Research Article

Solving the Fuzzy BiLevel Linear Programming with Multiple Followers through Structured Element Method

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The optimal solution of fuzzy bilevel linear programming with multiple followers (MFFBLP) model is shown to be equivalent to the optimal solution of the bilevel linear programming with multiple followers by using fuzzy structured element theory. The optimal solution to this model is found out by adopting the Kuhn-Tucker approach. Finally, an illustrative numerical example for this model is also provided to demonstrate the feasibility and efficiency of the proposed method.

1. Introduction

Bilevel programming introduced by Stackelberg in 1952 [1] has been developed to solve the decentralized planning problem in which decision makers are often arranged within a hierarchical administrative structure. A bilevel programming problem occurs when two decision makers are located at different hierarchical levels. In general, a decision maker at the upper level is termed as the leader and the lower level is termed as the follower [2, 3]. In the context of bilevel programming, the leader first specifies a strategy; the follower then specifies a strategy so as to optimize the objective with full knowledge of the action of the leader.

Many researches on bilevel programming so far have centered on the linear version of the problem [2–7]. Also two fundamental issues in theory and practice of both bilevel programming problems are mostly concerned: one is how to model a real world bilevel programming and the other is how to find properties and an optimal solution to the bilevel programming problem. There are many such hierarchical optimization problems in the fields of industry, agriculture, finance transportation, and so on [8–11]. But in many practical hierarchical decision making systems, the coefficients of objective functions and constraints sometimes cannot be described by precise values. Hence, it is necessary for us to formulate the decentralized decision making problem with uncertainty as fuzzy models.

At present, the linear bilevel programming in which the coefficients are characterized by fuzzy numbers is called fuzzy linear bilevel programming [12, 13]. Sakawa et al. [4, 5, 14–17] formulated cooperative fuzzy bilevel programming problems and proposed an interactive fuzzy programming approach to solve the problems. From this approach, the concept of a bilevel programming was introduced based on fuzzy number λ-level sets. At the same time, some researches applied fuzzy set technique to deal with bilevel programming problems. Shih and Stanely Lee [18] applied fuzzy set theory to overcome the computational difficulties in solving bilevel problems. Sinha [19] started from the fuzzy mathematical programming approach to obtain the solution of multilevel linear programming problems. Recently, Zhang et al. [13, 20–22] studied fuzzy bilevel programming problem, which focuses on the situation where the leader or the follower has multiple objectives with fuzzy parameters and all followers share their decision variables, and provided related algorithms based on the y function in fuzzy set theory. Moreover, they have first solved the fuzzy linear bilevel programming problems with a specialized form of membership functions, triangular form, in the fuzzy parameters [6, 12]. Nevertheless so far the fuzzy linear bilevel programming problem remains the focus of fuzzy multilevel programming problem.

This paper discusses the fuzzy bilevel linear programming with multiple followers (MFFBLP) model. Based on the homeomorphism properties between the bounded real fuzzy

number and the monotone functions on $[-1, 1]$, the comparison of a fuzzy number is changed into a new comparison of monotone function by the definition of fuzzy numbers structured element weighted order. Then the optimal solutions of new derived model is proved equivalent to the optimal solution of the MFFBLP model. The feasibility of the proposed approach is proved by giving a numerical example.

The following of this paper is arranged as follows. In Section 2, some concepts and properties of the fuzzy numbers structured element weighted order are introduced. In Section 3, we study the optimal solution of the MFFBLP model. In Section 4, one numerical example is shown for illustrating the proposed models and approach. Finally, we give some conclusions in Section 5.

2. Preliminaries

In this section, some necessary backgrounds and notions of fuzzy structured element theory are presented.

Definition 1 (see [23]). Let E be a fuzzy set on R and $E(x)$ the membership function of E. Then, E is called a fuzzy structured element, if (i) $E(0) = 1$; (ii) $E(x)$ is a function of monotonous increasing and right continuous on $[-1, 0]$ and monotone decreasing and left continuous on $(0,1]$; (iii) $E(x) = 0$ $(-\infty < x < -1$ or $1 < x < +\infty)$.

Definition 2 (see [23]). E is called a canonical fuzzy structured element, if (i) $\forall x \in (-1, 1), E(x) > 0$; (ii) $E(x)$ is continuous and strictly monotone increasing on $[-1, 0]$ and strictly monotone decreasing and continuous on $(0, 1]$.

Definition 3 (see [23]). E is called a symmetrical fuzzy structured element, if $E(-x) = E(x)$.

Lemma 4 (see [23]). *Let E be a fuzzy structured element and $E(x)$ is its membership function; the function $f(x)$ is continuous and monotone on $[-1, 1]$; then $f(E)$ is a fuzzy number, and the membership function of $f(E)$ is $E(f^{-1}(x))$, where $f^{-1}(x)$ is rotational symmetry function for variables x and y, if f is a strictly monotone function, then $f^{-1}(x)$ is the inverse function of $f(x)$.*

Lemma 5 (see [24]). *For a given canonical fuzzy structured element E and any finite fuzzy number $\widetilde{A}$, there always exists a monotone bounded function f on $[-1, 1]$, having the form $\widetilde{A} = f(E)$.*

Lemma 6 (see [25]). *Let the triangular fuzzy number $\widetilde{A} = (a, b, c)$, E is a fuzzy structured element, and its membership function is*

$$
E(x) = \begin{cases} 1 + x, & -1 \le x \le 0, \\ 1 - x, & 0 \le x \le 1, \\ 0, & others. \end{cases} \tag{1}
$$

Then arbitrary bounded triangular fuzzy number can be generated by E and the monotone bounded function is

$$
f(x) = \begin{cases} (b - a) x + b, & -1 \le x \le 0, \\ (c - b) x + b, & 0 \le x \le 1, \\ 0, & others. \end{cases} \tag{2}
$$

Therefore, we easily get the conclusion $\widetilde{A} = f(E)$.

Remark 7. The class of all bounded fuzzy numbers is denoted by $\widetilde{N}_C(R)$, and let E be a canonical fuzzy structured element.

Definition 8 (see [23]). Suppose $\widetilde{A}_1, \widetilde{A}_2 \in \widetilde{N}_C(R)$. Its structured element representation is $\widetilde{A}_i = f_i(E)$, $i = 1, 2$, respectively, where E is given a canonical fuzzy structured element and its membership function is $E(x)$, $f_1(x)$ and $f_2(x)$ are the same sequence monotonic functions on $[-1, 1]$, respectively. By the following formula

$$
\begin{aligned}
\widetilde{A}_1 \le \widetilde{A}_2 &\Longleftrightarrow F\left(\widetilde{A}_1, \widetilde{A}_2\right) \\
&= \int_{-1}^{1} E(x)\left(f_1(x) - f_2(x)\right) dx \\
&= \int_{-1}^{1} E(x) f_1(x) dx - \int_{-1}^{1} E(x) f_2(x) dx \le 0
\end{aligned} \tag{3}
$$

to determine the binary relation "$\le$" is a total order on $\widetilde{N}_C(R)$ and then called fuzzy numbers structured element weighted order.

Lemma 9 (see [23]). *Let E be a symmetrical fuzzy structured element, $f_1(x)$ and $f_2(x)$ are the same sequence monotonic functions on $[-1, 1]$, the fuzzy number $\widetilde{A}_1 = f_1(E)$, and $\widetilde{A}_2 = f_2(E)$, then*

$$
\begin{aligned}
\widetilde{A}_1 + \widetilde{A}_2 &= f_1(E) + f_2(E), \\
\widetilde{A}_1 - \widetilde{A}_2 &= f_1(E) + f_2^{\tau}(E), \\
k\widetilde{A}_1 &= |k| f_1^{\tau}(E), \\
k\widetilde{A}_2 &= |k| f_2^{\tau}(E),
\end{aligned} \tag{4}
$$

when $k \ge 0$, $f_1^{\tau}(E) = f_1(E)$, and $f_2^{\tau}(E) = f_2(E)$; when $k < 0$, $f_1^{\tau}(E) = -f_1(-E)$, and $f_2^{\tau}(E) = -f_2(-E)$.

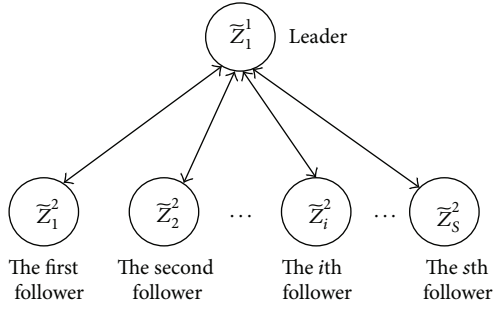*The proof of above Lemmas can be found in reference [23–25].*

FIGURE 1: Fuzzy bilevel linear programming with multiple followers.

3. Fuzzy Bilevel Linear Programming with Multiple Followers

Figure 1 shows the fuzzy bilevel linear programming with multiple followers model (MFFBLP), and this mathematical model is defined as follows:

$$\min_{x_i} \quad \widetilde{Z}_1^1 = \sum_{i=1}^{N} \widetilde{c}_i^1 x_i + \sum_{s=1}^{S}\sum_{j=1}^{M} \widetilde{d}_{sj}^1 y_{sj};$$

s.t.   where $y_{sj}\,(s = 1, 2, \ldots, S; j = 1, 2, \ldots, M)$

is the solution of the lower level problem:

$$\min_{y_{sj}} \quad \widetilde{Z}_s^2 = \sum_{i=1}^{N} \widetilde{c}_{si}^2 x_i + \sum_{j=1}^{M} \widetilde{d}_{sj}^2 y_{sj} \tag{5}$$

s.t $$\sum_{i=1}^{N} \widetilde{a}_{ti}^s x_i + \sum_{j=1}^{M} \widetilde{b}_{tj}^s y_{sj} \leq \widetilde{e}_t^s;$$

$$x_i \geq 0; \quad y_{sj} \geq 0;$$

$$i = 1, 2, \ldots, N; \quad j = 1, 2, \ldots, M; \quad s = 1, 2, \ldots, S;$$

$$t = 1, 2, \ldots, T,$$

where $\widetilde{Z}_1^1$ is the leader's objective function and $\widetilde{Z}_s^2$ is the sth follower's objective function; $x_i, \ y_{sj} \ \in R; \widetilde{c}_i^1, \widetilde{c}_{si}^2, \widetilde{d}_{sj}^1, \widetilde{d}_{sj}^2, \widetilde{a}_{ti}^s, \widetilde{b}_{tj}^s, \widetilde{e}_t^s \in \widetilde{N}_C(R)$.

Theorem 10. *Suppose that $\widetilde{Z}_1^1 = G_1^1(E)$, $\widetilde{Z}_s^2 = G_s^2(E)$, $\widetilde{c}_i^1 = f_i^1(E)$, $\widetilde{c}_{si}^2 = f_{si}^2(E)$, $\widetilde{d}_{sj}^1 = F_{sj}^1(E)$, $\widetilde{d}_{sj}^2 = F_{sj}^2(E)$, $\widetilde{a}_{ti}^s = \varphi_{ti}^s(E)$, $\widetilde{b}_{tj}^s = g_{tj}^s(E)$, $\widetilde{e}_t^s = \psi_t^s(E)$, and $M_1^1 = \int_{-1}^{1} E(t)G_1^1(t)dt$, if E is a canonical fuzzy structured element, $G_1^1(t)$, $G_s^2(t)$, $f_i^1(t)$, $f_{si}^2(t)$, $F_{sj}^1(t)$, $F_{sj}^2(t)$, $\varphi_{ti}^s(t)$, $g_{tj}^s(t)$, and $\psi_t^s(t)$ are monotonous*

increasing functions, then the model (5) is equivalent to the following model:

$$\min_{x_i} M_1 = \sum_{i=1}^{N} x_i \int_{-1}^{1} E(t) f_i^1(t)\, dt + \sum_{s=1}^{S}\sum_{j=1}^{M} y_{sj} \int_{-1}^{1} E(t) F_{sj}^1(t)\, dt;$$

s.t.   where $y_{sj}\,(s = 1, 2, \ldots, S; j = 1, 2, \ldots, M)$ is the

solution of the lower level problem:

$$\min_{y_{sj}} M_s^2 = \sum_{i=1}^{N} x_i \int_{-1}^{1} E(t) f_{si}^2(t)\, dt + \sum_{j=1}^{M} y_{sj} \int_{-1}^{1} E(t) F_{sj}^2(t)\, dt$$

s.t $$\sum_{i=1}^{N} x_i \int_{-1}^{1} E(t) \varphi_{ti}^s(t)\, dt + \sum_{j=1}^{M} y_{sj} \int_{-1}^{1} E(t) g_{tj}^s(t)\, dt$$

$$\leq \int_{-1}^{1} E(t) \psi_t^s(t)\, dt;$$

$$x_i \geq 0; \quad y_{sj} \geq 0;$$

$$i = 1, 2, \ldots, N; \quad j = 1, 2, \ldots, M; \quad s = 1, 2, \ldots, S;$$

$$t = 1, 2, \ldots, T. \tag{6}$$

Proof. By Definition 8, we know that comparing the size of fuzzy number $\widetilde{Z}_1^1$ by $M_1^1 = \int_{-1}^{-1} E(t)G_1^1(t)\, dt$ in model (5), then

$$\widetilde{Z}_1^1 = G_1^1(E) = \sum_{i=1}^{N} \widetilde{c}_i^1 x_i + \sum_{s=1}^{S}\sum_{j=1}^{M} \widetilde{d}_{sj}^1 y_{sj}$$

$$= \sum_{i=1}^{N} f_i^1(E) x_i + \sum_{s=1}^{S}\sum_{j=1}^{M} F_{sj}^1(E) y_{sj}; \tag{7}$$

because $f_i^1(t)$ and $F_{sj}^1(t)$ are monotonous increasing functions; then

$$M_1^1 = \int_{-1}^{1} E(t) G_1^1(t)\, dt$$

$$= \int_{-1}^{1} E(t) \left[\sum_{i=1}^{N} f_i^1(t) x_i + \sum_{s=1}^{S}\sum_{j=1}^{M} F_{sj}^1(t) y_{sj} \right] dt$$

$$= \int_{-1}^{1} E(t) \sum_{i=1}^{N} f_i^1(t) x_i\, dt + \int_{-1}^{1} E(t) \sum_{s=1}^{S}\sum_{j=1}^{M} F_{sj}^1(t) y_{sj}\, dt$$

$$= \sum_{i=1}^{N} x_i \int_{-1}^{1} E(t) f_i^1(t)\, dt + \sum_{s=1}^{S}\sum_{j=1}^{M} y_{sj} \int_{-1}^{1} E(t) F_{sj}^1(t)\, dt. \tag{8}$$

In the same way, we get

$$M_s^2 = \int_{-1}^{1} E(t) G_s^2(t)\, dt$$

$$= \sum_{i=1}^{N} x_i \int_{-1}^{1} E(t) f_{si}^2(t)\, dt + \sum_{j=1}^{M} y_{sj} \int_{-1}^{1} E(t) F_{sj}^2(t)\, dt. \tag{9}$$

By Lemmas 6 and 9, we obtain that

$$\sum_{i=1}^{N} x_i \int_{-1}^{1} E(t)\,\varphi_{ti}^{s}(t)\,dt + \sum_{j=1}^{M} y_{sj} \int_{-1}^{1} E(t)\,g_{tj}^{s}(t)\,dt$$

$$\le \int_{-1}^{1} E(t)\,\psi_{t}^{s}(t)\,dt. \tag{10}$$

The proof is completed.   □

4. Algorithm and Numerical Example

4.1. Algorithm. By using Theorem 10, we give all steps of the approach for the proposed MFFBLP model.

Step 1. If the fuzzy number is triangular fuzzy number, according to Lemma 6 and E expression, we have $\mu_{\widetilde{A}}(t) = E(f^{-1}(t))$ and then get $f_i^1(t)$, $f_{si}^2(t)$, $F_{sj}^1(t)$, $F_{sj}^2(t)$, $\varphi_{ti}^s(t)$, $g_{tj}^s(t)$, and $\psi_t^s(t)$.

Step 2. Computation formula is as follows: $\int_{-1}^{1} E(t) f_i^1(t) dt$, $\int_{-1}^{1} E(t) f_{si}^2(t) dt$, $\int_{-1}^{1} E(t) F_{sj}^1(t) dt$, $\int_{-1}^{1} E(t) F_{sj}^2(t) dt$, $\int_{-1}^{1} E(t) \varphi_{ti}^s(t) dt$, $\int_{-1}^{1} E(t) g_{tj}^s(t) dt$, and $\int_{-1}^{1} E(t) \psi_t^s(t) dt$, and plug in model (5).

Step 3. According to Theorem 10, the MFFBLP model is transformed into the classical bilevel linear programming with multiple followers model whose optimal solution of model (6) can be derived from Kuhn-Tucker's approach [7].

Step 4. After the optimal solution of model (6) is plugged into model (5), we get the optimal solution of the MFFBLP model.

4.2. Numerical Example. Consider the following MFFBLP problem with $x_1 \in R^1$, $x_2 \in R^1$, $y_{11} \in R^1$, and $y_{21} \in R^1$. Consider

$$\min_{x_1, x_2} \quad \widetilde{Z}_1^1(x_1, x_2, y_{11}, y_{21}) = \widetilde{3.1}x_1 + \widetilde{8}x_2 + \widetilde{6.9}y_{11} + \widetilde{11}y_{21};$$

s.t.   where given x_1, x_2, and y_{11}, y_{21} solve

the following problem.

$$\min_{y_{11}} \quad \widetilde{Z}_1^2(x_1, x_2, y_{11}) = \widetilde{2}x_1 + \widetilde{1}x_2 - \widetilde{1}y_{11};$$

s.t. $\widetilde{6}x_1 - \widetilde{1}x_2 + \widetilde{13}y_{11} \le \widetilde{15},$

$\widetilde{5}x_1 + \widetilde{6.9}y_{11} \le \widetilde{15},$

$- \widetilde{4}x_2 + \widetilde{25}y_{11} \le \widetilde{3.1};$

$$\min_{y_{21}} \quad \widetilde{Z}_2^2(x_1, x_2, y_{21}) = \widetilde{15}x_1 - \widetilde{1}x_2 + \widetilde{80}y_{21};$$

s.t. $\widetilde{1}x_1 + \widetilde{1}x_2 - \widetilde{6.9}y_{21} \le \widetilde{10},$

$\widetilde{40}x_1 + \widetilde{1}y_{21} \le \widetilde{5};$

$x_1 \ge 0, \ x_2 \ge 0, \ y_{11} \ge 0, \ y_{21} \ge 0,$

$$\tag{11}$$

where the triangular fuzzy numbers $\widetilde{1} = (0, 1, 2)$, $\widetilde{2} = (1.5, 2, 2.5)$, $\widetilde{3.1} = (2, 3.1, 3.6)$, $\widetilde{4} = (2, 4, 6)$, $\widetilde{5} = (4.7, 5, 5.3)$, $\widetilde{6} = (5, 6, 7)$, $\widetilde{6.9} = (6.5, 6.9, 7.9)$, $\widetilde{8} = (7.5, 8, 8.5)$, $\widetilde{10} = (8, 10, 12)$, $\widetilde{11} = (10.7, 11, 11.3)$, $\widetilde{13} = (12, 13, 14)$, $\widetilde{15} = (14.5, 15, 15.5)$, $\widetilde{25} = (23, 25, 27)$, $\widetilde{40} = (39, 40, 41)$, and $\widetilde{80} = (77, 80, 83)$.

Step 1. By Lemma 6, we have

$$f_1^1(t) = \psi_3^1(t) = \begin{cases} 1.1t + 3.1, & -1 \le t \le 0 \\ 0.5t + 3.1, & 0 \le t \le 1 \\ 0, & \text{others,} \end{cases}$$

$$f_2^1(t) = \begin{cases} 0.5t + 8, & -1 \le t \le 0 \\ 0.5t + 8, & 0 \le t \le 1 \\ 0, & \text{others,} \end{cases}$$

$$F_{11}^1(t) = g_{21}^1(t) = \begin{cases} 0.4t + 6.9, & -1 \le t \le 0 \\ t + 6.9, & 0 \le t \le 1 \\ 0, & \text{others,} \end{cases}$$

$$F_{21}^1(t) = \begin{cases} 0.3t + 11, & -1 \le t \le 0 \\ 0.3t + 11, & 0 \le t \le 1 \\ 0, & \text{others,} \end{cases}$$

$$f_{11}^2(t) = \begin{cases} 0.5t + 2, & -1 \le t \le 0 \\ 0.5t + 2, & 0 \le t \le 1 \\ 0, & \text{others,} \end{cases}$$

$$f_{12}^2(t) = \varphi_{11}^2(t) = \varphi_{12}^2(t) = g_{21}^2(t) = \begin{cases} t + 1, & -1 \le t \le 0 \\ t + 1, & 0 \le t \le 1 \\ 0, & \text{others,} \end{cases}$$

$$F_{11}^2(t) = \varphi_{12}^1(t) = f_{22}^2(t) = \begin{cases} t - 1, & -1 \le t \le 0 \\ t - 1, & 0 \le t \le 1 \\ 0, & \text{others,} \end{cases}$$

$$\varphi_{11}^1(t) = \begin{cases} t + 6, & -1 \le t \le 0 \\ t + 6, & 0 \le t \le 1 \\ 0, & \text{others,} \end{cases}$$

$$g_{11}^1(t) = \begin{cases} t + 13, & -1 \le t \le 0 \\ t + 13, & 0 \le t \le 1 \\ 0, & \text{others,} \end{cases}$$

$$\psi_1^1(t) = \psi_2^1(t) = f_{21}^2(t) = \begin{cases} 0.5t + 15, & -1 \le t \le 0 \\ 0.5t + 15, & 0 \le t \le 1 \\ 0, & \text{others,} \end{cases}$$

$$\varphi_{21}^1(t) = \psi_1^2(t) = \begin{cases} 0.3t + 5, & -1 \le t \le 0 \\ 0.3t + 5, & 0 \le t \le 1 \\ 0, & \text{others,} \end{cases}$$

$$\varphi_{32}^1(t) = \begin{cases} 2t - 4, & -1 \le t \le 0 \\ 2t - 4, & 0 \le t \le 1 \\ 0, & \text{others,} \end{cases}$$

$$g_{31}^1(t) = \begin{cases} 2t + 25, & -1 \le t \le 0 \\ 2t + 25, & 0 \le t \le 1 \\ 0, & \text{others,} \end{cases}$$

$$F_{21}^2(t) = \begin{cases} 3t + 80, & -1 \le t \le 0 \\ 3t + 80, & 0 \le t \le 1 \\ 0, & \text{others,} \end{cases}$$

$$g_{11}^2(t) = \begin{cases} t - 6.9, & -1 \le t \le 0 \\ 0.4t - 6.9, & 0 \le t \le 1 \\ 0, & \text{others,} \end{cases}$$

$$\psi_1^2(t) = \begin{cases} 2t + 10, & -1 \le t \le 0 \\ 2t + 10, & 0 \le t \le 1 \\ 0, & \text{others,} \end{cases}$$

$$\varphi_{11}^2(t) = \begin{cases} t + 40, & -1 \le t \le 0 \\ t + 40, & 0 \le t \le 1 \\ 0, & \text{others.} \end{cases} \tag{12}$$

Step 2. Compute $\int_{-1}^1 f_1^1(t)E(t)dt = 3$, $\int_{-1}^1 f_2^1(t)E(t)dt = 8$, $\int_{-1}^1 F_{11}^1(t)E(t)dt = 7$, $\int_{-1}^1 F_{21}^1(t)E(t)dt = 11$, $\int_{-1}^1 f_{11}^1(t)E(t)dt = 2$, $\int_{-1}^1 f_{12}^1(t)E(t)dt = 1$, $\int_{-1}^1 F_{11}^2(t)E(t)dt = -1$, $\int_{-1}^1 \varphi_{11}^1(t)E(t)dt = 6$, $\int_{-1}^1 g_{11}^1(t)E(t)dt = 13$, $\int_{-1}^1 \psi_1^1(t)E(t)dt = 15$, $\int_{-1}^1 \varphi_{21}^1(t)E(t)dt = 5$, $\int_{-1}^1 \varphi_{32}^1(t)E(t)dt = -4$, $\int_{-1}^1 g_{31}^1(t)E(t)dt = 25$, $\int_{-1}^1 F_{21}^2(t)E(t)dt = 80$, $\int_{-1}^1 g_{11}^2(t)E(t)dt = -7$, $\int_{-1}^1 \psi_1^2(t)E(t)dt = 10$, and $\int_{-1}^1 \varphi_{11}^2(t)E(t)dt = 40$.

Step 3. By Theorem 10, the original problem is equivalent to the following bilevel linear programming with multiple followers' problem:

$$\min_{x_1,x_2} \quad M_1^1(x_1, x_2, y_{11}, y_{21}) = 3x_1 + 8x_2 + 7y_{11} + 11y_{21};$$

s.t.   where given x_1, x_2, and y_{11}, y_{21} solve

the following problem.

$$\min_{y_{11}} \quad M_1^2(x_1, x_2, y_{11}) = 2x_1 + x_2 - y_{11};$$

s.t. $6x_1 - x_2 + 13y_{11} \le 15,$

$5x_1 + 7y_{11} \le 15,$

$-4x_2 + 25y_{11} \le 3;$

$$\min_{y_{21}} \quad M_2^2(x_1, x_2, y_{21}) = 15x_1 - x_2 + 80y_{21};$$

s.t. $x_1 + x_2 - 7y_{21} \le 10,$

$40x_1 + y_{21} \le 5;$

$$x_1 \ge 0, \quad x_2 \ge 0, \quad y_{11} \ge 0, \quad y_{21} \ge 0. \tag{13}$$

Step 4. We use the Kuhn-Tucker approach to get an optimal solution; the solution of the problem is as follows:

$$(x_1, x_2, y_{11}, y_{21}) = (0.1182, 11.7878, 2.0061, 0.2723),$$

$$M_1^1 = 111.6946, \qquad M_1^2 = 10.0181, \qquad M_2^2 = 11.7692$$

$$\widetilde{Z}_1^1 = \left(\underline{Z}_1^1, Z_1^1, \overline{Z}_1^1\right) = (104.5982, 111.5062, 119.5470),$$

$$\widetilde{Z}_1^2 = \left(\underline{Z}_1^2, Z_1^2, \overline{Z}_1^2\right) = (-3.8349, 10.0181, 23.8711),$$

$$\widetilde{Z}_2^2 = \left(\underline{Z}_2^2, Z_2^2, \overline{Z}_2^2\right) = (-0.8946, 11.7692, 24.4330). \tag{14}$$

The example illustrates how to solve a MFFBLP problem and obtains an optimal solution by using the proposed approach.

5. Conclusions

A real world bilevel decision problem may be modeled to have fuzzy coefficients. In this paper, we investigated the MFFBLP model and solved this complex problem by using the fuzzy structured element method. Further study includes the development of models and methods for fuzzy multilevel programming. We will also explore effective applications of the proposed techniques.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgments

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