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Research Article

Passivity Analysis of Complex Delayed Dynamical Networks with Output Coupling

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A new complex dynamical network model with output coupling is proposed. This paper is concerned with input passivity and output passivity of the proposed network model. By constructing new Lyapunov functionals, some sufficient conditions ensuring the input passivity and output passivity are obtained. Finally, two numerical examples are provided to demonstrate the effectiveness of the proposed results.

1. Introduction

Recently, there has been increasing interest in the study of complex dynamical networks. The main reason is that many practical systems can be characterized by various models of complex networks. It is well known that one of the most significant and interesting dynamical phenomena of complex networks is the synchronization of systems. Many interesting results on synchronization have been derived for various complex networks [1–10]. But, it should be noticed that the complex networks with state coupling were considered in these papers.

To our knowledge, Jiang et al. [11] first introduced a complex network model with output coupling without time delays. Some conditions for synchronization were established based on the Lyapunov stability theory. However, time delays always exist in complex networks due to the finite speeds of transmission and/or the traffic congestion, and most of delays are notable. So it is crucial for us to take the delay into the consideration when we study complex networks. Practically, many phenomena in nature can be modeled as complex networks with output coupling. The cooperative control problem of multiple agents has received much attention recently since it has challenging features and many applications, for example, large object moving, formation control, rescue mission, and satellite clustering. It is well known that the state of agent is difficult to be observed or measured because of technology limitations and environmental disturbances. For instance, the measuring of velocity is more difficult than that of position. In some circumstances, the information of velocity is unavailable for agents [12]. Therefore, it is quite necessary to design protocols based on the output variables. In this case, the closed-loop systems can be described by the complex networks with output coupling. Hence, study of complex networks with output coupling is very interesting and important in both theory and application. A complex delayed dynamical network model with output coupling was proposed in [13, 14]. Wang and Wu [13] investigated the output synchronization of the proposed network model, and some criteria on local and global exponential output synchronization were derived.

Passivity [15–33] is an important concept of system theory and provides a nice tool for analyzing the stability of systems and has found applications in diverse areas such as stability, complexity, signal processing, chaos control and synchronization, and fuzzy control. Many researchers have studied the passivity of fuzzy systems [19–22] and neural networks [23–28]. Liang et al. [20] discussed the passivity and passification problems for a class of uncertain stochastic fuzzy systems with time-varying delays. In [26] Song et al.

investigated the passivity for a class of discrete-time stochastic neural networks with time-varying delays, and a delay-dependent passivity condition was obtained by constructing proper Lyapunov-Krasovskii functional. However, there are few work on the passivity of complex networks [29, 30, 32, 33]. In [29, 30], Yao et al. obtained some sufficient conditions on passivity properties for linear (or linearized) complex networks with and without coupling delays (constant delay). However, in practical evolutionary processes of the networks, absolute constant delay may be scarce and delays are frequently varied with time. Therefore, it is important to further study the passivity of complex networks with time-varying delays. Wang et al. [32] considered input passivity and output passivity for a generalized complex network with nonlinear, time-varying, nonsymmetric, and delayed coupling. By constructing some suitable Lyapunov functionals, several sufficient conditions ensuring input passivity and output passivity were derived. To the best of our knowledge, the input passivity and output passivity of complex delayed dynamical network model with output coupling have not yet been established. Therefore, it is interesting to study the input passivity and output passivity of complex delayed dynamical network model with output coupling.

Motivated by the above discussions, we propose a new complex delayed dynamical network model with output coupling. The objective of this paper is to study the input and output passivity of the proposed network model. Some sufficient conditions ensuring input passivity and output passivity are obtained by Lyapunov functional method.

The rest of this paper is organized as follows. A new complex network model is introduced and some useful preliminaries are given in Section 2. Several input and output passivity criteria are established in Section 3. In Section 4, two numerical examples are given to illustrate the effectiveness of the proposed results. Conclusions are finally given in Section 5.

2. Network Model and Preliminaries

Let R^n be the n-dimensional Euclidean space, and let $R^{n\times m}$ be the space of $n \times m$ real matrices. $P \geqslant 0$ ($P \leqslant 0$) means that matrix P is real symmetric and semipositive (seminegative) definite. $P > 0$ ($P < 0$) means that matrix P is real symmetric and positive (negative) definite. I_n denotes the $n\times n$ identity matrix. B^T denotes the transpose of a square matrix B. $C([-\tau, 0], R^n)$ is a Banach space of continuous functions mapping the interval $[-\tau, 0]$ into R^n with the norm $\|\phi\|_\tau = \sup_{-\tau\leqslant\theta\leqslant 0}\|\phi(\theta)\|$, where $\|\cdot\|$ is the Euclidean norm.

In this paper, we consider a complex delayed dynamical network consisting of N identical nodes with diffusive and output coupling. The mathematical model of the coupled network can be described as follows:

$$\begin{aligned}\dot{x}_i(t) &= f(x_i(t)) + \frac{a}{k_i^{\beta_\omega}}\sum_{j=1}^{N} L_{ij}\Gamma y_j(t-\tau(t)) + B_i u_i(t),\\ y_i(t) &= C_i x_i(t) + D_i u_i(t),\end{aligned} \tag{1}$$

where $i = 1, 2, \ldots, N$. $\tau(t)$ is the time-varying delay with $0 \leqslant \tau(t) \leqslant \tau$.

The function $f(\cdot)$, describing the local dynamics of the nodes, is continuously differentiable and capable of producing various rich dynamical behaviors, $x_i(t) = (x_{i1}(t), x_{i2}(t), \ldots, x_{in}(t))^T \in R^n$ is the state variable of node i, $y_i(t) \in R^n$ is the output of node i, $u_i(t) \in R^n$ is the input vector of node i, B_i, C_i, and D_i are known matrices with appropriate dimensions, $\Gamma \in R^{n\times n}$ is inner-coupling matrix, which describes the individual coupling between two connected nodes of the network, $a > 0$ represents the overall coupling strength, k_i is the degree of node i, β_ω is a tunable weight parameter, and the real matrix $L = (L_{ij})_{N\times N}$ is a symmetric matrix with diagonal entries $L_{ii} = -k_i$ and off-diagonal entries $L_{ij} = 1$ if node i and node j are connected by a link, and $L_{ij} = 0$ otherwise.

In this paper, we always assume that complex network (1) is connected. Let $x(t) = (x_1^T(t), x_2^T(t), \ldots, x_N^T(t))^T$, $y(t) = (y_1^T(t), y_2^T(t), \ldots, y_N^T(t))^T$, and $u(t) = (u_1^T(t), u_2^T(t), \cdots, u_N^T(t))^T$. The initial condition associated with the complex network (1) is given in the form

$$\begin{gathered} x(0) = x_0, \quad x_0 = \left(x_{10}^T, x_{20}^T, \ldots, x_{N0}^T\right)^T, \quad x_{i0} \in R^n,\\ y(s) = \Phi(s), \quad s \in [-\tau, 0], \quad \phi_i \in C([-\tau, 0], R^n),\\ \Phi(s) = \left(\phi_1^T(s), \phi_2^T(s), \ldots, \phi_N^T(s)\right)^T, \quad i = 1, 2, \ldots, N.\end{gathered} \tag{2}$$

Next, we give several useful definitions.

Definition 1 (see [32]). Complex network (1) is called input passive if there exist two constants $\gamma > 0$ and $\beta \in R$ such that

$$2\int_0^{t_p} y^T(s)u(s)\,ds \geqslant -\beta^2 + \gamma\int_0^{t_p} u^T(s)u(s)\,ds, \tag{3}$$

for all $t_p \geqslant 0$.

Definition 2 (see [32]). Complex network (1) is called output passive if there exist two constants $\gamma > 0$ and $\beta \in R$ such that

$$2\int_0^{t_p} y^T(s)u(s)\,ds \geqslant -\beta^2 + \gamma\int_0^{t_p} y^T(s)y(s)\,ds, \tag{4}$$

for all $t_p \geqslant 0$.

Definition 3 (see [34]). Let $A = (a_{ij})_{m\times n} \in R^{m\times n}$, and let $B = (b_{ij})_{p\times q} \in R^{p\times q}$. Then the Kronecker product (or tensor product) of A and B is defined as the matrix

$$A \otimes B = \begin{bmatrix} a_{11}B & a_{12}B & \cdots & a_{1n}B\\ a_{21}B & a_{22}B & \cdots & a_{2n}B\\ \vdots & \vdots & \cdots & \vdots\\ a_{m1}B & a_{m2}B & \cdots & a_{mn}B\end{bmatrix} \in R^{mp\times nq}. \tag{5}$$

The Kronecker product has the following properties:

$$\begin{aligned}&(1)\ (A\otimes B)^T = A^T\otimes B^T;\\&(2)\ (\alpha A)\otimes B = A\otimes(\alpha B);\\&(3)\ (A+B)\otimes C = A\otimes C + B\otimes C;\\&(4)\ (A\otimes B)(C\otimes D) = (AC)\otimes(BD),\end{aligned}\tag{6}$$

where $\alpha\in R$, C, and D are matrices with suitable dimensions.

3. Main Results

In this section, we shall investigate the input passivity and output passivity of the complex delayed dynamical networks with output coupling.

In [32, 35], authors make the assumption that the function $f(\cdot)$ is in the QUAD function class. In this paper, we make similar assumptions.

(A1) There exist a positive definite diagonal matrix $P = \operatorname{diag}(p_1, p_2, \ldots, p_n)$ and a diagonal matrix $\Delta = \operatorname{diag}(\delta_1, \delta_2, \ldots, \delta_n)$ such that f satisfies the following inequality:

$$x^T P\left(f(x) - \Delta x\right) \leqslant -\eta x^T x, \tag{7}$$

for some $\eta > 0$ and all $x \in R^n$.

For the convenience, we denote

$$\begin{gathered}\widehat{P} = \operatorname{diag}(P, P, \ldots, P), \qquad \widehat{\Delta} = \operatorname{diag}(\Delta, \Delta, \ldots, \Delta),\\ B = \operatorname{diag}(B_1, B_2, \ldots, B_N), \qquad C = \operatorname{diag}(C_1, C_2, \ldots, C_N),\\ D = \operatorname{diag}(D_1, D_2, \ldots, D_N).\end{gathered}\tag{8}$$

In the following, we first give two input passivity criteria.

Theorem 4. *Let (A1) hold, and let $\dot{\tau}(t) \leqslant \sigma < 1$. The complex network (1) is input passive if there exist matrix $Q \geqslant 0$ and a scalar $\gamma > 0$ such that*

$$\begin{pmatrix} W_1 & a\widehat{P}(G\otimes\Gamma) & W_2\\ a(G\otimes\Gamma)^T\widehat{P} & -(1-\sigma)Q & 0\\ W_2^T & 0 & W_3\end{pmatrix} \leqslant 0, \tag{9}$$

where

$$\begin{gathered}W_1 = -2\eta I_{nN} + 2\widehat{P}\widehat{\Delta} + C^T QC,\\ W_2 = \widehat{P}B + C^T QD - C^T,\\ W_3 = -\left(D + D^T - \gamma I_{nN} - D^T QD\right).\end{gathered}\tag{10}$$

Proof. For convenient analysis, we let

$$G_{ij} = \frac{L_{ij}}{k_i^{\beta_\omega}}. \tag{11}$$

Then, complex network (1) can be rewritten as follows:

$$\begin{gathered}\dot{x}_i(t) = f\left(x_i(t)\right) + a\sum_{j=1}^{N} G_{ij}\Gamma y_j(t-\tau(t)) + B_i u_i(t),\\ y_i(t) = C_i x_i(t) + D_i u_i(t),\end{gathered}\tag{12}$$

where $i = 1, 2, \ldots, N$. $G = (G_{ij})_{N\times N}$ is a coupling matrix, accounting for the topology of complex dynamical network. We can rewrite system (12) in a compact form as follows:

$$\begin{gathered}\dot{x}(t) = F(x(t)) + a(G\otimes\Gamma)\,y(t-\tau(t)) + Bu(t),\\ y(t) = Cx(t) + Du(t),\end{gathered}\tag{13}$$

where

$$\begin{aligned}x(t) &= \left(x_1^T(t), x_2^T(t), \ldots, x_N^T(t)\right)^T,\\ u(t) &= \left(u_1^T(t), u_2^T(t), \ldots, u_N^T(t)\right)^T,\\ y(t) &= \left(y_1^T(t), y_2^T(t), \ldots, y_N^T(t)\right)^T,\\ F(x(t)) &= \left(f^T(x_1(t)), f^T(x_2(t)), \ldots, f^T(x_N(t))\right)^T,\\ y(t-\tau(t)) &\\ &= \left(y_1^T(t-\tau(t)), y_2^T(t-\tau(t)), \ldots, y_N^T(t-\tau(t))\right)^T.\end{aligned}\tag{14}$$

Construct Lyapunov functional for system (13) as follows:

$$V(t) = x^T(t)\widehat{P}x(t) + \int_{t-\tau(t)}^{t} y^T(\alpha)Qy(\alpha)\,d\alpha. \tag{15}$$

The derivative of $V(t)$ satisfies

$$\begin{aligned}\dot{V}(t) &= 2x^T(t)\widehat{P}\dot{x}(t) + y^T(t)Qy(t)\\ &\quad - (1-\dot{\tau}(t))\,y^T(t-\tau(t))\,Qy(t-\tau(t))\\ &\leqslant 2x^T(t)\widehat{P}\dot{x}(t) + y^T(t)Qy(t)\\ &\quad - (1-\sigma)\,y^T(t-\tau(t))\,Qy(t-\tau(t))\\ &= 2x^T(t)\widehat{P}F(x(t)) + 2ax^T(t)\widehat{P}(G\otimes\Gamma)\,y(t-\tau(t))\\ &\quad + 2x^T(t)\widehat{P}Bu(t) + y^T(t)Qy(t)\\ &\quad - (1-\sigma)\,y^T(t-\tau(t))\,Qy(t-\tau(t)).\end{aligned}\tag{16}$$

Then, we can get

$$\begin{aligned}&\dot{V}(t) - 2y^T(t)u(t) + \gamma u^T(t)u(t)\\ &\quad\leqslant 2x^T(t)\widehat{P}F(x(t)) + 2ax^T(t)\widehat{P}(G\otimes\Gamma)\,y(t-\tau(t))\end{aligned}$$

$$+\,2x^T(t)\,\widehat{P}Bu(t)+y^T(t)\,Qy(t)$$
$$-\,(1-\sigma)\,y^T(t-\tau(t))\,Qy(t-\tau(t)) \tag{17}$$
$$-\,2x^T(t)\,C^Tu(t)-u^T(t)\left(D+D^T-\gamma I_{nN}\right)u(t).$$

According to (A1), we can obtain

$$\begin{aligned} x^T(t)\,\widehat{P}F(x(t)) &= \sum_{i=1}^{N} x_i^T(t)\,Pf(x_i(t)) \\ &\leqslant \sum_{i=1}^{N}\left[-\eta x_i^T(t)\,x_i(t)+x_i^T(t)\,P\Delta x_i(t)\right] \\ &= x^T(t)\left(-\eta I_{nN}+\widehat{P}\widehat{\Delta}\right)x(t). \end{aligned} \tag{18}$$

It follows from (9) and (18) that

$$\begin{aligned} &\dot{V}(t)-2y^T(t)\,u(t)+\gamma u^T(t)\,u(t) \\ &\quad\leqslant x^T(t)\left(-2\eta I_{nN}+2\widehat{P}\widehat{\Delta}+C^TQC\right)x(t) \\ &\qquad+2ax^T(t)\,\widehat{P}\,(G\otimes\Gamma)\,y(t-\tau(t)) \\ &\qquad+2x^T(t)\left(\widehat{P}B+C^TQD-C^T\right)u(t) \\ &\qquad-(1-\sigma)\,y^T(t-\tau(t))\,Qy(t-\tau(t)) \\ &\qquad-u^T(t)\left(D+D^T-\gamma I_{nN}-D^TQD\right)u(t) \\ &\quad=\xi^T(t)\begin{pmatrix} W_1 & a\widehat{P}(G\otimes\Gamma) & W_2 \\ a(G\otimes\Gamma)^T\widehat{P} & -(1-\sigma)Q & 0 \\ W_2^T & 0 & W_3 \end{pmatrix}\xi(t) \\ &\quad\leqslant 0, \end{aligned} \tag{19}$$

where $\xi(t)=(x^T(t),y^T(t-\tau(t)),u^T(t))^T$.

By integrating (19) with respect to t over the time period 0 to t_p, we get

$$\begin{aligned} &2\int_0^{t_p} y^T(s)\,u(s)\,ds \\ &\quad\geqslant V\left(t_p\right)-V(0)+\gamma\int_0^{t_p}u^T(s)\,u(s)\,ds. \end{aligned} \tag{20}$$

From the definition of $V(t)$, we have $V(t_p)\geqslant 0$ and $V(0)\geqslant 0$. Thus,

$$2\int_0^{t_p} y^T(s)\,u(s)\,ds\geqslant-\beta^2+\gamma\int_0^{t_p}u^T(s)\,u(s)\,ds, \tag{21}$$

for all $t_p\geqslant 0$, $\beta=\sqrt{V(0)}$. The proof is completed. □

Theorem 5. *Let (A1) hold, and let $\dot{\tau}(t)\leqslant\sigma<1$. The complex network (1) is input passive if there exist two matrices $Z\geqslant 0$ and $Q\geqslant 0$ and a scalar $\gamma>0$ such that*

$$\begin{pmatrix} S_1 & a\widehat{P}(G\otimes\Gamma) & S_2 \\ a(G\otimes\Gamma)^T\widehat{P} & -(1-\sigma)Q & 0 \\ S_2^T & 0 & S_3 \end{pmatrix}\leqslant 0, \tag{22}$$

where

$$\begin{aligned} S_1 &= -2\eta I_{nN}+2\widehat{P}\widehat{\Delta}+C^T(Q+\tau Z)\,C, \\ S_2 &= \widehat{P}B-C^T+C^T(Q+\tau Z)\,D, \\ S_3 &= -\left[D+D^T-\gamma I_{nN}-D^T(Q+\tau Z)\,D\right]. \end{aligned} \tag{23}$$

Proof. Define the following Lyapunov functional for system (13):

$$\begin{aligned} V(t) &= x^T(t)\,\widehat{P}x(t)+\int_{-\tau(t)}^{0}\int_{t+\beta}^{t}y^T(\alpha)\,Zy(\alpha)\,d\alpha\,d\beta \\ &\quad+\int_{t-\tau(t)}^{t}y^T(\alpha)\,Qy(\alpha)\,d\alpha. \end{aligned} \tag{24}$$

The derivative of $V(t)$ satisfies

$$\begin{aligned} \dot{V}(t) &= 2x^T(t)\,\widehat{P}\dot{x}(t)+y^T(t)\,Qy(t) \\ &\quad-\int_{t-\tau(t)}^{t}y^T(\alpha)\,Zy(\alpha)\,d\alpha+\tau(t)\,y^T(t)\,Zy(t) \\ &\quad+\dot{\tau}(t)\int_{t-\tau(t)}^{t}y^T(\alpha)\,Zy(\alpha)\,d\alpha \\ &\quad-(1-\dot{\tau}(t))\,y^T(t-\tau(t))\,Qy(t-\tau(t)) \\ &\leqslant 2x^T(t)\,\widehat{P}\dot{x}(t)+y^T(t)\,(Q+\tau Z)\,y(t) \\ &\quad-(1-\sigma)\,y^T(t-\tau(t))\,Qy(t-\tau(t)) \\ &= 2x^T(t)\,\widehat{P}F(x(t))+2ax^T(t)\,\widehat{P}\,(G\otimes\Gamma)\,y(t-\tau(t)) \\ &\quad+2x^T(t)\,\widehat{P}Bu(t)+y^T(t)\,(Q+\tau Z)\,y(t) \\ &\quad-(1-\sigma)\,y^T(t-\tau(t))\,Qy(t-\tau(t)). \end{aligned} \tag{25}$$

Then, we can get

$$\begin{aligned} &\dot{V}(t)-2y^T(t)\,u(t)+\gamma u^T(t)\,u(t) \\ &\quad\leqslant 2x^T(t)\,\widehat{P}F(x(t))+2ax^T(t)\,\widehat{P}\,(G\otimes\Gamma)\,y(t-\tau(t)) \\ &\qquad+2x^T(t)\left(\widehat{P}B-C^T\right)u(t)+y^T(t)\,(Q+\tau Z)\,y(t) \\ &\qquad-(1-\sigma)\,y^T(t-\tau(t))\,Qy(t-\tau(t)) \\ &\qquad-u^T(t)\left(D+D^T-\gamma I_{nN}\right)u(t). \end{aligned} \tag{26}$$

It follows from (18) and (22) that

$$\begin{aligned} &\dot{V}(t)-2y^T(t)\,u(t)+\gamma u^T(t)\,u(t) \\ &\quad\leqslant x^T(t)\left[-2\eta I_{nN}+2\widehat{P}\widehat{\Delta}+C^T(Q+\tau Z)\,C\right]x(t) \\ &\qquad+2ax^T(t)\,\widehat{P}\,(G\otimes\Gamma)\,y(t-\tau(t)) \\ &\qquad+2x^T(t)\left[\widehat{P}B-C^T+C^T(Q+\tau Z)\,D\right]u(t) \end{aligned}$$

$$
\begin{aligned}
& - u^T(t)\left[D + D^T - \gamma I_{nN} - D^T(Q + \tau Z)D\right]u(t) \\
& - (1-\sigma)y^T(t-\tau(t))Qy(t-\tau(t)) \\
& = \xi^T(t)\begin{pmatrix} S_1 & a\widehat{P}(G\otimes\Gamma) & S_2 \\ a(G\otimes\Gamma)^T\widehat{P} & -(1-\sigma)Q & 0 \\ S_2^T & 0 & S_3 \end{pmatrix}\xi(t) \\
& \leqslant 0,
\end{aligned}
\tag{27}
$$

where $\xi(t) = (x^T(t), y^T(t-\tau(t)), u^T(t))^T$.

By integrating (27) with respect to t over the time period 0 to t_p, we can get

$$
\begin{aligned}
& 2\int_0^{t_p} y^T(s)u(s)\,ds \\
& \quad \geqslant V(t_p) - V(0) + \gamma\int_0^{t_p} u^T(s)u(s)\,ds.
\end{aligned}
\tag{28}
$$

From the definition of $V(t)$, we have $V(t_p) \geqslant 0$ and $V(0) \geqslant 0$. Thus,

$$
2\int_0^{t_p} y^T(s)u(s)\,ds \geqslant -\beta^2 + \gamma\int_0^{t_p} u^T(s)u(s)\,ds, \tag{29}
$$

for all $t_p \geqslant 0$, $\beta = \sqrt{V(0)}$. The proof is completed. □

In the above, two sufficient conditions are given to ensure the input passivity of complex network (1). In the following, we shall discuss the output passivity of complex network (1).

Theorem 6. *Let (A1) hold, and let $\dot{\tau}(t) \leqslant \sigma < 1$. The complex network (1) is output passive if there exist matrix $Q \geqslant 0$ and scalar $\gamma > 0$ such that*

$$
\begin{pmatrix} M_1 & a\widehat{P}(G\otimes\Gamma) & M_2 \\ a(G\otimes\Gamma)^T\widehat{P} & -(1-\sigma)Q & 0 \\ M_2^T & 0 & M_3 \end{pmatrix} \leqslant 0, \tag{30}
$$

where

$$
\begin{aligned}
M_1 &= -2\eta I_{nN} + 2\widehat{P}\widehat{\Delta} + C^T(Q + \gamma I_{nN})C, \\
M_2 &= \widehat{P}B + C^T(Q + \gamma I_{nN})D - C^T, \\
M_3 &= -\left[D + D^T - D^T(Q + \gamma I_{nN})D\right].
\end{aligned}
\tag{31}
$$

Proof. Construct the same Lyapunov functional as (15) for system (13). Then, we can get

$$
\begin{aligned}
\dot{V}(t) \leqslant{} & 2x^T(t)\widehat{P}F(x(t)) + 2ax^T(t)\widehat{P}(G\otimes\Gamma)y(t-\tau(t)) \\
& + 2x^T(t)\widehat{P}Bu(t) + y^T(t)Qy(t) \\
& - (1-\sigma)y^T(t-\tau(t))Qy(t-\tau(t)).
\end{aligned}
\tag{32}
$$

Therefore, we have

$$
\begin{aligned}
& \dot{V}(t) - 2y^T(t)u(t) + \gamma y^T(t)y(t) \\
& \quad \leqslant 2x^T(t)\widehat{P}F(x(t)) + 2ax^T(t)\widehat{P}(G\otimes\Gamma)y(t-\tau(t)) \\
& \qquad + 2x^T(t)\left(\widehat{P}B - C^T\right)u(t) + y^T(t)(Q + \gamma I_{nN})y(t) \\
& \qquad - (1-\sigma)y^T(t-\tau(t))Qy(t-\tau(t)) \\
& \qquad - u^T(t)\left(D + D^T\right)u(t).
\end{aligned}
\tag{33}
$$

It follows from (18) and (30) that

$$
\begin{aligned}
& \dot{V}(t) - 2y^T(t)u(t) + \gamma y^T(t)y(t) \\
& \quad \leqslant x^T(t)\left[-2\eta I_{nN} + 2\widehat{P}\widehat{\Delta} + C^T(Q + \gamma I_{nN})C\right]x(t) \\
& \qquad + 2ax^T(t)\widehat{P}(G\otimes\Gamma)y(t-\tau(t)) \\
& \qquad + 2x^T(t)\left[\widehat{P}B + C^T(Q + \gamma I_{nN})D - C^T\right]u(t) \\
& \qquad - (1-\sigma)y^T(t-\tau(t))Qy(t-\tau(t)) \\
& \qquad - u^T(t)\left[D + D^T - D^T(Q + \gamma I_{nN})D\right]u(t) \\
& \quad = \xi^T(t)\begin{pmatrix} M_1 & a\widehat{P}(G\otimes\Gamma) & M_2 \\ a(G\otimes\Gamma)^T\widehat{P} & -(1-\sigma)Q & 0 \\ M_2^T & 0 & M_3 \end{pmatrix}\xi(t) \\
& \quad \leqslant 0,
\end{aligned}
\tag{34}
$$

where $\xi(t) = (x^T(t), y^T(t-\tau(t)), u^T(t))^T$.

By integrating (34) with respect to t over the time period 0 to t_p, we get

$$
\begin{aligned}
& 2\int_0^{t_p} y^T(s)u(s)\,ds \\
& \quad \geqslant V(t_p) - V(0) + \gamma\int_0^{t_p} y^T(s)y(s)\,ds.
\end{aligned}
\tag{35}
$$

From the definition of $V(t)$, we have $V(t_p) \geqslant 0$ and $V(0) \geqslant 0$. Thus,

$$
2\int_0^{t_p} y^T(s)u(s)\,ds \geqslant -\beta^2 + \gamma\int_0^{t_p} y^T(s)y(s)\,ds, \tag{36}
$$

for all $t_p \geqslant 0$, $\beta = \sqrt{V(0)}$. The proof is completed. □

Theorem 7. *Let (A1) hold, and let $\dot{\tau}(t) \leqslant \sigma < 1$. The complex network (1) is output passive if there exist two matrices $Z \geqslant 0$ and $Q \geqslant 0$ and a scalar $\gamma > 0$ such that*

$$
\begin{pmatrix} H_1 & a\widehat{P}(G\otimes\Gamma) & H_2 \\ a(G\otimes\Gamma)^T\widehat{P} & -(1-\sigma)Q & 0 \\ H_2^T & 0 & H_3 \end{pmatrix} \leqslant 0, \tag{37}
$$

where

$$H_1 = -2\eta I_{nN} + 2\widehat{P}\widehat{\Delta} + C^T\left(Q + \tau Z + \gamma I_{nN}\right)C,$$
$$H_2 = \widehat{P}B - C^T + C^T\left(Q + \tau Z + \gamma I_{nN}\right)D, \quad (38)$$
$$H_3 = -\left[D + D^T - D^T\left(Q + \tau Z + \gamma I_{nN}\right)D\right].$$

Proof. Construct the same Lyapunov functional as (24) for system (13). Then, we can obtain

$$\begin{aligned}\dot{V}(t) &\leqslant 2x^T(t)\widehat{P}F(x(t)) + 2ax^T(t)\widehat{P}(G\otimes\Gamma)y(t-\tau(t))\\ &\quad + 2x^T(t)\widehat{P}Bu(t) + y^T(t)(Q+\tau Z)y(t)\\ &\quad - (1-\sigma)y^T(t-\tau(t))Qy(t-\tau(t)).\end{aligned} \quad (39)$$

Therefore, we have

$$\begin{aligned}&\dot{V}(t) - 2y^T(t)u(t) + \gamma y^T(t)y(t)\\ &\quad\leqslant 2x^T(t)\widehat{P}F(x(t)) + 2ax^T(t)\widehat{P}(G\otimes\Gamma)y(t-\tau(t))\\ &\quad + 2x^T(t)\left(\widehat{P}B - C^T\right)u(t) + y^T(t)\left(Q + \tau Z + \gamma I_{nN}\right)y(t)\\ &\quad - (1-\sigma)y^T(t-\tau(t))Qy(t-\tau(t))\\ &\quad - u^T(t)\left(D + D^T\right)u(t).\end{aligned} \quad (40)$$

It follows from (18) and (37) that

$$\begin{aligned}&\dot{V}(t) - 2y^T(t)u(t) + \gamma y^T(t)y(t)\\ &\quad\leqslant x^T(t)\left[-2\eta I_{nN} + 2\widehat{P}\widehat{\Delta} + C^T\left(Q + \tau Z + \gamma I_{nN}\right)C\right]x(t)\\ &\quad + 2ax^T(t)\widehat{P}(G\otimes\Gamma)y(t-\tau(t))\\ &\quad + 2x^T(t)\left[\widehat{P}B - C^T + C^T\left(Q + \tau Z + \gamma I_{nN}\right)D\right]u(t)\\ &\quad - (1-\sigma)y^T(t-\tau(t))Qy(t-\tau(t))\\ &\quad - u^T(t)\left[D + D^T - D^T\left(Q + \tau Z + \gamma I_{nN}\right)D\right]u(t)\\ &\quad = \xi^T(t)\begin{pmatrix} H_1 & a\widehat{P}(G\otimes\Gamma) & H_2\\ a(G\otimes\Gamma)^T\widehat{P} & -(1-\sigma)Q & 0\\ H_2^T & 0 & H_3\end{pmatrix}\xi(t)\\ &\quad\leqslant 0,\end{aligned} \quad (41)$$

where $\xi(t) = (x^T(t), y^T(t-\tau(t)), u^T(t))^T$.

By integrating (41) with respect to t over the time period 0 to t_p, we get

$$\begin{aligned}&2\int_0^{t_p} y^T(s)u(s)\,ds\\ &\quad\geqslant V\left(t_p\right) - V(0) + \gamma\int_0^{t_p} y^T(s)y(s)\,ds.\end{aligned} \quad (42)$$

From the definition of $V(t)$, we have $V(t_p) \geqslant 0$ and $V(0) \geqslant 0$. Thus,

$$2\int_0^{t_p} y^T(s)u(s)\,ds \geqslant -\beta^2 + \gamma\int_0^{t_p} y^T(s)y(s)\,ds, \quad (43)$$

for all $t_p \geqslant 0$, $\beta = \sqrt{V(0)}$. The proof is completed. □

Remark 8. In recent years, some researchers have studied the input passivity and output passivity of the complex networks with state coupling, and many interesting results have been derived. To the best of our knowledge, this is the first paper to investigate the input passivity and output passivity of complex delayed dynamical networks with output coupling. By constructing new Lyapunov functionals, some sufficient conditions ensuring the input passivity and output passivity are established in this paper.

4. Examples

In this section, two illustrative examples are provided to verify the effectiveness of the proposed theoretical results.

Example 1. Consider a three-order dynamical system as the dynamical node of the complex network (1) which is described by

$$\begin{pmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{pmatrix} = \begin{pmatrix}-10x_1 + 2x_2\\ 2x_1 - 10x_2 - x_1x_3\\ x_1x_2 - 6x_3\end{pmatrix}. \quad (44)$$

Clearly, we can take $\eta = 6$, $P = \mathrm{diag}(1,1,1)$, and $\Delta = \mathrm{diag}(0,0,0)$. Take

$$\Gamma = \begin{pmatrix}0.4 & 0.2 & 0.2\\ 0.3 & 0.2 & 0.3\\ 0.3 & 0.1 & 0.2\end{pmatrix}, \quad C_i = \begin{pmatrix}0.3 & 0.2 & 0.1\\ 0.4 & 0.1 & 0.5\\ 0.4 & 0.3 & 0.2\end{pmatrix},$$
$$B_i = \begin{pmatrix}0.3 & 0.1 & 0.1\\ 0.4 & 0.1 & 0.3\\ 0.7 & 0 & 0.2\end{pmatrix}, \quad D_i = \begin{pmatrix}2.5 & 0 & 0\\ 0 & 3.6 & 0\\ 0 & 0 & 2.6\end{pmatrix}, \quad (45)$$

$a = 0.2$, $\beta_\omega = 1$, and $i = 1, 2, \ldots, 10$. The matrix L is chosen as follows:

$$\begin{pmatrix}-3 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1\\ 1 & -4 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0\\ 1 & 1 & -5 & 1 & 0 & 1 & 0 & 0 & 0 & 1\\ 0 & 0 & 1 & -3 & 1 & 0 & 0 & 0 & 1 & 0\\ 0 & 1 & 0 & 1 & -3 & 1 & 0 & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & 1 & -3 & 1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 & 0 & 1 & -4 & 1 & 0 & 1\\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -2 & 1 & 0\\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & -2 & 0\\ 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & -3\end{pmatrix}. \quad (46)$$

Obviously, network (1) is connected, and matrix G is

$$\begin{pmatrix} -1 & \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} \\ \frac{1}{4} & -1 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & 0 \\ \frac{1}{5} & \frac{1}{5} & -1 & \frac{1}{5} & 0 & \frac{1}{5} & 0 & 0 & 0 & \frac{1}{5} \\ 0 & 0 & \frac{1}{3} & -1 & \frac{1}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & -1 & \frac{1}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{3} & 0 & \frac{1}{3} & -1 & \frac{1}{3} & 0 & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{4} & -1 & \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & -1 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & -1 & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & -1 \end{pmatrix}. \tag{47}$$

Next, we analyze the input passivity of complex network (1).

Setting $\tau(t) = 0.5 - 0.5e^{-t}$, then $0 \leqslant \tau(t) \leqslant \tau = 0.5$ and $\dot{\tau}(t) = 0.5e^{-t} \leqslant 0.5$, for $t \geqslant 0$.

We can find the following matrix Q satisfying (9) with $\gamma = 0.2$. Consider the following:

$$\begin{aligned} Q = \operatorname{diag}(&0.5341, 0.4135, 0.3429, 0.5341, 0.4135, 0.3429, \\ &0.5341, 0.4135, 0.3429, 0.5341, 0.4135, 0.3429, \\ &0.5341, 0.4135, 0.3429, 0.5341, 0.4135, 0.3429, \\ &0.5341, 0.4135, 0.3429, 0.5341, 0.4135, 0.3429, \\ &0.5341, 0.4135, 0.3429, 0.5341, 0.4135, 0.3429). \end{aligned} \tag{48}$$

Hence, it follows from Theorem 4 that complex network (1) with above given parameters is input passive.

Example 2. Consider a three-order dynamical system as the dynamical node of the complex network (1) which is described by

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{pmatrix} -9x_1 + 2x_2 \\ x_1 - 9x_2 - x_1x_3 \\ x_1x_2 - 7x_3 \end{pmatrix}. \tag{49}$$

Clearly, we can take $\eta = 7$, $P = \operatorname{diag}(1, 1, 1)$, and $\Delta = \operatorname{diag}(0, 0, 0)$. Take

$$\Gamma = \begin{pmatrix} 0.1 & 0.4 & 0.6 \\ 0.3 & 0.4 & 0.1 \\ 0.3 & 0.5 & 0.2 \end{pmatrix}, \qquad C_i = \begin{pmatrix} 0.6 & 0.3 & 0.4 \\ 0.2 & 0.1 & 0.3 \\ 0.1 & 0.3 & 0.7 \end{pmatrix},$$

$$B_i = \begin{pmatrix} 0.1 & 0.5 & 0.3 \\ 0.5 & 0.3 & 0.1 \\ 0.2 & 0.7 & 0.2 \end{pmatrix}, \qquad D_i = \begin{pmatrix} 2.9 & 0 & 0 \\ 0 & 3.3 & 0 \\ 0 & 0 & 2.8 \end{pmatrix}, \tag{50}$$

$a = 0.3$, $\beta_\omega = 1$, and $i = 1, 2, \ldots, 10$. The matrix L is chosen as follows:

$$\begin{pmatrix} -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -2 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & -4 & 1 \\ 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & -5 \end{pmatrix}. \tag{51}$$

Obviously, network (1) is connected, and matrix G is

$$\begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & -1 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & -1 & \frac{1}{2} & 0 \\ \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & -1 & \frac{1}{4} \\ \frac{1}{5} & 0 & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & 0 & 0 & 0 & \frac{1}{5} & -1 \end{pmatrix}. \tag{52}$$

In the following, we analyze the output passivity of complex network (1).

Setting $\tau(t) = 0.5 - 0.5e^{-t}$, then $0 \leqslant \tau(t) \leqslant \tau = 0.5$ and $\dot{\tau}(t) = 0.5e^{-t} \leqslant 0.5$, for $t \geqslant 0$.

We can find the following matrix Q satisfying (30) with $\gamma = 0.1$. Consider the following:

$$\begin{aligned} Q = \operatorname{diag}(&0.3548, 0.3893, 0.1592, 0.3548, 0.3893, 0.1592, \\ &0.3548, 0.3893, 0.1592, 0.3548, 0.3893, 0.1592, \\ &0.3548, 0.3893, 0.1592, 0.3548, 0.3893, 0.1592, \\ &0.3548, 0.3893, 0.1592, 0.3548, 0.3893, 0.1592, \\ &0.3548, 0.3893, 0.1592, 0.3548, 0.3893, 0.1592). \end{aligned} \tag{53}$$

By Theorem 6, we know that complex network (1) with above given parameters is output passive.

5. Conclusion

A new complex delayed dynamical network model with output coupling has been introduced. We have considered the input passivity and output passivity of the proposed network model. Some input passivity and output passivity criteria have been established by constructing new Lyapunov functionals. Moreover, two illustrative examples have been provided to verify the correctness and effectiveness of the obtained results. In future work, we shall study the input passivity and output passivity of impulsive complex delayed dynamical networks with output coupling.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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