Research Article

Distributed Node-to-Node Consensus of Multiagent Systems with Delayed Nonlinear Dynamics and Intermittent Communications

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The paper is concerned with the problem of distributed node-to-node consensus of multiagent systems with delayed nonlinear dynamics and communication constraints. A new kind of consensus protocol based only on the intermittent measurements of neighboring agents is proposed to make each follower track the corresponding leader asymptotically. Based on the Lyapunov stability theory and M-matrix theory, some novel and simple criteria are derived for node-to-node consensus of multiagent systems. It is shown that consensus can be reached if the communication time duration is larger than the corresponding threshold value. Finally, a numerical example is provided to demonstrate the effectiveness of the obtained theoretical results.

1. Introduction

Distributed control of multiagent systems has recently attracted various scientific communities ranging from mathematics to electronic engineering [1, 2], which is partly due to its broad applications in sensor networks, formation of unmanned vehicles, clustering of satellites, and so on. An important issue in distributed control of multiagent systems is consensus control, which can be described as only using the local relative information to make the whole system reach a state agreement [3].

Recently, there are many fruitful results on distributed consensus of multiagent systems, according to the presence or absence of a leader in the considered multiagent systems; within the context of multiagent systems, the commonly studied consensus problems can be classified into three categories: leaderless consensus, leader-following, and containment control. Leaderless consensus means that the states of considered multiple agents achieve a prior unknown agreement [4–6]. Leader-following consensus is also called distributed consensus tracking, which will converge to a designed state trajectory which is generated by a virtual or real leader [7–10]. Distributed containment control of multiagent systems with multiagent systems with multiple

leaders has been addressed in the literature [11–15], where the control goal is to make the states of each follower converge to a convex combination of the states of all leaders. It should be noted that there is a common assumption in existing works on containment control that there is no interaction among the leaders. However, in some real applications, there do exist interactions between the neighboring leaders and the state of a given follower needs to track that of a particular leader not the convex combination of the states of all leaders. Motivated by this observation, the notion of distributed node-to-node consensus is introduced in [16, 17]. Specifically, it is assumed that there are two levels in the multiagent systems: the leader's and the follower's levels; the dynamics of each leader may only be affected by those of its neighbors in the leader's levels, while the dynamics of each follower may be affected by those of its neighbors in both leader's and follower's levels. It should be noticed that most of the aforementioned works in multiagent systems are derived on a key assumption that information is transmitted continuously among the agents. However, this assumption may not be ensured in some practical cases due to communication disturbances and sensor range limitations [18, 19]. Thus it is reasonable to assume that each agent can sense its neighbors only intermittently. It is worth noting that both impulsive

control and intermittent control are discontinuous controls. The difference of them is that impulsive control is activated only at some isolated points [20–22], while intermittent control is activated during some intervals and does not work during the other intervals. On the other hand, impulsive control and intermittent control have attracted more interest recently due to their easy implementation in engineering control and being more efficient and useful in real-life applications. On the other hand, in order to guarantee stability in complex networks, adaptive strategies to appropriately tune the strengths of the interconnections among network nodes have been proposed [23–25]; it can be proved that the fixed coupling strength is larger than those needed in practice. Therefore, it may be conservative; adaptive strategies are proposed and can effectively overcome these shortcomings, which can appropriately tune the strength with the dynamic evolution of the network. Finally, the characteristic of time-delayed coupling is very common in biological and physical systems; some of the time delays are trivial and so can be ignored, while some others cannot be ignored, such as in long distance communication and traffic congestions. Therefore, time delays should be modeled in order to simulate more realistic networks [26, 27].

Motivated by these observations, this paper is devoted to investigate distributed node-to-node consensus problem for a class of multiagent systems with delayed nonlinear dynamics by adaptive intermittent control. The main contributions of this paper can be summarized as follows. (1) The multiagent systems in the present framework consist of two levels, that is, the leader's level and the follower's level. (2) The agent dynamics are extended to delayed nonlinear form. (3) A novel adaptive intermittent control protocol is designed, where only a small fraction of followers can sense their corresponding leaders on some disconnected intervals.

Notation. The notation used here is fairly standard except where otherwise stated. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space and $\mathbb{R}^{n \times m}$ is a set of real $n \times m$ matrices. I_n and 1_N represent the $n \times n$ identity matrices and the n-dimensional column vector with all entries equal to one. $\| \cdot \|$ refers to the Euclidean norm. Let diag$\{\cdots\}$ stand for a block-diagonal matrix. Symbol $\| \cdot \|$ denotes the Euclidean norm. M^T represents the transpose of the matrix M. For a real symmetric matrix A, $\lambda_{\max}(A)$ and $\lambda_{\min}(A)$ represent its largest and smallest eigenvalues, respectively. Notation $\otimes$ denotes the Kronecker product. In this paper, if not explicitly stated, matrices are assumed to have compatible dimensions.

2. Problem Formulation and Preliminaries

Let $g = (v, \varepsilon, G)$ be a directed graph with the set of nodes $v = (v_1, v_2, \ldots, v_N)$, the set of directed links $\varepsilon \subseteq v \times v$, and a weighted adjacency matrix $G = (G_{ij})_{N \times N}$ with nonnegative adjacency elements G_{ij}. A directed graph has or contains a directed spanning tree if there exists a node called root such that there exists a directed path from this node to every other node. The Laplacian matrix $L = (l_{ij})_{N \times N}$ of the weighted directed network is defined as $L_{ij} = -G_{ij}$ ($i \neq j$); $L_{ii} = \sum_{j=1, j \neq i}^{N} G_{ij}$.

Definition 1. A nonsingular real square matrix A is called an M-matrix if all of its off-diagonal elements are nonpositive, and all elements of A^{-1} are nonnegative.

Lemma 2. *Suppose that matrix $A \in \mathbb{R}^{n \times n}$ has nonpositive off-diagonal elements; then A is an M-matrix if and only if there exists a positive diagonal matrix $\Xi = \text{diag}\{1/\theta_1, 1/\theta_2, \ldots, 1/\theta_n\}$, such that $A^T \Xi + \Xi A > 0$, where $A\theta = 1_N$ and $\theta = [\theta_1, \theta_2, \ldots, \theta_n]^T$.*

Lemma 3. *Suppose that nonnegative function $y(t)$, $t \in [-\tau, +\infty)$, satisfies*

$$\dot{y}(t) \leq -c_1 y(t) + c_2 y(t - \tau) \quad (t \geq 0); \tag{1}$$

then

$$y(t) \leq \|y(0)\|_\tau e^{-rt} \quad (t \geq 0), \tag{2}$$

where $c_1 > c_2 > 0$, $\|y(0)\|_\tau = \max_{-\tau \leq s \leq 0}\{y(s)\}$, and r is the unique solution of $-r = -c_1 + c_2 e^{r\tau}$.

Lemma 4. *Suppose that nonnegative function $y(t)$, $t \in [-\tau, +\infty)$, satisfies*

$$\dot{y}(t) \leq c_1 y(t) + c_2 y(t - \tau) \quad (t \geq 0); \tag{3}$$

then

$$y(t) \leq \|y(0)\|_\tau e^{(c_1 + c_2)t} \quad (t \geq 0), \tag{4}$$

where c_1 and c_2 are two positive constants and $\|y(0)\|_\tau = \max_{-\tau \leq s \leq 0}\{y(s)\}$.

Consider that multiagent systems consist of the leader's and the follower's level; it is assumed that each level contains N coupled agents. The dynamics of the ith agent in the leader's level are described as follows:

$$\dot{x}_i(t) = f(x_i(t), x_i(t - \tau), t) - c \sum_{j=1}^{N} L_{ij} x_j(t) \tag{5}$$

$$(i = 1, 2, \ldots, N),$$

where $x_i(t) \in \mathbb{R}^n$ is the position state of the ith leader. $f(\cdot, \cdot, t) : \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^+ \to \mathbb{R}^n$ is a continuously differentiable vector-valued function representing the inherent delayed nonlinear dynamics of agent i. $\tau > 0$ is the time delay. $c > 0$ represents the coupling strength. $L = [l_{ij}]_{N \times N}$ is the Laplacian matrix of communication topology. Furthermore, the evolution of the ith follower is given as

$$\dot{\hat{x}}_i(t) = f(\hat{x}_i(t), \hat{x}_i(t - \tau), t) - c \sum_{j=1}^{N} l_{ij} \hat{x}_j(t) + u_i(t) \tag{6}$$

$$(i = 1, 2, \ldots, N),$$

where $\hat{x}_i(t) \in \mathbb{R}^n$ is the position state of the ith follower. $u_i(t)$ is the control input acting agent i.

It is commonly assumed in most existing works that all information is transmitted continuously among the agents.

However, in some real situations agents may only communicate with their neighbors over some disconnected time intervals due to the unreliability of communication channels, failure of physical devices; thus the adaptive intermittent consensus protocol is designed as follows:

$$
u_i(t)
$$
$$
= \begin{cases} cd_i(t)\left(x_i(t) - \widehat{x}_i(t)\right) & t \in [kT, kT + \delta) \\ 0 & t \in [kT + \delta, (k+1)T), \end{cases} \tag{7}
$$

where the pinning link $d_i(t) > 0$ if and only if the ith follower can directly sense the ith leader; otherwise $d_i(t) = 0$; its updating law is given later. The communication time duration δ satisfies $\tau < \delta \leq T$.

Remark 5. What kind of agents and how many agents should be pinned? The selection criteria are as follows. Firstly, divide the digraph of the network into m components, where each component contains a directed tree; select the root nodes in these components as pinned nodes. There are two reasons: (1) the related message of the leader can be transmitted to each follower; (2) the consensus can be achieved if and only if the root node of each component is pinned, which means that the minimum number of pinning controllers is m. If we select the root nodes in these components as pinned nodes, the following consensus is not achieved; we need to rearrange the remaining nodes in descending order according to the differences of out-degrees and in-degrees and gradually increase the amount of control node, until the

consensus is achieved. More detailed discussion can be found in the literatures [28, 29].

Definition 6. The distributed node-to-node consensus in multiagent systems (5) and (6) is said to be achieved if, for any initial conditions, the following holds:

$$
\lim_{t \to \infty} \left\| x_i(t) - \widehat{x}_i(t) \right\| = 0 \quad (i = 1, 2, \ldots, N). \tag{8}
$$

3. Main Result

In this section, we deal with node-to-node consensus problem for multiagent systems with delayed nonlinear dynamics by adaptive intermittent control. By defining a Lyapunov function and utilizing a combination of stability theory and Kronecker products, some simple conditions are established to ensure node-to-node consensus of multiagent systems (5) and (6). The following assumption is needed to derive the main results.

Assumption 7. For the nonlinear function $f(\cdot, \cdot, t) : \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^+ \to \mathbb{R}^n$ in (5) and (6), there exist two positive constants $L_1, L_2 > 0$ such that

$$
\left\| f(x_1, x_2, t) - f(y_1, y_2, t) \right\|^2
$$
$$
\leq L_1 \left\| x_1 - y_1 \right\|^2 + L_2 \left\| x_2 - y_2 \right\|^2, \tag{9}
$$

where $x_1, x_2, y_1, y_2 \in \mathbb{R}^n$.

Substituting (7) into (6), one has

$$
\dot{\widehat{x}}_i(t) = \begin{cases} f\left(\widehat{x}_i(t), \widehat{x}_i(t-\tau), t\right) - c\sum_{j=1}^{N} l_{ij}\widehat{x}_j(t) + cd_i(t)\left(x_i(t) - \widehat{x}_i(t)\right) & t \in [kT, kT + \delta) \\ f\left(\widehat{x}_i(t), \widehat{x}_i(t-\tau), t\right) - c\sum_{j=1}^{N} l_{ij}\widehat{x}_j(t) & t \in [kT + \delta, (k+1)T). \end{cases} \tag{10}
$$

For convenience, systems (5) and (10) can be rewritten as

$$
\dot{x}(t) = f\left(x(t), x(t-\tau), t\right) - c\left(L \otimes I_n\right)x(t),
$$
$$
\dot{\widehat{x}}(t) = \begin{cases} f\left(\widehat{x}(t), \widehat{x}(t-\tau), t\right) - c\left(L \otimes I_n\right)\widehat{x}(t) - c\left(D(t) \otimes I_n\right)\left(\widehat{x}(t) - x(t)\right) & t \in [kT, kT + \delta) \\ f\left(\widehat{x}(t), \widehat{x}(t-\tau), t\right) - c\left(L \otimes I_n\right)\widehat{x}(t) & t \in [kT + \delta, (k+1)T), \end{cases} \tag{11}
$$

where

$$
x(t) = \left[x_1^T(t), x_2^T(t), \ldots, x_N^T(t)\right]^T,
$$
$$
\widehat{x}(t) = \left[\widehat{x}_1^T(t), \widehat{x}_2^T(t), \ldots, \widehat{x}_N^T(t)\right]^T,
$$
$$
f(x(t), x(t-\tau), t) = \left[f^T(x_1(t), x_1(t-\tau), t),\right.
$$
$$
\left. f^T(x_2(t), x_2(t-\tau), t), \ldots,\right.
$$

$$
\left. f^T(x_N(t), x_N(t-\tau), t)\right]^T,
$$
$$
f(\widehat{x}(t), \widehat{x}(t-\tau), t) = \left[f^T(\widehat{x}_1(t), \widehat{x}_1(t-\tau), t),\right.
$$
$$
f^T(\widehat{x}_2(t), \widehat{x}_2(t-\tau), t), \ldots,
$$
$$
\left. f^T(\widehat{x}_N(t), \widehat{x}_N(t-\tau), t)\right]^T,
$$
$$
D(t) = \text{diag}\left\{d_1(t), d_2(t), \ldots, d_l(t), 0, \ldots, 0\right\}. \tag{12}
$$

Let $e(t) = \hat{x}(t) - x(t)$; then the following error system can be obtained from (11) such that

$$\dot{e}(t) = \begin{cases} f(e(t), e(t-\tau), t) - c(L \otimes I_n) e(t) - c(D(t) \otimes I_n) e(t) & t \in [kT, kT+\delta) \\ f(e(t), e(t-\tau), t) - c(L \otimes I_n) e(t) & t \in [kT+\delta, (k+1)T), \end{cases} \tag{13}$$

where

$$f(e(t), e(t-\tau), t) = f(\hat{x}(t), \hat{x}(t-\tau), t) \\ - f(x(t), x(t-\tau), t). \tag{14}$$

It is not hard to verify that node-to-node consensus for multiagent systems (5) and (6) will be achieved if and only if zero equilibrium point of error system (13) is globally attractive.

Theorem 8. *Suppose that Assumption 7 holds; then distributed node-to-node consensus problem for multiagent systems (5) and (6) is achieved under the following distributed adaptive intermittent control protocol:*

$$u(t) = \begin{cases} -c(D(t) \otimes I_n) e(t) & t \in [kT, kT+\delta) \\ 0 & t \in [kT+\delta, (k+1)T) \end{cases} \tag{15}$$

and the updating law $d_i(t)$ $(i = 1, 2, \ldots, l)$ is defined as

$$\dot{d}_i(t) \\ = \begin{cases} ca_{0i}e^{a_{1i}t}e^T(t)(\Xi \otimes I_n) e(t) & t \in [kT, kT+\delta) \\ 0 & t \in [kT+\delta, (k+1)T) \end{cases} \tag{16}$$

if the following conditions hold:

$$1 + L_1 - 2c\lambda_{\min}(\overline{L})\theta_{\min} + a_1 < 0, \tag{17}$$

$$1 + L_1 - c\lambda_{\min}(S) - b_1 < 0, \tag{18}$$

$$\delta > \frac{r\tau + (b_1 + L_2)T}{r + b_1 + L_2}, \tag{19}$$

where

$$\Xi = \text{diag}\left\{\frac{1}{\theta_1}, \frac{1}{\theta_2}, \ldots, \frac{1}{\theta_N}\right\},$$

$$\theta = [\theta_1, \theta_2, \ldots, \theta_n]^T,$$

$$L\theta = 1_N,$$

$$\theta_{\max} = \max_{1 \le i \le N}\{\theta_i\},$$

$$\theta_{\min} = \min_{1 \le i \le N}\{\theta_i\},$$

$$\overline{L} = \frac{\Xi\hat{L} + \hat{L}^T\Xi}{2},$$

$$\hat{L} = L + D^*,$$

$$D^* = \text{diag}\{d_1^*, d_2^*, \ldots, d_l^*, 0, \ldots, 0\},$$

$$S = \Xi^{-1}(\Xi L + L^T\Xi),$$

$$-r = -a_1 + L_2 e^{r\tau},$$

$$a_1 = \min_{1 \le i \le N}\{a_{1i}\}. \tag{20}$$

Proof. Choose the following Lyapunov function candidate:

$$V(t) = \frac{1}{2}e^T(t)(\Xi \otimes I_n) e(t) \\ + \frac{1}{2}\sum_{i=1}^{l} e^{-a_{1i}t}\frac{(d_i(t) - d^*)^2}{a_{0i}}, \tag{21}$$

where $\Xi = \text{diag}\{1/\theta_1, 1/\theta_2, \ldots, 1/\theta_N\}$, $\theta = [\theta_1, \theta_2, \ldots, \theta_n]^T$ satisfies $L\theta = 1_N$. d_i^* $(i = 1, 2, \ldots, l)$ are some undetermined sufficiently large positive constants.

(1) For $t \in [kT, kT+\delta)$ and an arbitrarily given $k \in \mathbb{N}$, taking the derivative of $V(t)$ along the trajectories of system (13) can be given as follows:

$$\dot{V}(t) = e^T(t)(\Xi \otimes I_n)\dot{e}(t) - \frac{1}{2}$$

$$\cdot \sum_{i=1}^{l} a_{1i}e^{-a_{1i}t}\frac{(d_i(t) - d_i^*)^2}{a_{0i}}$$

$$+ \sum_{i=1}^{l} e^{-a_{1i}t}\frac{(d_i(t) - d_i^*)}{a_{0i}}\dot{d}_i(t) = e^T(t)(\Xi \otimes I_n)$$

$$\cdot f(e(t), e(t-\tau), t) - ce^T(t)\left(\frac{\Xi L + L^T\Xi}{2} \otimes I_n\right)$$

$$\cdot e(t) - ce^T(t)\left(\frac{\Xi D(t) + D(t)^T\Xi}{2} \otimes I_n\right) e(t) - \frac{1}{2}$$

$$\cdot \sum_{i=1}^{l} a_{1i} e^{-a_{1i}t} \frac{(d_i(t) - d_i^*)^2}{a_{0i}}$$

$$+ \sum_{i=1}^{l} (d_i(t) - d_i^*) e^T(t) (\Xi \otimes I_n) \dot{e}(t) = e^T(t)$$

$$\cdot (\Xi \otimes I_n) f(e(t), e(t-\tau), t) - c e^T(t) (\overline{L} \otimes I_n)$$

$$\cdot e(t) - \frac{1}{2} \sum_{i=1}^{l} a_{1i} e^{-a_{1i}t} \frac{(d_i(t) - d_i^*)^2}{a_{0i}} \le \frac{1}{2} e^T(t)$$

$$\cdot (\Xi \otimes I_n) e(t) + \frac{1}{2} f^T(e(t), e(t-\tau), t) (\Xi \otimes I_n)$$

$$\cdot f(e(t), e(t-\tau), t) - c e^T(t) (\overline{L} \otimes I_n) e(t) - \frac{1}{2}$$

$$\cdot \sum_{i=1}^{l} a_{1i} e^{-a_{1i}t} \frac{(d_i(t) - d_i^*)^2}{a_{0i}} \le \frac{1}{2} e^T(t) (\Xi \otimes I_n) e(t)$$

$$+ \frac{1}{2} L_1 e^T(t) (\Xi \otimes I_n) e(t) + \frac{1}{2} L_2 e^T(t-\tau) (\Xi \otimes I_n)$$

$$\cdot e(t-\tau) - c \lambda_{\min}(\overline{L}) \theta_{\min} e^T(t) (\Xi \otimes I_n) e(t) - \frac{1}{2}$$

$$\cdot \sum_{i=1}^{l} a_{1i} e^{-a_{1i}t} \frac{(d_i(t) - d_i^*)^2}{a_{0i}}$$

$$= \frac{1}{2} \left(1 + L_1 - 2c \lambda_{\min}(\overline{L}) \theta_{\min}\right) e^T(t) (\Xi \otimes I_n) e(t)$$

$$- \frac{1}{2} \sum_{i=1}^{l} a_{1i} e^{-a_{1i}t} \frac{(d_i(t) - d_i^*)^2}{a_{0i}} + \frac{1}{2} L_2 e^T(t-\tau)$$

$$\cdot (\Xi \otimes I_n) e(t-\tau)$$

$$\le \frac{1}{2} \left(1 + L_1 - 2c \lambda_{\min}(\overline{L}) \theta_{\min} + a_1\right) e^T(t) (\Xi \otimes I_n)$$

$$\cdot e(t) - \frac{1}{2} a_1 e^T(t) (\Xi \otimes I_n) e(t) - \frac{1}{2}$$

$$\cdot a_1 \sum_{i=1}^{l} e^{-a_{1i}t} \frac{(d_i(t) - d_i^*)^2}{a_{0i}} + \frac{1}{2} L_2 e^T(t-\tau) (\Xi \otimes I_n)$$

$$\cdot e(t-\tau) + \frac{1}{2} L_2 \sum_{i=1}^{l} e^{-a_{1i}(t-\tau)} \frac{(d_i(t-\tau) - d_i^*)^2}{a_{0i}}$$

$$\le -a_1 V(t) + L_2 V(t-\tau),$$

(22)

where

$$a_1 = \min_{1 \le i \le N} \{a_{1i}\},$$

$$\overline{L} = \frac{\Xi \widehat{L} + \widehat{L}^T \Xi}{2},$$

$$\widehat{L} = L + D^*,$$

$$D^* = \operatorname{diag} \{d_1^*, d_2^*, \ldots, d_l^*, 0, \ldots, 0\},$$

$$\theta_{\min} = \min_{1 \le i \le N} \{\theta_i\}.$$

(23)

(2) For $t \in [kT + \delta, (k+1)T)$ with arbitrarily given $k \in \mathbb{N}$, taking the time derivative $V(t)$ along the trajectories of (13), we have

$$\dot{V}(t) = e^T(t) (\Xi \otimes I_n) \dot{e}(t) - \frac{1}{2}$$

$$\cdot \sum_{i=1}^{l} a_{1i} e^{-a_{1i}t} \frac{(d_i(t) - d_i^*)^2}{a_{0i}} = e^T(t) (\Xi \otimes I_n)$$

$$\cdot f(e(t), e(t-\tau), t) - c e^T(t) (\widetilde{L} \otimes I_n) e(t) - \frac{1}{2}$$

$$\cdot \sum_{i=1}^{l} a_{1i} e^{-a_{1i}t} \frac{(d_i(t) - d_i^*)^2}{a_{0i}} \le \frac{1}{2} e^T(t) (\Xi \otimes I_n) e(t)$$

$$+ \frac{1}{2} f^T(e(t), e(t-\tau), t) (\Xi \otimes I_n)$$

(24)

$$\cdot f(e(t), e(t-\tau), t) - c e^T(t) (\widetilde{L} \otimes I_n) e(t) - \frac{1}{2}$$

$$\cdot \sum_{i=1}^{l} a_{1i} e^{-a_{1i}t} \frac{(d_i(t) - d_i^*)^2}{a_{0i}} \le \frac{1}{2} \left(1 + L_1\right) e^T(t)$$

$$\cdot (\Xi \otimes I_n) e(t) + \frac{1}{2} L_2 e^T(t-\tau) (\Xi \otimes I_n) e(t-\tau)$$

$$- c e^T(t) (\widetilde{L} \otimes I_n) e(t) - \frac{1}{2} \sum_{i=1}^{l} a_{1i} e^{-a_{1i}t} \frac{(d_i(t) - d_i^*)^2}{a_{0i}},$$

where $\widetilde{L} = (\Xi L + L^T \Xi)/2$.

Note that, for an arbitrarily given $t \in [kT + \delta, (k+1)T)$, the matrix $\langle \Xi L + L^T \Xi, \Xi \rangle$ is normal; for any given $z \in \mathbb{R}^N$, it is easily derived from the generalized eigenvalue theory that

$$\lambda_{\min}(S) z^T \Xi z \le z^T (\Xi L + L^T \Xi) z,$$

(25)

where $S = \Xi^{-1}(\Xi L + L^T \Xi)$; then one has

$$\frac{1}{2} \lambda_{\min}(S) e^T(t) (\Xi \otimes I_n) e(t) \le e^T(t) (\widetilde{L} \otimes I_n) e(t).$$

(26)

By considering (18), (24), and (26), we can obtain

$$\dot{V}(t)$$

$$\le \frac{1}{2} \left(1 + L_1\right) e^T(t) (\Xi \otimes I_n) e(t)$$

$$+ \frac{1}{2} L_2 e^T(t-\tau) (\Xi \otimes I_n) e(t-\tau)$$

$$-\frac{1}{2}c\lambda_{\min}(S)e^{T}(t)(\Xi\otimes I_{n})e(t)$$

$$-\frac{1}{2}\sum_{i=1}^{l}a_{1i}e^{-a_{1i}t}\frac{(d_{i}(t)-d_{i}^{*})^{2}}{a_{0i}}$$

$$\leq\frac{1}{2}(1+L_{1}-c\lambda_{\min}(S)-b_{1})e^{T}(t)(\Xi\otimes I_{n})e(t)$$

$$+\frac{1}{2}b_{1}e^{T}(t)(\Xi\otimes I_{n})e(t)$$

$$+\frac{1}{2}b_{1}\sum_{i=1}^{l}e^{-a_{1i}t}\frac{(d_{i}(t)-d_{i}^{*})^{2}}{a_{0i}}$$

$$+\frac{1}{2}L_{2}e^{T}(t-\tau)(\Xi\otimes I_{n})e(t-\tau)$$

$$+\frac{1}{2}L_{2}\sum_{i=1}^{l}e^{-a_{1i}(t-\tau)}\frac{(d_{i}(t-\tau)-d_{i}^{*})^{2}}{a_{0i}}$$

$$\leq b_{1}V(t)+L_{2}V(t-\tau),$$

$$(27)$$

where $b_{1}>0$.

Based on the above analysis and according to Lemma 3, when $t\in[0,\delta)$, we can obtain

$$V(t)\leq\max_{-\tau\leq s\leq0}V(s)e^{-rt}=\|V(0)\|_{\tau}e^{-rt},\qquad(28)$$

where r is the positive solution of $-r=-a_{1}+L_{2}e^{-r\tau}$.

When $t\in[\delta,T)$, by using Lemma 4, one has

$$V(t)\leq\max_{\delta-\tau\leq s\leq\delta}V(s)e^{(b_{1}+L_{2})(t-\delta)}$$

$$\leq\max_{-\tau\leq s\leq0}V(s)\left\{\max_{\delta-\tau\leq t\leq\delta}e^{-rt}\right\}e^{(b_{1}+L_{2})(t-\delta)}\qquad(29)$$

$$=\|V(0)\|_{\tau}e^{-r(\delta-\tau)+(b_{1}+L_{2})(t-\delta)}.$$

Consider $T-\tau>\delta$; then

$$\|V(t)\|_{\tau}=\max_{T-\tau\leq t\leq T}V(t)$$

$$\leq\max_{\delta-\tau\leq s\leq\delta}V(s)\max_{T-\tau\leq t\leq T}\left\{e^{(b_{1}+L_{2})(t-\delta)}\right\}$$

$$\leq\max_{\delta-\tau\leq s\leq\delta}V(s)e^{(b_{1}+L_{2})(T-\delta)}\qquad(30)$$

$$\leq\max_{-\tau\leq s\leq0}V(s)e^{-r(\delta-\tau)+(b_{1}+L_{2})(T-\delta)}$$

$$=\|V(0)\|_{\tau}e^{-\Delta},$$

where $\Delta=r(\delta-\tau)-(b_{1}+L_{2})(T-\delta)$.

Next, for all $k\in\mathbb{N}$, we will prove that $\|V(kT)\|_{\tau}\leq\|V(0)\|_{\tau}e^{-k\Delta}$. One has

$$\|V(kT)\|_{\tau}=\max_{kT-\tau\leq t\leq kT}V(t)$$

$$\leq\max_{(k-1)T+\delta-\tau\leq s\leq(k-1)T+\delta}V(s)$$

$$\cdot\max_{kT-\tau\leq t\leq kT}e^{(b_{1}+L_{2})(t-(k-1)T-\delta)}$$

$$=\max_{(k-1)T+\delta-\tau\leq s\leq(k-1)T+\delta}V(s)e^{(b_{1}+L_{2})(T-\delta)}$$

$$\leq\max_{(k-1)T-\tau\leq s\leq(k-1)T}V(s)\qquad(31)$$

$$\cdot\left\{\max_{(k-1)T+\delta-\tau\leq t\leq(k-1)T+\delta}e^{-r(t-(k-1)T)}\right\}e^{(b_{1}+L_{2})(T-\delta)}$$

$$=\max_{(k-1)T-\tau\leq s\leq(k-1)T}V(s)e^{-r(\delta-\tau)+(b_{1}+L_{2})(T-\delta)}$$

$$=\max_{(k-1)T-\tau\leq s\leq(k-1)T}V(s)e^{-\Delta}=\cdots\leq\max_{-\tau\leq s\leq0}V(s)e^{-k\Delta}$$

$$=\|V(0)\|_{\tau}e^{-k\Delta}.$$

For any $t>0$, there exists a positive integer k_{0}, such that $k_{0}T\leq t\leq(k_{0}+1)T$; then one has the following.

When $t\in[k_{0}T,k_{0}T+\delta)$, we can obtain

$$V(t)\leq\max_{k_{0}T-\tau\leq s\leq k_{0}T}V(s)e^{-r(t-k_{0}T)}\leq\max_{-\tau\leq s\leq0}V(s)e^{-k_{0}\Delta}$$

$$\leq\max_{-\tau\leq s\leq0}V(s)e^{\Delta(1-t/T)}=\|V(0)\|_{\tau}e^{\Delta}e^{-(\Delta/T)t}.$$

$$(32)$$

When $t\in[k_{0}T+\delta,(k_{0}+1)T)$, we have

$$V(t)\leq\max_{k_{0}T+\delta-\tau\leq s\leq k_{0}T+\delta}V(s)e^{(b_{1}+L_{2})(t-k_{0}T-\delta)}$$

$$\leq\max_{k_{0}T-\tau\leq s\leq k_{0}T}V(s)\left\{\max_{k_{0}T+\delta-\tau\leq t\leq k_{0}T+\delta}e^{-r(t-k_{0}T)}\right\}$$

$$\cdot e^{(b_{1}+L_{2})(t-k_{0}T-\delta)}$$

$$=\max_{k_{0}T-\tau\leq s\leq k_{0}T}V(s)e^{-r(\delta-\tau)}e^{(t-(k_{0}+1)T)+T-\delta}\qquad(33)$$

$$\leq\|V(0)\|_{\tau}e^{-k_{0}\Delta}e^{-r(\delta-\tau)+(b_{1}+L_{2})(T-\delta)}=\|V(0)\|_{\tau}$$

$$\cdot e^{-(k_{0}+1)\Delta}\leq\|V(0)\|_{\tau}e^{-(\Delta/T)t}.$$

From (32) and (33), we can obtain

$$V(t)\leq K_{0}e^{-(\Delta/T)t},\qquad(34)$$

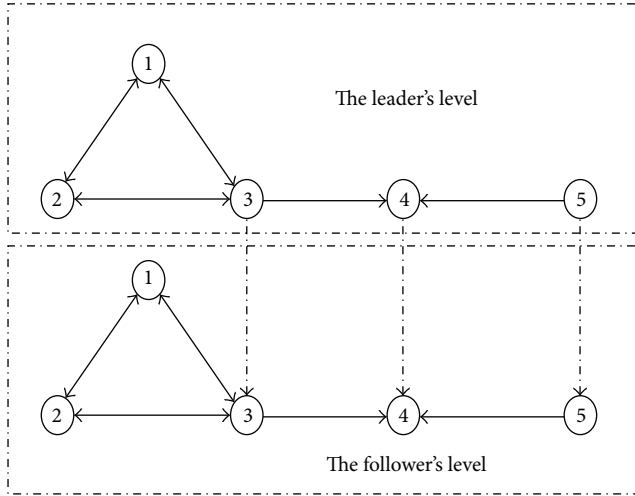FIGURE 1: Communication topology of multiagent systems (5) and (6).

FIGURE 2: The curves of position states $x_1(t)$ and $\hat{x}_1(t)$.

where $K_0 = \max\{1, e^{\Delta}\}$, which indicates that the states of agents exponentially converge to node-to-node consensus. This completes the proof. □

Remark 9. It can be seen from (34) that the convergence speed is related to the value of Δ; by some simple computation, we have

$$\Delta = r\delta - r\tau - (b_1 + L_2)T + (b_1 + L_2)\delta$$
$$= (r + b_1 + L_2)\delta - r\tau - (b_1 + L_2)T; \tag{35}$$

from the above equation, we can see that the convergence speed will accelerate with the increase of δ. Therefore, a trade-off has to be made between the convergence rate and the control cost.

4. A Numerical Example

In this section, a numerical example is provided to verify the analysis results.

Example 1. Suppose that there are five leaders in the considered multiagent systems; the communication topologies among the leaders and the followers are given in Figure 1; the solid lines indicate the neighboring relationship among the agents in the same level, while the dashed lines are the pinning links; the corresponding Laplacian matrix L and the pinning matrix D^* are given as follows:

$$L = \begin{bmatrix} 4 & -3 & -1 & 0 & 0 \\ -3 & 5 & -2 & 0 & 0 \\ -1 & -2 & 3 & 0 & 0 \\ 0 & 0 & -2 & 3 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$
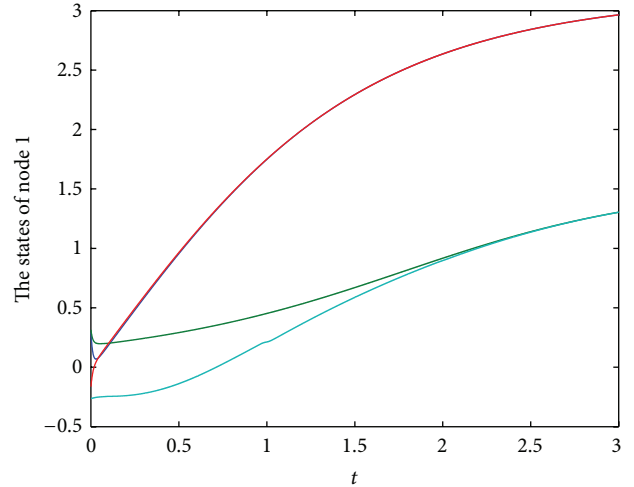
$$D^* = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 10 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}. \tag{36}$$

The nonlinear dynamics are described as $f(x_i(t), x_i(t-\tau), t) = [\cos(0.5x_{i1}(t)) + \cos(0.5x_{i1}(t-\tau)), 0.25\sin(2x_{i2}(t)) + 0.25\sin(2x_{i2}(t-\tau))]^T$; it can be verified that the intrinsic nonlinear dynamics of the agents obey Assumption 7 with $L_1 = L_2 = 0.5$.

By some simple computation, we have

$$\theta = [1.0273 \ 0.9364 \ 0.3 \ 0.1308 \ 0.1],$$
$$\Xi = \text{diag}\{0.9735 \ 1.068 \ 3.3333 \ 7.6471 \ 10\}; \tag{37}$$

then we can obtain $\theta_{\min} = 0.1$, $\lambda_{\min}(\overline{L}) = 0.8816$, and $\lambda_{\min}(S) = -0.8499$. Taking $c = 11.4$, $T = 1$, and $\tau = 0.1$, by using condition (17) in Theorem 8, we can obtain $a_1 < 0.51$; let $a_1 = 0.5099$. By using condition (18), we have $b_1 > 11.1319$, here taking $b_1 = 11.14$. Then we can obtain the communication $\delta > 0.9646$, taking $\delta = 0.9650$. It is assumed that the agents communicate with their neighbors only when $t \in \bigcup_{k \in N}[k, k + 0.9650)$, which indicates that the average communication rate is 0.9650. The position state trajectories of the leaders and the followers are shown in Figures 2–6, which indicate that node-to-node consensus in multiagent systems (5) and (6) is indeed guaranteed. The position state error trajectories of agents are shown in Figure 7, which further demonstrate the effectiveness of the obtained criteria. The curve of adaptive intermittent control is shown in Figure 8; it is noted that the control $u(t) = 0$ in time interval $\bigcup_{k \in N}[k + 0.9650, k + 1)$. The trajectory of pinning feedback gain $d_i(t)$ ($i = 3, 4, 5$) under the updating
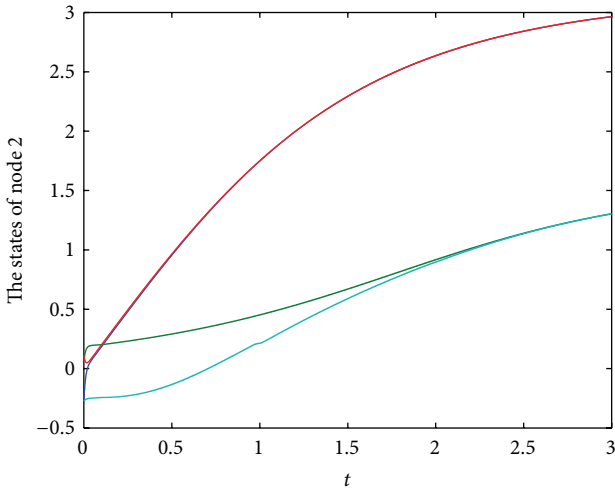
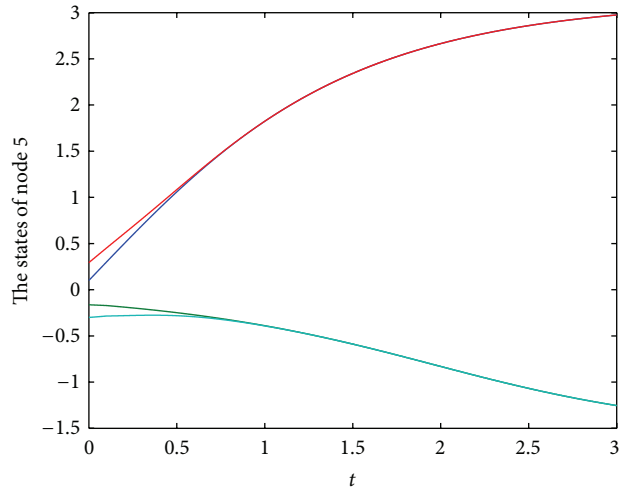FIGURE 3: The curves of position states $x_2(t)$ and $\hat{x}_2(t)$.

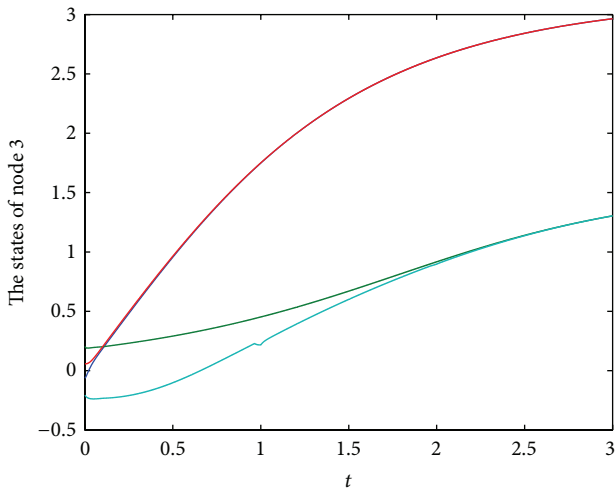FIGURE 4: The curves of position states $x_3(t)$ and $\hat{x}_3(t)$.

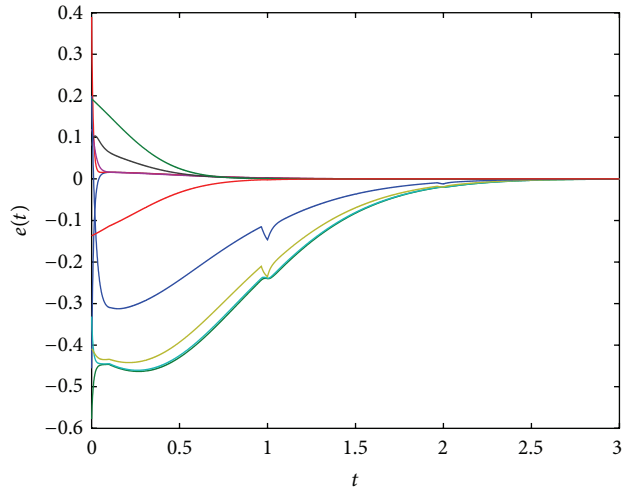FIGURE 5: The curves of position states $x_4(t)$ and $\hat{x}_4(t)$.

FIGURE 6: The curves of position states $x_5(t)$ and $\hat{x}_5(t)$.

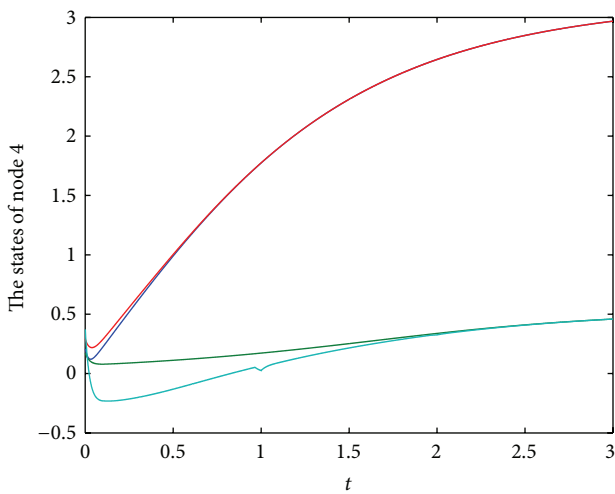FIGURE 7: The position state error trajectories of $e(t)$.

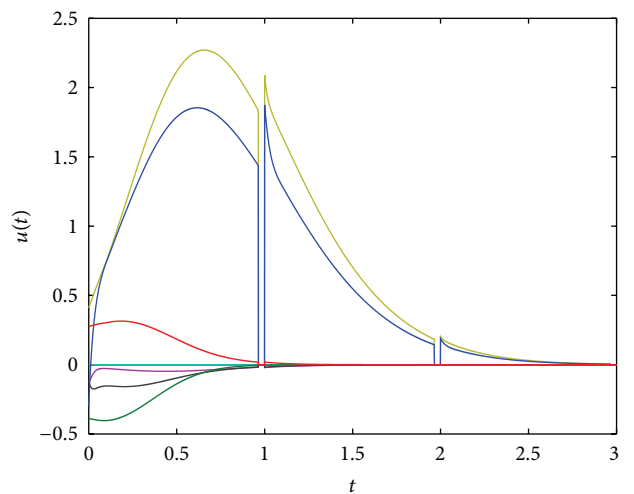FIGURE 8: The trajectory of adaptive intermit control $u(t)$.
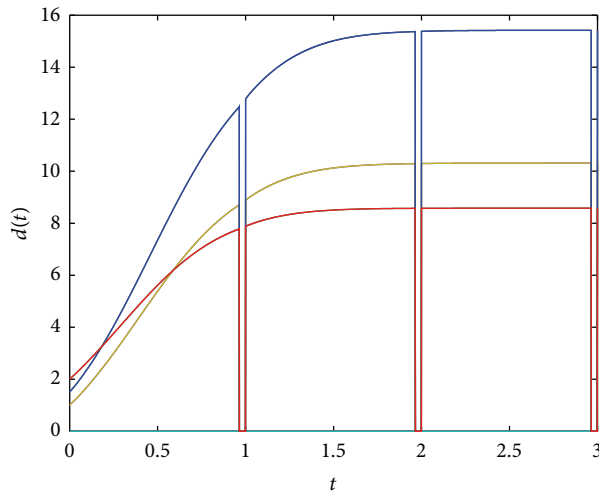
FIGURE 9: The trajectory of pinning feedback gains $d_i(t)$ ($i = 3, 4, 5$).

laws (16) is shown in Figure 9. The simulation has confirmed the effectiveness of our theoretical results.

5. Conclusions

In this paper, we have investigated node-to-node consensus of multiagent systems with delayed nonlinear dynamics. The coordination goal is to make each follower track the corresponding leader asymptotically, where only a small fraction of followers can sense the states of their corresponding leaders on some disconnected time intervals. Finally, a simulation example has been employed to demonstrate the effectiveness of the results. Further research topics include the extension of our results to second-order or high-order multiagent systems with delayed nonlinear dynamics and communication constraints.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgments

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