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Research Article

Soft Classes and Soft Rough Classes with Applications in Decision Making

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Rough set was defined by Pawlak in 1982. Concept of soft set was proposed as a mathematical tool to cope with uncertainty and vagueness by Molodtsov in 1999. Soft sets were combined with rough sets by Feng et al. in 2011. Feng et al. investigated relationships between a subset of initial universe of soft set and a soft set. Feng et al. defined the upper and lower approximations of a subset of initial universe over a soft set. In this study, we firstly define concept of soft class and soft class operations such as union, intersection, and complement. Then we give some properties of soft class operations. Based on definition and operations of soft classes, we define lower and upper approximations of a soft set. Subsequently, we introduce concept of soft rough class and investigate some properties of soft rough classes. Moreover, we give a novel decision making method based on soft class and present an example related to novel method.

1. Introduction

The concept of soft set was introduced by Molodtsov [1] in 1999 as a general mathematical tool for dealing with problems involving uncertain data. Maji et al. [2] defined some concepts and operations on soft sets such as soft subset, soft equality, soft union, soft intersection, and soft complement. Çağman and Enginoğlu [3] redefined soft set operations suggested by Maji et al. [2] and developed a decision making method called uni-int decision making method. Çağman [4] made some contributions to the theory of soft sets to fill gaps of former definition and operations.

Rough set theory was proposed by Pawlak [5] as an alternative approach to fuzzy sets theory and tolerance theory and has been applied successfully to a lot of fields such as machine learning, pattern recognition, and data mining. Dubois and Prade [6] defined lower and upper approximations of a fuzzy set to extend concept of rough set and they proposed the rough fuzzy sets. Soft sets were combined with fuzzy sets and rough sets by Feng et al. [7]. In 2011, Feng et al. [8] introduced soft rough approximation space and soft rough set based on the novel granulation structures called soft approximation spaces and presented basic properties of soft rough approximations supported by some illustrative example. They also defined some new types of soft sets such as full soft sets, intersection complement softs set, and partition soft sets. Meng et al. [9] proposed a new soft rough set model and derived its properties. They also established a more general model called soft rough fuzzy set. Irfan Ali [10] discussed concept of approximation space associated with each parameter in a soft set and defined an approximation space associated with the soft sets and established connection between soft set, fuzzy soft set, and rough sets. Feng [11] gave an application of soft rough approximations in multicriteria group decision making problems. Zhang [12] defined a new rough set model and investigated its some fundamental properties. He also presented a decision making method for intuitionistic fuzzy soft sets based on this new rough set approach. Zhang [13] studied parameter reduction of fuzzy soft sets based on soft fuzzy rough set and defined some new concepts such as lower soft fuzzy rough approximation operator and upper soft fuzzy rough approximation operator. To find approximation of a set, Shabir et al. [14] proposed modified soft rough sets. Sun and Ma [15] proposed a new concept of soft fuzzy rough set by combining the fuzzy soft set with the traditional fuzzy rough set. They also defined concept of the pseudofuzzy binary relation and based on this concept they defined the soft fuzzy rough lower and upper approximation operators of any fuzzy subset in the parameter set. In this paper, we define concept of soft class and soft class operations based on decision makers set and investigate some fundamental properties of soft class operations. Then, we define soft rough class approximations and soft rough class and investigate some properties of them. Furthermore, we present a method to evaluate the decision makers and give an example to illustrate the process of this method. Proposed method can be used in many areas such as industrial engineering, economy, and social sciences. In particular, in industrial engineering, it can be used effectively for Quality Lifecycle Management and Choosing Product.

2. Preliminary

Let U be an initial universe, let E be the universe of all possible parameters related to the objects in U, and let $\mathscr{P}(U)$ be power set of U.

Definition 1 (see [1]). Consider a nonempty set A such that $A \subseteq E$. A pair (f, A) is called a soft set over U, where f is a mapping given by $f : A \to \mathscr{P}(U)$.

In this paper, we will use the following definition given by Çağman [4] for basic set operations on soft sets.

Definition 2 (see [4]). A soft set f over U is a set valued function from E to $\mathscr{P}(U)$. It can be written as a set of ordered pairs:

$$f = \{(e, f(e)) : e \in E\}. \tag{1}$$

Note that if $f(e) = \emptyset$, then the element $(e, f(e))$ will not appear in soft set f. Set of all soft sets over U will be denoted by $\mathcal{S}(U)$.

Example 3. Let $U = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8\}$ be the universe containing eight houses and let $E = \{e_1, e_2, e_3, e_4, e_5, e_6\}$ be the set of parameters. Here, e_i $(i = 1, 2, 3, 4, 5, 6)$ stand for the parameters "modern," "with parking," "expensive," "cheap," "large," and "near to city," respectively. Then, the following soft sets are described by Mr. A and Mr. B who want to buy a house, respectively:

$$\begin{aligned} f &= \{(e_1, \{u_1, u_3, u_4\}), (e_2, \{u_1, u_4, u_7, u_8\}), \\ &\quad (e_3, \{u_1, u_2, u_3, u_8\})\}, \\ g &= \{(e_2 \{u_1, u_3, u_6\}), (e_3, U), (e_5, \{u_2, u_4, u_5, u_6\})\}. \end{aligned} \tag{2}$$

Definition 4 (see [4]). Let $f, g \in \mathcal{S}(U)$. Then,

(1) if $f(e) = \emptyset$, for all $e \in E$, f is said to be a null soft set, denoted by Φ;

(2) if $f(e) = U$, for all $e \in E$, f is said to be absolute soft set, denoted by $\widehat{U}$;

(3) f is soft subset of g, denoted by $f \mathrel{\tilde{\subseteq}} g$, if $f(e) \subseteq g(e)$ for all $e \in E$;

(4) $f = g$, if $f \mathrel{\tilde{\subseteq}} g$ and $g \mathrel{\tilde{\subseteq}} f$;

(5) soft union of f and g, denoted by $f \mathrel{\tilde{\cup}} g$, is a soft set over U and is defined by $f \mathrel{\tilde{\cup}} g : E \to \mathscr{P}(U)$ such that $(f \mathrel{\tilde{\cup}} g)(e) = f(e) \cup g(e)$ for all $e \in E$;

(6) soft intersection of f and g, denoted by $f \mathrel{\tilde{\cap}} g$, is a soft set over U and is defined by $f \mathrel{\tilde{\cap}} g : E \to \mathscr{P}(U)$ such that $(f \mathrel{\tilde{\cap}} g)(e) = f(e) \cap g(e)$ for all $e \in E$;

(7) soft complement of f is denoted by $f^{\tilde{c}}$ and is defined by $f^{\tilde{c}} : E \to \mathscr{P}(U)$ such that $f^{\tilde{c}}(e) = U \setminus f(e)$ for all $e \in E$.

Example 5. Let us consider soft sets f and g given in Example 3. Then,

$$\begin{aligned} f \mathrel{\tilde{\cup}} g &= \{(e_1, \{u_1, u_3, u_4\}), (e_2, \{u_1, u_3, u_4, u_6, u_7, u_8\}), \\ &\quad (e_3, U), (e_5, \{u_2, u_4, u_5, u_6\})\}, \\ f \mathrel{\tilde{\cap}} g &= \{(e_2 \{u_1\}), (e_3, \{u_1, u_2, u_3, u_8\})\}, \\ f^{\tilde{c}} &= \{(e_1, \{u_2, u_5, u_6, u_7, u_8\}), (e_2, \{u_2, u_3, u_5, u_6\}), \\ &\quad (e_3, \{u_4, u_5, u_6, u_7\}), (e_4, U), (e_5, U), (e_6, U)\}. \end{aligned} \tag{3}$$

Definition 6 (see [8]). Let $S = (f, A)$ be a soft set over U. Then, the pair $P = (U, S)$ is called soft approximation space. Based on the soft approximation space P, we define the two operations,

$$\begin{aligned} \underline{\mathrm{apr}}_P(X) &= \{u \in U : \exists a \in A,\ [u \in f(a) \subseteq X]\}, \\ \overline{\mathrm{apr}}_P(X) & \\ &= \{u \in U : \exists a \in A,\ [u \in f(a),\ f(a) \cap X \neq \emptyset]\}, \end{aligned} \tag{4}$$

assigning to every subset $X \subseteq U$ two sets $\underline{\mathrm{apr}}_P(X)$ and $\overline{\mathrm{apr}}_P(X)$, which are called the soft P-lower approximation and the soft P-upper approximation of X, respectively. In general, we refer to $\underline{\mathrm{apr}}_P(X)$ and $\overline{\mathrm{apr}}_P(X)$ as soft rough approximations of X with respect to P. Moreover, the sets

$$\begin{aligned} \mathrm{POS}_P(X) &= \underline{\mathrm{apr}}_P(X), \\ \mathrm{NEG}_P(X) &= -\overline{\mathrm{apr}}_P(X), \\ \mathrm{BND}_P(X) &= \overline{\mathrm{apr}}_P(X) - \underline{\mathrm{apr}}_P(X) \end{aligned} \tag{5}$$

are called the soft P-positive region, the soft P-negative region, and the soft P-boundary region of X, respectively. If $\underline{\mathrm{apr}}_P(X) = \overline{\mathrm{apr}}_P(X)$, X is said to be soft P-definable; otherwise X is called a soft P-rough set.

Example 7 (see [8]). Let $U = \{u_1, u_2, u_3, u_4, u_5, u_6\}$, let $E = \{e_1, e_2, e_3, e_4, e_5, e_6\}$, and let $A = \{e_1, e_2, e_3, e_4\} \subseteq E$. Let $S = (f, A)$ be a soft set over U given by Table 1 and the approximation space $P = (U, S)$.

For $X = \{u_3, u_4, u_5\} \subseteq U$, we have $\underline{\mathrm{apr}}_P(X) = \{u_3\}$ and $\overline{\mathrm{apr}}_P(X) = \{u_1, u_2, u_3, u_4\}$. Thus $\underline{\mathrm{apr}}_P(X) \neq \overline{\mathrm{apr}}_P(X) = \{u_3\}$ and X is a soft P-rough set. Note that $X = \{u_3, u_4, u_5\} \nsubseteq \overline{\mathrm{apr}}_P(X) = \{u_1, u_2, u_3, u_5\}$ in this case. Moreover, it is easy to see that $\mathrm{POS}_P(X) = \{u_3\}$, $\mathrm{NEG}_P(X_1) = \{u_3, u_4\}$, and

Table 1: The tabular representation of the soft set S.

	u_1	u_2	u_3	u_4	u_5	u_6
e_1	1	0	0	0	0	1
e_2	0	0	1	0	0	0
e_3	0	0	0	0	0	0
e_4	1	1	0	0	1	0

Table 2: The tabular representation of the soft class f_D.

f_D	f_{d_1}	f_{d_2}	f_{d_3}
e_1	$\{u_1, u_3, u_4\}$	$\{u_1, u_2\}$	$\{u_2, u_3, u_5\}$
e_2	$\{u_1, u_4, u_5, u_6\}$	$\{u_3, u_6\}$	$\{u_1, u_4, u_7\}$
e_3	$\{u_1, u_2, u_3, u_8\}$	U	$\{\}$
e_4	$\{\}$	$\{u_1, u_3, u_7, u_8\}$	$\{u_5, u_8\}$

$\mathrm{BND}_P(X) = \{u_1, u_2, u_3\}$. On the other hand, one can consider $X_1 = \{u_3, u_4\} \subseteq U$. Since $\underline{\mathrm{apr}}_P(X_1) = \{u_3\} = \overline{\mathrm{apr}}_P(X_1)$, by definition, X_1 is a soft P-definable set.

3. Soft Classes

In this section, we define concept of soft class and soft class operations. Also we obtain some basic properties of soft class operations.

Definition 8. Let E be a parameter set, let U be an initial universe, and let $D = \{d_i : i = 1, 2, \ldots, n\}$ be a set of decision makers. Indexed class of soft sets $\{f_{d_i} : f_{d_i} : E \to P(U),\ d_i \in D\}$ is called a soft class and is denoted by f_D.

If, for any $d_i \in D$, $f_{d_i} = \Phi$, the soft set f_{d_i} does not appear in soft class f_D.

Throughout this study E, U, and D denote parameter set, initial universe, and decision makers set, respectively.

From now on, all soft classes over parameter set E, initial universe U, and decision makers set D will be denoted by $\mathcal{SC}_D^E(U)$.

Example 9. Let $E = \{e_1, e_2, e_3, e_4\}$ be a parameter set, let $U = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8\}$ be an initial universe, and let $D = \{d_1, d_2, d_3\}$ be a set of decision makers. If we consider soft sets $f_{d_1}, f_{d_2}, f_{d_3}$ given as

$$\begin{aligned} f_{d_1} &= \{(e_1, \{u_1, u_3, u_4\}), (e_2, \{u_1, u_4, u_5, u_6\}), \\ &\quad (e_3, \{u_1, u_2, u_3, u_8\}), (e_4, \{\})\}, \\ f_{d_2} &= \{(e_1, \{u_1, u_2\}), (e_2, \{u_3, u_6\}), (e_3, U), \\ &\quad (e_4, \{u_1, u_3, u_7, u_8\})\}, \\ f_{d_3} &= \{(e_1, \{u_2, u_3, u_5\}), (e_2, \{u_1, u_4, u_7\}), (e_3, \{\}), \\ &\quad (e_4, \{u_5, u_8\})\}, \end{aligned} \tag{6}$$

then $f_D = \{f_{d_1}, f_{d_2}, f_{d_3}\}$ is a soft class. We can represent a soft class in tabular form as shown in Table 2.

Definition 10. Let $f_D \in \mathcal{SC}_D^E(U)$. If, for all $d_i \in D$, $f_{d_i} = \Phi$, then f_D is called an empty soft class and is denoted by $\emptyset$.

Definition 11. Let $f_D \in \mathcal{SC}_D^E(U)$. If, for all $d_i \in D$, $f_{d_i} = \widehat{U}$, then f_D is called a universal soft class and is denoted by $\mathcal{U}$.

Definition 12. Let $f_D, g_D \in \mathcal{SC}_D^E(U)$. Then, f_D is a soft subclass of g_D, denoted by $f_D \sqsubseteq g_D$, if, for all $d_i \in D$, $f_{d_i} \mathrel{\tilde{\subseteq}} g_{d_i}$.

Example 13. Let $E = \{e_1, e_2, e_3, e_4\}$ be a parameter set, let $U = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8\}$ be an initial universe, and let $D = \{d_1, d_2, d_3\}$ be a decision makers set. If

$$\begin{aligned} f_{d_1} &= \{(e_1, \{u_1, u_3, u_4\}), (e_2, \{u_1, u_4, u_5, u_6\}), \\ &\quad (e_3, \{u_1, u_2, u_3, u_8\}), (e_4, \{\})\}, \\ f_{d_2} &= \{(e_1, \{u_1, u_2\}), (e_2, \{u_3, u_6\}), (e_3, U), \\ &\quad (e_4, \{u_1, u_3, u_7, u_8\})\}, \\ f_{d_3} &= \{(e_1, \{u_2, u_3, u_5\}), (e_2, \{u_1, u_4, u_7\}), (e_3, \{\}), \\ &\quad (e_4, \{u_5, u_8\})\}, \\ g_{d_1} &= \{(e_1, \{u_1, u_3\}), (e_2, \{u_1, u_4, u_6\}), (e_3, \{u_1, u_2, u_8\}), \\ &\quad (e_4, \{\})\}, \\ g_{d_2} &= \{(e_1, \{u_1\}), (e_2, \{u_3, u_6\}), (e_3, \{u_1, u_2, u_3, u_6\}), \\ &\quad (e_4, \{u_1, u_3, u_7\})\}, \\ g_{d_3} &= \{(e_1, \{u_2, u_5\}), (e_2, \{u_1, u_4\}), (e_3, \{\}), \\ &\quad (e_4, \{u_5, u_8\})\}, \end{aligned} \tag{7}$$

then soft classes can be written as $f_D = \{f_{d_1}, f_{d_2}, f_{d_3}\}$ and $g_D = \{g_{d_1}, g_{d_2}, g_{d_3}\}$.

Note that, for all $d_i \in D$, since $g_{d_i} \mathrel{\tilde{\subseteq}} f_{d_i}$, $g_D \sqsubseteq f_D$.

Proposition 14. *If f_D, g_D, and $h_D \in \mathcal{SC}_D^E(U)$, then*

(1) $f_D \sqsubseteq \mathcal{U}$;
(2) $\emptyset \sqsubseteq f_D$;
(3) $f_D \sqsubseteq f_D$;
(4) $f_D \sqsubseteq g_D$ *and* $g_D \sqsubseteq h_D \Rightarrow f_D \sqsubseteq h_D$.

Proof. If f_D, g_D, and $h_D \in \mathcal{SC}_D^E(U)$, then, for all $d_i \in D$,

(1) $f_{d_i} \mathrel{\tilde{\subseteq}} \widehat{U} \Rightarrow f_D \sqsubseteq \mathcal{U}$;
(2) $\Phi \mathrel{\tilde{\subseteq}} f_{d_i} \Rightarrow \emptyset \sqsubseteq f_D$;
(3) $f_{d_i} \mathrel{\tilde{\subseteq}} f_{d_i} \Rightarrow f_D \sqsubseteq f_D$;
(4) $f_{d_i} \mathrel{\tilde{\subseteq}} g_{d_i}$ and $g_{d_i} \mathrel{\tilde{\subseteq}} h_{d_i} \Rightarrow f_{d_i} \mathrel{\tilde{\subseteq}} h_{d_i}$; then, $f_D \sqsubseteq g_D$ and $g_D \sqsubseteq h_D \Rightarrow f_D \sqsubseteq h_D$. □

Definition 15. Let $f_D, g_D \in \mathcal{SC}_D^E(U)$. Then, f_D and g_D are equal soft classes if and only if $f_D \sqsubseteq g_D$ and $f_D \sqsupseteq g_D$. This relation is denoted by $f_D = g_D$.

Definition 16. Let f and $g \in \mathcal{S}(U)$ and let $f \mathrel{\tilde{\subseteq}} g$. Then, according to the soft set g, degree of subsethood of soft set f, denoted by f_g°, is defined as follows:

$$f_g^\circ = \frac{1}{|E_g|}\sum_{e\in E}\frac{|f(e)|}{|g(e)|}, \quad g(e) \neq \emptyset. \tag{8}$$

Here, E_g is set of parameters such that $g(e) \neq \emptyset$.

Definition 17. Let $f_D, g_D \in \mathcal{SC}_D^E(U)$, let $f_D \neq \emptyset$, and let D_1 and D_2 be two subsets of D such that $D_1 \cup D_2 = D$ and $D_1 \cap D_2 = \emptyset$. If $\forall d_i \in D_1$, $f_{d_i} \mathrel{\tilde{\subseteq}} g_{d_i}$, and $\forall d_i \in D_2$, $f_{d_i} \mathrel{\tilde{\nsubseteq}} g_{d_i}$, then f_D is called almost-subclass of soft class g_D and is denoted by $f_D \sqsubseteq_a g_D$.

From now on, decision makers set D_1 will denote set of $d_i \in D$ such that $f_{d_i} \mathrel{\tilde{\subseteq}} g_{d_i}$.

Definition 18. Let $f_D, g_D \in \mathcal{SC}_D^E(U)$ and let $f_D \sqsubseteq_a g_D$. Then, according to the soft class g_D, degree of subclasshood of soft class f_D, denoted by $a(f_D, g_D)$, is defined as follows:

$$a(f_D, g_D) = \frac{|D_1|}{|D|}\sum_{d_i\in D_1} f_g^\circ(d_i). \tag{9}$$

Here, for all $d_i \in D_1$, $f_{d_i} \mathrel{\tilde{\subseteq}} g_{d_i}$ and $f_g^\circ(d_i) = (1/|E_g|)\sum_{e\in E}(|f_{d_i}(e)|/|g_{d_i}(e)|)$, such that $g_{d_i}(e) \neq \emptyset$.

Example 19. Let us consider soft class f_D given in Example 13 and soft class g_D given as follows:

$$\begin{aligned} g_{d_1} &= \{(e_1,\{u_1,u_3,u_4,u_7\}), \\ &\quad (e_2,\{u_1,u_2,u_4,u_5,u_6,u_7\}),(e_3,\{u_1,u_2,u_3,u_4,u_8\}), \\ &\quad (e_4,\{u_1\})\}, \\ g_{d_2} &= \{(e_1,\{u_1,u_2,u_5,u_6\}),(e_2,\{u_3,u_6,u_7\}),(e_3,U), \\ &\quad (e_4,\{u_1,u_2,u_3,u_6,u_7,u_8\})\}, \\ g_{d_3} &= \{(e_1,\{u_2,u_3\}),(e_2,\{u_1,u_4,u_7\}),(e_3,\{\}), \\ &\quad (e_4,\{u_5\})\}. \end{aligned} \tag{10}$$

Here, since $f_{d_1} \mathrel{\tilde{\subseteq}} g_{d_1}$, $f_{d_2} \mathrel{\tilde{\subseteq}} g_{d_2}$, and $f_{d_3} \mathrel{\tilde{\nsubseteq}} g_{d_3}$, $|D_1| = 2$. Then,

$$\begin{aligned} f_g^\circ(d_1) &= \frac{1}{|E_g|}\sum_{e\in E}\frac{|f_{d_1}(e)|}{|g_{d_1}(e)|} \\ &= \frac{1}{4}(0.75+0.66+0.80+0) = 0.55, \\ f_g^\circ(d_2) &= \frac{1}{|E_g|}\sum_{e\in E}\frac{|f_{d_2}(e)|}{|g_{d_2}(e)|} \\ &= \frac{1}{4}(0.50+0.66+1+0.66) = 0.71. \end{aligned} \tag{11}$$

Thus,

$$a(f_D, g_D) = \frac{2}{3}(0.55+0.71) = 0.84 \tag{12}$$

and $f_D \sqsubseteq_a g_D$.

Corollary 20. *Let $f_D, g_D \in \mathcal{SC}_D^E(U)$. Then*

(1) *if $\forall d_i \in D$, $f_{d_i} = g_{d_i}$, then $a(f_D, g_D) = 1$;*

(2) *if $f_D \sqsubseteq g_D$, f_D may be almost-subclass of soft class g_D;*

(3) *if $f_D \sqsubseteq_a g_D$, f_D may not be a subclass of soft class g_D.*

Definition 21. Let $f_D, g_D \in \mathcal{SC}_D^E(U)$. Then, union of soft classes f_D and g_D, denoted by $f_D \sqcup g_D$, is defined by class of soft sets as follows:

$$f_D \sqcup g_D = \{f_{d_i} \mathbin{\tilde{\cup}} g_{d_i} : d_i \in D\}. \tag{13}$$

Example 22. Let $E = \{e_1, e_2, e_3, e_4\}$ be a parameter set, let $U = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8\}$ be an initial universe, and let $D = \{d_1, d_2, d_3\}$ be a decision makers set. If

$$\begin{aligned} f_{d_1} &= \{(e_1,\{u_1,u_2,u_5\}),(e_2,\{u_3,u_4,u_7,u_8\}), \\ &\quad (e_3,\{u_1,u_3,u_5,u_6\}),(e_4,\{\})\}, \\ f_{d_2} &= \{(e_1,\{u_1,u_3\}),(e_2,\{u_3,u_5\}),(e_3,\{u_2,u_4\}), \\ &\quad (e_4,\{u_5,u_7,u_8\})\}, \\ f_{d_3} &= \{(e_1,\{u_4,u_5,u_6,u_7\}),(e_2,\{u_1,u_6,u_8\}),(e_3,\{\}), \\ &\quad (e_4,\{u_1,u_4,u_5\})\}, \\ g_{d_1} &= \{(e_1,\{u_1,u_3,u_4\}),(e_2,\{u_4,u_6\}),(e_3,\{u_3,u_5,u_8\}), \\ &\quad (e_4,\{u_3,u_7\})\}, \\ g_{d_2} &= \{(e_1,\{u_2,u_3\}),(e_2,\{u_3,u_6,u_8\}), \\ &\quad (e_3,\{u_1,u_2,u_3,u_6\}),(e_4,\{u_1,u_3,u_7,u_8\})\}, \\ g_{d_3} &= \{(e_1,\{u_2,u_5,u_6\}),(e_2,\{u_1,u_4\}),(e_3,\{\}), \\ &\quad (e_4,\{u_5,u_7,u_8\})\}, \end{aligned} \tag{14}$$

then soft classes can be written as $f_D = \{f_{d_1}, f_{d_2}, f_{d_3}\}$ and $g_D = \{g_{d_1}, g_{d_2}, g_{d_3}\}$.

Here,

$$f_D \sqcup g_D = \left\{ \begin{array}{l} f_{d_1} \tilde{\cup}\, g_{d_1} = \{(e_1, \{u_1, u_2, u_3, u_4, u_5\}), (e_2, \{u_3, u_4, u_6, u_7, u_8\}), (e_3, \{u_1, u_3, u_5, u_6, u_8\}), (e_4, \{u_3, u_7\})\} \\ f_{d_2} \tilde{\cup}\, g_{d_2} = \{(e_1, \{u_1, u_2, u_3\}), (e_2, \{u_3, u_5, u_6, u_8\}), (e_3, \{u_1, u_2, u_3, u_5, u_6\}), (e_4, \{u_1, u_3, u_5, u_7, u_8\})\} \\ f_{d_3} \tilde{\cup}\, g_{d_3} = \{(e_1, \{u_2, u_4, u_5, u_6, u_7\}), (e_2, \{u_1, u_4, u_6, u_8\}), (e_3, \{\}), (e_4, \{u_1, u_4, u_5, u_7, u_8\})\} \end{array} \right\}. \quad (15)$$

Proposition 23. *If f_D, g_D, and $h_D \in \mathcal{SC}_D^E(U)$, then*

(1) $f_D \sqcup f_D = f_D$;

(2) $f_D \sqcup \emptyset = f_D$;

(3) $f_D \sqcup \mathcal{U} = \mathcal{U}$;

(4) $f_D \sqcup f_D^{\tilde{c}} = \mathcal{U}$;

(5) $f_D \sqcup g_D = g_D \sqcup f_D$;

(6) $(f_D \sqcup g_D) \sqcup h_D = f_D \sqcup (g_D \sqcup h_D)$.

Proof. Let f_D, g_D, and $h_D \in \mathcal{SC}_D^E(U)$. Then, for all $d_i \in D$,

(1) $f_D \sqcup f_D = f_D$, since $f_{d_i} \tilde{\cup}\, f_{d_i} = f_{d_i}$;

(2) $f_D \sqcup \emptyset = f_D$, since $f_{d_i} \tilde{\cup}\, \Phi = f_{d_i}$;

(3) $f_D \sqcup \mathcal{U} = \mathcal{U}$, since $f_{d_i} \tilde{\cup}\, \widehat{U} = \widehat{U}$;

(4) $f_D \sqcup f_D^{\tilde{c}} = \mathcal{U}$, since $f_{d_i} \tilde{\cup}\, f_{d_i}^{\tilde{c}} = \widehat{U}$;

(5) $f_D \sqcup g_D = g_D \sqcup f_D$, since $f_{d_i} \tilde{\cup}\, g_{d_i} = g_{d_i} \tilde{\cup}\, f_{d_i}$;

(6) $(f_D \sqcup g_D) \sqcup h_D = f_D \sqcup (f_D \sqcup h_D)$, since $(f_{d_i} \tilde{\cup}\, g_{d_i}) \tilde{\cup}\, h_{d_i} = f_{d_i} \tilde{\cup}\, (g_{d_i} \tilde{\cup}\, h_{d_i})$.

□

Definition 24. Let $f_D, g_D \in \mathcal{SC}_D^E(U)$. Then, intersection of soft classes f_D and g_D, denoted by $f_D \sqcap g_D$, is defined by class of soft sets as follows:

$$f_D \sqcap g_D = \left\{ f_{d_i} \tilde{\cap}\, g_{d_i} : d_i \in D \right\}. \quad (16)$$

Example 25. Let us consider soft classes f_D and g_D given in Example 22. Then,

$$f_D \sqcap g_D = \left\{ \begin{array}{l} f_{d_1} \tilde{\cap}\, g_{d_1} = \{(e_1, \{u_1\}), (e_2, \{u_4\}), (e_3, \{u_3, u_5\}), (e_4, \{\})\} \\ f_{d_2} \tilde{\cap}\, g_{d_2} = \{(e_1, \{u_3\}), (e_2, \{u_3\}), (e_3, \{u_2\}), (e_4, \{u_7, u_8\})\} \\ f_{d_3} \tilde{\cap}\, g_{d_3} = \{(e_1, \{u_5, u_6\}), (e_2, \{u_1\}), (e_3, \{\}), (e_4, \{u_5\})\} \end{array} \right\}. \quad (17)$$

Proposition 26. *If f_D, g_D, and $h_D \in \mathcal{SC}_D^E(U)$, then*

(1) $f_D \sqcap f_D = f_D$;

(2) $f_D \sqcap \emptyset = \emptyset$;

(3) $f_D \sqcap \mathcal{U} = f_D$;

(4) $f_D \sqcap f_D^{\tilde{c}} = \emptyset$;

(5) $f_D \sqcap g_D = g_D \sqcap f_D$;

(6) $(f_D \sqcap g_D) \sqcap h_D = f_D \sqcap (g_D \sqcap h_D)$.

Proof. Let f_D, g_D, and $h_D \in \mathcal{SC}_D^E(U)$. Then, for all $d_i \in D$,

(1) $f_D \sqcap f_D = f_D$, since $f_{d_i} \tilde{\cap}\, f_{d_i} = f_{d_i}$;

(2) $f_D \sqcap \emptyset = \emptyset$, since $f_{d_i} \tilde{\cap}\, \Phi = \Phi$;

(3) $f_D \sqcap \mathcal{U} = f_D$, since $f_{d_i} \tilde{\cap}\, \widehat{U} = f_{d_i}$;

(4) $f_D \sqcap f_D^{\tilde{c}} = \emptyset$, since $f_{d_i} \tilde{\cap}\, f_{d_i}^{\tilde{c}} = \Phi$;

(5) $f_D \sqcap g_D = g_D \sqcap f_D$, since $f_{d_i} \tilde{\cap}\, g_{d_i} = g_{d_i} \tilde{\cap}\, f_{d_i}$;

(6) $(f_D \sqcap g_D) \sqcap h_D = f_D \sqcap (f_D \sqcap h_D)$, since $(f_{d_i} \tilde{\cap}\, g_{d_i}) \tilde{\cap}\, h_{d_i} = f_{d_i} \tilde{\cap}\, (g_{d_i} \tilde{\cap}\, h_{d_i})$.

□

Proposition 27. *If f_D, g_D, and $h_D \in \mathcal{SC}_D^E(U)$, then*

(1) $f_D \sqcup (g_D \sqcap h_D) = (f_D \sqcup g_D) \sqcap (f_D \sqcup h_D)$;

(2) $f_D \sqcap (g_D \sqcup h_D) = (f_D \sqcap g_D) \sqcup (f_D \sqcap h_D)$.

Proof. The proof can be easily obtained from Definitions 21 and 24. □

Definition 28. Let $f_D \in \mathcal{SC}_D^E(U)$. Then, soft complement of soft class f_D, denoted by $f_D^{\tilde{c}}$, is defined by class of soft sets as follows:

$$f_D^{\tilde{c}} = \left\{ f_{d_i}^{\tilde{c}} : d_i \in D \right\}. \quad (18)$$

Here, $f_{d_i}^{\tilde{c}} = \widehat{U} \,\tilde{\setminus}\, f_{d_i}$ for all $d_i \in D$.

Proposition 29. *If $f_D \in \mathcal{SC}_D^E(U)$, then*

(1) $(f_D^{\tilde{c}})^{\tilde{c}} = f_D$;

(2) $\emptyset^{\tilde{c}} = \mathcal{U}$.

Proof. The proof can be easily obtained from Definition 28. □

Proposition 30. *If $f_D, g_D \in \mathcal{SC}_D^E(U)$, then*

(1) $(f_D \sqcup g_D)^{\tilde{c}} = f_D^{\tilde{c}} \sqcap g_D^{\tilde{c}}$;

(2) $(f_D \sqcap g_D)^{\tilde{c}} = f_D^{\tilde{c}} \sqcup g_D^{\tilde{c}}$.

Proof. Let $f_D, g_D \in \mathcal{SC}_D^E(U)$. Then, for all $d_i \in D$,

(1) $(f_D \sqcup g_D)^{\tilde{c}} = f_D^{\tilde{c}} \sqcap g_D^{\tilde{c}}$, since $(f_{d_i} \tilde{\cup}\, g_{d_i})^{\tilde{c}} = f_{d_i}^{\tilde{c}} \tilde{\cap}\, g_{d_i}^{\tilde{c}}$;

(2) $(f_D \sqcap g_D)^{\tilde{c}} = f_D^{\tilde{c}} \sqcap g_D^{\tilde{c}}$, since $(f_{d_i} \tilde{\cap}\, g_{d_i})^{\tilde{c}} = f_{d_i}^{\tilde{c}} \tilde{\cup}\, g_{d_i}^{\tilde{c}}$.

□

Proposition 31. *If* $f_{j_D} \in \mathcal{SC}_D^E(U)$ $(j = 1, 2, \ldots, n)$, *then*

(1) $(\widetilde{\bigcup}_{j=1}^{n} f_{j_{d_i}})^{\tilde{c}} = \widetilde{\bigcap}_{j=1}^{n} (f_{j_{d_i}})^{\tilde{c}}$, *for all* $d_i \in D$;

(2) $(\widetilde{\bigcap}_{j=1}^{n} f_{j_{d_i}})^{\tilde{c}} = \widetilde{\bigcup}_{j=1}^{n} (f_{j_{d_i}})^{\tilde{c}}$, *for all* $d_i \in D$.

Proof. Since $f_{j_{d_i}}$ are soft sets for all $d_i \in D$ $(j = 1, 2, \ldots, n)$, the proof is clear. □

Proposition 32. *Let* $f_{j_D} \in \mathcal{SC}_D^E(U)$. *Then,*

(1) $(\bigsqcup_{j=1}^{n} f_{j_D})^{\tilde{c}} = \bigsqcap_{j=1}^{n} (f_{j_D})^{\tilde{c}}$;

(2) $(\bigsqcap_{j=1}^{n} f_{j_D})^{\tilde{c}} = \bigsqcup_{j=1}^{n} (f_{j_D})^{\tilde{c}}$.

Proof. The proof is obvious from Propositions 30 and 31. □

Definition 33. Let $f_D \in \mathcal{SC}_D^E(U)$ and let $g \in \mathcal{S}(U)$. Then, f_D is called soft partition of soft set g if and only if all of the following conditions hold:

(1) $\Phi \notin f_D$.

(2) $\bigcup_{d_i \in D} f_{d_i}(e) = g(e)$, for all $e \in E$.

(3) If $f_{d_i}, f_{d_j} \in f_D$ and $i \neq j$, then $f_{d_i}(e) \cap f_{d_j}(e) = \emptyset$, for all $e \in E$.

Definition 34. Let $f_D \in \mathcal{SC}_D^E(U)$ and let $g \in \mathcal{S}(U)$. If, for all $e \in E$,

$$g(e) \subseteq \bigcup_{d_i \in D} f_{d_i}(e), \tag{19}$$

then soft class f_D is called soft cover of soft set g.

Example 35. Let us consider soft class f_D given in Example 22. Then, soft class f_D is soft cover of soft set g given as follows:

$$g = \{(e_1, \{u_1, u_6\}), (e_2, \{u_4, u_5, u_8\}), (e_3, \{u_2, u_5\}), (e_4, \{u_4, u_5\})\}. \tag{20}$$

Definition 36. Let $f_D \in \mathcal{SC}_D^E(U)$. If, for all $e \in E$ and $d_i \in D$, $\bigcup_{d_i \in D} f_{d_i}(e) = \widehat{U}$, then soft class f_D is called full soft class and is denoted by $\widehat{f}_D$.

Proposition 37. *Let* $f_D, g_D \in \mathcal{SC}_D^E(U)$ *be two soft covers of soft set* $h \in \mathcal{S}(U)$. *Then,* $f_D \sqcap g_D$ *is a soft cover of soft set* h.

Proof. Assume that f_D and g_D be two soft covers of soft set h; then, for all $e \in E$, $h(e) \subseteq \bigcup_{d_i \in D} f_{d_i}(e)$ and $h(e) \subseteq \bigcup_{d_i \in D} g_{d_i}(e)$. Hence, $h(e) \subseteq (\bigcup_{d_i \in D} f_{d_i}(e)) \cap (\bigcup_{d_i \in D} g_{d_i}(e)) = \bigcup_{d_i \in D}(f_{d_i}(e) \cap g_{d_i}(e))$ for all $e \in E$. So, soft class $f_D \sqcap g_D$ is a soft cover of soft set h. □

Proposition 38. *Let* $f_D, g_D \in \mathcal{SC}_D^E(U)$ *be two soft covers of soft set* $h \in \mathcal{S}(U)$. *Then,* $f_D \sqcup g_D$ *is a soft cover of soft set* h.

Proof. Assume that f_D and g_D be two soft covers of soft set h; then, for all $e \in E$, $h(e) \subseteq \bigcup_{d_i \in D} f_{d_i}(e)$ and $h(e) \subseteq \bigcup_{d_i \in D} g_{d_i}(e)$. Hence, $h(e) \subseteq (\bigcup_{d_i \in D} f_{d_i}(e)) \cup (\bigcup_{d_i \in D} g_{d_i}(e)) = \bigcup_{d_i \in D}(f_{d_i}(e) \cup g_{d_i}(e))$ for all $e \in E$. So soft class $f_D \sqcup g_D$ is a soft cover of soft set h. □

Corollary 39. *Let* $\widehat{f}_D \in \mathcal{SC}_D^E(U)$. *Then, for all* $g \in \mathcal{S}(U)$, $\widehat{f}_D$ *is a soft cover of soft set* g.

4. Soft Rough Classes

In this section, we define soft rough class and investigate its some properties.

Definition 40. Let $f_D \in \mathcal{SC}_D^E(U)$. For $e_j \in E$, parameterized class (e_j-class) of soft class f_D, denoted by $C_{f_D}(e_j)$, is defined as follows:

$$C_{f_D}(e_j) = \{f_{d_i}(e_j) : d_i \in D\}. \tag{21}$$

Example 41. Let $E = \{e_1, e_2, e_3, e_4\}$ be a parameter set, let $U = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8\}$ be an initial universe, and let $\{d_1, d_2, d_3\}$ be a set of decision makers. Let us consider soft sets f_{d_1}, f_{d_2}, and f_{d_3} given as follows:

$$\begin{aligned} f_{d_1} &= \{(e_1, \{u_1, u_3, u_4\}), (e_2, \{u_1, u_4, u_5, u_6\}), \\ &\quad (e_3, \{u_1, u_2, u_3, u_8\}), (e_4, \{\})\}, \\ f_{d_2} &= \{(e_1, \{u_1, u_2\}), (e_2, \{u_3, u_6\}), (e_3, U), \\ &\quad (e_4, \{u_1, u_3, u_7, u_8\})\}, \\ f_{d_3} &= \{(e_1, \{u_2, u_3, u_5\}), (e_2, \{u_1, u_4, u_7\}), (e_3, \{\}), \\ &\quad (e_4, \{u_5, u_8\})\}; \end{aligned} \tag{22}$$

then all of parameterized classes of f_D are as follows:

$$\begin{aligned} C_{f_D}(e_1) &= \{\{u_1, u_3, u_4\}, \{u_1, u_2\}, \{u_2, u_3, u_5\}\}, \\ C_{f_D}(e_2) &= \{\{u_1, u_4, u_5, u_6\}, \{u_3, u_6\}, \{u_1, u_4, u_7\}\}, \\ C_{f_D}(e_3) &= \{\{u_1, u_2, u_3, u_8\}, U, \{\}\}, \\ C_{f_D}(e_4) &= \{\{\}, \{u_1, u_3, u_7, u_8\}, \{u_5, u_8\}\}. \end{aligned} \tag{23}$$

Definition 42. Let $f_D \in \mathcal{SC}_D^E(U)$. Then, for $g \in \mathcal{S}(U)$ and $e \in E$, e-lower approximation, denoted by $\underline{g}_{f_D}(e)$, is defined as follows:

$$\underline{g}_{f_D}(e) = \{u \in U : \exists f_{d_i}(e) \in C_{f_D}(e), u \in f_{d_i}(e) \subseteq g(e)\}. \tag{24}$$

e-upper approximation, denoted by $\overline{g}_{f_D}(e)$, is defined as follows:

$$\overline{g}_{f_D}(e) = \{u \in U : \exists f_{d_i}(e) \in C_{f_D}(e), u \in f_{d_i}(e), f_{d_i}(e) \cap g(e) \neq \emptyset\}. \tag{25}$$

Moreover, the sets

$$\begin{aligned} \mathrm{POS}_{f_D} g(e) &= \underline{g}_{f_D}(e), \\ \mathrm{NEG}_{f_D} g(e) &= -\overline{g}_{f_D}(e), \\ \mathrm{BND}_{f_D} g(e) &= \overline{g}_{f_D}(e) - \underline{g}_{f_D}(e) \end{aligned} \tag{26}$$

are called the e-positive region, the e-negative region, and e-boundary region of $g \in \mathcal{SC}_D^E(U)$. Here $-g_{f_D}(e)$ is complement of set $g_{f_D}(e)$. If $\underline{g}_{f_D}(e) = \overline{g}_{f_D}(e)$, g is said to be e-definable; otherwise g is called e-rough set.

Proposition 43. *Let $f_D \in \mathcal{SC}_D^E(U)$. Then we have*

$$\begin{aligned} \underline{g}_{f_D}(e) &= \bigcup_{d_i \in D} \left\{ f_{d_i}(e) : f_{d_i}(e) \subseteq g(e) \right\}, \\ \overline{g}_{f_D}(e) &= \bigcup_{d_i \in D} \left\{ f_{d_i}(e) : f_{d_i}(e) \cap g(e) \neq \emptyset \right\} \end{aligned} \tag{27}$$

for all $g \in \mathcal{S}(U)$.

Definition 44. Let $f_D \in \mathcal{SC}_D^E(U)$ and let $g \in \mathcal{S}(U)$. Then, soft f_D-lower approximation, denoted by $\underline{\mathrm{apr}}_{f_D} g$, is defined as follows:

$$\underline{\mathrm{apr}}_{f_D} g = \left\{ \left(e, \underline{g}_{f_D}(e) \right) : e \in E \right\}. \tag{28}$$

Also, soft f_D-upper approximation, denoted by $\overline{\mathrm{apr}}_{f_D} g$, is defined as follows:

$$\overline{\mathrm{apr}}_{f_D} g = \left\{ \left(e, \overline{g}_{f_D}(e) \right) : e \in E \right\}. \tag{29}$$

Moreover, the sets

$$\begin{aligned} \mathrm{POS}_{f_D} g &= \underline{\mathrm{apr}}_{f_D} g = \left\{ \left(e, \underline{g}_{f_D}(e) \right) : e \in E \right\}, \\ \mathrm{NEG}_{f_D} g &= -\overline{\mathrm{apr}}_{f_D} g = \left\{ \left(e, -\overline{g}_{f_D}(e) \right) : e \in E \right\}, \\ \mathrm{BND}_{f_D} g &= \overline{\mathrm{apr}}_{f_D} g - \underline{\mathrm{apr}}_{f_D} g \\ &= \left\{ \left(e, \underline{g}_{f_D}(e) - \overline{g}_{f_D}(e) \right) : e \in E \right\} \end{aligned} \tag{30}$$

are called the soft f_D-positive region, the soft f_D-negative region, and soft f_D-boundary region of $g \in \mathcal{S}(U)$, respectively. If $\underline{\mathrm{apr}}_{f_D} g = \overline{\mathrm{apr}}_{f_D} g$, f_D is said to be soft f_D-definable; otherwise g is called a soft f_D-rough class.

Example 45. Let us consider soft class f_D given in Example 41. Let $g = \{(e_1, \{u_1, u_3, u_4, u_5\}), (e_2, \{u_3, u_6, u_7\}), (e_3, U), (e_4, \{u_5, u_7, u_8\})\} \in \mathcal{S}(U)$. Then,

$$\begin{aligned} \underline{\mathrm{apr}}_{f_D} g &= \{(e_1, \{u_1, u_3, u_4\}), (e_2, \{u_3, u_6\}), (e_3, U), \\ &\quad (e_4, \{u_5, u_8\})\}, \\ \overline{\mathrm{apr}}_{f_D} g &= \{(e_1, \{u_1, u_2, u_3, u_4, u_5\}), \\ &\quad (e_2, \{u_1, u_3, u_4, u_5, u_6, u_7\}), (e_3, U), \\ &\quad (e_4, \{u_1, u_3, u_5, u_7, u_8\})\}, \\ \mathrm{POS}_{f_D} g &= \{(e_1, \{u_1, u_3, u_4\}), (e_2, \{u_3, u_6\}), (e_3, U), \\ &\quad (e_4, \{u_5, u_8\})\}, \\ \mathrm{NEG}_{f_D} g &= \{(e_1, \{u_6, u_7, u_8\}), (e_2, \{u_2, u_8\}), (e_3, \{\}), \\ &\quad (e_4, \{u_2, u_4, u_6\})\}, \\ \mathrm{BND}_{f_D} g &= \{(e_1, \{u_2, u_5\}), (e_2, \{u_1, u_4, u_5, u_7\}), (e_3, \{\}), \\ &\quad (e_4, \{u_1, u_3, u_7\})\}. \end{aligned} \tag{31}$$

Lemma 46. *Let $f_D, g_D \in \mathcal{SC}_D^E(U)$ and let $h, k \in \mathcal{S}(U)$. Then, for all $e \in E$ and for all $d_i \in D$,*

(1) *if $h \neq \widehat{U}$, $h \neq \Phi$, $\underline{h}_{\mathcal{U}}(e) = \emptyset$ and $\overline{h}_{\mathcal{U}}(e) = U$;*

(2) *if $h = \widehat{U}$, $\underline{h}_{\mathcal{U}}(e) = \overline{h}_{\mathcal{U}}(e) = U$;*

(3) *$\underline{h}_{\emptyset}(e) = \overline{h}_{\emptyset}(e) = \emptyset$;*

(4) *$\underline{h}_{f_D \sqcup g_D}(e) = \underline{h}_{f_D}(e) \cup \underline{h}_{g_D}(e)$;*

(5) *$\overline{h}_{f_D \sqcup g_D}(e) \supseteq \overline{h}_{f_D}(e) \cup \overline{h}_{g_D}(e)$;*

(6) *$\underline{h}_{f_D \sqcap g_D}(e) \supseteq \underline{h}_{f_D}(e) \cap \underline{h}_{g_D}(e)$;*

(7) *$\overline{h}_{f_D \sqcap g_D}(e) = \overline{h}_{f_D}(e) \cap \overline{h}_{g_D}(e)$.*

Proof. Let $f_D, g_D \in \mathcal{SC}_D^E(U)$ and let $h, k \in \mathcal{S}(U)$. The proofs of (1), (2), and (3) are clear from definitions of e-upper and e-lower approximations:

(4) Let $x \in \underline{h}_{f_D \sqcup g_D}(e)$. Then, $x \in f_{d_i}(e) \cup g_{d_i}(e) \subseteq h(e)$ for some $d_i \in D$. Thus $x \in f_{d_i}(e) \subseteq h(e)$ or $x \in g_{d_i}(e) \subseteq h(e)$. So $x \in \underline{h}_{f_D}(e) \cup \underline{h}_{g_D}(e)$ and $\underline{h}_{f_D \sqcup g_D}(e) \subseteq \underline{h}_{f_D}(e) \cup \underline{h}_{g_D}(e)$. To prove the reverse inclusion, assume that $x \in \underline{h}_{f_D}(e) \cup \underline{h}_{g_D}(e)$. Then, $x \in \underline{h}_{f_D}(e)$ or $x \in \underline{h}_{g_D}(e)$. So, for some $d_i \in D$, $x \in f_{d_i}(e) \subseteq h(e)$ or $x \in g_{d_i}(e) \subseteq h(e)$ and $x \in f_{d_i}(e) \cup g_{d_i}(e) \subseteq h(e)$. Thus, $x \in \underline{h}_{f_D \sqcup g_D}(e)$. Then we have $\underline{h}_{f_D \sqcup g_D}(e) \subseteq \underline{h}_{f_D}(e) \cup \underline{h}_{g_D}(e) \overline{h}_{f_D \sqcup g_D}(e) = \overline{h}_{f_D}(e) \cup \overline{h}_{g_D}(e)$.

(5) Let $x \in \overline{h}_{f_D}(e) \cup \overline{h}_{g_D}(e)$. Then we have that $x \in \overline{h}_{f_D}(e)$ or $x \in \overline{h}_{g_D}(e)$. By definition, there exists some $d_i \in D$ such that $x \in f_{d_i}(e)$ and $f_{d_i}(e) \cap h(e) \neq \emptyset$ or $x \in g_{d_i}$ and $g_{d_i}(e) \cap h(e) \neq \emptyset$. So $x \in f_{d_i}(e) \cup g_{d_i}(e)$ and $(f_{d_i}(e) \cup g_{d_i}(e)) \cap h(e) \neq \emptyset$. Thus, $x \in \overline{h}_{f_D \sqcup g_D}(e)$. We concluded that $\overline{h}_{f_D \sqcup g_D}(e) \supseteq \overline{h}_{f_D}(e) \cup \overline{h}_{g_D}(e)$.

(6) Let $x \in \underline{h}_{f_D}(e) \cap \underline{h}_{g_D}(e)$. By definition, there exists some $d_i \in D$ such that $x \in f_{d_i}(e) \subseteq h(e)$ and $x \in g_{d_i}(e) \subseteq h(e)$. So $x \in (f_{d_i}(e) \cap g_{d_i})(e) \subseteq h(e)$. Hence, $x \in \underline{h}_{f_D \sqcap g_D}(e)$. Thus, we conclude that $\underline{h}_{f_D \sqcap g_D}(e) \supseteq \underline{h}_{f_D}(e) \cap \underline{h}_{g_D}(e)$.

(7) Let $x \in \overline{h}_{f_D}(e) \cap \overline{h}_{g_D}(e)$. Then we have that $x \in \overline{h}_{f_D}(e)$ and $x \in \overline{h}_{g_D}(e)$. By definition, there exists some $d_i \in D$ such that $x \in f_{d_i}(e)$ and $f_{d_i}(e) \cap h(e) \neq \emptyset$ and $x \in g_{d_i}$ and $g_{d_i}(e) \cap h(e) \neq \emptyset$. So $x \in f_{d_i}(e) \cap g_{d_i}(e)$ and $(f_{d_i}(e) \cap d_{d_i}(e)) \cap h(e) \neq \emptyset$. Thus, $x \in \overline{h}_{f_D \sqcap g_D}(e)$. We concluded that $\overline{h}_{f_D \sqcap g_D}(e) \supseteq \overline{h}_{f_D}(e) \cap \overline{h}_{g_D}(e)$. To prove the reverse inclusion, assume that $x \in \overline{h}_{f_D \sqcap g_D}(e)$; then $x \in f_{d_i}(e) \cap g_{d_i}(e)$ such that $(f_{d_i}(e) \cap g_{d_i}(e)) \cap h(e) \neq \emptyset$ and $(f_{d_i}(e) \cap h_{d_i}(e)) \cap (g_{d_i}(e) \cap h(e)) \neq \emptyset$. Hence, $(f_{d_i}(e) \cap h_{d_i}(e)) \neq \emptyset$ and $(g_{d_i}(e) \cap h_{d_i}(e)) \neq \emptyset$. Since $x \in f_{d_i}(e)$ and $(f_{d_i}(e) \cap h_{d_i}(e)) \neq \emptyset$, $x \in \overline{h}_{f_D}(e)$ and in a similar way $x \in \overline{h}_{g_D}(e)$. We get that $\overline{h}_{f_D \sqcap g_D}(e) \subseteq \overline{h}_{f_D}(e) \cap \overline{h}_{g_D}(e)$. Then, $\overline{h}_{f_D \sqcap g_D}(e) = \overline{h}_{f_D}(e) \cap \overline{h}_{g_D}(e)$.

□

Theorem 47. *Let $f_D, g_D \in \mathcal{SC}_D^E(U)$ and $h \in \mathcal{S}(U)$. Then,*

(1) $\underline{\mathrm{apr}}_{f_D \sqcup g_D}(h) = \underline{\mathrm{apr}}_{f_D}(h) \tilde{\cup} \underline{\mathrm{apr}}_{g_D}(h)$;

(2) $\overline{\mathrm{apr}}_{f_D \sqcup g_D}(h) \supseteq \overline{\mathrm{apr}}_{f_D}(h) \tilde{\cup} \overline{\mathrm{apr}}_{g_D}(h)$;

(3) $\underline{\mathrm{apr}}_{f_D \sqcap g_D}(h) \supseteq \underline{\mathrm{apr}}_{f_D}(h) \tilde{\cap} \underline{\mathrm{apr}}_{g_D}(h)$;

(4) $\overline{\mathrm{apr}}_{f_D \sqcap g_D}(h) = \overline{\mathrm{apr}}_{f_D}(h) \tilde{\cap} \overline{\mathrm{apr}}_{g_D}(h)$;

(5) $\underline{\mathrm{apr}}_{\emptyset}(h) = \Phi$;

(6) $\overline{\mathrm{apr}}_{\emptyset}(h) = \Phi$;

(7) *if* $\forall d_i \in D, h \neq \widehat{U}$, $\underline{\mathrm{apr}}_{\mathcal{U}}(h) = \Phi$ *and* $\overline{\mathrm{apr}}_{\mathcal{U}}(h) = \widehat{U}$;

(8) *if* $\forall d_i \in D, h = \widehat{U}$, $\underline{\mathrm{apr}}_{\mathcal{U}}(h) = \widehat{U}$ *and* $\overline{\mathrm{apr}}_{\mathcal{U}}(h) = \widehat{U}$.

Proof. By using Lemma 46, the proof can be easily made. □

Theorem 48. *Let $f_D \in \mathcal{SC}_D^E(U)$ and let $h, k \in \mathcal{S}(U)$. Then,*

(1) $\underline{\mathrm{apr}}_{f_D}(\Phi) = \overline{\mathrm{apr}}_{f_D}(\Phi) = \Phi$;

(2) $\underline{\mathrm{apr}}_{f_D}(\widehat{U}) = \overline{\mathrm{apr}}_{f_D}(\widehat{U}) = \widehat{U}$;

(3) $h \tilde{\subseteq} k \Rightarrow \underline{\mathrm{apr}}_{f_D}(h) \tilde{\subseteq} \underline{\mathrm{apr}}_{f_D}(k)$;

(4) $h \tilde{\subseteq} k \Rightarrow \overline{\mathrm{apr}}_{f_D}(h) \tilde{\subseteq} \overline{\mathrm{apr}}_{f_D}(k)$;

(5) $\underline{\mathrm{apr}}_{f_D}(h \tilde{\cap} k) \tilde{\subseteq} \underline{\mathrm{apr}}_{f_D}(h) \tilde{\cap} \underline{\mathrm{apr}}_{f_D}(k)$;

(6) $\overline{\mathrm{apr}}_{f_D}(h \tilde{\cap} k) \tilde{\subseteq} \overline{\mathrm{apr}}_{f_D}(h) \tilde{\cap} \overline{\mathrm{apr}}_{f_D}(k)$;

(7) $\underline{\mathrm{apr}}_{f_D}(h \tilde{\cup} k) \tilde{\supseteq} \underline{\mathrm{apr}}_{f_D}(h) \tilde{\cup} \underline{\mathrm{apr}}_{f_D}(k)$;

(8) $\overline{\mathrm{apr}}_{f_D}(h \tilde{\cup} k) = \overline{\mathrm{apr}}_{f_D}(h) \tilde{\cup} \overline{\mathrm{apr}}_{f_D}(k)$.

Proof. (1) It is straightforward.

(2) It is straightforward.

(3) Let $x \in \underline{h}_{f_D}(e)$. Then, for some $d_i \in D$, $x \in f_{d_i}(e) \subseteq h(e)$. Since $(h) \tilde{\subseteq} k$, $h(e) \subseteq k(e)$, and $x \in f_{d_i}(e) \subseteq k(e)$. Therefore $x \in \underline{k}_{f_D}(e)$ and $\underline{h}_{f_D}(e) \subseteq \underline{k}_{f_D}(e)$; From definition of soft f_D-lower approximation $\underline{\mathrm{apr}}_{f_D}(h) \tilde{\subseteq} \underline{\mathrm{apr}}_{f_D}(k)$.

(4) Let $h \tilde{\subseteq} k$. Then, for all $d_i \in D$, $h(e) \cap f_{d_i}(e) \subseteq k(e) \cap f_{d_i}(e)$ and $h(e) \cap f_{d_i}(e) \neq \emptyset, k(e) \cap f_{d_i}(e) \neq \emptyset$ for some $d_i \in D$. Therefore $\overline{h}_{f_D}(e) \subseteq \overline{k}_{f_D}(e)$. From definition of soft f_D-upper approximation $\overline{\mathrm{apr}}_{f_D}(h) \tilde{\subseteq} \overline{\mathrm{apr}}_{f_D}(k)$.

(5) Since $h \tilde{\cap} k \tilde{\subseteq} h$ and $h \tilde{\cap} k \tilde{\subseteq} k$, from (3), $\underline{\mathrm{apr}}_{f_D}(h \tilde{\cap} k) \tilde{\subseteq} \underline{\mathrm{apr}}_{f_D}(k)$ and $\underline{\mathrm{apr}}_{f_D}(h \tilde{\cap} k) \tilde{\subseteq} \underline{\mathrm{apr}}_{f_D}(k)$, respectively. Therefore, $\underline{\mathrm{apr}}_{f_D}(h \tilde{\cap} k) \tilde{\subseteq} \underline{\mathrm{apr}}_{f_D}(h) \tilde{\cap} \underline{\mathrm{apr}}_{f_D}(k)$.

(6) This is similar to proof (5).

(7) Since $k \tilde{\subseteq} h \tilde{\cup} k$, from (3), $\underline{\mathrm{apr}}_{f_D}(k) \tilde{\subseteq} \underline{\mathrm{apr}}_{f_D}(h \tilde{\cup} k)$. Similarly, $\underline{\mathrm{apr}}_{f_D}(h) \tilde{\subseteq} \underline{\mathrm{apr}}_{f_D}(h \tilde{\cup} k)$. Therefore, $\underline{\mathrm{apr}}_{f_D}(h \tilde{\cup} k) \tilde{\supseteq} \underline{\mathrm{apr}}_{f_D}(k) \tilde{\cup} \underline{\mathrm{apr}}_{f_D}(h)$.

(8) Let $(e, f(e)) \in \overline{\mathrm{apr}}_{f_D}(h \tilde{\cup} k)$. By definition of soft f_D-upper approximation, there exist some $e \in E$ such that $u \in f(e)$ and $f(e) \cap (h \tilde{\cup} k)(e) \neq \emptyset$. Hence, we get that either $f(e) \cap h(e) \neq \emptyset$ or $f(e) \cap k(e) \neq \emptyset$. Then, $(e, f(e)) \in \overline{\mathrm{apr}}_{f_D}(h)$ or $(e, f(e)) \in \overline{\mathrm{apr}}_{f_D}(h)$. This shows that

$$\overline{\mathrm{apr}}_{f_D}(h \tilde{\cup} k) \tilde{\subseteq} \overline{\mathrm{apr}}_{f_D}(h) \tilde{\cup} \overline{\mathrm{apr}}_{f_D}(k). \tag{32}$$

To prove the reverse inclusion, note that $k \tilde{\subseteq} h \tilde{\cup} k$ and $h \tilde{\subseteq} h \tilde{\cup} k$; then from (3) $\overline{\mathrm{apr}}_{f_D}(k) \tilde{\subseteq} \overline{\mathrm{apr}}_{f_D}(h \tilde{\cup} k)$ and $\overline{\mathrm{apr}}_{f_D}(h) \tilde{\subseteq} \overline{\mathrm{apr}}_{f_D}(h \tilde{\cup} k)$, respectively. Thus,

$$\overline{\mathrm{apr}}_{f_D}(h) \tilde{\cup} \overline{\mathrm{apr}}_{f_D}(k) \tilde{\subseteq} \overline{\mathrm{apr}}_{f_D}(h \tilde{\cup} k). \tag{33}$$

From (32) and (33),

$$\overline{\mathrm{apr}}_{f_D}(h \tilde{\cup} k) = \overline{\mathrm{apr}}_{f_D}(h) \tilde{\cup} \overline{\mathrm{apr}}_{f_D}(k). \tag{34}$$

□

Definition 49. Let $f_D \in \mathcal{SC}_D^E(U)$ and let $h, k \in \mathcal{S}(U)$. We define

$$\begin{aligned} h \smile_{f_D} k &\Longleftrightarrow \underline{\mathrm{apr}}_{f_D}(h) = \underline{\mathrm{apr}}_{f_D}(k), \\ h \frown_{f_D} k &\Longleftrightarrow \overline{\mathrm{apr}}_{f_D}(h) = \overline{\mathrm{apr}}_{f_D}(k), \\ h \asymp_{f_D} k &\Longleftrightarrow h \smile_{f_D} k = h \frown_{f_D} k. \end{aligned} \tag{35}$$

These binary relations are called the lower soft class rough equal relation and the upper soft class rough equal relation, respectively.

Theorem 50. *Let $f_D \in \mathcal{SC}_D^E(U)$ and let h, k, h', and $k' \in \mathcal{S}(U)$. Then,*

(1) $h \frown_{f_D} k \Leftrightarrow h \frown_{f_D} (h \tilde{\cup} k) \frown_{f_D} k$;

(2) $h \frown_{f_D} h', k \frown_{f_D} k' \Rightarrow (h \tilde{\cup} k) \frown_{f_D} (h' \tilde{\cup} k')$;

(3) $h \frown_{f_D} k \Rightarrow h \tilde{\cup} (\widehat{U} \tilde{\setminus} k) \frown_{f_D} \widehat{U}$;

(4) $h \,\tilde{\subseteq}\, k, k \frown_{f_D} \Phi \Rightarrow h \frown_{f_D} \Phi$;

(5) $h \,\tilde{\subseteq}\, k, h \frown_{f_D} \widehat{U} \Rightarrow k \frown_{f_D} \widehat{U}$.

Proof. (1) Assume that $h \frown_{f_D} k$; then $\overline{\text{apr}}_{f_D}(h) = \overline{\text{apr}}_{f_D}(k)$. From Theorem 48, we know that $\overline{\text{apr}}_{f_D}(h \,\tilde{\cup}\, k) = \overline{\text{apr}}_{f_D}(h) \,\tilde{\cup}\, \overline{\text{apr}}_{f_D}(k)$. Thus, $\overline{\text{apr}}_{f_D}(h \,\tilde{\cup}\, k) = \overline{\text{apr}}_{f_D}(h) = \overline{\text{apr}}_{f_D}(k)$ and so $h \frown_{f_D} (h \,\tilde{\cup}\, k) \frown_{f_D} k$. Conversely, suppose that $h \frown_{f_D} (h \,\tilde{\cup}\, k) \frown_{f_D} k$. From transitivity of $\frown_{f_D}$, $h \frown_{f_D} k$.

(2) Suppose that $h \frown_{f_D} h'$ and $k \frown_{f_D} k'$; then, from definition, $\overline{\text{apr}}_{f_D}(h) = \overline{\text{apr}}_{f_D}(h')$ and $\overline{\text{apr}}_{f_D}(k) = \overline{\text{apr}}_{f_D}(k')$. From Theorem 48, $\overline{\text{apr}}_{f_D}(h \,\tilde{\cup}\, k) = \overline{\text{apr}}_{f_D}(h) \,\tilde{\cup}\, \overline{\text{apr}}_{f_D}(k)$ and $\overline{\text{apr}}_{f_D}(h' \,\tilde{\cup}\, k') = \overline{\text{apr}}_{f_D}(h') \,\tilde{\cup}\, \overline{\text{apr}}_{f_D}(k')$. Hence, we get $\overline{\text{apr}}_{f_D}(h \,\tilde{\cup}\, k) = \overline{\text{apr}}_{f_D}(h' \,\tilde{\cup}\, k')$ and so $(h \,\tilde{\cup}\, k) \frown_{f_D} (h' \,\tilde{\cup}\, k')$.

(3) Let $h \frown_{f_D} k$. Then, from definition, $\overline{\text{apr}}_{f_D}(h) = \overline{\text{apr}}_{f_D}(k)$. By Theorem 48, $\overline{\text{apr}}_{f_D}(h \,\tilde{\cup}\, (\widehat{U} \,\tilde{\setminus}\, k)) = \overline{\text{apr}}_{f_D}(h) \,\tilde{\cup}\, \overline{\text{apr}}_{f_D}(\widehat{U} \,\tilde{\setminus}\, k)$ and $\overline{\text{apr}}_{f_D}(\widehat{U}) = \overline{\text{apr}}_{f_D}(k) \,\tilde{\cup}\, \overline{\text{apr}}_{f_D}(\widehat{U} \,\tilde{\setminus}\, k)$. It follows that $\overline{\text{apr}}_{f_D}(\widehat{U}) = \overline{\text{apr}}_{f_D}(h \,\tilde{\cup}\, (\widehat{U} \,\tilde{\setminus}\, k))$. Therefore, $h \,\tilde{\cup}\, (\widehat{U} \,\tilde{\setminus}\, k) \frown_{f_D} \widehat{U}$.

(4) Let $h \,\tilde{\subseteq}\, k$ and let $k \frown_{f_D} \Phi$. From Theorem 48, we get $\overline{\text{apr}}_{f_D}(h) \,\tilde{\subseteq}\, \overline{\text{apr}}_{f_D}(k) = \overline{\text{apr}}_{f_D}\Phi = \Phi$. Thus, $\overline{\text{apr}}_{f_D}(h) = \Phi = \overline{\text{apr}}_{f_D}\Phi$ and so $h \frown_{f_D} \Phi$.

(5) Suppose that $h \,\tilde{\subseteq}\, k$ and $h \frown_{f_D} \widehat{U}$. By Theorem 48, we have $\overline{\text{apr}}_{f_D}(k) \,\tilde{\supseteq}\, \overline{\text{apr}}_{f_D} h = \overline{\text{apr}}_{f_D}\widehat{U}$. Also, since $k \subseteq \widehat{U}$, $\overline{\text{apr}}_{f_D}(k) \,\tilde{\subseteq}\, \overline{\text{apr}}_{f_D}\widehat{U}$. Hence, $\overline{\text{apr}}_{f_D}(k) = \overline{\text{apr}}_{f_D}(\widehat{U})$, and so $k \frown_{f_D} \widehat{U}$. □

Definition 51. Let $f_D, g_D \in \mathcal{SC}_D^E(U)$ and let $h \in \mathcal{S}(U)$. We define

$$
\begin{gathered}
f_D \smile_h g_D \Longleftrightarrow \underline{\text{apr}}_{f_D}(h) = \underline{\text{apr}}_{g_D}(h), \\
f_D \frown_h g_D \Longleftrightarrow \overline{\text{apr}}_{f_D}(h) = \overline{\text{apr}}_{g_D}(h), \\
f_D \asymp_h g_D \Longleftrightarrow f_D \smile_h g_D = f_D \frown_h g_D.
\end{gathered}
\tag{36}
$$

These binary relations are called the lower soft class rough h-equal relation and the upper soft class rough h-equal relation, respectively.

Theorem 52. *Let f_D, g_D, f'_D, and $g'_D \in \mathcal{SC}_D^E(U)$ and let $h \in \mathcal{S}(U)$. Then,*

(1) $f_D \frown_h g_D \Leftrightarrow f_D \frown_h (f_D \sqcap g_D) \frown_h g_D$;

(2) $f_D \frown_h f'_D, g_D \frown_h g'_D \Rightarrow (f_D \sqcap f'_D) \frown_h (g_D \sqcap g'_D)$;

(3) $f_D \frown_h g_D \Rightarrow f_D \sqcap (\mathcal{U} \,\tilde{\setminus}\, g_D) \frown_h \mathcal{U}$;

(4) $f_D \,\tilde{\sqsubseteq}\, g_D, g_D \frown_h \emptyset \Rightarrow f_D \frown_h \emptyset$;

(5) $f_D \,\tilde{\sqsubseteq}\, g_D, f_D \frown_h \mathcal{U} \Rightarrow g_D \frown_h \mathcal{U}$.

Proof. (1) Assume that $f_D \frown_h g_D$; then $\overline{\text{apr}}_{f_D}(h) = \overline{\text{apr}}_{g_D}(h)$. From Theorem 47, we know that $\overline{\text{apr}}_{f_D \sqcap g_D}(h) = \overline{\text{apr}}_{f_D}(h) \,\tilde{\cap}\, \overline{\text{apr}}_{g_D}(h)$. Thus, $\overline{\text{apr}}_{f_D \sqcap g_D}(h) = \overline{\text{apr}}_{f_D}(h) = \overline{\text{apr}}_{g_D}(h)$ and so $f_D \frown_h (f_D \sqcap g_D) \frown_h g_D$. Conversely, suppose that $f_D \frown_h (f_D \sqcap g_D) \frown_h g_D$. From transitivity of $\frown_h$, $f_D \frown_h g_D$.

(2) Suppose that $f_D \frown_h f'_D$ and $g_D \frown_h g'_D$; then, from definition, $\overline{\text{apr}}_{f_D}(h) = \overline{\text{apr}}_{f'_D}(h)$ and $\overline{\text{apr}}_{g_D}(h) = \overline{\text{apr}}_{g'_D}(h)$. From Theorem 47, $\overline{\text{apr}}_{f_D \sqcap g_D}(h) = \overline{\text{apr}}_{f_D}(h) \,\tilde{\cap}\, \overline{\text{apr}}_{g_D}(h) = \overline{\text{apr}}_{f'_D}(h) \,\tilde{\cap}\, \overline{\text{apr}}_{g'_D}(h)$. So $\overline{\text{apr}}_{f_D \sqcap g_D}(h) = \overline{\text{apr}}_{f'_D \sqcap g'_D}(h)$ and $(f_D \sqcap g_D) \frown_h (f'_D \sqcap g'_D)$.

(3) The proof can be made by similar way to proof of (1) and (2).

(4) Let $f_D \sqsubseteq g_D$ and $g_D \frown_h \emptyset$. Then, $\overline{\text{apr}}_{g_D}(h) = \overline{\text{apr}}_{\emptyset}(h)$ and since $f_D \sqsubseteq g_D$, $\overline{\text{apr}}_{f_D}(h) \,\tilde{\subseteq}\, \overline{\text{apr}}_{\emptyset}(h) = \Phi$. Therefore, $\overline{\text{apr}}_{f_D}(h) = \overline{\text{apr}}_{\emptyset}(h)$. We have $f_D \frown_h \emptyset$.

(5) The proof can be made by similar way to (4). □

5. Decision Making Using Soft Rough Class

In this section, some concepts are defined to construct a decision making method using soft rough class and a decision making algorithm is given. Then, an application of proposed decision making method is made for a real problem.

Definition 53. Let $f_D \in \mathcal{SC}_D^E(U)$ and let g be a soft set (reference soft set) over U. Then, consistency degree of soft set g related to parameter $e \in E$ and soft class f_D, denoted by $\gamma_{f_D}^g(e)$, is formulated as follows:

$$
\gamma_{f_D}^g(e) = \frac{\left|\underline{g}_{f_D}(e)\right|}{\left|\overline{g}_{f_D}(e)\right|}.
\tag{37}
$$

According to soft class f_D consistency degree of soft set g, denoted by $\Gamma_{f_D}^g$, is formulated as follows:

$$
\Gamma_{f_D}^g = \frac{1}{|E|} \sum_{e \in E} \gamma_{f_D}^g(e).
\tag{38}
$$

Definition 54. Let $f_D \in \mathcal{SC}_D^E(U)$ and let g be a soft set over U. Then, relative consistence degree (rcd) between soft class $f_D - \{d_i\}$ and soft set g related to parameter $e \in E$, denoted by $\gamma_{f_D - \{f_{d_i}\}}^g(e)$, is formulated as follows:

$$
\gamma_{f_D - \{f_{d_i}\}}^g(e) = \frac{\left|\underline{g}_{f_D - \{f_{d_i}\}}(e)\right|}{\left|\overline{g}_{f_D - \{f_{d_i}\}}(e)\right|}.
\tag{39}
$$

Between soft class $f_D - \{d_i\}$ and soft set g total relative consistency degree is formulated as follows:

$$
\Gamma_{f_D - \{f_{d_i}\}}^g = \frac{1}{|E|} \sum_{e \in E} \gamma_{f_D - \{f_{d_i}\}}^g(e).
\tag{40}
$$

Definition 55. Let $f_D \in \mathcal{SC}_D^E(U)$ and let g be a soft set over U. Then $\Gamma_{f_D}^g - \Gamma_{f_D - \{f_{d_i}\}}^g$ is called effectiveness of decision maker d_i and is denoted by $e(d_i)$.

Now we will give relations between two decision makers in decision maker set D.

Table 3: The tabular representation of the soft class f_D.

f_D	f_{d_1}	f_{d_2}	f_{d_3}	f_{d_4}	f_{d_5}
t_1	$\{u_1, u_3, u_4\}$	$\{u_1, u_2\}$	$\{u_2, u_3, u_5\}$	$\{u_1, u_3\}$	$\{u_1, u_3, u_5\}$
t_2	$\{u_1, u_4, u_5, u_6\}$	$\{u_3, u_6\}$	$\{u_1, u_4, u_7\}$	$\{u_1, u_2, u_8\}$	$\{\}$
t_3	$\{u_1, u_2, u_3, u_8\}$	$\{u_1, u_2, u_3, u_4, u_5, u_6\}$	$\{\}$	$\{u_1, u_3, u_4\}$	$\{u_1, u_8\}$
t_4	$\{\}$	$\{u_1, u_3, u_7, u_8\}$	$\{u_5, u_8\}$	$\{u_7, u_8\}$	$\{u_1, u_3, u_5\}$
t_5	$\{u_2, u_4, u_7\}$	$\{u_1, u_2\}$	$\{u_3, u_5, u_7\}$	$\{u_2, u_5, u_8\}$	$\{u_1, u_4, u_6\}$

Definition 56. Let $f_D \in \mathcal{SC}_D^E(U)$ and let g be a soft set over U. Effectiveness relations between d_i and d_j are defined as follows:

(1) If $e(d_i) >_g e(d_j)$, d_i is more effective than d_j.

(2) If $e(d_i) =_g e(d_j)$, d_i has same effect as d_j.

(3) If $e(d_i) <_g e(d_j)$, d_j is more effective than d_i.

Algorithm 57.

Step 1. Construct a soft class f_D and reference soft set g over U.

Step 2. Find the consistency degree of soft set g denoted by $\gamma_{f_D}^g(e)$ related to parameter $e \in E$.

Step 3. Find consistency degree of soft set g according to soft class f_D.

Step 4. Find total relative consistency degree between soft class $f_D - \{d_i\}$ and soft set g.

Step 5. Find effectiveness of each decision maker $d_i \in D$.

Step 6. Chose effective decision maker.

6. Applied Example

Assume that an investment company wants to employ stock market analysts. Five persons apply for this position in the company and the department of human resources wants to make appropriate choice among the applicants. Therefore, department of human resources wants some previous evaluations made by applicants d_1, d_2, d_3, d_4, and d_5 for firms $u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8$ in different times t_1, t_2, t_3, t_4, t_5 in the last two years.

Step 1. According to the appreciation criteria, evaluations of applicants d_1, d_2, d_3, d_4, and d_5 performed in different times specified by human resources department are represented by soft sets f_{d_1}, f_{d_2}, f_{d_3}, f_{d_4}, and f_{d_5} given as follows:

$$\begin{aligned}
f_{d_1} &= \{(t_1, \{u_1, u_3, u_4\}), (t_2, \{u_1, u_4, u_5, u_6\}), \\
&\quad (t_3, \{u_1, u_2, u_3, u_8\}), (t_4, \{\}), (t_5, \{u_2, u_4, u_7\})\}, \\
f_{d_2} &= \{(t_1, \{u_1, u_2\}), (t_2, \{u_3, u_6\}), \\
&\quad (t_3, \{u_1, u_2, u_3, u_4, u_5, u_6\}), (t_4, \{u_1, u_3, u_7, u_8\}), \\
&\quad (t_5, \{u_1, u_2\})\}, \\
f_{d_3} &= \{(t_1, \{u_2, u_3, u_5\}), (t_2, \{u_1, u_4, u_7\}), (t_3, \{\}), \\
&\quad (t_4, \{u_5, u_8\}), (t_5, \{u_3, u_5, u_7\})\}, \\
f_{d_4} &= \{(t_1, \{u_1, u_3\}), (t_2, \{u_1, u_2, u_8\}), (t_3, \{u_1, u_3, u_4\}), \\
&\quad (t_4, \{u_7, u_8\}), (t_5, \{u_2, u_5, u_8\})\}, \\
f_{d_5} &= \{(t_1, \{u_1, u_3, u_5\}), (t_2, \{\}), (t_3, \{u_1, u_8\}), \\
&\quad (t_4, \{u_1, u_3, u_5\}), (t_5, \{u_1, u_4, u_6\})\}.
\end{aligned} \tag{41}$$

Table 4: The tabular representation of the consistency degree of soft set g for t_i ($i = 1, 2, 3, 4, 5$).

	t_1	t_2	t_3	t_4	t_5
$\gamma_{f_D}^g$	0.800	0.666	1.000	0.600	0.500

Department of human resources has real results previously obtained in specified times: t_1, t_2, t_3, t_4, and t_5. These real results are represented by soft set g (reference soft set) as follows:

$$\begin{aligned}
g &= \{(t_1, \{u_1, u_3, u_4, u_5\}), (t_2, \{u_3, u_6, u_7\}), (t_3, U), \\
&\quad (t_4, \{u_5, u_7, u_8\}), (t_5, \{u_1, u_2, u_4, u_7\})\}.
\end{aligned} \tag{42}$$

Tabular representation of soft class $f_D = \{f_{d_1}, f_{d_2}, f_{d_3}, f_{d_4}, f_{d_5}\}$ is shown in Table 3.

Step 2. Using (37), consistency degree of soft set g for time parameters t_1, t_2, t_3, t_4, and t_5 is obtained as in Table 4.

Step 3. Using (38) and Table 4, consistency degree of soft set g related to soft class f_D is obtained as $\Gamma_{f_D}^g = 0.713$.

Step 4. Using (39), relative consistency degrees of soft set g with respect to soft class f_D are as in Table 5. And, from (40), total relative consistency degrees of soft set g with respect to soft class f_D are as in Table 6.

Table 5: Relative consistency degrees between soft set g and $f_D - \{d_i\}$.

f_D	t_1	t_2	t_3	t_4	t_5
$\gamma^g_{f_D-\{f_{d_1}\}}$	0.750	0.400	1.000	0.600	0.250
$\gamma^g_{f_D-\{f_{d_2}\}}$	0.750	0.000	1.000	0.600	0.375
$\gamma^g_{f_D-\{f_{d_3}\}}$	0.200	0.400	1.000	0.400	0.571
$\gamma^g_{f_D-\{f_{d_4}\}}$	0.800	0.500	1.000	0.400	0.666
$\gamma^g_{f_D-\{f_{d_5}\}}$	0.600	0.666	1.000	0.600	0.571

Table 6: Total relative consistency degrees between soft set g and soft class $f_D - \{d_i\}$.

Total relative consistency formula	Values
$\Gamma^g_{f_D-\{f_{d_1}\}}$	0.600
$\Gamma^g_{f_D-\{f_{d_2}\}}$	0.535
$\Gamma^g_{f_D-\{f_{d_3}\}}$	0.514
$\Gamma^g_{f_D-\{f_{d_4}\}}$	0.673
$\Gamma^g_{f_D-\{f_{d_5}\}}$	0.687

Step 5. Using Definition 55, effectiveness of the applicants d_1, d_2, d_3, d_4, and d_5 is obtained as follows:

$$\begin{aligned} e(d_1) &= 0.113, \\ e(d_2) &= 0.278, \\ e(d_3) &= 0.199, \\ e(d_4) &= 0.040, \\ e(d_5) &= 0.026. \end{aligned} \tag{43}$$

Step 6. From Definition 56, effectiveness of applicants can be ordered as follows:

$$e(d_2) >_g e(d_3) >_g e(d_1) >_g e(d_4) >_g e(d_5). \tag{44}$$

Then, d_2 is the most effective decision maker in soft class f_D by soft set g.

7. Conclusion

In this paper, we have defined concepts of soft class, soft class operations, and soft rough class. Then we have presented a decision making method based on the soft rough class. Finally, we have provided an example that demonstrated that this decision making method can successfully work. It can be applied to problems of many fields that contain uncertainty. Next, we can define fuzzy soft class and fuzzy soft rough class and their operations as generalization of soft classes and soft rough classes. Also a reduction method can be developed based on soft class and soft rough classes.

Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

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