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Research Article

H_∞ Fuzzy Control for Nonlinear Singular Markovian Jump Systems with Time Delay

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This paper investigates the problem of H_∞ fuzzy control for a class of nonlinear singular Markovian jump systems with time delay. This class of systems under consideration is described by Takagi-Sugeno (T-S) fuzzy models. Firstly, sufficient condition of the stochastic stabilization by the method of the augmented matrix is obtained by the state feedback. And a designed algorithm for the state feedback controller is provided to guarantee that the closed-loop system not only is regular, impulse-free, and stochastically stable but also satisfies a prescribed H_∞ performance for all delays not larger than a given upper bound in terms of linear matrix inequalities. Then H_∞ fuzzy control for this kind of systems is also discussed by the static output feedback. Finally, numerical examples are given to illustrate the validity of the developed methodology.

1. Introduction

Singular systems, also known as descriptor systems, have been widely studied in the past several decades. They have broad applications and can be found in many practical systems, such as electrical circuits, power systems, network, economics, and other systems [1, 2]. Due to their extensive applications, many research topics on singular systems have been extensively investigated such as the stability and stabilization [3, 4] and H_∞ control problem [5, 6]. A lot of attention has been paid to the investigation of Markovian jump systems (MJSs) over the past decades. Applications of such class of systems can be found representing many physical systems with random changes in their structures and parameters. Many important issues have been studied for this kind of physical systems, such as the stability analysis, stabilization, and H_∞ control [7–10]. When singular systems experience abrupt changes in their structures, it is natural to model them as singular Markovian jump systems (SMJSs) [11–13]. Time delay is one of the instability sources for dynamical systems and is a common phenomenon in many industrial and engineering systems such as those in communication networks, manufacturing, and biology [14]. So the study of SMJSs with time delay is of theoretical and practical importance [15, 16].

The fuzzy control has been proved to be a powerful method for the control problem of complex nonlinear systems. Specially, the Takagi-Sugeno (T-S) fuzzy model has attracted much attention due to the fact that it provides an efficient approach to take full advantage of the linear control theory to the nonlinear control. In recent years, this fuzzy-model-based technique has been used to deal with nonlinear time delay systems [17, 18] and nonlinear MJSs [19, 20]. But singular Markovian jump fuzzy systems (SMJFSs) are not fully studied [21, 22], which motivates the main purpose of our study. In this paper, a new method using the augmented matrix will be given to the control of SMJFSs. By this method the number of LMIs will be decreased, so the complexity of the calculation will be greatly reduced when the number of fuzzy rulers is relatively large. And, at the same time, some new relaxation matrices added will reduce the conservation of control conditions compared with previous literatures. And when using the augmented matrix to design the static output feedback control, there are not any crossing terms between system matrices and controller gains, so assumptions for the output matrix [23], the equality constraint for the output matrix [24], and the bounding technique for crossing terms are not necessary; therefore, the conservatism brought by them will not exist.

In this paper, the H_∞ fuzzy control problem for a class of nonlinear SMJSs with time delay which can be represented by T-S fuzzy models is considered. Our aim is to design fuzzy state feedback controllers and static output feedback controllers for SMJFSs with time delay, such that closed-loop systems are stochastically admissible (regular, impulse-free, and stochastically stable) with a prescribed H_∞ performance γ. Sufficient criterions are presented in forms of LMIs which are simple and easy to implement compared with previous literatures. Finally, numerical examples are given to illustrate the merit and usability of the approach proposed in this paper.

Notations. Throughout this paper, notations used are fairly standard; for real symmetric matrices A and B, the notation $A \geq B$ $(A > B)$ means that the matrix $A - B$ is positive semidefinite (positive definite). A^T represents the transpose of the matrix A, and A^{-1} represents the inverse of the matrix A. $\lambda_{\max} B$ $(\lambda_{\min} B)$ is the maximal (minimal) eigenvalue of the matrix B. diag$\{\cdot\}$ stands for a block-diagonal matrix. I is the unit matrix with appropriate dimensions, and, in a matrix, the term of symmetry is stated by the asterisk "$*$." Let $\mathbb{R}^n$ stand for the n-dimensional Euclidean space, $\mathbb{R}^{n\times m}$ is the set of all $n \times m$ real matrices, and $\|\cdot\|$ denotes the Euclidean norm of vectors. $\mathscr{E}\{\cdot\}$ denotes the mathematics expectation of the stochastic process or vector. $L_2^n[0,\infty)$ stands for the space of n-dimensional square integrable functions on $[0,\infty)$. $C_{n,d} = C([-d,0],\mathbb{R}^n)$ denotes Banach space of continuous vector functions mapping the interval $[-d,0]$ into $\mathbb{R}^n$ with the norm $\|\phi\|_d = \sup_{-d\leq s\leq 0}\|\phi(s)\|$.

2. Basic Definitions and Lemmas

Consider a SMJFS; its ith fuzzy rule is given by

R_i: if $\xi_1(t)$ is M_{i1}, $\xi_2(t)$ is $M_{i2}, \ldots,$ and $\xi_l(t)$ is M_{il}, then

$$\begin{aligned} E\dot{x}(t) &= A_i(r_t)x(t) + A_{d,i}(r_t)x(t-d) + B_i(r_t)u(t) \\ &\quad + B_{w,i}(r_t)w(t), \\ z(t) &= C_i(r_t)x(t) + C_{d,i}(r_t)x(t-d) + D_i(r_t)u(t) \\ &\quad + C_{w,i}(r_t)w(t), \\ x(t) &= \phi(t), \\ &\qquad \forall t \in [-\bar{d},0], \; i \in \mathscr{T} \triangleq \{1,2,\ldots,k\}, \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control input, $w(t) \in \mathbb{R}^v$ is the exogenous disturbance which belongs to $L_2^v[0,\infty)$, and $z(t) \in \mathbb{R}^p$ is the controlled output. $\phi(t) \in C_{n,\bar{d}}$ is a compatible vector-valued initial function, and d is an unknown but constant delay satisfying $d \in [0,\bar{d}]$. The scalar k is the number of If-Then rules. M_{ij} $(i \in \mathscr{T}, j = 1,2,\ldots,l)$ are fuzzy sets. $\xi_1(t) - \xi_l(t)$ are premise variables. $E \in \mathbb{R}^{n\times n}$ may be a singular matrix with rank$E = r \leq n$. $A_i(r_t)$, $A_{d,i}(r_t)$, $B_i(r_t)$, $B_{w,i}(r_t)$, $C_i(r_t)$, $C_{d,i}(r_t)$, $D_i(r_t)$, and $C_{w,i}(r_t)$ are known constant matrices with appropriate dimensions. $\{r_t, t \geq 0\}$ is a continuous-time Markovian process with right continuous trajectories taking values in a finite set given by $\mathscr{S} = \{1,2,\ldots,N\}$ with the transition rate matrix $\Pi \triangleq \{\pi_{pq}\}$ satisfying

$$\Pr\{r_{t+h} = q \mid r_t = p\} = \begin{cases} \pi_{pq}h + o(h) & p \neq q \\ 1 + \pi_{pp}h + o(h) & p = q, \end{cases} \tag{2}$$

where $h > 0$, $\lim_{h\to 0} o(h)/h = 0$, and $\pi_{pq} \geq 0$, for $q \neq p$, is the transition rate from mode p at time t to q at time $t+h$ and $\pi_{pp} = -\sum_{q=1,q\neq p}^{N} \pi_{pq}$.

By fuzzy blending, the overall fuzzy model is inferred as follows:

$$\begin{aligned} E\dot{x}(t) &= \sum_{i=1}^{k} \lambda_i(\xi(t))\big(A_i(r_t)x(t) + A_{d,i}(r_t)x(t-d) \\ &\quad + B_i(r_t)u(t) + B_{w,i}(r_t)w(t)\big), \\ z(t) &= \sum_{i=1}^{k} \lambda_i(\xi(t))\big(C_i(r_t)x(t) + C_{d,i}(r_t)x(t-d) \\ &\quad + D_i(r_t)u(t) + C_{w,i}(r_t)w(t)\big), \\ x(t) &= \phi(t), \\ &\qquad \forall t \in [-\bar{d},0], \; i \in \mathscr{T} \triangleq \{1,2,\ldots,k\}, \end{aligned} \tag{3}$$

where $\xi(t) = [\xi_1(t)\ \xi_2(t)\ \cdots\ \xi_l(t)]^T$, $\beta_i(\xi(t)) = \prod_{j=1}^{l} M_{ij}(\xi_j(t))$. Letting $\lambda_i(\xi(t)) = \beta_i(\xi(t))/\sum_{i=1}^{k}\beta_i(\xi(t))$, it follows that $\lambda_i(\xi(t)) \geq 0$, $\sum_{i=1}^{k}\lambda_i(\xi(t)) = 1$.

For the notational simplicity, in the sequel, for each possible $r_t = p \in \mathscr{S}$, $A_i(r_t) \triangleq A_{pi}$, $B_{d,i}(r_t) \triangleq B_{d,pi}$, $C_{d,i}(r_t) \triangleq C_{d,pi}$, $\lambda_i(\xi(t)) \triangleq \lambda_i$, and so on.

Definition 1 (see [15, 25]). (i) For a given scalar $\bar{d} > 0$, the SMJS with time delay

$$\begin{aligned} E\dot{x}(t) &= A(r_t)x(t) + A_d(r_t)x(t-d), \\ x(t) &= \phi(t), \\ &\qquad t \in [-\bar{d},0] \end{aligned} \tag{4}$$

is said to be regular and impulse-free for any constant time delay satisfying $d \in [0,\bar{d}]$, if pairs $(E, A(r_t))$ and $(E, A(r_t) + A_d(r_t))$ are regular and impulse-free.

(ii) System (4) is said to be stochastically stable if there exists a finite number $M(\phi(t), r_0)$ such that the following inequality holds:

$$\begin{aligned} &\lim_{t\to\infty} \mathscr{E}\left\{\int_0^t \|x(s)\|^2 \,\mathrm{d}s \mid r_0, x(s) = \phi(s), s \in [-\bar{d},0]\right\} \\ &\quad < M(\phi(t), r_0). \end{aligned} \tag{5}$$

(iii) System (4) is said to be stochastically admissible if it is regular, impulse-free, and stochastically stable.

Lemma 2 (see [26]). *Given matrices* $E, X > 0, Y$, *if* $E^TX + Y\Lambda^T$ *is nonsingular, there exist matrices* $S > 0, L$, *such that* $ES + L\Theta^T = (E^TX + Y\Lambda^T)^{-1}$, *where* $\Lambda, \Theta \in \mathbb{R}^{n\times(n-r)}$, *such that* $E^T\Lambda = 0$, $E\Theta = 0$, $\operatorname{rank}\Lambda = \operatorname{rank}\Theta = n - r$, $X, S \in \mathbb{R}^{n\times n}$, *and* $Y, L \in \mathbb{R}^{n\times(n-r)}$.

Lemma 3 (see [27]). *For matrices* $Q > 0$, P, *and* R *with appropriate dimensions, the following inequality holds:*

$$PR^T + RP^T \le RQR^T + PQ^{-1}P^T. \tag{6}$$

Lemma 4 (see [28]). *For any constant matrix* $X \in \mathbb{R}^{n\times n}$, $X = X^T > 0$, *scalar* $r > 0$, *and vector function* $\dot{x} : [-r, 0] \to \mathbb{R}^n$ *such that the following integration is well defined; then*

$$\begin{aligned} &-r\int_{-r}^{0}\dot{x}^T(t+s)X\dot{x}(t+s)\,ds \\ &\quad\le \begin{bmatrix} x^T(t) & x^T(t-r) \end{bmatrix}\begin{bmatrix} -X & X \\ X & -X \end{bmatrix}\begin{bmatrix} x(t) \\ x(t-r) \end{bmatrix}. \end{aligned} \tag{7}$$

Lemma 5 (see [29]). *Suppose there are piecewise continuous real square matrices* $A(t)$, X, *and* $Q > 0$ *satisfying* $A^T(t)X + X^TA(t) < 0$ *for all* t. *Then the following conditions hold:*

(i) $A(t)$ *and* X *are nonsingular.*

(ii) $\|A^{-1}(t)\| \le \delta$ *for some* $\delta > 0$.

Lemma 6 (see [30]). *If the following conditions hold:*

$$\begin{aligned} &M_{ii} < 0, \quad 1 \le i \le r; \\ &\frac{1}{r-1}M_{ii} + \frac{1}{2}\left(M_{ij} + M_{ji}\right) < 0, \quad 1 \le i \ne j \le r, \end{aligned} \tag{8}$$

then the following parameterized matrix inequality holds:

$$\sum_{i=1}^{r}\sum_{j=1}^{r}\alpha_i(t)\,\alpha_j(t)\,M_{ij} < 0, \tag{9}$$

where $\alpha_i(t) \ge 0$ *and* $\sum_{i=1}^{r}\alpha_i(t) = 1$.

Based on the parallel distributed compensation, the following state feedback controller will be considered here:

$$u_p(t) = \sum_{i=1}^{k}\lambda_i K_{pi}x(t), \tag{10}$$

where K_{pi} $(p \in \mathcal{S}, i \in \mathcal{T})$ are local controller gains, such that the closed-loop system

$$\begin{aligned} E\dot{x}(t) &= \sum_{i=1}^{k}\sum_{j=1}^{k}\lambda_i\lambda_j\Big(\big(A_{pi} + B_{pi}K_{pj}\big)x(t) \\ &\quad + A_{d,pi}x(t-d) + B_{w,pi}w(t)\Big), \\ z(t) &= \sum_{i=1}^{k}\sum_{j=1}^{k}\lambda_i\lambda_j\Big(\big(C_{pi} + D_{pi}K_{pj}\big)x(t) \\ &\quad + C_{d,pi}x(t-d) + C_{w,pi}w(t)\Big), \\ x(t) &= \phi(t), \\ &\qquad t \in \left[-\bar{d}, 0\right] \end{aligned} \tag{11}$$

is stochastically admissible.

3. The Design of the State Feedback H_∞ Controller

Firstly, the sufficient condition will be given such that system (11) is stochastically admissible. Combining (4) and (10), fuzzy closed-loop system (11) can be rewritten in the following form:

$$\begin{aligned} \widetilde{E}\dot{\widetilde{x}}(t) &= \sum_{i=1}^{k}\lambda_i\left(\widetilde{A}_{pi}\widetilde{x}(t) + \widetilde{A}_{d,pi}\widetilde{x}(t-d) + \widetilde{B}_{w,pi}w(t)\right), \\ z(t) &= \sum_{i=1}^{k}\lambda_i\left(\widetilde{C}_{pi}\widetilde{x}(t) + \widetilde{C}_{d,pi}\widetilde{x}(t-d) + C_{w,pi}w(t)\right), \\ \widetilde{x}(t) &= \widetilde{\phi}(t), \\ &\qquad t \in \left[-\bar{d}, 0\right], \end{aligned} \tag{12}$$

where

$$\begin{aligned} \widetilde{E} &= \begin{bmatrix} E & 0 \\ 0 & 0 \end{bmatrix} \in \mathbb{R}^{(n+m)\times(n+m)}, \\ \widetilde{A}_{pi} &= \begin{bmatrix} A_{pi} & B_{pi} \\ K_{pi} & -I_m \end{bmatrix} \in \mathbb{R}^{(n+m)\times(n+m)}, \\ \widetilde{x}(t) &= \begin{bmatrix} x(t) \\ u(t) \end{bmatrix} \in \mathbb{R}^{n+m}, \\ \widetilde{A}_{d,pi} &= \begin{bmatrix} A_{d,pi} & 0 \\ 0 & 0 \end{bmatrix} \in \mathbb{R}^{(n+m)\times(n+m)}, \\ \widetilde{B}_{w,pi} &= \begin{bmatrix} B_{w,pi} \\ 0 \end{bmatrix} \in \mathbb{R}^{(n+m)\times v}, \\ \widetilde{C}_{pi} &= \begin{bmatrix} C_{pi} & D_{pi} \end{bmatrix} \in \mathbb{R}^{p\times(n+m)}, \\ \widetilde{C}_{d,pi} &= \begin{bmatrix} C_{d,pi} & 0 \end{bmatrix} \in \mathbb{R}^{p\times(n+m)}, \\ \widetilde{\phi}(t) &= \sum_{i=1}^{k}\lambda_i\begin{bmatrix} \phi(t) \\ K_{pi}\phi(t) \end{bmatrix}. \end{aligned} \tag{13}$$

Remark 7. For systems (11) and (12), it can be seen that

$$\det\left(sE-\sum_{i=1}^{k}\sum_{j=1}^{k}\lambda_i\lambda_j\left(A_{pi}+B_{pi}K_{pj}\right)\right)$$
$$=\det\left(s\begin{bmatrix}E & 0\\ 0 & 0\end{bmatrix}-\sum_{i=1}^{k}\sum_{j=1}^{k}\lambda_i\lambda_j\begin{bmatrix}A_{pi} & B_{pi}\\ K_{pj} & -I_m\end{bmatrix}\right)$$
$$=\det\left(s\begin{bmatrix}E & 0\\ 0 & 0\end{bmatrix}-\sum_{i=1}^{k}\lambda_i\begin{bmatrix}A_{pi} & B_{pi}\\ K_{pi} & -I_m\end{bmatrix}\right)$$
$$=\det\left(s\widetilde{E}-\sum_{i=1}^{k}\lambda_i\widetilde{A}_{pi}\right),$$
$$\det\left(sE-\sum_{i=1}^{k}\sum_{j=1}^{k}\lambda_i\lambda_j\left(A_{pi}+B_{pi}K_{pj}+A_{d,pi}\right)\right)$$
$$=\det\left(s\begin{bmatrix}E & 0\\ 0 & 0\end{bmatrix}-\sum_{i=1}^{k}\lambda_i\begin{bmatrix}A_{pi}+A_{d,pi} & B_{pi}\\ K_{pi} & -I_m\end{bmatrix}\right)$$
$$=\det\left(s\widetilde{E}-\sum_{i=1}^{k}\lambda_i\left(\widetilde{A}_{pi}+\widetilde{A}_{d,pi}\right)\right). \tag{14}$$

By rank E = rank $\widetilde{E}$ and Definition 1, it can be obtained that the regularity and nonimpulse of system (11) are equal to the regularity and nonimpulse of system (12). So the stochastic admissibility of system (11) can be studied by system (12).

Theorem 8. *For a prescribed scalar $\overline{d}>0$, there exists a state feedback controller (10) with $u_p(t)=\sum_{i=1}^{k}\lambda_i L_{pi}Y_p^{-1}x(t)$ such that system (11) when $w(t)=0$ is stochastically admissible for any constant time delay d satisfying $d\in[0,\overline{d}]$, if there exist matrices $\overline{P}_p>0$, $\overline{Q}_p>0$, $\overline{Q}>0$, $\overline{Z}>0$, L_{pi}, $\overline{S}_p$, Y_{p2}, and Y_{p3}, $i\in\mathcal{T}$, $p\in\mathcal{S}$, such that*

$$\begin{bmatrix}
\Gamma_{1pi} & * & * & * & * & * & *\\
L_{pi}+Y_{p3}^TB_{pi}^T+Y_{p2} & -Y_{p3}^T-Y_{p3} & * & * & * & * & *\\
\begin{pmatrix}Y_p^TA_{d,pi}^T+EY_p\\ +Y_p^TE^T-\overline{Z}\end{pmatrix} & 0 & \begin{pmatrix}-Y_p^T-Y_p+\overline{Q}_p\\ -Y_p^TE^T-EY_p+\overline{Z}\end{pmatrix} & * & * & * & *\\
\overline{d}A_{pi}Y_p-\overline{d}B_{pi}Y_{p2} & \overline{d}B_{pi}Y_{p3} & \overline{d}A_{d,pi}Y_p & -\overline{Z} & * & * & *\\
Y_p & 0 & 0 & 0 & -\overline{Q}_p & * & *\\
\overline{d}Y_p & 0 & 0 & 0 & 0 & -\overline{d}\overline{Q} & *\\
\left[I_r\ \ 0\right]H^{-1}M_p^T & 0 & 0 & 0 & 0 & 0 & -J_p
\end{bmatrix}<0, \tag{15}$$

$$\begin{bmatrix}\left(\pi_{pp}-1\right)Y_p+\left(\pi_{pp}-1\right)Y_p^T+\overline{Q}-\pi_{pp}\overline{Q}_p & *\\ M_p^T & -T_p\end{bmatrix}<0, \tag{16}$$

where $\Gamma_{1pi}=\pi_{pp}Y_p^TE^T+A_{pi}Y_p+Y_p^TA_{pi}^T-Y_{p2}^TB_{pi}^T-B_{pi}Y_{p2}-Y_p^TE^T-EY_p+\overline{Z}$, $Y_p=(E\overline{P}_p+\overline{S}_p\overline{R}^T)^T$, $L_{pi}=K_{pi}Y_p$, $Y_{p2}=Y_{p3}\widetilde{Y}_{p2}Y_p$, $M_p=\left[\sqrt{\pi_{p1}}Y_p^T\ \cdots\ \sqrt{\pi_{pp-1}}Y_p^T\ \sqrt{\pi_{pp+1}}Y_p^T\ \cdots\ \sqrt{\pi_{pN}}Y_p^T\right]$, $T_p=\operatorname{diag}\{\overline{Q}_1,\ldots,\overline{Q}_{p-1},\overline{Q}_{p+1},\ldots,\overline{Q}_N\}$, $J_p=\operatorname{diag}\{\Phi_1,\ldots,\Phi_{p-1},\Phi_{p+1},\ldots,\Phi_N\}$, $\Phi_t=[I_r\ \ 0]GEY_tG^T\left[\begin{smallmatrix}I_r\\ 0\end{smallmatrix}\right]$, $\overline{R}\in\mathbb{R}^{n\times(n-r)}$ *is any matrix with full column rank and satisfies* $E\overline{R}=0$, *and* G,H *are nonsingular matrices that make* $GEH=\left[\begin{smallmatrix}I_r & 0\\ 0 & 0\end{smallmatrix}\right]$.

Proof. From **??**, it can be concluded that Y_p and Y_{p3} are nonsingular matrices. Because $Y_p=(E\overline{P}_p+\overline{S}_p\overline{R}^T)^T$,

$$Y_p^TE^T=EY_p=E\overline{P}_pE^T\geq 0. \tag{17}$$

Denote $H^{-1}Y_pG^T=\left[\begin{smallmatrix}Y_{p11} & Y_{p12}\\ Y_{p21} & Y_{p22}\end{smallmatrix}\right]$; from (17), it is easy to obtain that $Y_{p12}=0$ and Y_{p11} is symmetric; then $H^{-1}Y_pG^T=\left[\begin{smallmatrix}Y_{p11} & 0\\ Y_{p21} & Y_{p22}\end{smallmatrix}\right]$. So it can be concluded that Y_{p11} and Y_{p22} are nonsingular; furthermore, $G^{-T}Y_p^{-1}H=\left[\begin{smallmatrix}Y_{p11}^{-1} & 0\\ -Y_{p22}^{-1}Y_{p21}Y_{p11}^{-1} & Y_{p22}^{-1}\end{smallmatrix}\right]$. Let $\widetilde{Y}_p=\left[\begin{smallmatrix}Y_p & 0\\ -Y_{p2} & Y_{p3}\end{smallmatrix}\right]$. So $[I_r\ \ 0]\operatorname{diag}\{G,I_m\}\widetilde{E}\widetilde{Y}_q\operatorname{diag}\{G^T,I_m\}\left[\begin{smallmatrix}I_r\\ 0\end{smallmatrix}\right]=Y_{q11}$ is nonsingular. By Lemma 2, $X_p=Y_p^{-1}=(E^TP_p+S_pR^T)^T$, where $P_p>0$, $S_p\in\mathbb{R}^{n\times(n-r)}$, and $R\in\mathbb{R}^{n\times(n-r)}$ is a matrix with full column rank and satisfies $E^TR=0$. Denote $X_{p3}\triangleq Y_{p3}^{-1}$, $X_{p2}\triangleq\widetilde{Y}_{p2}=Y_{p3}^{-1}Y_{p2}Y_p^{-1}$, and $\widetilde{X}_p\triangleq\widetilde{Y}_p^{-1}=\left[\begin{smallmatrix}X_p & 0\\ X_{p2} & X_{p3}\end{smallmatrix}\right]$. So

$$\begin{bmatrix}H^{-T}\begin{bmatrix}I_r\\ 0\end{bmatrix}\\ 0_{m\times r}\end{bmatrix}\left(\left[I_r\ \ 0\right]GEY_qG^T\begin{bmatrix}I_r\\ 0\end{bmatrix}\right)^{-1}$$
$$\cdot\left[\left[I_r\ \ 0\right]H^{-1}\ \ 0_{r\times m}\right]=\operatorname{diag}\left\{H^{-T},I_m\right\}$$

$$\begin{aligned}&\cdot \begin{bmatrix} \begin{bmatrix} Y_{q11}^{-1} & 0 \\ 0 & 0 \end{bmatrix} & 0 \\ 0 & 0_m \end{bmatrix} \operatorname{diag}\left\{H^{-1}, I_m\right\} = \widetilde{E}^T \widetilde{Y}_q^{-1} \\ &= \widetilde{E}^T \widetilde{X}_q. \end{aligned} \tag{18}$$

Denote $Q_p \triangleq \overline{Q}_p^{-1}$, $Q \triangleq \overline{Q}^{-1}$, and $Z \triangleq \overline{Z}^{-1}$. By Lemma 3, it can be obtained that

$$\begin{aligned}&\begin{bmatrix} -E^T ZE & E^T ZE \\ E^T ZE & -E^T ZE \end{bmatrix} \\ &= \begin{bmatrix} I_n \\ -I_n \end{bmatrix} \left(-E^T ZE\right) \left[I_n \;\; -I_n\right] \\ &\leq \begin{bmatrix} I_n \\ -I_n \end{bmatrix} \left(-E^T X_p - X_p^T E + X_p^T \overline{Z} X_p\right) \left[I_n \;\; -I_n\right]. \end{aligned} \tag{19}$$

Now pre- and postmultiplying **??** by $\operatorname{diag}\{\widetilde{X}_p^T, X_p^T, Z, I_n, I_n, \underbrace{I_r, \ldots, I_r}_{N-1}\}$ and its transpose, by Schur complement lemma, and (18)-(19), it is easy to see that

$$\begin{bmatrix} \widetilde{\Gamma}_{1pi} & * & * \\ E^T \left[Z \;\; 0\right] \widetilde{E} + \left[A_{d,pi}^T \;\; 0\right] \widetilde{X}_p & -E^T ZE - Q_p & * \\ \overline{d} Z \left[A_{pi} \;\; B_{pi}\right] & \overline{d} Z A_{d,pi} & -Z \end{bmatrix} < 0, \tag{20}$$

where $\widetilde{\Gamma}_{1pi} = \sum_{q=1}^{N} \pi_{pq} \widetilde{E}^T \widetilde{X}_p + \widetilde{X}_p^T \widetilde{A}_{pi} + \widetilde{A}_{pi}^T \widetilde{X}_{pi} + \operatorname{diag}\{Q_p, 0\} + \overline{d} \operatorname{diag}\{Q, 0\} - \widetilde{E}^T \operatorname{diag}\{Z, 0\} \widetilde{E}$. Pre- and postmultiplying **??** by $\operatorname{diag}\{X_p^T, \underbrace{I_r, \ldots, I_r}_{N-1}\}$ and its transposition by Schur complement lemma, it can be seen that

$$\sum_{q=1}^{N} \pi_{pq} Q_q < Q. \tag{21}$$

From (20), it can be concluded that

$$\begin{aligned}&\sum_{i=1}^{k} \lambda_i \left(\pi_{pp} \widetilde{E}^T \widetilde{X}_p + \widetilde{A}_{pi}^T \widetilde{X}_p + \widetilde{X}_p^T \widetilde{A}_{pi} \right. \\ &\quad \left. - \widetilde{E}^T \operatorname{diag}\{Z, 0\} \widetilde{E}\right) < 0. \end{aligned} \tag{22}$$

On the other hand, $\operatorname{diag}\{G, I_m\} \widetilde{E} \operatorname{diag}\{H, I_m\} = \left[\begin{smallmatrix} \left[\begin{smallmatrix} I_r & 0 \\ 0 & 0 \end{smallmatrix}\right] & 0 \\ 0 & 0_m \end{smallmatrix}\right]$. Then

$$\begin{aligned}\widetilde{E}^T \widetilde{X}_p = \widetilde{X}_p^T \widetilde{E} &= \begin{bmatrix} E^T & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} X_p & 0 \\ X_{p2} & X_{p3} \end{bmatrix} = \begin{bmatrix} E^T X_p & 0 \\ 0 & 0 \end{bmatrix} \\ &= \begin{bmatrix} E^T P_p E & 0 \\ 0 & 0 \end{bmatrix} \geq 0. \end{aligned} \tag{23}$$

Denote $\widetilde{A}_p(t) \triangleq (\sum_{i=1}^{k} \lambda_i \widetilde{A}_{pi}) = \left[\begin{smallmatrix} \widetilde{A}_{p1}(t) & \widetilde{A}_{p2}(t) \\ \widetilde{A}_{p3}(t) & \widetilde{A}_{p4}(t) \end{smallmatrix}\right]$; from (22), it can be obtained that

$$X_{p3}^T \widetilde{A}_{p4}(t) + \widetilde{A}_{p4}^T(t) X_{p3} < 0, \tag{24}$$

for every $p \in \mathcal{S}$, which implies that $\widetilde{A}_{p4}(t)$ is nonsingular. Thus, the pair $(\widetilde{E}, \sum_{i=1}^{k} \lambda_i \widetilde{A}_{pi})$ is regular and impulse-free for every $p \in \mathcal{S}$. By (20), it is easy to see that

$$\begin{bmatrix} \widetilde{\Gamma}_{1pi} & * \\ E^T \left[Z \;\; 0\right] \widetilde{E} + \left[A_{d,pi}^T \;\; 0\right] \widetilde{X}_p & -E^T ZE - Q_p \end{bmatrix} < 0. \tag{25}$$

Pre- and postmultiplying (25) by $\left[\begin{smallmatrix} I_{n+m} & \left[\begin{smallmatrix} I_n \\ 0 \end{smallmatrix}\right] \\ 0 & I_n \end{smallmatrix}\right]$ and its transpose, it can be obtained that

$$\begin{aligned}&\sum_{q=1}^{N} \pi_{pq} \widetilde{E}^T \widetilde{X}_p + \widetilde{X}_p^T \left(\widetilde{A}_{pi} + \widetilde{A}_{d,pi}\right) \\ &\quad + \left(\widetilde{A}_{pi} + \widetilde{A}_{d,pi}\right)^T \widetilde{X}_p < 0. \end{aligned} \tag{26}$$

Hence,

$$\begin{aligned}&\sum_{i=1}^{k} \lambda_i \left(\pi_{pp} \widetilde{E}^T \widetilde{X}_p + \widetilde{X}_p^T \left(\widetilde{A}_{pi} + \widetilde{A}_{d,pi}\right) \right. \\ &\quad \left. + \left(\widetilde{A}_{pi} + \widetilde{A}_{d,pi}\right)^T \widetilde{X}_p\right) < 0. \end{aligned} \tag{27}$$

Equation (27) implies that the pair $(\widetilde{E}, \sum_{i=1}^{k} \lambda_i (\widetilde{A}_{pi} + \widetilde{A}_{d,pi}))$ is regular and impulse-free for every $p \in \mathcal{S}$. Thus, by Definition 1, system (12) is regular and impulse-free. By Remark 7, this implies that system (11) is regular and impulse-free.

Now, it will be shown that system (11) is stochastically stable. Define a new process $\{(x_t, r_t), t \geq 0\}$ by $\{x_t = x(t + \theta), -2d \leq \theta \leq 0\}$; then $\{(x_t, r_t), t \geq d\}$ is a Markovian process with the initial state $(\phi(\cdot), r_0)$. Now, for $t \geq d$, choose the following stochastic Lyapunov-Krasovskii candidate for this system:

$$V(x_t, p, t) = \sum_{m=1}^{4} V_m(x_t, p, t), \tag{28}$$

where

$$\begin{aligned} V_1(x_t, p, t) &= x^T(t) E^T P_p E x(t) = x^T(t) E^T X_p x(t) \\ &= \widetilde{x}^T(t) \widetilde{E}^T \widetilde{X}_p \widetilde{x}(t), \\ V_2(x_t, p, t) &= \int_{t-d}^{t} x^T(s) Q_p x(s) \, ds, \\ V_3(x_t, p, t) &= d \int_{-d}^{0} \int_{t+\theta}^{t} \dot{x}^T(s) E^T Z E \dot{x}(s) \, ds \, d\theta, \\ V_4(x_t, p, t) &= \int_{-d}^{0} \int_{t+\theta}^{t} x^T(s) Q x(s) \, ds \, d\theta. \end{aligned} \tag{29}$$

Let $\mathscr{L}$ be the weak infinitesimal generator of the random process $\{(x_t, p), t \geq 0\}$. Then, for each $p \in \mathcal{S}$,

$$\begin{aligned}\mathscr{L}V(x_t, p, t) \leq{}& 2\tilde{x}^T(t)\widetilde{X}_p^T\widetilde{E}\dot{\tilde{x}}(t) \\ &+ \tilde{x}^T(t)\left(\sum_{q=1}^{N}\pi_{pq}\widetilde{E}^T\widetilde{X}_q\right)\tilde{x}(t) \\ &+ x^T(t)Q_p x(t) \\ &- x^T(t-d)Q_p x(t-d) \\ &+ \int_{t-d}^{t} x^T(s)\left(\sum_{q=1}^{N}\pi_{pq}Q_q\right)x(s)\,\mathrm{d}s \\ &+ \bar{d}x^T(t)Qx(t) \\ &- \int_{t-d}^{t} x^T(s)Qx(s)\,\mathrm{d}s \\ &+ \bar{d}^2\dot{x}^T(t)E^TZE\dot{x}(t) \\ &- d\int_{t-d}^{t}\dot{x}^T(s)E^TZE\dot{x}(s)\,\mathrm{d}s.\end{aligned} \tag{30}$$

From (21), it is clear that

$$\begin{aligned}&\int_{t-d}^{t} x^T(s)\left(\sum_{q=1}^{N}\pi_{pq}Q_q\right)x(s)\,\mathrm{d}s \\ &\quad < \int_{t-d}^{t} x^T(s)Qx(s)\,\mathrm{d}s.\end{aligned} \tag{31}$$

From Lemma 4, it follows that

$$\begin{aligned}&-d\int_{t-d}^{t}\dot{x}^T(s)E^TZE\dot{x}(s)\,\mathrm{d}s \\ &\quad \leq \begin{bmatrix} x(t) \\ x(t-d)\end{bmatrix}^T \begin{bmatrix} -E^TZE & E^TZE \\ E^TZE & -E^TZE\end{bmatrix} \begin{bmatrix} x(t) \\ x(t-d)\end{bmatrix}.\end{aligned} \tag{32}$$

So it can be concluded that

$$\mathscr{L}V(x_t, p, t) \leq \sum_{i=1}^{k}\eta^T(t)\Phi_{pi}\eta(t), \tag{33}$$

where

$$\begin{aligned}\eta^T(t) &= \left[\tilde{x}^T(t) \;\; x^T(t-d)\right], \\ \Phi_{pi} &= \begin{bmatrix} \Upsilon_{1pi} & * \\ \Upsilon_{2pi} & \Upsilon_{3pi}\end{bmatrix}, \\ \Upsilon_{1pi} &= \widetilde{\Gamma}_{1pi} + \begin{bmatrix} A_{pi}^T \\ B_{pi}^T\end{bmatrix}\bar{d}^2 Z\left[A_{pi} \;\; B_{pi}\right], \\ \Upsilon_{3pi} &= -Q_p - E^TZE + A_{d,pi}^T\bar{d}^2 ZA_{d,pi}, \\ \Upsilon_{2pi} &= E^T\left[Z \;\; 0\right]\widetilde{E} + \left[A_{d,pi}^T \;\; 0\right]\widetilde{X}_p \\ &\quad + A_{d,pi}^T\bar{d}^2 Z\left[A_{pi} \;\; B_{pi}\right].\end{aligned} \tag{34}$$

Using (20), it is easy to see that there exists a scalar $\kappa > 0$ such that, for every $p \in \mathcal{S}$, $\mathscr{L}V(x_t, p, t) \leq -\kappa\|x(t)\|^2$, where $\kappa = \min_{i\in\mathcal{T}, p\in\mathcal{S}}(\lambda_{\min}(-\Phi_{pi}))$.

So, for $t \geq d$, by Dynkin's formula, it can be obtained that

$$\begin{aligned}&\mathscr{E}\{V(x_t, p, t)\} - \mathscr{E}\{V(x_d, r_d, d)\} \\ &\quad \leq -\kappa\mathscr{E}\left\{\int_{d}^{t}\|x(s)\|^2\,\mathrm{d}s\right\},\end{aligned} \tag{35}$$

which yields

$$\mathscr{E}\left\{\int_{d}^{t}\|x(s)\|^2\,\mathrm{d}s\right\} \leq \kappa^{-1}\mathscr{E}\{V(x_d, r_d, d)\}. \tag{36}$$

Because $GEH = \left[\begin{smallmatrix} I_r & 0 \\ 0 & 0\end{smallmatrix}\right]$, denote

$$\begin{aligned}A_p(t) &\triangleq \sum_{i=1}^{k}\sum_{j=1}^{k}\lambda_i\lambda_j\left(A_{pi} + B_{pi}K_{pj}\right) \\ &= \begin{bmatrix} A_{p1}(t) & A_{p2}(t) \\ A_{p3}(t) & A_{p4}(t)\end{bmatrix} \\ &= \begin{bmatrix} \sum_{i=1}^{k}\sum_{j=1}^{k}\lambda_i\lambda_j\widehat{A}_{pij1} & \sum_{i=1}^{k}\sum_{j=1}^{k}\lambda_i\lambda_j\widehat{A}_{pij2} \\ \sum_{i=1}^{k}\sum_{j=1}^{k}\lambda_i\lambda_j\widehat{A}_{pij3} & \sum_{i=1}^{k}\sum_{j=1}^{k}\lambda_i\lambda_j\widehat{A}_{pij4}\end{bmatrix}, \\ A_{d,p}(t) &\triangleq \sum_{i=1}^{k}\lambda_i A_{d,pi} = \begin{bmatrix} A_{d,p1}(t) & A_{d,p2}(t) \\ A_{d,p3}(t) & A_{d,p4}(t)\end{bmatrix} \\ &= \begin{bmatrix} \sum_{i=1}^{k}\lambda_i A_{d,pi1} & \sum_{i=1}^{k}\lambda_i A_{d,pi2} \\ \sum_{i=1}^{k}\lambda_i A_{d,pi3} & \sum_{i=1}^{k}\lambda_i A_{d,pi4}\end{bmatrix}.\end{aligned} \tag{37}$$

By the regularity and nonimpulse of system (11), $A_{p4}(t)$ is nonsingular; for each $p \in \mathcal{S}$, set $\overline{G}_p = \left[\begin{smallmatrix} I_r & -A_{p2}(t)A_{p4}^{-1}(t) \\ 0 & A_{p4}^{-1}(t)\end{smallmatrix}\right]G$. It is easy to obtain

$$\begin{aligned}\overline{G}_p EH &= \begin{bmatrix} I_r & 0 \\ 0 & 0\end{bmatrix}, \\ \overline{G}_p A_p(t)H &= \begin{bmatrix} \overline{A}_{p1}(t) & 0 \\ \overline{A}_{p3}(t) & I_{n-r}\end{bmatrix}, \\ \overline{G}_p A_p(t)H &= \begin{bmatrix} \overline{A}_{p1}(t) & 0 \\ \overline{A}_{p3}(t) & I_{n-r}\end{bmatrix},\end{aligned} \tag{38}$$

where

$$\begin{aligned}\overline{A}_{p1}(t) &= A_{p1}(t) - A_{p2}(t)A_{p4}^{-1}(t)A_{p3}(t),\\ \overline{A}_{p3}(t) &= A_{p4}^{-1}(t)A_{p3}(t),\\ \overline{A}_{d,p1}(t) &= A_{d,p1}(t) - A_{p2}(t)A_{p4}^{-1}(t)A_{d,p3}(t),\\ \overline{A}_{d,p2}(t) &= A_{d,p2}(t) - A_{p2}(t)A_{p4}^{-1}(t)A_{d,p4}(t),\\ \overline{A}_{d,p3}(t) &= A_{p4}^{-1}(t)A_{d,p3}(t),\\ \overline{A}_{d,p4}(t) &= A_{p4}^{-1}(t)A_{d,p4}(t).\end{aligned} \tag{39}$$

Then, for each $p \in \mathcal{S}$, system (11) is equal to

$$\begin{aligned}\dot{\psi}_1(t) &= \overline{A}_{p1}(t)\psi_1(t) + \overline{A}_{d,p1}(t)\psi_1(t-d)\\ &\quad + \overline{A}_{d,p2}(t)\psi_2(t-d),\\ -\psi_2(t) &= \overline{A}_{p3}(t)\psi_1(t) + \overline{A}_{d,p3}(t)\psi_1(t-d)\\ &\quad + \overline{A}_{d,p4}(t)\psi_2(t-d),\\ \psi(t) &= \varphi(t) = H^{-1}x(t),\\ &\qquad t \in \left[-\overline{d}, 0\right],\end{aligned} \tag{40}$$

where $\psi(t) = \left[\begin{smallmatrix}\psi_1(t)\\ \psi_2(t)\end{smallmatrix}\right] = H^{-1}x(t)$.

For any $t \geq 0$, using Lemma 5, there exists a scalar $\delta_p > 0$ such that $\|A_{p4}(t)\| < \delta_p$, and $\lambda_i(\xi(t)) \geq 0$, and $\sum_{i=1}^{k}\lambda_i(\xi(t)) = 1$; it follows from (40) that

$$\begin{aligned}&\|\psi_1(t)\|\\ &\quad \leq \|\psi_1(0)\|\\ &\qquad + k_1\int_0^t \left[\|\psi_1(s)\| + \|\psi_1(s-d)\| + \|\psi_2(s-d)\|\right]\mathrm{d}s,\end{aligned} \tag{41}$$

where

$$\begin{aligned}k_1 = \max_{p\in\mathcal{S}}&\left\{\max_{i,j\in\mathcal{T}}\left\|\widehat{A}_{pij1}\right\|\right.\\ &+ \delta_p\max_{i,j\in\mathcal{T}}\left\|\widehat{A}_{pij2}\right\|\max_{i,j\in\mathcal{T}}\left\|\widehat{A}_{pij3}\right\|, \max_{i\in\mathcal{T}}\left\|A_{d,pi1}\right\|\\ &+ \delta_p\max_{i,j\in\mathcal{T}}\left\|\widehat{A}_{pij2}\right\|\max_{i,j\in\mathcal{T}}\left\|A_{d,pi3}\right\|, \max_{i\in\mathcal{T}}\left\|A_{d,pi2}\right\|\\ &\left.+ \delta_p\max_{i,j\in\mathcal{T}}\left\|\widehat{A}_{pij2}\right\|\max_{i,j\in\mathcal{T}}\left\|A_{d,pi4}\right\|\right\}.\end{aligned} \tag{42}$$

Then, for any $0 \leq t \leq d$,

$$\|\psi_1(t)\| \leq \left(2k_1\overline{d} + 1\right)\|\varphi\|_{\overline{d}} + k_1\int_0^t \|\psi_1(s)\|\,\mathrm{d}s. \tag{43}$$

Applying the Gronwall-Bellman lemma, it can be obtained, for any $0 \leq t \leq d$, that

$$\|\psi_1(t)\| \leq \left(2k_1\overline{d} + 1\right)\|\varphi\|_{\overline{d}}\, e^{k_1\overline{d}}. \tag{44}$$

Thus,

$$\sup_{0\leq s\leq d}\|\psi_1(s)\|^2 \leq \left(2k_1\overline{d} + 1\right)^2\|\varphi\|_{\overline{d}}^2\, e^{2k_1\overline{d}}. \tag{45}$$

It can be seen from (40) that

$$\sup_{0\leq s\leq d}\|\psi_2(s)\|^2 \leq k_2^2\left[\left(2k_1\overline{d} + 1\right)e^{k_1\overline{d}} + 2\right]^2\|\varphi\|_{\overline{d}}^2, \tag{46}$$

where $k_2 = \max_{p\in\mathcal{S}}\{\delta_p\max_{i,j\in\mathcal{T}}\|\widehat{A}_{pij3}\|, \delta_p\max_{i\in\mathcal{T}}\|A_{d,pi3}\|, \delta_p\max_{i\in\mathcal{T}}\|A_{d,pi4}\|\}$. Hence,

$$\begin{aligned}\sup_{0\leq s\leq d}\|\psi(s)\|^2 &\leq \sup_{0\leq s\leq d}\|\psi_1(s)\|^2 + \sup_{0\leq s\leq d}\|\psi_2(s)\|^2\\ &\leq k_3\|\varphi\|_{\overline{d}}^2,\end{aligned} \tag{47}$$

where $k_3 = (2k_1\overline{d}+1)^2e^{2k_1d} + k_2^2[(2k_1\overline{d}+1)e^{k_1d}+2]^2$. Therefore,

$$\sup_{0\leq s\leq d}\|x(s)\|^2 \leq k_3\|H\|^2\left\|H^{-1}\right\|^2\|\phi\|_{\overline{d}}^2. \tag{48}$$

Note that

$$\begin{aligned}&\int_{-d}^{0}\int_{t+\theta}^{t} x^T(s)Qx(s)\,\mathrm{d}s\,\mathrm{d}\theta \leq \overline{d}\int_{t-d}^{t} x^T(s)Qx(s)\,\mathrm{d}s,\\ &\int_{-d}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)E^TZE\dot{x}(s)\,\mathrm{d}s\,\mathrm{d}\theta\\ &\quad \leq \overline{d}\int_{t-kd}^{t} \dot{x}^T(s)E^TZE\dot{x}(s)\,\mathrm{d}s.\end{aligned} \tag{49}$$

Then, from (48) and (28), it can be obtained that there exists a scalar ρ such that

$$V(x_d, r_d, d) \leq \rho\|\phi\|_{\overline{d}}^2. \tag{50}$$

This together with (36) and (48) implies that there exists a scalar ν such that

$$\begin{aligned}\mathcal{E}\left\{\int_0^t\|x(s)\|^2\mathrm{d}s\right\} &= \mathcal{E}\left\{\int_0^d\|x(s)\|^2\mathrm{d}s\right\}\\ &\quad + \mathcal{E}\left\{\int_d^t\|x(s)\|^2\mathrm{d}s\right\}\\ &\leq \nu\mathcal{E}\left\{\|\phi\|_{\overline{d}}^2\right\}.\end{aligned} \tag{51}$$

Considering this and Definition 1, system (11) is stochastically stable for any constant delay d satisfying $d \in [0, \overline{d}]$. Therefore, system (11) is stochastically admissible. This completes the proof. □

In the following, a set of sufficient conditions will be developed under which the considered system is guaranteed to be stochastically admissible with an H_∞ performance.

Definition 9. System (11) is said to be stochastically admissible with an H_∞ performance γ, if it is stochastically admissible when $w(t) = 0$, and under zero initial condition, for nonzero $w(t) \in L_2^{\nu}[0, \infty)$,

$$\mathcal{E}\left\{\int_0^\infty z^T(t)z(t)\,\mathrm{d}t\right\} \leq \gamma^2\int_0^\infty w^T(t)w(t)\,\mathrm{d}t. \tag{52}$$

The following result can be presented.

Theorem 10. *For a prescribed scalar* $\overline{d} > 0$, *there exists a state feedback controller (10) with* $u_p(t) = \sum_{i=1}^{k} \lambda_i L_{pi} Y_p^{-1} x(t)$ *such that system (11) is stochastically admissible with an* H_∞ *performance* γ *for any constant time delay* d *satisfying* $d \in [0, \overline{d}]$, *if there exist matrices* $\overline{P}_p > 0$, $\overline{Q}_p > 0$, $\overline{Q} > 0$, $\overline{Z} > 0$, L_{pi}, $\overline{S}_p$, Y_{p2}, *and* Y_{p3}, $i \in \mathcal{T}$, $p \in \mathcal{S}$, *such that* **??** *and*

$$\begin{bmatrix} \Xi_{pi1} & * \\ \Xi_{pi2} & \Xi_{p3} \end{bmatrix} < 0, \qquad (53)$$

where

$$\Xi_{pi1} = \begin{bmatrix} \Gamma_{1pi} & * & * & * & * & * \\ L_{pi} + Y_{p3}^T B_{pi}^T + Y_{p2} & -Y_{p3}^T - Y_{p3} & * & * & * & * \\ \begin{pmatrix} Y_p^T A_{d,pi}^T + EY_p \\ +Y_p^T E^T - \overline{Z} \end{pmatrix} & 0 & \begin{pmatrix} -Y_p^T - Y_p + \overline{Q}_p \\ -Y_p^T E^T - EY_p + \overline{Z} \end{pmatrix} & * & * & * \\ \overline{d} A_{pi} Y_p - \overline{d} B_{pi} Y_{p2} & \overline{d} B_{pi} Y_{p3} & \overline{d} A_{d,pi} Y_p & -\overline{Z} & * & * \\ B_{w,pi}^T & 0 & 0 & 0 & -\gamma^2 I & * \\ C_{pi} Y_p - D_{pi} Y_{p2} & D_{pi} Y_{p3} & C_{d,pi} Y_p & 0 & C_{w,pi} & -I \end{bmatrix},$$

$$\Xi_{p2} = \begin{bmatrix} Y_p & 0 & 0 & 0 & 0 & 0 \\ \overline{d} Y_p & 0 & 0 & 0 & 0 & 0 \\ \left[I_r \ \ 0\right] H^{-1} M_p^T & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Xi_{p3} = \operatorname{diag}\left\{-\overline{Q}_p, -\overline{d}\overline{Q}, -J_p\right\}, \qquad (54)$$

and the other notations are the same as in Theorem 8.

Proof. From Theorem 8 when $w(t) = 0$ system (11) is stochastically admissible. Let

$$J_{zw}(t) = \mathscr{E}\left\{\int_0^t \left[z^T(s) z(s) - \gamma^2 w^T(s) w(s)\right] ds\right\}. \qquad (55)$$

Under zero initial condition, it is easy to see that

$$\begin{aligned} J_{zw}(t) &\leq \mathscr{E}\left\{\int_0^t \left[z^T(s) z(s) - \gamma^2 w^T(s) w(s) \right.\right. \\ &\quad \left.\left. + \mathscr{L}V(x_s, p, s)\right] ds\right\} \qquad (56) \\ &\leq \mathscr{E}\left\{\int_0^t \sum_{i=1}^{k} \lambda_i \left[\varsigma^T(s) \left(\Omega_{pi} + \Theta_{pi}^T \Theta_{pi}\right) \varsigma(s)\right] ds\right\}, \end{aligned}$$

where

$$\varsigma^T(t) = \left[\tilde{x}^T(t) \ \ x^T(t-d) \ \ w^T(t)\right],$$

$$\Omega_{pi} = \begin{bmatrix} \Upsilon_{1pi} & * & * \\ \Upsilon_{2pi} & \Upsilon_{3pi} & * \\ \left[B_{w,pi}^T \ \ 0\right] X_p & 0 & -\gamma^2 I \end{bmatrix}, \qquad (57)$$

$$\Theta_{pi} = \left[\widetilde{C}_{pi} \ \ C_{d,pi} \ \ C_{w,pi}\right],$$

and notations of Υ_{1pi}, Υ_{2pi}, and Υ_{3pi} are the same as in Theorem 8. Hence, by Schur complement lemma and using the similar method in the proof of Theorem 8, from **??** and (53), it can be obtained that $J_{zw}(t) < 0$ for all $t > 0$. Therefore, for any nonzero $w(t) \in L_2^v[0, \infty)$, (52) holds. Hence, according to Definition 9, the system is stochastically admissible with an H_∞ performance γ. This completes the proof. □

Remark 11. Compared with methods in [21, 22], because of the method of the augmented matrix adopted in Theorems 8 and 10, the number of LMIs needed to solve is relatively small in this paper. When the value of k is relatively large, the quality of the computation is greatly reduced. some new relaxation matrices added will reduce the conservatism of control conditions compared with previous literatures, which can be seen from Example 2.

4. The Design of the Static Output Feedback Controller

When $r_t = p \in \mathcal{S}$, consider the overall SMJFS as follows:

$$\begin{aligned} E\dot{x}(t) &= \sum_{i=1}^{k} \lambda_i \left(A_{pi} x(t) + A_{d,pi} x(t-d) + B_{pi} u(t) \right. \\ &\quad \left. + B_{w,pi} w(t)\right), \\ y(t) &= \sum_{i=1}^{k} \lambda_i C_{y,pi} x(t), \end{aligned}$$

$$
\begin{aligned}
z(t) &= \sum_{i=1}^{k}\lambda_i \big(C_{pi}x(t) + C_{d,pi}x(t-d) + D_{pi}u(t) \\
&\quad + C_{w,pi}w(t)\big), \\
x(t) &= \phi(t), \\
&\qquad \forall t \in [-d,0],\ i \in \mathcal{T} \triangleq \{1,2,\dots,k\},
\end{aligned}
\tag{58}
$$

where $y(t) \in \mathbb{R}^{p_1}$ is the system output, $C_{d,pi}$ $(i \in \mathcal{S})$ are known constant matrices with appropriate dimensions, and the other notations are the same as in (3).

The following static output feedback controller will be considered here:

$$
u_p(t) = \sum_{i=1}^{k}\lambda_i K_{pi} y(t), \tag{59}
$$

where K_{pi} $(p \in \mathcal{S}, i \in \mathcal{T})$ are local controller gains, such that the closed-loop system is

$$
\begin{aligned}
E\dot{x}(t) &= \sum_{i=1}^{k}\sum_{j=1}^{k}\sum_{s=1}^{k}\lambda_i\lambda_j\lambda_s \Big(\big(A_{pi} + B_{pi}K_{pj}C_{y,ps}\big)x(t) + A_{d,pi}x(t-d) + B_{w,pi}w(t)\Big), \\
z(t) &= \sum_{i=1}^{k}\sum_{j=1}^{k}\sum_{s=1}^{k}\lambda_i\lambda_j\lambda_s \Big(\big(C_{pi} + D_{pi}K_{pj}C_{y,ps}\big)x(t) + C_{d,pi}x(t-d) + C_{w,pi}w(t)\Big).
\end{aligned}
\tag{60}
$$

It is difficult to drive LMI-based conditions of the stochastic stabilization by employing the static output feedback control approach due to the appearance of crossing terms between system matrices and control gains. And system (60) can be rewritten in the following form:

$$
\begin{aligned}
\widehat{E}\widehat{x}(t) &= \sum_{i=1}^{k}\sum_{j=1}^{k}\lambda_i\lambda_j \big(\Lambda_{p,ij}\widehat{x}(t) + \widehat{B}_{w,pi}w(t)\big), \\
z(t) &= \sum_{i=1}^{k}\sum_{j=1}^{k}\lambda_i\lambda_j \big(\widehat{C}_{p,ij}\widehat{x}(t) + C_{w,pi}w(t)\big),
\end{aligned}
\tag{61}
$$

where

$$
\begin{aligned}
\widehat{E} &= \begin{bmatrix} E & 0 \\ 0 & 0 \end{bmatrix} \in \mathbb{R}^{(n+p_1)\times(n+p_1)}, \\
\widehat{C}_{p,ij} &= \begin{bmatrix} C_{pi} & D_{pi}K_{pj} \end{bmatrix}, \\
\widehat{A}_{d,pi} &= \begin{bmatrix} A_{d,pi} & 0 \\ 0 & 0 \end{bmatrix} \in \mathbb{R}^{(n+p_1)\times(n+p_1)}, \\
\widehat{x}(t) &= \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}, \\
\widehat{B}_{w,pi} &= \begin{bmatrix} B_{w,pi} \\ 0 \end{bmatrix} \in \mathbb{R}^{(n+p_1)\times v}, \\
\Lambda_{p,ij} &= \begin{bmatrix} A_{pi} & B_{pi}K_{pj} \\ C_{y,pi} & -I \end{bmatrix}.
\end{aligned}
\tag{62}
$$

Remark 12. For systems (60) and (61), it can be seen that

$$
\begin{aligned}
&\det\left(sE - \sum_{i=1}^{k}\sum_{j=1}^{k}\sum_{s=1}^{k}\lambda_i\lambda_j\lambda_s \big(A_{pi} + B_{pi}K_{pj}C_{y,ps}\big)\right) \\
&\quad = \det\left(s\widehat{E} - \sum_{i=1}^{k}\sum_{j=1}^{k}\lambda_i\lambda_j\Lambda_{p,ij}\right), \\
&\det\left(sE \vphantom{\sum}\right. \\
&\quad \left. - \sum_{i=1}^{k}\sum_{j=1}^{k}\sum_{s=1}^{k}\lambda_i\lambda_j\lambda_s \big(A_{pi} + B_{pi}K_{pj}C_{y,ps} + A_{d,pi}\big)\right) \\
&\quad = \det\left(s\widehat{E} - \sum_{i=1}^{k}\sum_{j=1}^{k}\lambda_i\lambda_j \big(\Lambda_{p,ij} + \widehat{A}_{d,pi}\big)\right).
\end{aligned}
\tag{63}
$$

As the discussion in Remark 7, the stochastic admissibility of system (60) can be studied by means of system (61).

Theorem 13. *There exists an output feedback controller (59) with controller gains $K_{pi} = L_{pi}Y_{p2}^{-1}$ $(p \in \mathcal{S}, i \in \mathcal{T})$ such that system (60) with $w(t) = 0$ is stochastically admissible, if there exist matrices $\overline{P}_p > 0$, $\overline{Q}_p > 0$, $\overline{Q} > 0$, $\overline{Z} > 0$, L_{pi}, $\overline{S}_p$, and Y_{p2}, $p \in \mathcal{S}$, $1 \le i \ne j \le k$, such that* **??** *and*

$$
\begin{aligned}
&\Theta_{p,ii} < 0, \\
&\frac{1}{k-1}\Theta_{p,ii} + \frac{1}{2}\big(\Theta_{p,ij} + \Theta_{p,ji}\big) < 0,
\end{aligned}
\tag{64}
$$

where

$$\Theta_{p,ij} = \begin{bmatrix} \widehat{\Gamma}_{p,ij} & * & * & * & * & * & * \\ L_{pj}^T B_{pi}^T + C_{y,pi} Y_p & -Y_{p2}^T - Y_{p2} & * & * & * & * & * \\ \begin{pmatrix} Y_p^T A_{d,pi}^T + EY_p \\ Y_p^T E^T - \overline{Z} \end{pmatrix} & 0 & \Sigma_p & * & * & * & * \\ \overline{d} A_{pi} Y_p & \overline{d} B_{pi} L_{pj} & \overline{d} A_{d,pi} Y_p & -\overline{Z} & * & * & * \\ Y_p & 0 & 0 & 0 & -\overline{Q}_p & * & * \\ \overline{d} Y_p & 0 & 0 & 0 & 0 & -\overline{d}\overline{Q} & * \\ [I_r \;\; 0] H^{-1} M_p^T & 0 & 0 & 0 & 0 & 0 & -J_p \end{bmatrix}, \quad (65)$$

$\widehat{\Gamma}_{p,ij} = \pi_{pp} Y_p^T E^T + A_{pi} Y_p + Y_p^T A_{pi}^T - Y_p^T E^T - EY_p + \overline{Z}$, $\Sigma_p = -Y_p^T - Y_p + \overline{Q}_p - Y_p^T E^T - EY_p + \overline{Z}$, $L_{pi} = K_{pi} Y_{p2}$, $Y_p = (E\overline{P}_p + \overline{S}_p \overline{R}^T)^T$, $\overline{R} \in \mathbb{R}^{n \times (n-r)}$ is any matrix with full column rank and satisfies $E\overline{R} = 0$, G, H are nonsingular matrices that make $GEH = \left[\begin{smallmatrix} I_r & 0 \\ 0 & 0 \end{smallmatrix}\right]$, and the other notations are the same as in Theorem 8.

Proof. Let $\widetilde{Y}_p = \left[\begin{smallmatrix} Y_p & 0 \\ 0 & Y_{p2} \end{smallmatrix}\right]$. Using Lemma 6, the proof process is similar to Theorem 8. ☐

Theorem 14. *For a prescribed scalar $\overline{d} > 0$, there exists an output feedback controller (59) with controller gains $K_{pi} = L_{pi} Y_{p2}^{-1}$ $(p \in \mathcal{S}, i \in \mathcal{T})$ such that system (60) is stochastically admissible with an H_∞ performance γ for any constant time delay d satisfying $d \in [0, \overline{d}]$, if there exist matrices $\overline{P}_p > 0$, $\overline{Q}_p > 0$, $\overline{Q} > 0$, $\overline{Z} > 0$, L_{pi}, $\overline{S}_p$, and Y_{p2}, $p \in \mathcal{S}$, $1 \le i \ne j \le k$, such that* **??** *and*

$$\begin{gathered} \Delta_{p,ii} < 0, \\ \frac{1}{k-1} \Delta_{p,ii} + \frac{1}{2} \left(\Delta_{p,ij} + \Delta_{p,ji} \right) < 0, \end{gathered} \quad (66)$$

where

$$\Delta_{p,ij} = \begin{bmatrix} \widehat{\Gamma}_{p,ij} & * & * & * & * & * & * & * & * \\ L_p^T B_{pi}^T + C_{y,pi} Y_p & -Y_{p2}^T - Y_{p2} & * & * & * & * & * & * & * \\ \begin{pmatrix} Y_p^T A_{d,pi}^T + EY_p \\ +Y_p^T E^T - \overline{Z} \end{pmatrix} & 0 & \Sigma_p & * & * & * & * & * & * \\ \overline{d} A_{pi} Y_p & \overline{d} B_{pi} L_{pj} & \overline{d} A_{d,pi} Y_p & -\overline{Z} & * & * & * & * & * \\ B_{w,pi}^T & 0 & 0 & 0 & -\gamma^2 I & * & * & * & * \\ C_{pi} Y_p & D_{pi} L_{pj} & C_{d,pi} Y_p & 0 & C_{w,pi} & -I & * & * & * \\ Y_p & 0 & 0 & 0 & 0 & 0 & -\overline{Q}_p & * & * \\ \overline{d} Y_p & 0 & 0 & 0 & 0 & 0 & 0 & -\overline{d}\overline{Q} & * \\ [I_r \;\; 0] H^{-1} M_p^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -J_p \end{bmatrix}, \quad (67)$$

and the other notations are the same as in Theorem 13.

Remark 15. Compared with the method in [31, 32], because of the augmented matrix adopted in Theorems 13 and 14, the number of LMIs needed to solve is greatly decreased. When the value of k is relatively large, the computational complexity will be reduced. On the other hand, by the augmented matrix, there are not any crossing terms between system matrices and controller gains, so assumptions for the output matrix [23], the equality constraint for the output matrix [24], and the bounding technique for crossing terms are not necessary here; therefore, the conservatism brought by them will not happen.

5. Numerical Examples

Two examples will be given to illustrate the validity of developed methods.

Example 1. To illustrate the H_∞ controller synthesis, the following nonlinear time delay system is considered:

$$\begin{aligned}(1+a\cos\theta(t))\ddot{\theta}(t)&=-b\dot{\theta}^3(t)+c\theta(t)+c_d(t-d)\\&\quad+\delta(r_t)eu(t)+fw(t).\end{aligned}\tag{68}$$

The range of $\dot{\theta}(t)$ is assumed to satisfy $|\dot{\theta}(t)|<\psi$, $\psi=2$, $a=b=e=f=1$, $c=-1$, $c_d=0.8$, $d\in[0,\overline{d}]$, $\overline{d}=0.3$, and $u(t)$ is the control input. $w(t)=\cos(0.5t)e^{-0.01t}$ is the disturbance input. r_t is a Markovian process taking values in a finite set $\{1,2,3\}$, $\delta(1)=1$, $\delta(2)=0.8$, $\delta(3)=0.5$, and the output vector $z(t)=\theta(t)$.

Choose the vector $x(t)=[x_1(t)\ \ x_2(t)\ \ x_3(t)]^T$ with $x_1(t)=\theta(t)$, $x_2(t)=\dot{\theta}(t)$, and $x_3(t)=\ddot{\theta}(t)$. Then, the system is described by

$$\begin{aligned}&\begin{bmatrix}1&0&0\\0&1&0\\0&0&0\end{bmatrix}\dot{x}(t)\\&=\begin{bmatrix}0&1&0\\0&0&1\\c&-bx_2^2(t)&-1-a\cos x_1(t)\end{bmatrix}x(t)\\&\quad+\begin{bmatrix}0&0&0\\0&0&0\\c_d&0&0\end{bmatrix}x(t-d)+\begin{bmatrix}0\\0\\\delta(r_t)e\end{bmatrix}u(t)\\&\quad+\begin{bmatrix}0\\0\\f\end{bmatrix}w(t).\end{aligned}\tag{69}$$

It can be expressed exactly by the following fuzzy singular Markovian jump form:

$$\begin{aligned}E\dot{x}(t)&=\sum_{i=1}^{3}\lambda_i\left(A_{pi}x(t)+A_{d,pi}x(t-d)+B_{pi}u(t)\right.\\&\quad\left.+B_{w,pi}w(t)\right),\\z(t)&=\sum_{i=1}^{3}\lambda_iC_{pi}x(t),\\x(t)&=\phi(t),\\&\qquad t\in[-\overline{d},0],\ p\in\{1,2,3\},\end{aligned}\tag{70}$$

where

$$\begin{aligned}E&=\begin{bmatrix}1&0&0\\0&1&0\\0&0&0\end{bmatrix},\\A_{p1}&=\begin{bmatrix}0&1&0\\0&0&1\\c&-b\left(\psi^2+2\right)&a-1\end{bmatrix},\\A_{p2}&=\begin{bmatrix}0&1&0\\0&0&1\\c&0&-a-1-a\psi^2\end{bmatrix},\\A_{p3}&=\begin{bmatrix}0&1&0\\0&0&1\\c&0&a-1\end{bmatrix},\\A_{d,p1}&=A_{d,p2}=A_{d,p3}=\begin{bmatrix}0&0&0\\0&0&0\\c_d&0&0\end{bmatrix},\\B_{11}&=B_{12}=B_{13}=\begin{bmatrix}0\\0\\e\end{bmatrix},\\B_{21}&=B_{22}=B_{23}=\begin{bmatrix}0\\0\\0.8e\end{bmatrix},\\B_{31}&=B_{32}=B_{33}=\begin{bmatrix}0\\0\\0.5e\end{bmatrix},\\B_{w,p1}&=B_{w,p2}=B_{w,p3}=\begin{bmatrix}0\\0\\f\end{bmatrix},\\C_{p1}&=C_{p2}=C_{p3}=\begin{bmatrix}1&0&0\end{bmatrix},\\\lambda_1&=\frac{x_2^2(t)}{\psi^2+2},\\\lambda_2&=\frac{1+\cos x_1(t)}{\psi^2+2},\\\lambda_3&=\frac{\psi^2-x_2^2(t)+1-\cos x_1(t)}{\psi^2+2}.\end{aligned}\tag{71}$$

It is seen that $0\le\lambda_i\le1$, $\sum_{i=1}^{3}\lambda_i=1$. Let $\Pi=\begin{bmatrix}-0.2&0.2&0\\0.1&-0.3&0.2\\0.2&0.3&-0.5\end{bmatrix}$, $\gamma=1$; by solving **??** and (53) in Theorem 10, controller gains are given by

$$K_{11}=\begin{bmatrix}-14.2939&-14.0620&-3.6426\end{bmatrix},$$

$$K_{12}=\begin{bmatrix}-14.2911&-14.4082&-3.2860\end{bmatrix},$$

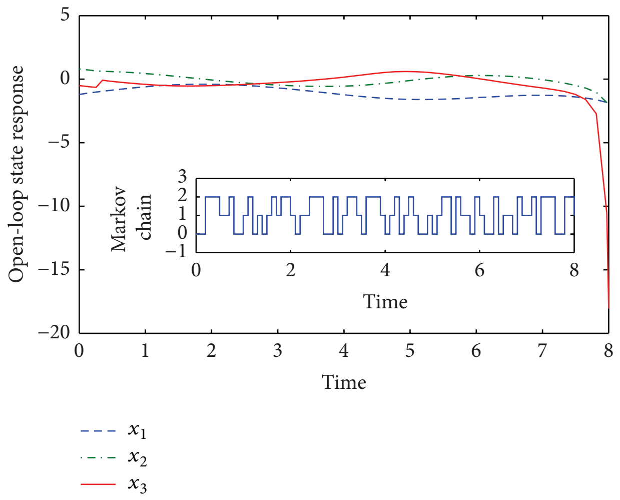

FIGURE 1: State responses of the open-loop system.

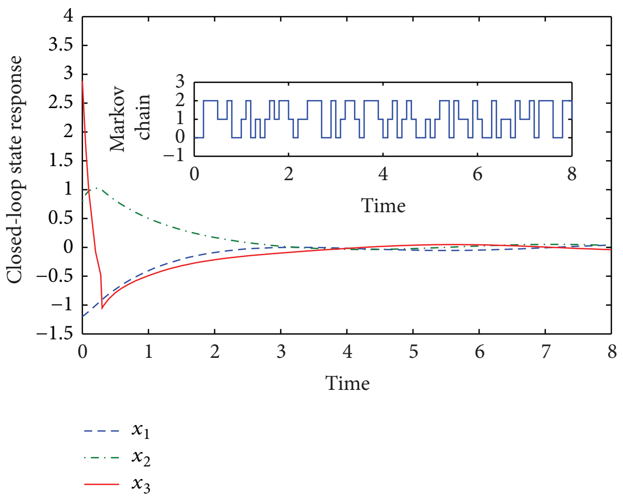

FIGURE 2: State responses of the closed-loop system.

$$
\begin{aligned}
K_{13} &= \begin{bmatrix} -14.2988 & -14.4111 & -3.6425 \end{bmatrix}, \\
K_{21} &= \begin{bmatrix} -17.9535 & -17.7012 & -4.4471 \end{bmatrix}, \\
K_{22} &= \begin{bmatrix} -17.9509 & -17.9806 & -4.1585 \end{bmatrix}, \\
K_{23} &= \begin{bmatrix} -17.9578 & -17.9828 & -4.4470 \end{bmatrix}, \\
K_{31} &= \begin{bmatrix} -28.6360 & -28.3641 & -6.9941 \end{bmatrix}, \\
K_{32} &= \begin{bmatrix} -28.6344 & -28.5420 & -6.8118 \end{bmatrix}, \\
K_{33} &= \begin{bmatrix} -28.6377 & -28.5436 & -6.9941 \end{bmatrix}.
\end{aligned}
\tag{72}
$$

To demonstrate the effectiveness, assuming the initial condition $\phi(t) = \begin{bmatrix} -1.2 & 0.8 & -0.5 \end{bmatrix}^T$, Figures 1 and 2 show state responses of the open-loop system and the closed-loop system controlled by (10), respectively. From Figure 1, it can be seen that the open-loop system is not stochastically admissible, and Figure 2 shows that when the controller obtained by Theorem 10 is exerted to this system it is stochastically admissible.

Example 2. Consider the example without uncertainties in [6].

Mode 1: $A_1 = \begin{bmatrix} 1.5 & 1.4 \\ -3.5 & -4.5 \end{bmatrix}$, $A_{d1} = \begin{bmatrix} 0.2 & a \\ -0.24 & -0.4 \end{bmatrix}$, $B_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $B_{w1} = \begin{bmatrix} 1.5 \\ 1.4 \end{bmatrix}$, $C_1 = \begin{bmatrix} 0.5 & 1 \end{bmatrix}$, $C_{d1} = \begin{bmatrix} -0.2 & 0.2 \end{bmatrix}$, $D_1 = 0.2$, and $C_{w1} = 0.2$.

Mode 2: $A_2 = \begin{bmatrix} 1.7 & 1.5 \\ -1.3 & -2.5 \end{bmatrix}$, $A_{d2} = \begin{bmatrix} b & 1.1 \\ -0.21 & -0.3 \end{bmatrix}$, $B_2 = \begin{bmatrix} 0.9 \\ 0.9 \end{bmatrix}$, $B_{w2} = \begin{bmatrix} 1.4 \\ 1.5 \end{bmatrix}$, $C_2 = \begin{bmatrix} 0.4 & 0.3 \end{bmatrix}$, $C_{d2} = \begin{bmatrix} -0.1 & 0.2 \end{bmatrix}$, $D_2 = 0.3$, and $C_{w2} = 0.3$.

$\Pi = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$, $E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $d = 0.3$, $\gamma = 2.6$, and in [6] $a = -0.5$, $b = 2.1$, but in this paper $-2.4 \le a \le 2$, $-2 \le b \le 4.8$ are taken.

In Figure 3, "o" represents the range of the feasible solutions using Theorem 10 in this paper, and "$*$" represents the range of the feasible solutions using Theorem 3 in [6].

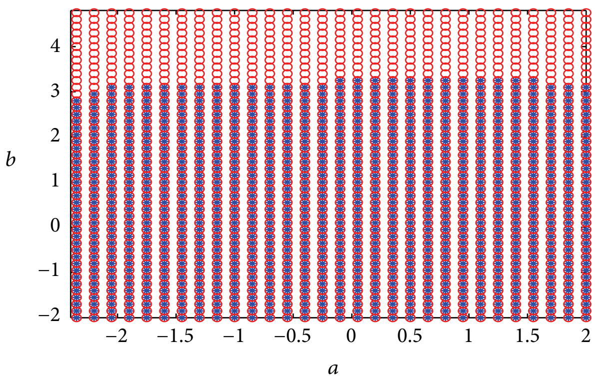

FIGURE 3: Comparison of the feasible regions.

This illustrates that the method obtained in this paper has less conservatism.

6. Conclusions

In this paper, the problem of mode-dependent H_∞ control for singular Markovian jump fuzzy systems with time delay is considered. This class of systems under consideration is described by T-S fuzzy models. The main contribution of this paper is to design state feedback controllers and static output feedback controllers which can guarantee that resulting closed-loop systems are stochastically admissible with an H_∞ performance γ by the method of the augmented matrix. Finally, two examples are given to demonstrate the effectiveness of main results obtained here.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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