOAs are examined. Consider that the directions of the three signals are uniformly generated within direction intervals $[20^\circ, 40^\circ]$, and the other parameters stay consistent. The RMSE of different DOA estimation algorithms versus different SNR is plotted in Figure 3. It is seen from the figure that when the certain predefined threshold $\Delta = 0.1$, the estimation accuracy of the proposed algorithm is less than the

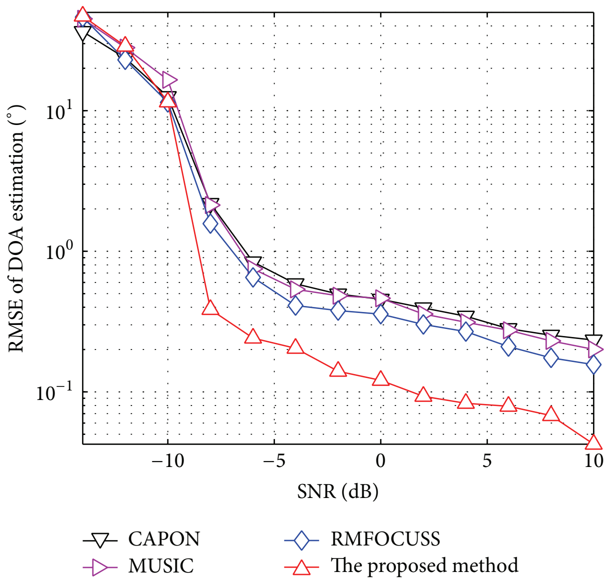

Figure 2: RMSE of the DOA estimates versus input SNR.

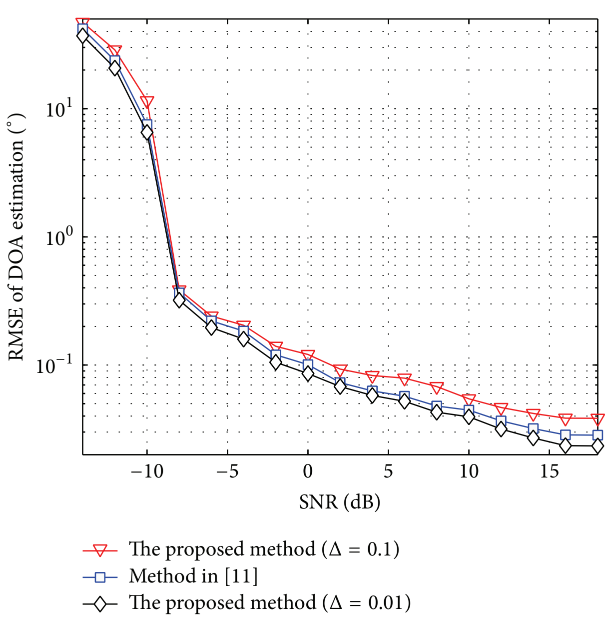

Figure 3: RMSE of the DOA estimates versus input SNR.

method in [11], but when the certain predefined threshold $\Delta = 0.01$, the proposed algorithm can achieve higher estimation accuracy compared with other CS-DOA methods which deal with off-grid targets.

In the last experiment, the different angle resolution cells are considered to examine the ability of the proposed method to represent the true signals. The angle resolution cells are selected as $\varphi = [1^\circ, 3^\circ, 5^\circ]$. The other parameters stay consistent. The RMSE versus different angle resolution cells is depicted in Figure 4, which demonstrate that the performance of the proposed method increases with the decrease of the angle resolution cell.

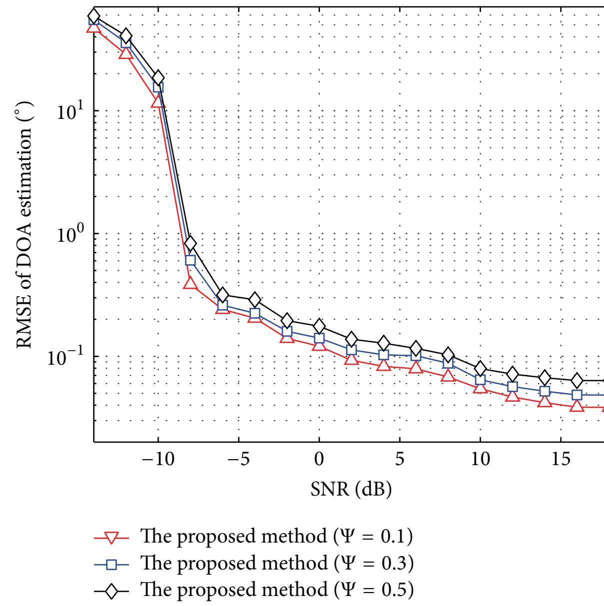

Figure 4: RMSE of the DOA estimates versus input SNR with different angle resolution cells.

5. Conclusion

In this paper, a novel CS-based DOA estimation algorithm is proposed to solve the problem that the CS-based DOA estimation performance deteriorates in the presence of sensing matrix mismatching. The proposed algorithm reduces the estimation error of target angle information through calibrating sensing matrix. The algorithm proposed in this paper is characterized by great value in practical applications, since it improves the performance of CS-based DOA estimation algorithm and achieves high resolution on DOA estimation.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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