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ALMOST γ-CONTINUOUS FUNCTIONS

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ABSTRACT. In this paper, a new class of functions called "almost γ-continuous" is introduced and their several properties are investigated. This new class is also utilized to improve some published results concerning weak continuity [6] and γ-continuity [2].

KEY WORDS AND PHRASES. γ-continuity, weak-continuity, faint-continuity, u-weak-continuity, γ-open (γ-closed), θ-open (θ-closed), weakly-compact.

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1. INTRODUCTION. Levine [6] introduced the notion of weak continuity as a weakened form of continuity in topological spaces. In [5], Joseph defined the notion of u-weak continuity and utilized it to obtain a necessary and sufficient condition for a Urysohn space to be Urysohn-closed. On the other hand, Lo Faro [2] and the first author introduced γ-continuous functions. The purpose of the present paper is to introduce a new class of functions called "almost γ-continuous functions". Almost γ-continuity implies u-weak continuity and is implied by both weak continuity and γ-continuity which are independent of each other. In Section 2, we obtain some characterizations of almost γ-continuous functions. In Section 3, in order to sharpen the positive results in this paper, we will compare almost γ-continuous functions with other related functions. In Section 4, we deal with some basic properties of almost γ-continuous functions, that is, restriction, composition, product, etc. In the last section, almost γ-continuity will be utilized to improve some published results concerning weak continuity and γ-continuity.

1. PRELIMINARIES.

Throughout the present paper, X and Y denote topological spaces on which no separation is assumed unless explicitly stated. Let S be a subset and x a point of a topological space. The closure and the interior of S are denoted by $Cl(S)$ and $Int(S)$, respectively. A subset S is said to be <u>regular closed</u> (resp. <u>regular open</u>) if $Cl(Int(S)) = S$ (resp. $Int(Cl(S)) = S$). A point x is said to be in the θ-<u>closure</u> of S [15] (denoted by $\theta - Cl(S)$) if $S \cap Cl(V) \neq \emptyset$ for each open set V containing x. A subset S is said to be θ-<u>closed</u> if $\theta - Cl(S) = S$. The complement of a θ-closed set is said to be θ-<u>open</u>. It is shown in [9, Theorem 1] that if V is θ-open in X and $x \varepsilon V$ then there exists a regular open set U such that $x \in U \subset Cl(U) \subset V$. Open sets G and H will be called <u>an ordered pair of</u>

open sets containing x [4] (denoted by (G,H)) if $x \varepsilon G \subset Cl(G) \subset H$. A point x is said to be in the γ-closure of S (denoted by $\gamma - Cl(S)$) if $S \cap H \neq \emptyset$ for each ordered pair (G,H) of open sets containing x. A subset S is said to be γ-closed if $\gamma - Cl(S) = S$. The family $\mathfrak{U}_x$ of all neighborhoods of x is called the neighborhood filterbase of x. We denote by $\bar{\mathfrak{U}}_x$ the closed filter on x having $\{Cl(V) \mid V \varepsilon \mathfrak{U}_x\}$ as a basis. Moreover, we denote by $\mathfrak{U}(\bar{\mathfrak{U}}_x)$ the neighborhood filter of $\bar{\mathfrak{U}}_x$. A point $x \varepsilon X$ is said to be in the γ-adherence of a filter base $\mathfrak{F}$ [3] (denoted by γ-ad $\mathfrak{F}$) if $F \cap A_x \neq \emptyset$ for every $F \varepsilon \mathfrak{F}$ and every neighborhood A_x of $\bar{\mathfrak{U}}_x$, or equivalently, $F \cap H \neq \emptyset$ for every $F \varepsilon \mathfrak{F}$ and every ordered pair (G,H) of open sets containing x.

2. CHARACTERIZATIONS.

DEFINITION 2.1. A function $f: X \to Y$ is said to be almost γ-continuous (briefly *a.γ.c.*) if for each $x \varepsilon X$ and each $V \varepsilon \mathfrak{U}(\bar{\mathfrak{U}}_{f(x)})$, there exists $U \varepsilon \mathfrak{U}_x$ such that $f(U) \subset V$.

THEOREM 2.2. For a function $f: X \to Y$, the following are equivalent:

(a) f is *a.γ.c.*

(b) For each $x \varepsilon X$ and each ordered pair (G,H) of open sets containing $f(x)$, there exists an open set U containing x such that $f(U) \subset H$.

(c) $Cl(f^{-1}(B)) \subset f^{-1}(\gamma - Cl(B))$ for every subset B of Y.

(d) $f(Cl(A)) \subset \gamma - Cl(f(A))$ for every subset A of X.

(e) $f(ad\mathfrak{F}) \subset \gamma - adf(\mathfrak{F})$ for every filter base $\mathfrak{F}$ on X.

PROOF. (a) $\Rightarrow$ (b): Let $x \varepsilon X$ and (G,H) any ordered pair of open sets containing $f(x)$. Then $f(x) \varepsilon G \subset Cl(G) \subset H$ and hence $H \varepsilon \mathfrak{U}(\bar{\mathfrak{U}}_{f(x)})$. There exists an open neighborhood U of x such that $f(U) \subset H$.

(b) $\Rightarrow$ (c): Let B be a subset of Y and suppose that $x \notin f^{-1}(\gamma - Cl(B))$. Then $f(x) \notin \gamma - Cl(B)$ and there exists an ordered pair (G,H) of open sets containing $f(x)$ such that $B \cap H = \emptyset$. By (b), there exists an open set U containing x such that $f(U) \subset H$. Therefore, we have $B \cap f(U) = \emptyset$ and hence $U \cap f^{-1}(B) = \emptyset$. This shows that $x \notin Cl(f^{-1}(B))$. This implies that $Cl(f^{-1}(B)) \subset f^{-1}(\gamma - Cl(B))$.

(c) $\Rightarrow$ (d): Let A be any subset of X. By (c), we have

$$Cl(A) \subset Cl(f^{-1}(f(A))) \subset f^{-1}(\gamma - Cl(f(A)))$$

and hence $f(Cl(A)) \subset \gamma - Cl(f(A))$.

(d) $\Rightarrow$ (e): Let $\mathfrak{F} = \{F_\alpha \mid \alpha \in \nabla\}$ be any filter base on X. We have $f(Cl(F_\alpha)) \subset \gamma - Cl(f(F_\alpha))$ for each $\alpha \in \nabla$ and hence

$$f(ad\mathfrak{F}) = f(\bigcap_{\alpha \in \nabla} Cl(F_\alpha)) \subset \bigcap_{\alpha \in \nabla} f(Cl(F_\alpha)) \subset \bigcap_{\alpha \in \nabla} \gamma - Cl(f(F_\alpha)) = \gamma - adf(\mathfrak{F}).$$

(e) $\Rightarrow$ (a): Suppose that there exist $x \in X$ and $V \in \mathfrak{U}(\bar{\mathfrak{U}}_{f(x)})$ such that $f(U_x) \not\subset V$ for every $U_x \in \mathfrak{U}_x$. Then $U_x \cap (X - f^{-1}(V)) \neq \emptyset$ for every $U_x \in \mathfrak{U}_x$ and hence $\mathfrak{F} = \{U_x \cap (X - f^{-1}(V)) \mid U_x \in \mathfrak{U}_x\}$ is a filter base on X. By (e), we have $f(ad\mathfrak{F}) \subset \gamma - adf(\mathfrak{F})$. This is a contradiction, since $x \in ad\mathfrak{F}$ but $f(x) \notin \gamma - adf(\mathfrak{F})$.

3. COMPARISON.

DEFINITION 3.1. A function $f: X \to Y$ is said to be

(a) weakly continuous [6] if for each $x \in X$ and each open neighborhood V of $f(x)$, there exists an open neighborhood U of x such that $f(U) \subset Cl(V)$.

(b) γ-continuous [2] if for each $x \in X$ and each open neighborhood V of $f(x)$ containing a nonempty regular closed set, there exists an open neighborhood U of x such that $f(U) \subset V$.

(c) u-weakly continuous [5] if for each $x \in X$ and each ordered pair (G,H) of open sets containing $f(x)$, there exists an open neighborhood U of x such that $f(U) \subset Cl(H)$.

(d) faintly continuous [9] if for each $x \in X$ and each θ-open set V containing $f(x)$, there exists an open neighborhood U of x such that $f(U) \subset V$.

THEOREM 3.2. For properties on a function, we have the following implications:

$$\begin{tikzcd}
& \text{weak continuity} \arrow[dr, Rightarrow] & & & \text{faint continuity} \\
\text{continuity} \arrow[ur, Rightarrow] \arrow[dr, Rightarrow] & & \text{almost } \gamma\text{-continuity} \arrow[urr, Rightarrow] \arrow[drr, Rightarrow] & & \\
& \gamma\text{-continuity} \arrow[ur, Rightarrow] & & & \text{u-weak continuity}
\end{tikzcd}$$

PROOF. We shall only show that if $f: X \to Y$ is $a.\gamma.c.$ then it is faintly continuous. Let V be any θ-open set of Y and x any point of $f^{-1}(V)$. There exists an open set W of Y containing $f(x)$ such that $f(x) \in W \subset Cl(W) \subset V$. Therefore, (W, V) is an ordered pair of open sets containing $f(x)$. Since f is $a.\gamma.c.$, there exists an open set U of X containing x such that $f(U) \subset V$; hence $x \in U \subset f^{-1}(V)$. Therefore, $f^{-1}(V)$ is open in X and hence f is faintly continuous [9, Theorem 9].

REMARK 3.3. None of the implications in Theorem 3.2 is reversible as the following three examples show.

EXAMPLE 3.4. Let (R, τ) be the topological space of real numbers with the usual topology. Let $X = \{a, b, c\}$, $\sigma = \{\emptyset, X, \{a\}, \{c\}, \{a, c\}\}$ and $f: (R, \tau) \to (X, \sigma)$ be the function defined as follows: $f(x) = a$ if x is rational; $f(x) = c$ if x is rational. Then f is γ-continuous [2, Example 1] and hence $a.\gamma.c.$ but it is not weakly continuous.

EXAMPLE 3.5. Let $X = \{a, b, c, d, e\}$, $\tau = \{\emptyset, X, \{c\}, \{a, d, e\}, \{a, d, e, c\}\}$ and $\sigma = \{\emptyset, X, \{a\}, \{c\}, \{a, c\}, \{a, b, c\}, \{a, d, e\}, \{a, c, d, e\}\}$. The identity function $f: (X, \tau) \to (X, \sigma)$ is weakly continuous and hence $a.\gamma.c.$ but it is not γ-continuous [2, Example 2].

EXAMPLE 3.6. Let $X = \{a, b, c, d\}$, $\tau = \{\emptyset, X, \{a\}, \{c\}, \{a, c\}, \{c, d\}, \{a, c, d\}, \{a, b, c\}\}$, and $f: (X, \tau) \to (X, \tau)$ be the function defined as follows: $f(a) = b, f(b) = c, f(c) = d$, and $f(d) = a$. Then f is u-weakly continuous and faintly continuous but it is not $a.\gamma.c.$ at $d \in X$ [1, Example 3].

4. BASIC PROPERTIES. In this section, we shall investigate some basic properties of $a.\gamma.c.$ functions, that is, restriction, composition, product, etc.

PROPOSITION 4.1. If $f: X \to Y$ is $a.\gamma.c.$ and A is a subset of X, then the restriction $f \mid A: A \to Y$ is $a.\gamma.c.$.

PROOF. Let $x \in A$ and (G, H) be any ordered pair of open sets containing $(f \mid A)(x) = f(x)$. Since f is $a.\gamma.c.$, there exists an open set U containing x such that $f(U) \subset H$. Then $U \cap A$ is an open set of the subspace $A, x \in U \cap A$ and $(f \mid A)(U \cap A) \subset H$. Therefore, $f \mid A$ is $a.\gamma.c.$

The composition of $a.\gamma.c.$ functions is not necessarily $a.\gamma.c.$ There exist weakly continuous functions whose composition is not $a.\gamma.c.$ as the following example shows.

EXAMPLE 4.2. Let $X = \{a, b, c, d\}$, $\tau = \{\emptyset, X, \{b\}, \{c\}, \{b, c\}, \{a, b\}, \{a, b, c\}, \{b, c, d\}\}$, and $f: (X, \tau) \to (X, \tau)$ be the function defined as follows: $f(a) = c, f(b) = d$, $f(c) = b$ and $f(d) = a$. Then f is weakly continuous [10, Example] and hence $a.\gamma.c.$ However, $fof: (X, \tau) \to (X, \tau)$ is not $a.\gamma.c.$ at $d \in X$.

PROPOSITION 4.3. If $f: X \to Y$ is $a.\gamma.c.$ and $g: Y \to Z$ is continuous, then $gof: X \to Z$ is $a.\gamma.c.$

PROOF. Let $x \in X$ and (G, H) be any ordered pair of open sets in Z containing $(gof)(x)$. Since g is continuous, $(g^{-1}(G),\ g^{-1}(H))$ is an ordered pair of open sets containing $f(x)$. Since f is *a.γ.c.*, there exists an open set U containing x such that $f(U) \subset g^{-1}(H)$. Therefore, we have $(gof)(U) \subset H$. This shows that gof is *a.γ.c.*

Let $\{X_\alpha \mid \alpha \in \nabla\}$ and $\{Y_\alpha \mid \alpha \in \nabla\}$ be two families of topological spaces with the same index set ∇. We denote their product spaces by $\prod X_\alpha$ and $\prod Y_\alpha$. Let $f_\alpha: X_\alpha \to Y_\alpha$ be a function for each $\alpha \in \nabla$. We denote by $f: \prod X_\alpha \to \prod Y_\alpha$ the product function defined by $f(\{x_\alpha\}) = \{f_\alpha(x_\alpha)\}$ for each $\{x_\alpha\} \varepsilon \prod X_\alpha$.

COROLLARY 4.4. If $f: X \to \prod X_\alpha$ is *a.γ.c.* and $p_\beta: \prod X_\alpha \to X_\beta$ is the βth projection, then $p_\beta o f: X \to X_\beta$ is *a.γ.c.* for each $\beta \varepsilon \nabla$.

PROOF. Since f is *a.γ.c.* and p_β is continuous, by Proposition 4.3 $p_\beta o f$ is *a.γ.c.*

COROLLARY 4.5. Let $f: X \to Y$ be a function and $g: X \to X \times Y$ the graph function defined by $g(x) = (x, f(x))$ for each $x \varepsilon X$. If g is *a.γ.c.*, then f is *a.γ.c.*

PROOF. Let $p_y: X \times Y \to Y$ be the projection. Then we have $p_y o g = f$. It follows from Corollary 4.4 that f is *a.γ.c.*

PROPOSITION 4.6. If $f: X \to Y$ is continuous and $g: Y \to Z$ is *a.γ.c.*, then $gof: X \to Z$ is *a.γ.c.*

PROOF. Let $x \varepsilon X$ and (G, H) be any ordered pair of open sets in Z containing $(gof)(x)$. Since g is *a.γ.c.*, there exists an open set V containing $f(x)$ such that $g(V) \subset H$. Let $U = f^{-1}(V)$, then U is an open set of X containing x, since f is continuous. We have $(gof)(U) \subset H$ and hence gof is *a.γ.c.*

PROPOSITION 4.7. Let $f: X \to Y$ be an open surjection. Then $g: Y \to Z$ is *a.γ.c.* if $gof: X \to Z$ is *a.γ.c.*

PROOF. Let $y \varepsilon Y$ and (G, H) be any ordered pair of open sets in Z containing $g(y)$. Since f is surjective, there exists $x \varepsilon X$ such that $f(x) = y$. Since *g.o.f.* is *a.γ.c.*, there exists an open set U containing x such that $(gof)(U) \subset H$. By openness of f, $f(U)$ is an open set containing y and $g(f(U)) \subset H$.

COROLLARY 4.8. If the product function $f: \prod X_\alpha \to \prod Y_\alpha$ is *a.γ.c.* then $f_\alpha: X_\alpha \to Y_\alpha$ is *a.γ.c.* for each $\alpha \varepsilon \nabla$.

PROOF. Let β be an arbitrarily chosen index of ∇. Let $p_\beta: \prod X_\alpha \to X_\beta$ and $q_\beta: \prod Y_\alpha \to Y_\beta$ be the βth projections. Then we have $q_\beta o f = f_\beta o p_\beta$ for each $\beta \varepsilon \nabla$. Since f is *a.γ.c.* and q_β is continuous, by Proposition 4.3 $q_\beta o f$ is *a.γ.c.* and hence $f_\beta o p_\beta$ is *a.γ.c.* Since p_β is an open surjection, it follows from Proposition 4.7 that f_β is *a.γ.c.*

COROLLARY 4.9. Let $f: X \to Y$ be an open continuous surjection. Then $g: Y \to Z$ is *a.γ.c.* if and only if $gof: X \to Z$ is *a.γ.c.*

PROOF. This follows immediately from Propositions 4.6 and 4.7.

5. FURTHER PROPERTIES. In this section, we shall improve some known results concerning weak continuity and γ-continuity. We shall recall that a space X is said to be Urysohn if for distinct points x_1, x_2 in X, there exist open sets U_1, U_2 of X such that $x_1 \varepsilon U_1, x_2 \varepsilon U_2$ and $Cl(U_1) \cap Cl(U_2) = \emptyset$.

THEOREM 5.1. If $f_1: X_1 \to Y$ is weakly continuous, $f_2: X_2 \to Y$ is *a.γ.c.* and Y is Urysohn, then $\{(x_1, x_2) \mid f_1(x_1) = f_2(x_2)\}$ is closed in $X_1 \times X_2$.

PROOF. Let A denote the set $\{x_1, x_2) \mid f_1(x_1) = f_2(x_2)\}$. Let $(x_1, x_2) \notin A$, then $f_1(x_1) \neq f_2(x_2)$. There exist open sets V_1 and V_2 of Y such that $f_1(x_1) \varepsilon V_1, f_2(x_2) \varepsilon V_2$ and $Cl(V_1) \cap Cl(V_2 = \emptyset$. Then we have $f_2(x_2) \varepsilon V_2 \subset Cl(V_2) \subset Y - Cl(V_1)$ and hence $(V_2, Y - Cl(V_1))$ is an ordered pair of open sets in Y containing $f_2(x_2)$. Since f_2 is *a.γ.c.*, there exists an open set U_2 containing x_2 such that $f_2(U_2) \subset Y - Cl(V_1)$. On the other hand, since f_1 is weakly continuous, there exists an open set U_1 containing x_1 such that $f_1(U_1) \subset Cl(V_1)$. Therefore, we obtain $f_1(U_1) \cap f_2(U_2) = \emptyset$ which implies that $[U_1 \times U_2] \cap A = \emptyset$. This shows that A is closed in $X_1 \times X_2$.

COROLLARY 5.2. (Prakash and Srivistava [14]). If $f_1: X_1 \to Y$ and $f_2: X_2 \to Y$ are weakly continuous and Y is Urysohn, then the set $\{x_1, x_2) \mid f_1(x_1) = f_2(x_2)\}$ is closed in $X_1 \times X_2$.

PROOF. This follows immediately from Theorem 3.2 and 5.1.

A function $f: X \to Y$ is said to be weakly quasi continuous [13] at $x \varepsilon X$ if for each open set V containing $f(x)$ and each open set U containing x, there exists an open set G of X such that $\emptyset \neq G \subset U$ and $f(G) \subset Cl(V)$. If f is weakly quasi continuous at every $x \varepsilon X$, then it is said to be weakly quasi continuous. A subset S of X is said to be semi-open [7] if there exists an open set U of X such that $U \subset S \subset Cl(U)$.

It is shown in [12, Theorem 4.1] that a function $f: X \to Y$ is weakly quasi continuous if and only if each $x \varepsilon X$ and each open set V containing $f(x)$, there exists a semi-open set U containing x such that $f(U) \subset Cl(V)$. It follows from this result and [12, Example 5.2] that weak continuity implies weak quasi continuity but not conversely.

THEOREM 5.3. Let $f: X \to Y$ be weakly quasi continuous and $g: X \to Y$ *a.γ.c.* If Y is Urysohn, D is dense in X and $f = g$ on D, then $f = g$.

PROOF. Let $A = \{x \varepsilon X \mid f(x) = g(x)\}$ then $D \subset A$ and hence $Cl(D) = Cl(A) = X$. Assume that $x \notin X - A$. Assume that $x \notin X - A$. Then $f(x) \neq g(x)$. There exist open sets V and W in Y such that $f(x) \varepsilon V, g(x) \varepsilon W$ and $Cl(V) \cap Cl(W) = \emptyset$. We have $g(x) \varepsilon W \subset Cl(W) \subset Y - Cl(V)$ and hence $(W, Y - Cl(V))$ is an ordered pair of open sets containing $g(x)$. Since g is *a.γ.c.*, there exists an open set U containing x such that $g(U) \subset X - Cl(V)$. On the other hand, f is weakly quasi continuous, there exists a semi-open set G of X containing x such that $f(G) \subset Cl(V)$ [12, Theorem 4.1]. Therefore, we have $f(G) \cap g(U) = \emptyset$ which implies that $(G \cap U) \cap A = \emptyset$. Since $U \cap G$ is a semi-open set containing x, $\mathrm{Int}(U \cap G) \neq \emptyset$ and $\mathrm{Int}(G \cap U) \cap A = \emptyset$. This contradicts that $Cl(A) = X$. Therefore, we obtain $A = X$ and hence $f = g$.

COROLLARY 5.4. (Noiri [11]). Let $f_1, f_2: X \to Y$ be weakly continuous. If Y is Urysohn, D is dense in X and $f_1 = f_2$ on D, then $f_1 = f_2$..

PROOF. This is an immediate consequence of Theorem 5.3.

For a function $f: X \to Y$, the subset $\{(x, f(x)) \mid x \varepsilon X\}$ is called the graph of f and is denoted by $G(f)$. The graph $G(f)$ is said to be strongly-closed [8] if for each $(x,y) \notin G(f)$, there exists open sets U and V containing x and y, respectively, such that $[U \times Cl(V)] \cap G(f) = \emptyset$.

THEOREM 5.5. If $f: X \to Y$ is *a.γ.c.* and Y is Urysohn, then $G(f)$ is strongly-closed.

PROOF. Let $(x,y) \varepsilon X \times Y$ and $y \neq f(x)$. There exist open sets V and W such that $y \varepsilon V, f(x) \varepsilon W$ and $Cl(V) \cap Cl(W) = \emptyset$. Therefore, $(W, X - Cl(V))$ is an ordered pair of open sets containing $f(x)$. Since f is *a.γ.c.*, there exists an open set U containing x such that $f(U) \subset X - Cl(V)$; hence $f(U) \cap Cl(V) = \emptyset$. It follows from [8, Lemma 1] that $G(f)$ is strongly-closed.

COROLLARY 5.6. (Long and Herrington [8]). If $f: X \to Y$ is weakly continuous and Y is Urysohn, then $G(f)$ is strongly-closed.

THEOREM 5.7. If $f: X \to Y$ is an *a.γ.c.* surjection and X is connected, then Y is connected.

PROOF. Suppose that Y is not connected. There exist nonempty disjoint open sets V and W such that $Y = V \cup W$. Since V and W are open and closed, V and W are γ-closed in Y. By Theorem 2.2, we have $Cl(f^{-1}(V)) \subset f^{-1}(\gamma - Cl(V)) = f^{-1}(V)$ and hence $f^{-1}(V)$ is closed in X. Similarly, $f^{-1}(W)$ is closed in X. Moreover, $f^{-1}(V)$ and $f^{-1}(W)$ are nonempty disjoint and $f^{-1}(V) \cup f^{-1}(W) = X$. This shows that X is not connected.

COROLLARY 5.8. (Noiri [11]). If $f: X \to Y$ is a weakly continuous surjection and X is connected, then Y is connected.

DEFINITION 5.9. (1) An open cover $\{V_\alpha \mid \alpha \varepsilon \nabla\}$ of a space X is said to be regular [3] if for each $\alpha \varepsilon \nabla$, there exists a nonempty regular closed set F_α of X such that $F_\alpha \subset V_\alpha$, and $X = \cup\{\text{Int}(F_\alpha) \mid \alpha \varepsilon \nabla\}$; (2) A space X is said to be weakly compact [3] if every regular cover of X has a finite subfamily whose closures cover X.

THEOREM 5.10. If $f: X \to Y$ is a u-weakly continuous surjection and X is compact, then Y is weakly compact.

PROOF. Let $\{V_\alpha \mid \alpha \varepsilon \nabla\}$ be any regular cover of Y. Then for each $\alpha \varepsilon \nabla$, there exists a regular closed set F_α such that $\emptyset \neq F_\alpha \subset V_\alpha$ and $\cup\{\text{ Int }(F_\alpha) \mid \alpha \varepsilon \nabla\} = Y$. For each $x \varepsilon X$, there exists $\alpha(x) \varepsilon \nabla$ such that $f(x) \varepsilon$ $\text{Int}(F_{\alpha(x)}) \subset Cl(\text{Int}(F_{\alpha(x)})) = F_{\alpha(x)} \subset V_{\alpha(x)}$. Therefore, (Int $(F_{\alpha(x)}, V_{\alpha(x)})$ is an ordered pair of open sets containing $f(x)$. Since f is u-weakly continuous, there exists an open set U_x of X containing x such that $f(U_x) \subset Cl(V_{\alpha(x)})$. Since X is compact, there exist a finite number of points $x_1, x_2, ..., x_n$ in X such that $X = \cup\{U_{x_i} \mid i = 1, 2, ..., n\}$. Therefore, we have $Y = \cup\{Cl(V_{\alpha(x_i)}) \mid i = 1, 2, ..., n\}$. This shows that Y is weakly compact.

COROLLARY 5.11. (Cammaroto and Lo Faro [2]). If $f: X \to Y$ is a γ-continuous surjection and X is compact, then Y is weakly compact.

PROOF. This follows immediately from Theorems 3.2 and 5.10.

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