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Research Article

Output Feedback Control of Discrete Impulsive Switched Systems with State Delays and Missing Measurements

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This paper is concerned with the problem of dynamic output feedback (DOF) control for a class of uncertain discrete impulsive switched systems with state delays and missing measurements. The missing measurements are modeled as a binary switch sequence specified by a conditional probability distribution. The problem addressed is to design an output feedback controller such that for all admissible uncertainties, the closed-loop system is exponentially stable in mean square sense. By using the average dwell time approach and the piecewise Lyapunov function technique, some sufficient conditions for the existence of a desired DOF controller are derived, then an explicit expression of the desired controller is given. Finally, a numerical example is given to illustrate the effectiveness of the proposed method.

1. Introduction

Due to their wide applications, switched systems which are an important class of hybrid systems have drawn considerable attention in the last decade [1, 2]. During these years, there have been increasing research activities in the field of stability analysis for such systems (see [3–6], and the references cited therein). Recently, impulsive switched systems as a class of special switched systems have gained research attention. This is because impulsive switched systems can represent some practical switched systems that exhibit impulsive dynamical behavior due to sudden changes in the state of the system at certain instants of switching. Some problems on impulsive switched systems with and without delays have been successfully investigated, and a rich body of the literatures is now available [7–10].

On the other hand, control synthesis is one of the important issues in system theory. State feedback control as an effective control strategy has been widely used in various complex dynamical systems. For instance, some state feedback control problems for switched systems have been extensively studied in [11, 12]. The adaptive control for a class of nonlinear systems via backstepping method was studied in [13]. The authors in [14] considered an optimal state feedback control problem for impulsive switched systems. In [15–18], some controller design methods for impulsive switched systems were developed. In addition, output feedback control has been considered as an effective control method when the states of the system are not all measurable in practice. At present, many results on the output feedback controller design for nonlinear systems or switched systems have been obtained (see [19–24]), and less work has been done for impulsive switched systems.

In almost all the works mentioned above, the assumption of consecutive measurements has been made implicitly. Unfortunately, in many practical applications, such an assumption does not hold. For example, due to sensor temporal failure or network transmission delay/loss, at certain time points, the system measurement may contain noise only, indicating that the real signal is missing. One of the most popular ways to describe the missing measurement is to view it as a Bernoulli distributed (binary switching) white sequence specified by a conditional probability distribution in the output equation. The Bernoulli distribution description was first proposed in [25] to deal with the optimal recursive filtering problem and then has been used in [26–29] for various control and filtering problems of linear systems with probabilistic missing measurements. It is worth pointing out that the references mentioned above did not consider the effect of impulse. However, the missing measurements and impulsive jumps happening simultaneously in the systems will bring some challenges and difficulties for the analysis and synthesis. To the best of our knowledge, the issue of dynamic output feedback controller design with missing measurements for impulsive switched systems has not been fully investigated, which motivates our present study.

In this paper, we will focus our interest on the problem of dynamic output feedback (DOF) control for a class of uncertain discrete impulsive switched systems with state delays and missing measurements. The main contributions of the paper are as follows: (1) a DOF controller is proposed for discrete impulsive switched systems, and the controller contains impulsive jumps, which is different from most of the existing ones, for example, those in [20–23]; (2) sufficient conditions for the existence of a DOF controller are developed such that the resulting closed-loop system is exponentially stable in mean square sense.

The remainder of the paper is organized as follows. In Section 2, problem formulation and some necessary preliminaries are given. In Section 3, the main results are presented. Section 4 gives a numerical example to illustrate the effectiveness of the proposed approach. Concluding remarks are given in Section 5.

Notations. Throughout this paper, the superscript "T" denotes the transpose, and the notation $X \geq Y$ ($X > Y$) means that matrix $X - Y$ is positive semidefinite (positive definite, resp.). $\|\cdot\|$ denotes the Euclidean norm. $\varepsilon\{\cdot\}$ stands for the mathematical expectation, and Prob$\{\cdot\}$ means the occurrence probability of the event "$\cdot$". I represents the identity matrix and diag$\{a_i\}$ denotes a diagonal matrix with the diagonal elements a_i, $i = 1, 2, \ldots, n$. X^{-1} denotes the inverse of X. The asterisk $*$ in a matrix is used to denote a term that is induced by symmetry. The set of all positive integers is represented by Z^+.

2. Problem Formulation and Preliminaries

Consider the following uncertain discrete impulsive switched systems with state delay:

$$x(k+1) = \widehat{A}_{\sigma(k)} x(k) + \widehat{A}_{d\sigma(k)} x(k-d) + B_{\sigma(k)} u(k), \quad k \neq k_b - 1, \ b \in Z^+, \tag{1a}$$

$$x(k+1) = E_{\sigma(k+1)\sigma(k)} x(k), \quad k = k_b - 1, \ b \in Z^+, \tag{1b}$$

$$y(k) = C_{\sigma(k)} x(k), \tag{1c}$$

$$x(k_0 + \theta) = \phi(\theta), \quad \theta \in [-d, 0], \tag{1d}$$

where $x(k) \in R^n$ is the state vector, $u(k) \in R^m$ is the control input, $y(k) \in R^p$ is the output vector, and $\phi(\theta)$ is a discrete vector-valued initial function on interval $[-d, 0]$. d is the discrete time delay. $\sigma(k)$ is a switching signal which takes its values in the finite set $\underline{N} := \{1, \ldots, N\}$, N denotes the number of subsystems. k_0 is the initial time and k_b ($b \in Z^+$) denotes the bth switching instant. Moreover, $\sigma(k) = i \in \underline{N}$ means that the ith subsystem is activated.

The measurement output which may contain missing data is described by

$$\widetilde{y}(k) = r(k) y(k) = r(k) C_{\sigma(k)} x(k), \tag{2}$$

where $\widetilde{y}(k) \in R^q$ is the measurement output vector and C_i ($i \in \underline{N}$) are known real constant matrices with appropriate dimensions. The stochastic variable $r(k) \in R$ is a Bernoulli distributed white sequence taking the values of 0 and 1 with

$$\text{Prob}\{r(k) = 1\} = \varepsilon\{r(k)\} = \overline{r}, \tag{3a}$$

$$\text{Prob}\{r(k) = 0\} = 1 - \varepsilon\{r(k)\} = 1 - \overline{r}, \tag{3b}$$

where $\overline{r} \in R$ is a known positive scalar. From (3a)-(3b), we obtain that

$$\tau = \varepsilon\left\{(r(k) - \overline{r})^2\right\} = (1 - \overline{r})\overline{r}. \tag{4}$$

For each $i \in \underline{N}$, $\widehat{A}_i$ and $\widehat{A}_{di}$ are uncertain real-valued matrices with appropriate dimensions. We assume that these uncertainties are norm-bounded and satisfy

$$\begin{bmatrix} \widehat{A}_i & \widehat{A}_{di} \end{bmatrix} = \begin{bmatrix} A_i & A_{di} \end{bmatrix} + H_i F_i(k) \begin{bmatrix} M_{1i} & M_{2i} \end{bmatrix}, \tag{5}$$

where A_i, A_{di}, H_i, M_{1i}, and M_{2i}, $i \in \underline{N}$, are known real constant matrices with appropriate dimensions. $F_i(k)$ are unknown and are possibly time-varying matrices with Lebesgue measurable elements and satisfy

$$F_i^T(k) F_i(k) \leq I. \tag{6}$$

Here, we are interested in designing a DOF switched controller described by

$$x_c(k+1) = A_{c\sigma(k)} x_c(k) + B_{c\sigma(k)} \widetilde{y}(k), \quad k \neq k_b - 1, \quad b \in Z^+, \tag{7a}$$

$$x_c(k+1) = G_{\sigma(k+1)\sigma(k)} x_c(k), \quad k = k_b - 1, \quad b \in Z^+, \tag{7b}$$

$$u(k) = C_{c\sigma(k)} x_c(k), \tag{7c}$$

$$x_c(k_0 + \theta) = 0, \quad \theta \in [-d, 0], \tag{7d}$$

where $x_c(k) \in R^{n_c}$ is the controller state vector; A_{ci}, B_{ci}, and C_{ci} are constant matrices to be determined later.

Remark 1. Different from the existing DOF controllers proposed in [20–23], the controller proposed here contains (7b), which coincides with the structure of system (1a), (1b), (1c), and (1d).

Now, define a new state vector,

$$\xi(k) = \begin{bmatrix} x^T(k) & x_c^T(k) \end{bmatrix}^T \in R^{n+n_c}. \tag{8}$$

The combination of the previous DOF controller (7a), (7b), (7c), and (7d) and system (1a), (1b), (1c), and (1d) yields the following closed-loop system:

$$\xi(k+1) = \overline{A}_{\sigma(k)}\xi(k) + (r(k) - \overline{r})\overline{A}_{m\sigma(k)}\xi(k) + \overline{A}_{d\sigma(k)}\xi(k-d), \quad k \neq k_b - 1, \tag{9a}$$

$$\xi(k+1) = \overline{E}_{\sigma(k+1)\sigma(k)}\xi(k), \quad k = k_b - 1, \tag{9b}$$

$$\xi(k_0+\theta) = \varphi(\theta), \quad \theta \in [-d, 0], \tag{9c}$$

where

$$\overline{A}_i = \begin{bmatrix} \widehat{A}_i & B_i C_{ci} \\ \overline{r} B_{ci} C_i & A_{ci} \end{bmatrix}, \qquad \overline{A}_{di} = \begin{bmatrix} \widehat{A}_{di} & 0 \\ 0 & 0 \end{bmatrix},$$
$$\overline{A}_{mi} = \begin{bmatrix} 0 & 0 \\ B_{ci} C_i & 0 \end{bmatrix}, \qquad \overline{E}_{ij} = \begin{bmatrix} E_{ij} & 0 \\ 0 & G_{ij} \end{bmatrix}, \tag{10}$$
$$\varphi(\theta) = \begin{bmatrix} \phi(\theta) \\ 0 \end{bmatrix}, \quad i, j \in \underline{N}, \ i \neq j.$$

The following definitions and lemmas will be essential for our later development.

Definition 2 (see [30]). For any $k > k_0 \geq 0$, let $N_\sigma(k_0, k)$ denote the switching number of $\sigma(k)$ during the interval $[k_0, k)$. If there exist $N_0 \geq 0$ and $\tau_a \geq 0$ such that

$$N_\sigma(k_0, k) \leq N_0 + \frac{k - k_0}{\tau_a}, \quad \forall k \geq k_0, \tag{11}$$

then τ_a and N_0 are called the average dwell time and the chatter bound, respectively.

Remark 3. In this paper, the average dwell time method is used to restrict the switching number during a time interval such that the stability of system (9a), (9b), and (9c) can be guaranteed.

Definition 4 (see [27]). System (9a), (9b), and (9c) is said to be exponentially stable in mean square sense under the switching signal $\sigma(k)$, if there exist constants $\gamma \geq 0$ and $\rho \in (0, 1)$, such that the trajectory of system (9a), (9b), and (9c) satisfies

$$\varepsilon\left\{\|\xi(k)\|^2\right\} \leq \gamma\rho^{k-k_0} \sup_{-d\leq\theta\leq 0} \varepsilon\left\{\|\xi(k_0+\theta)\|^2\right\}, \quad k \geq k_0. \tag{12}$$

Lemma 5 (see [31]). *For a given matrix* $S = \begin{bmatrix} S_{11} & S_{12} \\ S_{12}^T & S_{22} \end{bmatrix}$, *where* S_{11} *and* S_{22} *are square matrices, the following conditions are equivalent:*

(i) $S < 0$;

(ii) $S_{11} < 0$, $S_{22} - S_{12}^T S_{11}^{-1} S_{12} < 0$;

(iii) $S_{22} < 0$, $S_{11} - S_{12} S_{22}^{-1} S_{12}^T < 0$.

Lemma 6 (see [32]). *Let* U, V, W, *and* X *be real matrices of appropriate dimensions with* X *satisfying* $X = X^T$, *then for all* $V^T V \leq I$, $X + UVW + W^T V^T U^T < 0$, *if and only if there exists a scalar* β *such that* $X + \beta U U^T + \beta^{-1} W^T W < 0$.

3. Main Results

3.1. Stability Analysis. The following theorem provides sufficient conditions under which the exponential stability of system (9a), (9b), and (9c) can be guaranteed in mean square sense.

Theorem 7. *Consider system* (9a), (9b), *and* (9c), *for a given scalar* $0 < \alpha < 1$, *if there exist positive definite symmetric matrices* R_i *and* P_i ($i \in \underline{N}$) *with appropriatedimensions, such that*

$$\begin{bmatrix} R_i - \alpha P_i & 0 & \overline{A}_i^T P_i & \tau \overline{A}_{mi}^T P_i \\ * & -\alpha^d R_i & \overline{A}_{di}^T P_i & 0 \\ * & * & -P_i & 0 \\ * & * & * & -\tau P_i \end{bmatrix} < 0, \tag{13}$$

where $\tau = (1-\overline{r})\overline{r}$, *then, under the following average dwell time scheme*

$$\tau_a > \tau_a^* = -\frac{\ln\mu}{\ln\alpha} + 1, \tag{14}$$

system (9a), (9b), *and* (9c) *is exponentially stable in mean square sense, where* $\mu \geq 1$ *satisfies*

$$\begin{bmatrix} R_i - \mu P_j & \overline{E}_{ij}^T P_i \\ * & -P_i \end{bmatrix} < 0,$$
$$\alpha R_i < \mu R_j, \quad (i, j \in \underline{N}, i \neq j). \tag{15}$$

Proof. Choose a piecewise Lyapunov function candidate for system (9a), (9b), and (9c) of the form

$$V(k) = V_{\sigma(k)}(x(k)) = V_{\sigma(k)}(k), \tag{16}$$

the form of $V_{\sigma(k)}(k)$ is given by

$$V_{\sigma(k)}(k) = V_{1\sigma(k)}(k) + V_{2\sigma(k)}(k), \tag{17}$$

where

$$V_{1i}(k) = \xi^T(k) P_i \xi(k),$$
$$V_{2i}(k) = \sum_{r=k-d}^{k-1} \xi^T(r) R_i \xi(r) \alpha^{k-r-1}, \quad i \in \underline{N}. \tag{18}$$

When $k \in [k_b, k_{b+1}-1)$, we let $\sigma(k) = \sigma(k+1) = i \ (i \in \underline{N})$. Then along the trajectory of system (9a), (9b), and (9c), we have

$$\begin{aligned}
\Delta V_{1i}(k) &= \varepsilon\{V_{1i}(k+1)\} - \alpha\varepsilon\{V_{1i}(k)\} \\
&= \xi^T(k)\left(\overline{A}_i^T P_i \overline{A}_i - \alpha P_i\right)\xi(k) \\
&\quad + 2\varepsilon\{(r(k)-\overline{r})\}\xi^T(k)\overline{A}_i^T P_i \overline{A}_{mi}\xi(k) \\
&\quad + 2\xi^T(k)\overline{A}_i^T P_i \overline{A}_{di}\xi(k-d) \\
&\quad + \varepsilon\left\{(r(k)-\overline{r})^2\right\}\xi^T(k)\overline{A}_{mi}^T P_i \overline{A}_{mi}\xi(k) \\
&\quad + \xi^T(k-d)\overline{A}_{di}^T P_i \overline{A}_{di}\xi(k-d) \\
&\quad + 2\varepsilon\{(r(k)-\overline{r})\}\xi^T(k)\overline{A}_{mi}^T P_i \overline{A}_{di}\xi(k-d), \\
\Delta V_{2i}(k) &= \varepsilon\{V_{2i}(k+1)\} - \alpha\varepsilon\{V_{2i}(k)\} \\
&= \sum_{r=k+1-d}^{k}\xi^T(r)R_i\xi(r)\alpha^{k-r} \\
&\quad - \sum_{r=k-d}^{k-1}\xi^T(r)R_i\xi(r)\alpha^{k-r} \\
&= \xi^T(k)R_i\xi(k) - \alpha^d\xi^T(k-d)R_i\xi(k-d).
\end{aligned} \tag{19}$$

Notice that

$$\varepsilon\{r(k)-\overline{r}\} = 0, \qquad \tau = \varepsilon\left\{(r(k)-\overline{r})^2\right\} = (1-\overline{r})\overline{r}. \tag{20}$$

Thus we obtain

$$\begin{aligned}
\Delta V_i(k) &= \varepsilon\{V_i(k+1)\} - \alpha\varepsilon\{V_i(k)\} = X^T(k)\varphi_i X(k), \\
\varphi_i &= \begin{bmatrix} R_i - \alpha P_i & 0 \\ 0 & -\alpha^d R_i \end{bmatrix} + \begin{bmatrix} \overline{A}_i^T \\ \overline{A}_{di}^T \end{bmatrix} P_i \begin{bmatrix} \overline{A}_i & \overline{A}_{di} \end{bmatrix} \\
&\quad + \begin{bmatrix} \tau\overline{A}_{mi}^T P_i \overline{A}_{mi} & 0 \\ 0 & 0 \end{bmatrix},
\end{aligned} \tag{21}$$

where $X^T(k) = \begin{bmatrix} \xi^T(k) & \xi^T(k-d) \end{bmatrix}$.

Applying Lemma 5, it is easy to get that inequality (13) is equivalent to $\varphi_i < 0$. Thus we can obtain from (13) that

$$\varepsilon\{V_i(k+1)\} < \alpha\varepsilon\{V_i(k)\}, \quad 0 < \alpha < 1. \tag{22}$$

When $k = k_b - 1$, we let $\sigma(k_b - 1) = j$. Along the trajectory of system (9a), (9b), and (9c), we have

$$\begin{aligned}
&\varepsilon\{V_i(x(k_b))\} - \mu\varepsilon\left\{V_j(x(k_b-1))\right\} \\
&\quad = x^T(k_b-1)\left(\overline{E}_{ij}^T P_i \overline{E}_{ij} - \mu P_j\right)x(k_b-1) \\
&\qquad + \sum_{r=k_b-d}^{k_b-1} x^T(r)R_i x(r)\alpha^{k_b-r-1} \\
&\qquad - \mu\sum_{r=k_b-1-d}^{k_b-2} x^T(r)R_j x(r)\alpha^{k_b-r-2} \\
&\quad = x^T(k_b-1)\left(\overline{E}_{ij}^T P_i \overline{E}_{ij} - \mu P_j + R_i\right)x(k_b-1) \\
&\qquad - \mu x^T(k_b-1-d)R_j x(k_b-1-d)\alpha^{d-1} \\
&\qquad + \sum_{r=k_b-d}^{k_b-2}\alpha^{k_b-r-2}x^T(r)\left(\alpha R_i - \mu R_j\right)x(r).
\end{aligned} \tag{23}$$

It can be obtained from (15) that

$$\begin{aligned}
\overline{E}_{ij}^T P_i \overline{E}_{ij} - \mu P_j + R_i &< 0, \\
\alpha R_i - \mu R_j &< 0.
\end{aligned} \tag{24}$$

It follows that

$$\varepsilon\left\{V_{\sigma(k_b)}(x(k_b))\right\} < \mu\varepsilon\left\{V_{\sigma(k_b-1)}(x(k_b-1))\right\}. \tag{25}$$

Thus, for $k \in [k_b, k_{b+1})$, we have

$$\begin{aligned}
\varepsilon\left\{V_{\sigma(k)}(x(k))\right\} &< \alpha^{k-k_b}\varepsilon\left\{V_{\sigma(k_b)}(x(k_b))\right\} \\
&< \mu\alpha^{k-k_b}\varepsilon\left\{V_{\sigma(k_b-1)}(x(k_b-1))\right\}.
\end{aligned} \tag{26}$$

Repeating the previous manipulation, one has that

$$\begin{aligned}
&\varepsilon\left\{V_{\sigma(k)}(x(k))\right\} \\
&\quad < \alpha^{k-k_b}\varepsilon\left\{V_{\sigma(k_b)}(x(k_b))\right\} \\
&\quad < \mu\alpha^{k-k_b}\varepsilon\left\{V_{\sigma(k_b-1)}(x(k_b-1))\right\} \\
&\quad < \mu\alpha^{k-k_{b-1}-1}\varepsilon\left\{V_{\sigma(k_{b-1})}(x(k_{b-1}))\right\} \\
&\quad < \mu^2\alpha^{k-k_{b-1}-1}\varepsilon\left\{V_{\sigma(k_{b-1}-1)}(x(k_{b-1}-1))\right\} \\
&\quad < \cdots \\
&\quad < \mu^b\alpha^{k-k_0-b}\varepsilon\left\{V_{\sigma(k_0)}(x(k_0))\right\}.
\end{aligned} \tag{27}$$

From Definition 2, we get that $b = N_\sigma(k_0, k) \leq N_0 + (k - k_0)/\tau_a$, where b denotes the switching number of $\sigma(k)$ during the interval $[k_0, k)$, then it follows that

$$\begin{aligned} &\varepsilon\left\{V_{\sigma(k)}\left(x\left(k\right)\right)\right\} \\ &\quad < \left(\mu\alpha^{-1}\right)^{N_0+(k-k_0)/\tau_a}\alpha^{k-k_0}\varepsilon\left\{V_{\sigma(k_0)}\left(x\left(k_0\right)\right)\right\} \\ &\quad < \left(\mu\alpha^{-1}\right)^{N_0}e^{((k-k_0)/\tau_a)(\ln\mu-\ln\alpha)}e^{(k-k_0)\ln\alpha}\varepsilon\left\{V_{\sigma(k_0)}\left(x\left(k_0\right)\right)\right\} \\ &\quad < \left(\mu\alpha^{-1}\right)^{N_0}e^{((\ln\mu-\ln\alpha)/\tau_a+\ln\alpha)(k-k_0)}\varepsilon\left\{V_{\sigma(k_0)}\left(x\left(k_0\right)\right)\right\}. \end{aligned} \tag{28}$$

Notice that

$$\begin{aligned} &\min_{i\in\underline{N}}\left\{\lambda_{\min}\left(P_i\right)\right\}\varepsilon\left\{\left\|\xi\left(k\right)\right\|^2\right\} \leq \varepsilon\left\{V_{\sigma(k)}\left(x\left(k\right)\right)\right\}, \\ &\varepsilon\left\{V\left(x\left(k_0\right)\right)\right\} \\ &\quad \leq \max_{i\in\underline{N}}\left\{\lambda_{\max}\left(P_i\right) + d\lambda_{\max}\left(R_i\right)\right\}\sup_{-d\leq\theta\leq0}\varepsilon\left\{\left\|\xi\left(k_0+\theta\right)\right\|^2\right\}. \end{aligned} \tag{29}$$

Then, one obtains

$$\varepsilon\left\{\left\|\xi\left(k\right)\right\|^2\right\} < \gamma\rho^{(k-k_0)}\sup_{-d\leq\theta\leq0}\varepsilon\left\{\left\|\xi\left(k_0+\theta\right)\right\|^2\right\}, \quad \forall k \geq k_0, \tag{30}$$

where

$$\begin{aligned} \gamma &= \left(\mu\alpha^{-1}\right)^{N_0}\frac{\max_{i\in\underline{N}}\left\{\lambda_{\max}\left(P_i\right) + d\lambda_{\max}\left(R_i\right)\right\}}{\min_{i\in\underline{N}}\left\{\lambda_{\min}\left(P_i\right)\right\}}, \\ \rho &= e^{(\ln\mu-\ln\alpha)/\tau_a+\ln\alpha}. \end{aligned} \tag{31}$$

Then under the average dwell time scheme (14), it is easy to get that $0 < \rho < 1$, which implies that system (9a), (9b), and (9c) is exponentially stable in mean square sense.

This completes the proof. □

Remark 8. Compared with the existing results presented in [16–18], we get sufficient conditions of exponential stability in mean square sense. In addition, the paper takes the missing measurement into consideration, which yields different results from those of [16–18], where the missing measurement is not considered.

3.2. Design of DOF Controller. This section will give some LMIs conditions for the controller design.

Theorem 9. *Consider system* (1a), (1b), (1c), *and* (1d) *for a given positive scalar* $0 < \alpha < 1$, *if there exist positive-definite symmetric matrices* $\overline{S}_{11i}$, P_{11i}, R_{11i}, *and* $\widetilde{Y}_i'$, *any matrices* Σ_i, Y_i', *and* $\widetilde{Z}_i'$ *with appropriate dimensions, and positive scalars* ε_i *and* δ_i $(i \in \underline{N})$, *such that*

$$\begin{bmatrix} \Phi_i & \Lambda_i & 0 & 0 & \theta_i & \Sigma_i & 0 & \tau C_i^T Z_i^T & \varepsilon_i M_{1i}^T & 0 & \delta_i M_{1i}^T & 0 \\ * & \Upsilon_i & 0 & 0 & A_i^T\overline{S}_{11i} & \Theta_i & 0 & \tau C_i^T Z_i^T & \varepsilon_i M_{1i}^T & 0 & \delta_i M_{1i}^T & 0 \\ * & * & \Xi_i & \Omega_i & A_{di}^T\overline{S}_{11i} & A_{di}^T P_{11i} & 0 & 0 & \varepsilon_i M_{2i}^T & 0 & \delta_i M_{2i}^T & 0 \\ * & * & * & -\alpha^d R_{11i} & A_{di}^T\overline{S}_{11i} & A_{di}^T P_{11i} & 0 & 0 & \varepsilon_i M_{2i}^T & 0 & \delta_i M_{2i}^T & 0 \\ * & * & * & * & -\overline{S}_{11i} & -I & 0 & 0 & 0 & \overline{S}_{11i}H_i & 0 & 0 \\ * & * & * & * & * & -P_{11i} & 0 & 0 & 0 & 0 & 0 & P_{11i}H_i \\ * & * & * & * & * & * & -\tau\overline{S}_{11i} & -\tau\overline{S}_{11i} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -\tau P_{11i} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & -\varepsilon_i I & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & -\varepsilon_i I & 0 & 0 \\ * & * & * & * & * & * & * & * & * & * & -\delta_i I & 0 \\ * & * & * & * & * & * & * & * & * & * & * & -\delta_i I \end{bmatrix} < 0, \tag{32a}$$

where $\tau = (1-\overline{r})\overline{r}$, $\Phi_i = R_{11i} + Y_i' + (Y_i')^T + \widetilde{Y}_i' - \alpha\overline{S}_{11i}$, $\Upsilon_i = R_{11i} - \alpha P_{11i}$, $\Lambda_i = R_{11i} + Y_i' - \alpha\overline{S}_{11i}$, $\theta_i = A_i^T\overline{S}_{11i} + (\widetilde{Z}_i')^T$, $\Theta_i = A_i^T P_{11i} + \overline{r}C_i^T Z_i^T$, $\Xi_i = -\alpha^d R_{11i} - \alpha^d Y_i' - \alpha^d (Y_i')^T - \alpha^d \widetilde{Y}_i'$, $\Omega_i = -\alpha^d R_{11i} - \alpha^d Y_i'$.

Then, there exists a DOF controller (7a), (7b), (7c), *and* (7d) *such that the closed-loop system* (9a), (9b), *and* (9c) *is exponentially stable in mean square sense for any switching signals with average dwell time scheme* (14), *where* $\mu \geq 1$ *satisfies*

$$\begin{bmatrix} R_i - \mu P_j & \overline{E}_{ij}^T P_i \\ * & -P_i \end{bmatrix} < 0, \tag{32b}$$

$$\alpha R_i < \mu R_j, \quad \left(i, j \in \underline{N}\right), \tag{32c}$$

where

$$P_i = \begin{bmatrix} P_{11i} & P_{12i} \\ * & P_{22i} \end{bmatrix}, \qquad R_i = \begin{bmatrix} R_{11i} & R_{12i} \\ * & R_{22i} \end{bmatrix}, \tag{33}$$

$$P_{11i}\overline{S}_{11i}^{-1} + P_{12i}S_{12i}^T = I, \qquad P_{12i}^T\overline{S}_{11i}^{-1} + P_{22i}S_{12i}^T = 0, \tag{34a}$$

$$R_{12i} = \left(Y_i'\right)^T \overline{S}_{11i}^{-1} S_{12i}^{-T}, \qquad R_{22i} = S_{12i}^{-1}\overline{S}_{11i}^{-1}\widetilde{Y}_i'\overline{S}_{11i}^{-1}S_{12i}^{-T}. \tag{34b}$$

Moreover, if the previous LMI conditions are feasible, then the desired dynamic output feedback controller parameters can be designed as

$$A_{ci} = P_{12i}^{-1}\left(\Sigma_i^T - P_{11i}A_i - \overline{r}Z_iC_i - P_{11i}\overline{S}_{11i}^{-1}\widetilde{Z}_i'\right)\overline{S}_{11i}^{-1}S_{12i}^{-T}, \tag{35a}$$

$$B_{ci} = P_{12i}^{-1}Z_i, \qquad C_{ci} = B_i^{-1}\overline{S}_{11i}^{-1}\widetilde{Z}_i'\overline{S}_{11i}^{-1}S_{12i}^{-T}. \tag{35b}$$

Proof. Let the matrix P_i^{-1} be partitioned as follows:

$$P_i^{-1} = \begin{bmatrix} S_{11i} & S_{12i} \\ * & S_{22i} \end{bmatrix}, \tag{36}$$

where $S_{11i} \in R^{n\times n}$.

By $P_iP_i^{-1} = I$ and from (34a) and (36), we have

$$P_{11i}S_{11i} + P_{12i}S_{12i}^T = I, \qquad P_{12i}^TS_{11i} + P_{22i}S_{12i}^T = 0. \tag{37}$$

Define the following matrices:

$$J_i = \begin{bmatrix} S_{11i} & I \\ S_{12i}^T & 0 \end{bmatrix}, \qquad \widetilde{J}_i = \begin{bmatrix} I & P_{11i} \\ 0 & P_{12i}^T \end{bmatrix}. \tag{38}$$

Then we have

$$\begin{gathered}
P_iJ_i = \widetilde{J}_i, \\
J_i^TP_i\overline{A}_iJ_i = \begin{bmatrix} \widehat{A}_iS_{11i} + B_iC_{ci}S_{12i}^T & \widehat{A}_i \\ \Psi_{1i} & P_{11i}\widehat{A}_i + \overline{r}P_{12i}B_{ci}C_i \end{bmatrix}, \\
J_i^TP_i\overline{A}_{mi}J_i = \begin{bmatrix} 0 & 0 \\ P_{12i}B_{ci}C_iS_{11i} & P_{12i}B_{ci}C_i \end{bmatrix}, \\
J_i^TP_i\overline{A}_{di}J_i = \begin{bmatrix} \widehat{A}_{di}S_{11i} & \widehat{A}_{di} \\ P_{11i}\widehat{A}_{di}S_{11i} & P_{11i}\widehat{A}_{di} \end{bmatrix}, \\
J_i^TP_iJ_i = \begin{bmatrix} S_{11i} & I \\ I & P_{11i} \end{bmatrix}, \\
J_i^TR_iJ_i = \begin{bmatrix} \Psi_{2i} & S_{11i}R_{11i} + S_{12i}R_{12i}^T \\ R_{11i}S_{11i} + R_{12i}S_{12i}^T & R_{11i} \end{bmatrix}, \\
\Psi_{1i} = P_{11i}\widehat{A}_iS_{11i} + \overline{r}P_{12i}B_{ci}C_iS_{11i} + P_{11i}B_iC_{ci}S_{12i}^T + P_{12i}A_{ci}S_{12i}^T, \\
\Psi_{2i} = S_{11i}R_{11i}S_{11i} + S_{12i}R_{12i}^TS_{11i} + S_{11i}R_{12i}S_{12i}^T + S_{12i}R_{22i}S_{12i}^T.
\end{gathered} \tag{39}$$

Use $\mathrm{diag}\{J_i^T, J_i^T, J_i^T, J_i^T\}$ to premultiply and $\mathrm{diag}\{J_i, J_i, J_i, J_i\}$ to postmultiply the left-hand term of (13), and denote

$$Z_i = P_{12i}B_{ci}, \qquad \widetilde{Z}_i = C_{ci}S_{12i}^T, \qquad Y_i = S_{12i}R_{12i}^T, \qquad \widetilde{Y}_i = S_{12i}R_{22i}S_{12i}^T. \tag{40}$$

Then, we can obtain that (13) is equivalent to the following inequality:

$$\begin{bmatrix}
\Phi_{i1} & \Lambda_{i1} & 0 & 0 & \widehat{\theta}_{i1} & \widehat{\Sigma}_{i1} & 0 & \tau S_{11i}C_i^TZ_i^T \\
* & \Upsilon_{i1} & 0 & 0 & \widehat{A}_i^T & \widehat{\Theta}_{i1} & 0 & \tau C_i^TZ_i^T \\
* & * & \Xi_{i1} & \Omega_{i1} & S_{11i}\widehat{A}_{di}^T & S_{11i}\widehat{A}_{di}^TP_{11i} & 0 & 0 \\
* & * & * & -\alpha^dR_{11i} & \widehat{A}_{di}^T & \widehat{A}_{di}^TP_{11i} & 0 & 0 \\
* & * & * & * & -S_{11i} & -I & 0 & 0 \\
* & * & * & * & * & -P_{11i} & 0 & 0 \\
* & * & * & * & * & * & -\tau S_{11i} & -\tau I \\
* & * & * & * & * & * & * & -\tau P_{11i}
\end{bmatrix} < 0, \tag{41}$$

where

$$\Phi_{i1} = S_{11i}R_{11i}S_{11i} + Y_i S_{11i} + S_{11i}Y_i^T + \widetilde{Y}_i - \alpha S_{11i},$$
$$\Lambda_{i1} = S_{11i}R_{11i} + Y_i - \alpha I,$$
$$\Upsilon_{i1} = R_{11i} - \alpha P_{11i},$$
$$\widehat{\Sigma}_{i1} = S_{11i}\widehat{A}_i^T P_{11i} + \bar{r}S_{11i}C_i^T Z_i^T + \widetilde{Z}_i^T B_i^T P_{11i} + \left(P_{12i}A_{ci}S_{12i}^T\right)^T, \quad (42)$$
$$\widehat{\theta}_{i1} = S_{11i}\widehat{A}_i^T + \widetilde{Z}_i^T B_i^T,$$
$$\Xi_{i1} = -\alpha^d S_{11i}R_{11i}S_{11i} - \alpha^d Y_i S_{11i} - \alpha^d S_{11i}Y_i^T - \alpha^d \widetilde{Y}_i,$$
$$\widehat{\Theta}_{i1} = \widehat{A}_i^T P_{11i} + \bar{r}C_i^T Z_i^T,$$
$$\Omega_{i1} = -\alpha^d S_{11i}R_{11i} - \alpha^d Y_i.$$

Using $\mathrm{diag}\{S_{11i}^{-1}, I, S_{11i}^{-1}, I, S_{11i}^{-1}, I, S_{11i}^{-1}, I\}$ to pre- and post-multiply the left-hand term of (41) and denoting

$$\widetilde{Z}_i' = S_{11i}^{-1}B_i\widetilde{Z}_iS_{11i}^{-1}, \qquad Y_i' = S_{11i}^{-1}Y_i, \qquad \widetilde{Y}_i' = S_{11i}^{-1}\widetilde{Y}_iS_{11i}^{-1}, \quad (43)$$

one obtains

$$\begin{bmatrix} \Phi_i & \Lambda_i & 0 & 0 & \widehat{\theta}_i & \widehat{\Sigma}_i & 0 & \tau C_i^T Z_i^T \\ * & \Upsilon_i & 0 & 0 & \widehat{A}_i^T S_{11i}^{-1} & \widehat{\Theta}_i & 0 & \tau C_i^T Z_i^T \\ * & * & \Xi_i & \Omega_i & \widehat{A}_{di}^T S_{11i}^{-1} & \widehat{A}_{di}^T P_{11i} & 0 & 0 \\ * & * & * & -\alpha^d R_{11i} & \widehat{A}_{di}^T S_{11i}^{-1} & \widehat{A}_{di}^T P_{11i} & 0 & 0 \\ * & * & * & * & -S_{11i}^{-1} & -I & 0 & 0 \\ * & * & * & * & * & -P_{11i} & 0 & 0 \\ * & * & * & * & * & * & -\tau S_{11i}^{-1} & -\tau S_{11i}^{-1} \\ * & * & * & * & * & * & * & -\tau P_{11i} \end{bmatrix} < 0, \quad (44)$$

where

$$\Phi_i = R_{11i} + Y_i' + \left(Y_i'\right)^T + \widetilde{Y}_i' - \alpha S_{11i}^{-1},$$
$$\widehat{\Theta}_i = \widehat{A}_i^T P_{11i} + \bar{r}C_i^T Z_i^T,$$
$$\Lambda_i = R_{11i} + Y_i' - \alpha S_{11i}^{-1},$$
$$\widehat{\theta}_i = \widehat{A}_i^T S_{11i}^{-1} + \left(\widetilde{Z}_i'\right)^T,$$
$$\widehat{\Sigma}_i = \widehat{A}_i^T P_{11i} + \bar{r}C_i^T Z_i^T + \left(\widetilde{Z}_i'\right)^T S_{11i}P_{11i} + S_{11i}^{-1}\left(P_{12i}A_{ci}S_{12i}^T\right)^T,$$
$$\Upsilon_i = R_{11i} - \alpha P_{11i},$$
$$\Xi_i = -\alpha^d R_{11i} - \alpha^d Y_i' - \alpha^d \left(Y_i'\right)^T - \alpha^d \widetilde{Y}_i',$$
$$\Omega_i = -\alpha^d R_{11i} - \alpha^d Y_i'. \quad (45)$$

Then combining (5) with (44), one has

$$\widehat{T}_i = T_i + \Delta T_i < 0$$
$$T_i = \begin{bmatrix} \Phi_i & \Lambda_i & 0 & 0 & \theta_i & \Sigma_i & 0 & \tau C_i^T Z_i^T \\ * & \Upsilon_i & 0 & 0 & A_i^T S_{11i}^{-1} & \Theta_i & 0 & \tau C_i^T Z_i^T \\ * & * & \Xi_i & \Omega_i & A_{di}^T S_{11i}^{-1} & A_{di}^T P_{11i} & 0 & 0 \\ * & * & * & -\alpha^d R_{11i} & A_{di}^T S_{11i}^{-1} & A_{di}^T P_{11i} & 0 & 0 \\ * & * & * & * & -S_{11i}^{-1} & -I & 0 & 0 \\ * & * & * & * & * & -P_{11i} & 0 & 0 \\ * & * & * & * & * & * & -\tau S_{11i}^{-1} & -\tau S_{11i}^{-1} \\ * & * & * & * & * & * & * & -\tau P_{11i} \end{bmatrix}, \quad (46)$$

where

$$\Theta_i = A_i^T P_{11i} + \bar{r}C_i^T Z_i^T,$$
$$\theta_i = A_i^T S_{11i}^{-1} + \left(\widetilde{Z}_i'\right)^T,$$
$$\Sigma_i = A_i^T P_{11i} + \bar{r}C_i^T Z_i^T + \left(\widetilde{Z}_i'\right)^T S_{11i}P_{11i} + S_{11i}^{-1}\left(P_{12i}A_{ci}S_{12i}^T\right)^T,$$
$$\Delta T_i = \overline{M}_i F_i(k)^T \overline{H}_i + \left(\overline{M}_i F_i(k)^T \overline{H}_i\right)^T + \overline{M}_i F_i(k)^T \widetilde{H}_i + \left(\overline{M}_i F_i(k)^T \overline{H}_i\right)^T,$$
$$\overline{M}_i^T = \left[M_{1i} \;\; M_{1i} \;\; M_{2i} \;\; M_{2i} \;\; 0 \;\; 0 \;\; 0 \;\; 0\right],$$
$$\overline{H}_i = \left[0 \;\; 0 \;\; 0 \;\; 0 \;\; H_i^T S_{11i}^{-1} \;\; 0 \;\; 0 \;\; 0\right],$$
$$\widetilde{H}_i = \left[0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; H_i^T P_{11i} \;\; 0 \;\; 0\right]. \quad (47)$$

By Lemma 6, (46) is equivalent to

$$T_i + \varepsilon_i \overline{M}_i \overline{M}_i^T + \varepsilon_i^{-1} \overline{H}_i^T \overline{H}_i + \delta_i \overline{M}_i \overline{M}_i^T + \delta_i^{-1} \widetilde{H}_i^T \widetilde{H}_i < 0, \quad (48)$$

where ε_i and δ_i are positive scalars.

Using Lemma 5, we have

$$\begin{bmatrix} \Phi_i & \Lambda_i & 0 & 0 & \theta_i & \Sigma_i & 0 & \tau C_i^T Z_i^T & M_{1i}^T & 0 & M_{1i}^T & 0 \\ * & \Upsilon_i & 0 & 0 & A_i^T S_{11i}^{-1} & \Theta_i & 0 & \tau C_i^T Z_i^T & M_{1i}^T & 0 & M_{1i}^T & 0 \\ * & * & \Xi_i & \Omega_i & A_{di}^T S_{11i}^{-1} & A_{di}^T P_{11i} & 0 & 0 & M_{2i}^T & 0 & M_{2i}^T & 0 \\ * & * & * & -\alpha^d R_{11i} & A_{di}^T S_{11i}^{-1} & A_{di}^T P_{11i} & 0 & 0 & M_{2i}^T & 0 & M_{2i}^T & 0 \\ * & * & * & * & -S_{11i}^{-1} & -I & 0 & 0 & 0 & S_{11i}^{-1} H_i & 0 & 0 \\ * & * & * & * & * & -P_{11i} & 0 & 0 & 0 & 0 & 0 & P_{11i} H_i \\ * & * & * & * & * & * & -\tau S_{11i}^{-1} & -\tau S_{11i}^{-1} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -\tau P_{11i} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & -\varepsilon_i^{-1} I & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & -\varepsilon_i I & 0 & 0 \\ * & * & * & * & * & * & * & * & * & 0 & -\delta_i^{-1} I & 0 \\ * & * & * & * & * & * & * & * & * & 0 & * & -\delta_i I \end{bmatrix} < 0. \quad (49)$$

Using $\operatorname{diag}\{I, I, I, I, I, I, I, I, \varepsilon_i, I, \delta_i, I\}$ to pre- and postmultiply the left-hand term of (49) and denoting $\overline{S}_{11i} = S_{11i}^{-1}$ we can obtain that (32a) is equivalent to (49), that is to say, (32a) guarantees that (13) is tenable.

The proof is completed. □

Remark 10. From Theorem 9, it is easy to see that a larger α will be favorable to the solvability of inequality (32a), (32b), and (32c) which leads to a larger value of τ_a^*. Considering these, we can first select a larger α to guarantee the feasibility of inequality (32a), (32b), and (32c), and then decrease α to obtain a smaller τ_a^*.

Based on Theorem 9, we present an algorithm for the design of dynamic output controller.

Algorithm 11.

Step 1. Given the system matrices and a constant $0 < \alpha < 1$; by solving (32a), we can get the feasible solution of positive definite symmetric matrices $\overline{S}_{11i}$, P_{11i}, R_{11i}, $\widetilde{Y}_i'$, matrices Σ_i, Y_i', $\widetilde{Z}_i'$, and positive scalars ε_i, δ_i.

Step 2. Applying singular value decomposition to the first equation of (34a), we can obtain square and nonsingular matrices P_{12i} and S_{12i}. Then we can get P_{22i}, R_{12i}, and R_{22i} by (34a) and (34b).

Step 3. By substituting matrices P_{11i}, P_{12i}, P_{22i}, R_{11i}, R_{12i}, and R_{22i} into (32b)-(32c) and solving them, we can get μ and τ_a^* by (14).

Step 4. Determine the DOF controller parameters A_{ci}, B_{ci}, and C_{ci} based on (35a) and (35b).

4. Numerical Example

In this section, we present an example to illustrate the effectiveness of the proposed approach. Consider system (1a), (1b), (1c), and (1d) with parameters as follows:

$$\begin{gathered} A_1 = \begin{bmatrix} -0.5 & 0.6 \\ -0.19 & -0.6 \end{bmatrix}, \qquad A_{d1} = \begin{bmatrix} 0.03 & -0.053 \\ -0.044 & 0.012 \end{bmatrix}, \\ B_1 = \begin{bmatrix} 0.025 & -0.012 \\ -0.041 & 0.051 \end{bmatrix}, \qquad C_1 = \begin{bmatrix} -0.3 & 0.14 \\ -0.2 & -0.5 \end{bmatrix}, \\ H_1 = \begin{bmatrix} 0.033 & 0.052 \\ -0.041 & -0.06 \end{bmatrix}, \qquad M_{11} = \begin{bmatrix} 0.012 & -0.04 \\ 0.025 & -0.06 \end{bmatrix}, \\ M_{21} = \begin{bmatrix} -0.25 & 0.08 \\ -0.077 & 0.055 \end{bmatrix}, \\ F_1(k) = \begin{bmatrix} \dfrac{e^{-0.1k}}{1+0.5k}\cos(k) & 0 \\ 0 & \sin(k) \end{bmatrix}, \\ A_2 = \begin{bmatrix} -0.8 & -0.43 \\ 0.35 & -0.4 \end{bmatrix}, \qquad A_{d2} = \begin{bmatrix} -0.035 & 0.037 \\ 0.027 & -0.063 \end{bmatrix}, \\ B_2 = \begin{bmatrix} -0.041 & 0.051 \\ 0.025 & -0.012 \end{bmatrix}, \qquad C_2 = \begin{bmatrix} -0.1 & 0.25 \\ 0.18 & 0.25 \end{bmatrix}, \\ H_2 = \begin{bmatrix} -0.01 & -0.033 \\ 0.073 & 0.03 \end{bmatrix}, \qquad M_{12} = \begin{bmatrix} -0.025 & 0.02 \\ -0.015 & 0.074 \end{bmatrix}, \\ M_{22} = \begin{bmatrix} 0.4 & -0.05 \\ 0.046 & -0.062 \end{bmatrix}, \\ F_2(k) = \begin{bmatrix} \sin(k) & 0 \\ 0 & \dfrac{e^{-0.1k}}{1+0.5k}\cos(k) \end{bmatrix}. \end{gathered} \quad (50)$$

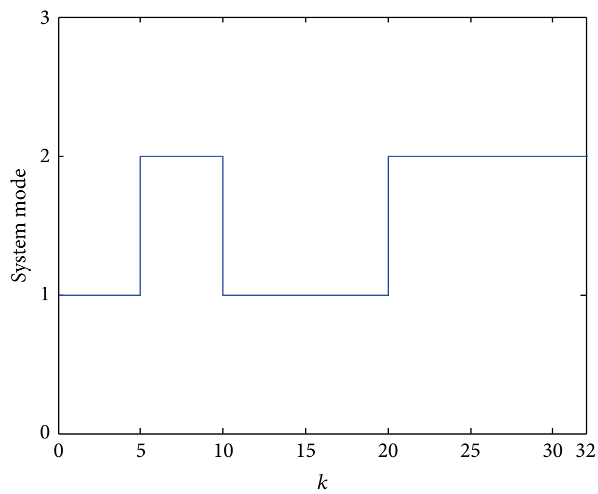

Figure 1: Switching signal.

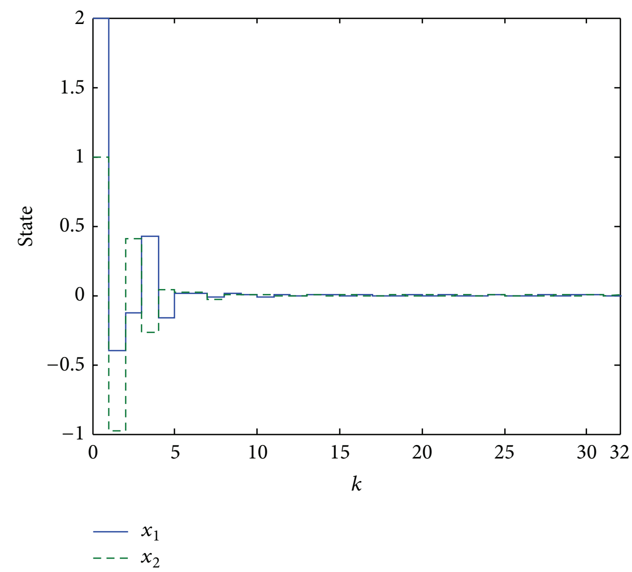

Figure 2: State responses of the resulting closed-loop system.

Let $\alpha = 0.788$, $d = 2$, and $\bar{r} = 0.7$, then by solving the matrix inequalities in Theorem 9, we can get the DOF controller parameters

$$A_{c1} = \begin{bmatrix} 0.2203 & 0.0818 \\ -0.1621 & 0.1568 \end{bmatrix}, \qquad B_{c1} = \begin{bmatrix} 0.4072 & -2.0364 \\ -2.1032 & 0.7452 \end{bmatrix},$$
$$C_{c1} = \begin{bmatrix} 10.2067 & -13.1427 \\ 5.4233 & -11.6044 \end{bmatrix}, \qquad A_{c2} = \begin{bmatrix} 0.1868 & -0.0881 \\ 0.1339 & 0.1329 \end{bmatrix},$$
$$B_{c2} = \begin{bmatrix} 1.4037 & 2.2041 \\ -2.6439 & 3.7189 \end{bmatrix}, \qquad C_{c2} = \begin{bmatrix} -16.7083 & 19.4157 \\ -22.2824 & 11.7057 \end{bmatrix}. \tag{51}$$

Let

$$\overline{E}_{1,2} = \begin{bmatrix} 0.8 & 0 & 0 & 0 \\ 0 & 1.15 & 0 & 0 \\ 0 & 0 & 1.2 & 0 \\ 0 & 0 & 0 & 1.08 \end{bmatrix},$$
$$\overline{E}_{2,1} = \begin{bmatrix} 1.16 & 0 & 0 & 0 \\ 0 & 0.9 & 0 & 0 \\ 0 & 0 & 0.9 & 0 \\ 0 & 0 & 0 & 1.12 \end{bmatrix}. \tag{52}$$

According to (32a), (32b), and (32c), we get $\mu = 5.4319$. From (14), it can be obtained that $\tau_a^* = 7.1028$. Choosing $\tau_a = 8$, simulation results are shown in Figures 1 and 2, where the initial conditions are $x(0) = \begin{bmatrix} 2 & 1 \end{bmatrix}^T$, $x(\theta) = 0$, $\theta \in [-d, 0)$, and $x_c(\theta) = 0$, $\theta \in [-d, 0]$. Figure 1 depicts the switching signal. Under this switching signal and dynamic output feedback controller, the state responses of the resulting closed-loop system are shown in Figure 2. From Figure 1, we can see that the switching signal satisfies $\tau_a = 8$. Furthermore, it can be observed from Figure 2 that the resulting closed-loop system is exponentially stable in mean square sense. This indicates that the designed controller is effective although there exist missing measurements.

5. Conclusions

This paper has presented a solution to the problem of dynamic output feedback controller design for a class of uncertain discrete impulsive switched systems with state delay and missing measurements. By employing the average dwell time approach, a sufficient condition for the existence of a DOF controller is presented such that the exponential stability in mean square sense of the resulting closed-loop system is ensured. An example is given to illustrate the applicability of the proposed approach. Our future work will focus on studying the problem of asynchronous control for discrete impulsive switched systems with state delay and missing measurements.

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