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Research Article

Weighted Fusion Robust Steady-State Kalman Filters for Multisensor System with Uncertain Noise Variances

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A direct approach of designing weighted fusion robust steady-state Kalman filters with uncertain noise variances is presented. Based on the steady-state Kalman filtering theory, using the minimax robust estimation principle and the unbiased linear minimum variance (ULMV) optimal estimation rule, the six robust weighted fusion steady-state Kalman filters are designed based on the worst-case conservative system with the conservative upper bounds of noise variances. The actual filtering error variances of each fuser are guaranteed to have a minimal upper bound for all admissible uncertainties of noise variances. A Lyapunov equation method for robustness analysis is proposed. Their robust accuracy relations are proved. A simulation example verifies their robustness and accuracy relations.

1. Introduction

The multisensor information fusion (multisource information fusion or multisensor data fusion) has been applied widely in many fields including guidance, defense, robotics, target tracking, signal processing, GPS positioning, unmanned aerial vehicle (UAV), communication, command, control, computer, and intelligent systems (C^4I) and has attracted significant interest in recent years. Over the past two decades, many fused Kalman filtering algorithms have been developed to handle the state and signal estimation problems for the multisensor systems. The aim is how to combine the local estimators or local measurements obtained from each sensor to obtain a fused estimator, whose accuracy is higher than that of each local estimator [1]. The basic fused filtering algorithms include the centralized and distributed algorithms depending on whether the measurements' information is directly communicated to the fusion center or not [2]. For the centralized fusion algorithm, all the measurement data from local sensor are carried to the fusion center which can give the globally optimal fused state estimation, but its disadvantage is requiring a larger computation and communication burden. The distributed fusion algorithms can give the globally optimal or suboptimal state estimation by combing or weighting the local state estimators, whose advantages are that they can reduce the communication burden, and is more robust and reliable, and also has stronger fault tolerance. Under the ULMV rule, there are three optimal distributed fusion algorithms weighted by matrices, diagonal matrices, and scalars, respectively [3–5]. The optimal weighted measurement fusion algorithms can give the global optimal state estimation by weighting the local measurements to obtain a fusion measurement equation, accompanied with the state equation; based on a single Kalman filter, two optimal weighted measurements fusion algorithms were presented in [6–8].

The classical Kalman filtering is only suitable to handle the state estimation problems for the systems that the model parameters and noises variances are precisely known. However, in many application problems, the uncertainties of model parameters and noise variances are widely found. In the presence of uncertainties, the filter performance is degraded and may yield the filter divergence. In order to solve the filtering problems for uncertain systems, in recent years, several results have been derived on the design of robust Kalman filters. The so-called robust Kalman filtering is to design a filter to guarantee a minimal upper bound of the actual filtering error variances for all admissible uncertainties. There are two basic approaches to solve this problem, which are the Riccati equation approach [9–15] and the linear matrix inequality (LMI) approach [16–20]. These two methods have been applied to design the robust Kalman filter for uncertain systems with the uncertainties of model parameters [9–20], where the noise variances are assumed to be exactly known. However, the design of the robust Kalman filters with uncertain noise variances is seldom reported [21–24].

Several robust Kalman filters only consider the stochastic systems with single sensor, while the design of the multisensor information fusion robust Kalman filters is seldom considered and the robustness of the fused Kalman filters was not proved [25–27].

The robust Kalman filters design includes the finite-horizon (time-varying) robust Kalman filters design and the infinite-horizon (steady-state) robust Kalman filters design. The steady-state robust Kalman filters can be designed by taking the limits for the time-varying robust Kalman filters [9, 10, 14], and this is called the indirect design method. However, it is seldom reported that applying the steady-state Kalman filtering theory can directly design the steady-state robust Kalman filters, and this is called the direct design method.

For the systems with uncertain model parameters and/or noise variances, the covariance intersection (CI) fusion robust filtering method was presented in [28–31]. Its basic principle is that the robust CI fusion filter can be obtained by the convex combination of the robust local filters. Its advantage is that the cross-covariance of the local filtering errors is avoided, and it is suitable to handle the systems with unknown cross-covariance. Its disadvantage is that the local robust filters are assumed to be known, and the upper bound of the actual fused estimates has larger conservativeness because the information of cross-covariance is ignored. The geometric principle of the CI fusion is that the variance ellipse of the upper bound of actual fusion estimation error variance tightly encloses the intersection region of all variances ellipses of the upper bounds of actual local estimation error variances [32]. Recently, the ellipsoidal intersection (EI) fusion method with the cross-covariance information was presented in [33], which improves the robust accuracy of the CI fusion estimate. The comparison of the CI fusion method with several weighted fusion methods was given in [5]. The CI fusion method has been applied to many fields including remote sensing [34], simultaneous localization and mapping (SLAM) [35], rocket tracking and prediction [36], and vehicle localization [37].

Recently, the robust weighted fusion Kalman filters for multisensor time-varying systems with uncertain noise variances were presented by our team in [24], where the Lyapunov equation method of designing robust Kalman filters, the five robust weighted fusion time-varying Kalman filters based on the minimax robust estimation principle, and the concept of robust accuracy have been presented. By taking the limits for the time-varying robust weighted fusion Kalman filters, the corresponding robust steady-state Kalman filters have been also presented. However, based on the steady-state Kalman filtering theory, the problem to design directly the robust steady-state Kalman filters is not solved, and, based on the cross-covariance information, the problem to reduce the conservativeness of the upper bound of the CI fusion estimate is not solved. In addition, only one robust weighted measurement fuser was presented in [24].

In this paper, based on the steady-state Kalman filtering theory [38, 39], for the multisensor time-invariant system with noise variances uncertainties, using the minimax robust estimation principle, for the worst-case conservative system with the conservative upper bound of noise variances, three weighted state fusion robust steady-state Kalman filters will be presented. In order to improve the robust accuracy of the CI fuser and to reduce its conservativeness, a modified robust CI fuser with the cross-covariance information will be presented. In addition, two weighted measurement fusion robust steady-state Kalman filters will be presented. The proposed robust weighted fusion steady-state Kalman filtering approach is different from that in [24]. Our approach avoids finding the time-varying robust weighted fusers and their limits.

Finally, the robustness of the local and weighting fused robust Kalman filters is proved based on the Lyapunov equation method, which is completely different from the Riccati equation method and LMI method [9–20]. Their robust accuracy relations are strictly proved. In order to verify the correctness of theoretical accuracy relations, a Monte-Carlo simulation example for a three-sensor tracking system with uncertain noise variances is given.

The remainder of the paper is organized as follows. The problem formulation is given in Section 2. The local robust steady-state Kalman filters and their robustness analysis are presented in Section 3. Six weighted fusion robust steady-state Kalman filters and their robustness analysis are proposed in Section 4. The comparison of the robust accuracies of the local and fused robust Kalman filters is given in Section 5. Section 6 gives a simulation example. The conclusions are presented in Section 7.

2. Problem Formulation

Consider the multisensor time-invariant system with uncertain noise variances

$$x(t+1) = \Phi x(t) + \Gamma w(t), \tag{1}$$

$$y_i(t) = H_i x(t) + v_i(t), \quad i = 1, \dots, L, \tag{2}$$

where t is the discrete time, $x(t) \in R^n$ is the state to be estimated, L is the number of sensors, $y_i(t) \in R^{m_i}$ and $v_i(t) \in R^{m_i}$ are the measurement and measurement noise of the ith subsystem, and $w(t) \in R^r$ is the input noise. Φ, Γ, and H_i are known constant matrices with appropriate dimensions.

Assumption 1. $w(t)$ and $v_i(t)$ are uncorrelated white noises with zero mean and unknown uncertain actual variances $\overline{Q}$ and $\overline{R}_i$, and Q and R_i are known conservative upper bounds of $\overline{Q}$ and $\overline{R}_i$, respectively; that is,

$$\overline{Q} \le Q, \qquad \overline{R}_i \le R_i, \quad i = 1, \dots, L, \tag{3}$$

in the sense that $A \le B$ means that $B - A \ge 0$ is a positive semidefinite matrix.

Assumption 2. (Φ, Γ) is a completely controllable pair and (Φ, H_i) is a completely observable pair.

The problem is to design the local or fused robust steady-state Kalman filter $\hat{x}(t \mid t)$ such that the variances of actual filtering errors are guaranteed to have a minimal upper bound S for all admissible uncertain noise variances $\overline{Q}$ and $\overline{R}_i$ satisfying (3); that is, the actual filtering error variance satisfies

$$E\left[\left(x(t)-\hat{x}(t \mid t)\right)\left(x(t)-\hat{x}(t \mid t)\right)^T\right] \leq S, \tag{4}$$

where E is the mathematical expectation operator and the superscript T is the transpose.

Definition 3. The measurements $y_i(t)$ generated from the systems (1) and (2) with unknown actual noise variances $\overline{Q}$ and $\overline{R}_i$, $i = 1, \ldots, L$, are called the actual measurements $y_i(t)$, which are obtained via the sensors, and are available (known).

Definition 4. The measurements $y_i(t)$ generated from the systems (1) and (2) with the conservative upper bounds Q and R_i of noise variances are called the conservative measurement which are unavailable (unknown).

Definition 5. The Kalman filters with conservative measurements $y_i(t)$ are called the conservative Kalman filter which is unrealizable. The Kalman filters with the actual measurements $y_i(t)$ are called the actual Kalman filters.

3. Local Robust Steady-State Kalman Filters

According to the minimax robust optimal estimation principle [40], consider the worst-case conservative systems (1) and (2) with Assumptions 1–2 and with the conservative upper bounds Q and R_i of noise variances; the conservative local steady-state Kalman filters are given as [38, 39]

$$\hat{x}_i(t \mid t) = \Psi_i \hat{x}_i(t-1 \mid t-1) + K_i y_i(t), \quad i = 1, \ldots, L, \tag{5}$$

$$\Psi_i = \left[I_n - K_i H_i\right]\Phi, \qquad K_i = \Sigma_i H_i^T\left(H_i \Sigma_i H_i^T + R_i\right)^{-1}, \tag{6}$$

where I_n is an $n \times n$ identity matrix, Ψ_i is a stable matrix, and K_i is the steady-state filtering gain matrix. Here, the measurements $y_i(t)$ are unavailable as in Definition 4.

The conservative one-step prediction error variances Σ_i satisfy the Riccati equations

$$\Sigma_i = \Phi\left[\Sigma_i - \Sigma_i H_i^T\left(H_i \Sigma_i H_i^T + R_i\right)^{-1} H_i \Sigma_i\right]\Phi^T + \Gamma Q \Gamma^T. \tag{7}$$

The conservative local steady-state filtering error variances satisfy the Lyapunov equations

$$P_i = \Psi_i P_i \Psi_i^T + \left[I_n - K_i H_i\right]\Gamma Q \Gamma^T\left[I_n - K_i H_i\right]^T + K_i R_i K_i^T, \tag{8}$$

and the conservative local steady-state filtering error cross-covariances also satisfy the Lyapunov equations [4]

$$P_{ij} = \Psi_i P_{ij} \Psi_j^T + \left[I_n - K_j H_j\right]\Gamma Q \Gamma^T\left[I_n - K_j H_j\right]^T. \tag{9}$$

Notice that the conservative local Kalman filters (5) are unrealizable, because the conservative measurements $y_i(t)$ given in Definition 4 are unavailable. Only the actual measurements $y_i(t)$ measured via sensors are available, which are generated from systems (1) and (2) with the actual noise variances $\overline{Q}$ and $\overline{R}_i$, $i = 1, \ldots, L$. Therefore, replacing the conservative measurements $y_i(t)$ with the actual measurements $y_i(t)$ in (5), we obtain the actual local Kalman filters $\hat{x}_i(t \mid t)$.

Define the actual local steady-state filtering error variance as

$$\overline{P}_i = E\left[\tilde{x}_i(t \mid t)\tilde{x}_i^{\mathrm{T}}(t \mid t)\right], \qquad \tilde{x}_i(t \mid t) = x(t) - \hat{x}_i(t \mid t). \tag{10}$$

Substituting (1) and (5) into $\tilde{x}_i(t \mid t) = x(t) - \hat{x}_i(t \mid t)$, we obtain that

$$\begin{aligned}\tilde{x}_i(t \mid t) = {} & \Phi x(t-1) + \Gamma w(t-1) \\ & - \Psi_i \hat{x}(t-1 \mid t-1) - K_i y_i(t).\end{aligned} \tag{11}$$

Substituting the actual measurements (2) into (11) yields

$$\begin{aligned}\tilde{x}_i(t \mid t) = {} & \Psi_i \tilde{x}(t-1 \mid t-1) \\ & + \left(I_n - K_i H_i\right)\Gamma w(t-1) - K_i v_i(t).\end{aligned} \tag{12}$$

Substituting (12) into (10) yields the actual steady-state filtering error variances as

$$\overline{P}_i = \Psi_i \overline{P}_i \Psi_i^T + \left[I_n - K_i H_i\right]\Gamma \overline{Q} \Gamma^T\left[I_n - K_i H_i\right]^T + K_i \overline{R}_i K_i^T. \tag{13}$$

Applying (12), the actual steady-state filtering error cross-covariances are obtained as

$$\overline{P}_{ij} = \Psi_i \overline{P}_{ij} \Psi_j^T + \left[I_n - K_i H_i\right]\Gamma \overline{Q} \Gamma^T\left[I_n - K_j H_j\right]^T, \quad i \neq j. \tag{14}$$

Lemma 6 (see [38]). *Consider the Lyapunov equation with U being a symmetric matrix,*

$$P = \Psi P \Psi^T + U. \tag{15}$$

If Ψ is a stable matrix (all its eigenvalues are inside the unit circle) and $U \geq 0$, then P is unique and symmetric and $P \geq 0$.

Theorem 7. *Formultisensor uncertainsystems* (1) *and* (2) *with Assumptions 1–2, the actual local Kalman filters* (5) *with conservative upper bounds Q and R_i of noise variances are robust in the sense that, for all admissible actual variances $\overline{Q}$ and $\overline{R}_i$ satisfying* (3)*, one has*

$$\overline{P}_i \leq P_i, \quad i = 1, \ldots, L, \tag{16}$$

and they are called the local robust steady-state Kalman filters, and P_i is the minimal upper bound of $\overline{P}_i$.

Proof. Defining $\Delta P_i = P_i - \overline{P}_i$, subtracting (13) from (8) yields the Lyapunov equation

$$\Delta P_i = \Psi_i \Delta P_i \Psi_i^T + U_i, \tag{17}$$

where

$$U_i = [I_n - K_i H_i]\,\Gamma\left(Q - \overline{Q}\right)\Gamma^T [I_n - K_i H_i]^T + K_i\left(R_i - \overline{R}_i\right)K_i^T. \quad (18)$$

Applying (3) and (18) yields $U_i \geq 0$, and noting that Ψ_i is a stable matrix and applying Lemma 6, we have $\Delta P_i \geq 0$; that is $\overline{P}_i \leq P_i$ holds. Taking $\overline{Q} = Q$, $\overline{R}_i = R_i$, then the constraints (3) are satisfied, and $U_i = 0$. Applying Lemma 6 to (17) yields $\Delta P_i = 0$; that is, $P_i = \overline{P}_i$. For arbitrary other upper bound P_i^*, we have $P_i = \overline{P}_i \leq P_i^*$, which yields that P_i is the minimal upper bound of $\overline{P}_i$. The proof is completed. □

4. Weighted Fusion Robust Steady-State Kalman Filters

4.1. Four Robust Weighted State Fusion Steady-State Kalman Filters. For the worst-case conservative multisensor systems (1) and (2) with Assumptions 1–2, and with conservative upper bounds Q and R_i, under the ULMV fusion rule, the four conservative steady-state optimal weighted fusion Kalman filters are given by [3–5]

$$\hat{x}_\theta(t \mid t) = \sum_{i=1}^{L} \Omega_i^\theta \hat{x}_i(t \mid t), \quad \theta = m, s, d, \mathrm{CI} \quad (19)$$

with the constraint of unbiasedness

$$\sum_{i=1}^{L} \Omega_i^\theta = I_n, \quad (20)$$

where $\theta = m, s, d$ and CI denote the fusers weighted by matrices, scalars, diagonal matrices, and the CI fuser, respectively.

The optimal weighted matrices are computed as [3–5]

$$[\Omega_1^m \cdots \Omega_L^m] = \left(e^T P^{-1} e\right)^{-1} e^T P^{-1}, \quad e = [I_n \cdots I_n]^T, \quad (21)$$

$$P = \left(P_{ij}\right)_{nL\times nL} \quad (22)$$

with the definition $P_{ii} = P_i$.

The conservative fused filtering error variance is given as

$$P_m = \left(e^T P^{-1} e\right)^{-1}. \quad (23)$$

The optimal scalars weights are computed as

$$[\omega_1, \ldots, \omega_L] = \left(e^T P_{\mathrm{tr}}^{-1} e\right)^{-1} e^T P_{\mathrm{tr}}^{-1}, \quad (24)$$

$$\Omega_i^s = \omega_i I_n, \quad i = 1, \ldots, L, \quad (25)$$

where $e = [1 \cdots 1]^T$ and the $L \times L$ matrix $P_{\mathrm{tr}}(t \mid t)$ is defined as

$$P_{\mathrm{tr}} = \left(\mathrm{tr}\, P_{ij}\right)_{L\times L}, \quad (26)$$

where $\mathrm{tr}\, P_{ij}$ denotes the trace of P_{ij}. The conservative fused error variance is given as

$$P_s = \sum_{i=1}^{L}\sum_{j=1}^{L} \omega_i \omega_j P_{ij}. \quad (27)$$

The optimal diagonal matrix weights are computed as

$$\Omega_i^d = \mathrm{diag}\left(\omega_{i1}, \ldots, \omega_{in}\right),$$
$$[\omega_{1i} \cdots \omega_{Li}] = \left(e^T \left(P^{ii}\right)^{-1} e\right)^{-1} e^T \left(P^{ii}\right)^{-1}, \quad i = 1, \ldots, n,$$
$$e = [1 \cdots 1]^T, \qquad P^{ii} = \left(P_{sk}^{ii}\right)_{L\times L}, \quad (28)$$

where P_{sk}^{ii} is the (i, i)th diagonal element of P_{sk}, $s, k = 1, \ldots, L$.

The conservative fused error variance is given as

$$P_d = \sum_{i=1}^{L}\sum_{j=1}^{L} \Omega_i^d P_{ij} \Omega_j^{dT}. \quad (29)$$

The CI fusion weights are computed as [5, 31, 32]

$$\Omega_{\mathrm{CI}} = \left[\Omega_1^{\mathrm{CI}}, \ldots, \Omega_L^{\mathrm{CI}}\right], \qquad \Omega_i^{\mathrm{CI}} = \omega_i P_{\mathrm{CI}}^* P_i^{-1}, \quad i = 1, \ldots, L, \quad (30)$$

$$P_{\mathrm{CI}}^* = \left[\sum_{i=1}^{L} \omega_i P_i^{-1}\right]^{-1}. \quad (31)$$

The optimal weighting coefficients ω_i are obtained by minimizing the performance index

$$\min_{\omega_i} \mathrm{tr}\, P_{\mathrm{CI}}^* = \min_{\substack{\omega_i \in [0,1] \\ \omega_1 + \cdots + \omega_L = 1}} \mathrm{tr}\left\{\left[\sum_{i=1}^{L} \omega_i P_i^{-1}\right]^{-1}\right\}. \quad (32)$$

This needs to solve L-dimension nonlinear convex optimization problem, which can be solved by "fmincon" function in MATLAB toolbox.

Define

$$\Omega_\theta = \left[\Omega_1^\theta, \ldots, \Omega_L^\theta\right], \quad \theta = m, s, d, \mathrm{CI}. \quad (33)$$

From (20), we have

$$x(t) = \sum_{i=1}^{L} \Omega_i^\theta x(t), \quad \theta = m, s, d, \mathrm{CI}. \quad (34)$$

Subtracting (19) from (34) yields the conservative fused filtering errors as

$$\tilde{x}_\theta(t \mid t) = \sum_{i=1}^{L} \Omega_i^\theta \tilde{x}_i(t \mid t), \quad \theta = m, s, d, \mathrm{CI}. \quad (35)$$

Applying (34) and (35) yields the conservative fused filtering error variances having a unified form

$$P_\theta = \Omega_\theta P \Omega_\theta^T, \quad \theta = m, s, d, \mathrm{CI} \quad (36)$$

with P defined in (22). Replacing the conservative local Kalman filters $\hat{x}_i(t \mid t)$ in (19) by the actual local Kalman filters $\hat{x}_i(t \mid t)$, we obtain the actual weighted fusion Kalman filters.

Define the actual weighted fusion filtering error variance $\overline{P}_\theta(t \mid t)$ as

$$\overline{P}_\theta(t \mid t) = E\left[\tilde{x}_\theta(t \mid t)\tilde{x}_\theta^{\mathrm{T}}(t \mid t)\right], \quad \theta = m, s, d, \mathrm{CI}, \quad (37)$$

where $\tilde{x}_\theta(t \mid t) = x(t) - \hat{x}_\theta(t \mid t)$ and $\hat{x}_\theta(t \mid t)$ is the actual fused filters (19) with $\hat{x}_i(t \mid t)$ $(i = 1, \ldots, L)$ being the actual local Kalman filters. From (35) and (37), we obtain the actual fused filtering error variances as

$$\overline{P}_\theta = \Omega_\theta \overline{P} \Omega_\theta^T, \quad \theta = m, s, d, \mathrm{CI}, \quad (38)$$

with the definition

$$\overline{P} = (\overline{P}_{ij})_{nL\times nL}. \quad (39)$$

In particular, from (30), (36), and (38), taking $\theta = \mathrm{CI}$, the CI fuser has the conservative and actual steady-state fused error variances as

$$P_{\mathrm{CI}} = P_{\mathrm{CI}}^* \left[\sum_{i=1}^{L}\sum_{j=1}^{L} \omega_i P_i^{-1} P_{ij} P_j^{-1} \omega_j\right] P_{\mathrm{CI}}^*, \quad (40)$$

$$\overline{P}_{\mathrm{CI}} = P_{\mathrm{CI}}^* \left[\sum_{i=1}^{L}\sum_{j=1}^{L} \omega_i P_i^{-1} \overline{P}_{ij} P_j^{-1} \omega_j\right] P_{\mathrm{CI}}^*. \quad (41)$$

Notice that P_{CI} is defined with the conservative cross-covariance P_{ij}.

Lemma 8 (see [24]). *Let Λ be the $r \times r$ positive semidefinite matrix; that is, $\Lambda \geq 0$; then the following $rL \times rL$ matrix Λ_δ is also positive semidefinite; that is,*

$$\Lambda_\delta = \begin{bmatrix} \Lambda & \cdots & \Lambda \\ \vdots & \ddots & \vdots \\ \Lambda & \cdots & \Lambda \end{bmatrix}_{rL\times rL} \geq 0. \quad (42)$$

Lemma 9 (see [24]). *Let R_i be the $m_i \times m_i$ positive semidefinite matrix; that is, $R_i \geq 0$; the following $m \times m$ block-diagonal matrix R_δ is also positive semidefinite; that is,*

$$R_\delta = \operatorname{diag}(R_1, \ldots, R_L) \geq 0 \quad (43)$$

with $m = m_1 + \cdots + m_L$.

Theorem 10. *For multisensor uncertain systems* (1) *and* (2) *with Assumptions 1–2 and with conservative upper bounds Q and R_i of noise variances, the actual four steady-state weighted Kalman fusers are robust in the sense that, for all admissible actual variances $\overline{Q}$ and $\overline{R}_i$ satisfying* (3)*, one has*

$$\overline{P}_\theta \leq P_\theta, \quad \theta = m, s, d, CI, \quad (44)$$

and they are called the robust weighted fusion steady-state Kalman filters, and P_θ is the minimal upper bound of $\overline{P}_\theta$.

Proof. Defining $\Delta P_\theta = P_\theta - \overline{P}_\theta$, subtracting (36) from (38) yields

$$\Delta P_\theta = \Omega_\theta \left(P - \overline{P}\right) \Omega_\theta^T. \quad (45)$$

In order to prove the robustness $\Delta P_\theta = P_\theta - \overline{P}_\theta \geq 0$, we only need to prove that the inequality $P - \overline{P} \geq 0$ holds.

Applying (8) and (9) yields the following Lyapunov equation

$$P = \Psi P \Psi^T + U Q_a U^T + K R K^T, \quad (46)$$

where we define

$$\Psi = \begin{bmatrix} \Psi_1 & & 0 \\ & \ddots & \\ 0 & & \Psi_L \end{bmatrix},$$

$$U = \begin{bmatrix} (I_n - K_1 H_1)\Gamma & & 0 \\ & \ddots & \\ 0 & & (I_n - K_L H_L)\Gamma \end{bmatrix},$$

$$Q_a = \begin{bmatrix} Q & & Q \\ & \ddots & \\ Q & & Q \end{bmatrix}, \quad (47)$$

$$K = \begin{bmatrix} K_1 & & 0 \\ & \ddots & \\ 0 & & K_L \end{bmatrix},$$

$$R = \begin{bmatrix} R_1 & & 0 \\ & \ddots & \\ 0 & & R_L \end{bmatrix}.$$

Similarly, applying (13) and (14), $\overline{P}$ can be expressed as

$$\overline{P} = \Psi \overline{P} \Psi^T + U \overline{Q}_a U^T + K \overline{R} K^T. \quad (48)$$

with the definitions

$$\overline{Q}_a = \begin{bmatrix} \overline{Q} & & \overline{Q} \\ \vdots & \ddots & \vdots \\ \overline{Q} & & \overline{Q} \end{bmatrix}, \qquad \overline{R} = \begin{bmatrix} \overline{R}_1 & & 0 \\ & \ddots & \\ 0 & & \overline{R}_L \end{bmatrix}. \quad (49)$$

Since Ψ_i is a stable matrix, then the eigenvalues of the matrix Ψ_i are all within the unit circle and are determined from its characteristic equation $\det(\lambda I_n - \Psi_i) = 0$. The eigenvalues of the matrix Ψ are determined from the characteristic equation

$$\det(\lambda I_{nL} - \Psi) = \det(\lambda I_n - \Psi_1) \cdots \det(\lambda I_n - \Psi_L) = 0, \quad (50)$$

which yields that Ψ is also a stable matrix, because the eigenvalues of Ψ_i are also the eigenvalues of Ψ.

Denoting $\Delta P = P - \overline{P}$, subtracting (48) from (46) yields the Lyapunov equation

$$\Delta P = \Psi \Delta P \Psi^T + U\left(Q_a - \overline{Q}_a\right)U^T + K\left(R - \overline{R}\right)K^T. \quad (51)$$

Noting that $\overline{Q} \leq Q, \overline{R}_i \leq R_i$, applying Lemmas 8 and 9 yields $\overline{Q}_a \leq Q_a, \overline{R} \leq R$; therefore $U(Q_a - \overline{Q}_a)U^T + K(R -$

$\overline{R})K^T \geq 0$. Noting that Ψ is a stable matrix and applying Lemma 6 to (51), we have $\Delta P \geq 0$; that is,

$$\overline{P} \leq P. \tag{52}$$

From (45) and (52), we have $\Delta P_\theta \geq 0$, so (44) holds. If P_θ^* is other upper bound of $\overline{P}_\theta$, taking $\overline{Q} = Q, \overline{R}_i = R_i$, we have $\overline{Q}_a = Q_a, \overline{R} = R$, so applying Lemma 6 to (51) yields $\Delta P = 0$; that is, $\overline{P} = P$. Hence, applying (45) yields $\Delta P_\theta = 0$, so $P_\theta = \overline{P}_\theta \leq P_\theta^*$, which yields that P_θ is the minimal upper bound of $\overline{P}_\theta$. The proof is completed. □

Remark 11. From (19) and (30), the CI fusion Kalman filter can be rewritten as

$$\hat{x}_{\mathrm{CI}}(t \mid t) = P_{\mathrm{CI}}^* \sum_{i=1}^{L} \omega_i P_i^{-1} \hat{x}_i(t \mid t), \tag{53}$$

which can be reviewed as a special fuser weighted by matrices with weights $\Omega_i^{\mathrm{CI}} = \omega_i P_{\mathrm{CI}}^* P_i^{-1}$.

From (31) and (53), the CI fuser has the convex combination form as

$$(P_{\mathrm{CI}}^*)^{-1} \hat{x}_{\mathrm{CI}}(t \mid t) = \sum_{i=1}^{L} \omega_i P_i^{-1} \hat{x}_i(t \mid t), \tag{54}$$

$$(P_{\mathrm{CI}}^*)^{-1} = \sum_{i=1}^{L} \omega_i P_i^{-1}, \tag{55}$$

and it is proved [5] that P_{CI}^* is a conservative upper bound of $\overline{P}_{\mathrm{CI}}$,

$$\overline{P}_{\mathrm{CI}} \leq P_{\mathrm{CI}}^*. \tag{56}$$

From (31) or (55), the upper bound P_{CI}^* is defined without the cross-covariance information and is only determined by the conservative local variances P_i, so that P_{CI}^* has certain conservativeness; that is, P_{CI}^* is not a minimal upper bound of $\overline{P}_{\mathrm{CI}}$ for all admissible uncertainties of noise variances. From Theorem 10, we have the robustness

$$\overline{P}_{\mathrm{CI}} \leq P_{\mathrm{CI}} \tag{57}$$

and P_{CI} defined by (40) with the conservative cross-covariance P_{ij}, is the minimal upper bound of $\overline{P}_{\mathrm{CI}}$. Hence, we have

$$P_{\mathrm{CI}} \leq P_{\mathrm{CI}}^*; \tag{58}$$

that is, the upper bound P_{CI} has less conservativeness than P_{CI}^*.

4.2. Two Robust Weighted Measurement Fusion Steady-State Kalman Filters. For the worst-case conservative systems (1) and (2) with Assumptions 1–2, and with the conservative upper bounds Q and R_i of noise variances, if H_i have the common $m \times n$ right factor H, that is,

$$H_i = M_i H, \quad i = 1, \ldots, L, \tag{59}$$

where M_i is $m_i \times n$ matrix, and the matrix $M^{(0)T} R^{(0)-1} M^{(0)}$ or $H^{(0)T} R^{(0)-1} H^{(0)}$ is assumed to be invertible, with the definition

$$M^{(0)} = \left[M_1^T, \ldots, M_L^T\right]^T, \qquad H^{(0)} = \left[H_1^T, \ldots, H_L^T\right]^T, \tag{60}$$

we have the conservative centralized fusion measurement equation

$$y^{(0)}(t) = H^{(0)} x(t) + v^{(0)}(t), \tag{61}$$

$$y^{(0)}(t) = \left[y_1^T(t), \ldots, y_L^T(t)\right]^T, \quad v^{(0)}(t) = \left[v_1^T(t), \ldots, v_L^T(t)\right]^T, \tag{62}$$

Where, according to Assumption 1, the fused noise $v^{(0)}(t)$ has the conservative variance matrix

$$R^{(0)} = \mathrm{diag}(R_1, \ldots, R_L). \tag{63}$$

If (59) holds, then (61) becomes

$$y^{(0)}(t) = M^{(0)} H x(t) + v^{(0)}(t). \tag{64}$$

If $M^{(0)T} R^{(0)-1} M^{(0)}$ is invertible, applying the weighted least squares (WLS) method [34], $Hx(t)$ has the WLS estimate

$$y^{(1)}(t) = \left[M^{(0)T} R^{(0)-1} M^{(0)}\right]^{-1} M^{(0)T} R^{(0)-1} y^{(0)}(t). \tag{65}$$

Substituting (64) into (65) yields the first conservative weighted measurement fusion equation

$$y^{(1)}(t) = H^{(1)} x(t) + v^{(1)}(t), \quad H^{(1)} = H, \tag{66}$$

$$v^{(1)}(t) = \left[M^{(0)T} R^{(0)-1} M^{(0)}\right]^{-1} M^{(0)T} R^{(0)-1} v^{(0)}(t), \tag{67}$$

Where, from (67), the fused noise $v^{(1)}(t)$ has the variance matrix

$$R^{(1)} = \left[M^{(0)T} R^{(0)-1} M^{(0)}\right]^{-1}. \tag{68}$$

If $H^{(0)T} R^{(0)-1} H^{(0)}$ is invertible, from (61), $x(t)$ has the WLS estimate

$$y^{(2)}(t) = \left[H^{(0)T} R^{(0)-1} H^{(0)}\right]^{-1} H^{(0)T} R^{(0)-1} y^{(0)}(t). \tag{69}$$

Substituting (61) into (69) yields the second conservative measurement fusion equation

$$y^{(2)}(t) = H^{(2)} x(t) + v^{(2)}(t), \quad H^{(2)} = I_n, \tag{70}$$

$$v^{(2)}(t) = \left[H^{(0)T} R^{(0)-1} H^{(0)}\right]^{-1} H^{(0)T} R^{(0)-1} v^{(0)}(t), \tag{71}$$

where from (71) the fused noise $v^{(2)}(t)$ has the variance matrix

$$R^{(2)} = \left[H^{(0)T} R^{(0)-1} H^{(0)}\right]^{-1}. \tag{72}$$

Hence, the centralized fusion system and two measurement fusion systems have a unified form as

$$x(t+1) = \Phi x(t) + \Gamma w(t), \quad y^{(j)}(t) = H^{(j)} x(t) + v^{(j)}(t), \quad j = 0, 1, 2. \tag{73}$$

Theorem 12. *For the worst-case multisensor uncertain system with Assumptions 1–2, and with the conservative upper bounds* Q *and* R_i *of noise variances, the robust centralized fusion Kalman filter and two robust weighted measurement fusion Kalman filters have a unified form as*

$$\hat{x}^{(j)}(t \mid t) = \Psi^{(j)}\hat{x}^{(j)}(t-1 \mid t-1) + K^{(j)}y^{(j)}(t), \quad j = 0,1,2$$
$$\Psi^{(j)} = \left[I_n - K^{(j)}H^{(j)}\right]\Phi,$$
$$K^{(j)} = \Sigma^{(j)}H^{(j)T}\left[H^{(j)}\Sigma^{(j)}H^{(j)T} + R^{(j)}\right]^{-1}$$
$$\Sigma^{(j)} = \Phi\left[\Sigma^{(j)} - \Sigma^{(j)}H^{(j)T}\left(H^{(j)}\Sigma^{(j)}H^{(j)T} + R^{(j)}\right)^{-1} \times H^{(j)}\Sigma^{(j)}\right]\Phi^T + \Gamma Q\Gamma^T, \tag{74}$$

and the conservative and actual fused error variances are, respectively, given as

$$P^{(j)} = \Psi^{(j)}P^{(j)}\Psi^{(j)T} + \left[I_n - K^{(j)}H^{(j)}\right]\Gamma Q\Gamma^T\left[I_n - K^{(j)}H^{(j)}\right]^T + K^{(j)}R^{(j)}K^{(j)T},$$
$$\overline{P}^{(j)} = \Psi^{(j)}\overline{P}^{(j)}\Psi^{(j)T} + \left[I_n - K^{(j)}H^{(j)}\right]\Gamma\overline{Q}\Gamma^T\left[I_n - K^{(j)}H^{(j)}\right]^T + K^{(j)}\overline{R}^{(j)}K^{(j)T}. \tag{75}$$

From (62), (67), *and* (71), *applying Assumption 1 yields*

$$\overline{R}^{(0)} = \operatorname{diag}\left(\overline{R}_1, \ldots, \overline{R}_L\right),$$
$$\overline{R}^{(1)} = \left[M^{(0)T}\overline{R}^{(0)-1}M^{(0)}\right]^{-1}M^{(0)T}R^{(0)-1} \times \overline{R}^{(0)-1}R^{(0)-1}M^{(0)}\left[M^{(0)T}\overline{R}^{(0)-1}M^{(0)}\right]^{-1}, \tag{76}$$
$$\overline{R}^{(2)} = \left[H^{(0)T}\overline{R}^{(0)-1}H^{(0)}\right]^{-1}H^{(0)T}R^{(0)-1} \times \overline{R}^{(0)-1}R^{(0)-1}H^{(0)}\left[H^{(0)T}\overline{R}^{(0)-1}H^{(0)}\right]^{-1}.$$

We have three equivalent robust Kalman filters

$$\hat{x}^{(0)}(t \mid t) = \hat{x}^{(1)}(t \mid t) = \hat{x}^{(2)}(t \mid t) \tag{77}$$

with the robustness

$$\overline{P}^{(j)} \le P^{(j)}, \quad j = 0,1,2, \tag{78}$$

and we have the accuracy relations

$$P^{(0)} = P^{(1)} = P^{(2)}, \qquad \overline{P}^{(0)} = \overline{P}^{(1)} = \overline{P}^{(2)}, \tag{79}$$

$$\operatorname{tr}P^{(0)} = \operatorname{tr}P^{(1)} = \operatorname{tr}P^{(2)}, \qquad \operatorname{tr}\overline{P}^{(0)} = \operatorname{tr}\overline{P}^{(1)} = \operatorname{tr}\overline{P}^{(2)}. \tag{80}$$

Proof. The robust Kalman filters (74) have the equivalent information filter form [38]

$$\hat{x}^{(j)}(t \mid t) = \Psi^{(j)}\hat{x}^{(j)}(t-1 \mid t-1) + K^{(j)}y^{(j)}(t), \quad j = 0,1,2,$$
$$\Psi^{(j)} = P^{(j)}\Sigma^{(j)}\Phi,$$
$$K^{(j)} = P^{(j)}H^{(j)T}R^{(j)-1},$$
$$\Sigma^{(j)} = \Phi P^{(j)}\Phi^T + \Gamma Q\Gamma^T,$$
$$P^{(j)} = \Sigma^{(j)-1} + H^{(j)}R^{(j)-1}H^{(j)T}. \tag{81}$$

From (81), we see that, in order to prove (77) and (79), we only need to prove

$$H^{(0)T}R^{(0)-1}H^{(0)} = H^{(1)T}R^{(1)-1}H^{(1)} = H^{(2)T}R^{(2)-1}H^{(2)},$$
$$H^{(0)T}R^{(0)-1}y^{(0)}(t) = H^{(1)T}R^{(1)-1}y^{(1)}(t) = H^{(2)T}R^{(2)-1}y^{(2)}(t). \tag{82}$$

Applying (59), (60), (63), (65), (68), (69), and (72), we easily verify that (82) hold. In order to prove (78), from (79), we only need to prove $\overline{P}^{(0)} \le P^{(0)}$; that is, the centralized fusion Kalman filter is robust. In fact, applying (3) and Lemma 9 yields $\overline{R}^{(0)} \le R^{(0)}$, so applying the derivation similar to Theorem 7 yields $\overline{P}^{(0)} \le P^{(0)}$. The proof is completed. □

5. Robust Accuracy Analysis

Theorem 13. *For multisensor uncertain systems* (1) *and* (2) *with Assumptions 1–2, the local and fused Kalman filters have the following accuracy relations:*

$$\overline{P}_\theta \le P_\theta, \quad \theta = 1, \ldots, L, m, s, d, CI, \tag{83}$$
$$\overline{P}^{(j)} \le P^{(j)}, \quad j = 0,1,2, \tag{84}$$
$$P^{(0)} = P^{(1)} = P^{(2)}, \qquad \overline{P}^{(0)} = \overline{P}^{(1)} = \overline{P}^{(2)}, \tag{85}$$
$$P^{(0)} \le P_m, P_m \le P_\theta, \quad \theta = 1, \ldots, L, m, s, d, \tag{86}$$
$$P_{CI} \le P_{CI}^*, \qquad \overline{P}_{CI} \le P_{CI}^*, \tag{87}$$
$$\operatorname{tr}\overline{P}_\theta \le \operatorname{tr}P_\theta, \quad \theta = 1, \ldots, L, m, s, d, CI, \tag{88}$$
$$\operatorname{tr}\overline{P}^{(j)} \le \operatorname{tr}P^{(j)}, \quad j = 0,1,2, \tag{89}$$
$$\operatorname{tr}P^{(0)} = \operatorname{tr}P^{(1)} = \operatorname{tr}P^{(2)}, \qquad \operatorname{tr}\overline{P}^{(0)} = \operatorname{tr}\overline{P}^{(1)} = \operatorname{tr}\overline{P}^{(2)}, \tag{90}$$
$$\operatorname{tr}P^{(0)} \le \operatorname{tr}P_m, \qquad \operatorname{tr}P_m \le \operatorname{tr}P_\theta, \quad \theta = 1, \ldots, L, m, s, d, \tag{91}$$
$$\operatorname{tr}P_{CI} \le \operatorname{tr}P_{CI}^*, \qquad \operatorname{tr}\overline{P}_{CI} \le \operatorname{tr}P_{CI}^*, \tag{92}$$
$$\operatorname{tr}P_m \le \operatorname{tr}P_d \le \operatorname{tr}P_s \le \operatorname{tr}P_i, \quad i = 1, \ldots, L. \tag{93}$$

TABLE 1: The actual and conservative filtering error variances of the local and fused robust Kalman filters.

P_1	P_2	P_3	P_m	P_d
$\begin{bmatrix} 0.2492 & 0.1855 \\ 0.1855 & 0.3046 \end{bmatrix}$	$\begin{bmatrix} 0.4035 & 0.0645 \\ 0.0645 & 0.121 \end{bmatrix}$	$\begin{bmatrix} 0.2087 & 0.1642 \\ 0.1642 & 0.2865 \end{bmatrix}$	$\begin{bmatrix} 0.0775 & 0.0416 \\ 0.0416 & 0.1167 \end{bmatrix}$	$\begin{bmatrix} 0.1039 & 0.0438 \\ 0.0438 & 0.1173 \end{bmatrix}$
$\overline{P}_1$	$\overline{P}_2$	$\overline{P}_3$	$\overline{P}_m$	$\overline{P}_d$
$\begin{bmatrix} 0.2019 & 0.1497 \\ 0.1497 & 0.2447 \end{bmatrix}$	$\begin{bmatrix} 0.2922 & 0.0448 \\ 0.0448 & 0.0892 \end{bmatrix}$	$\begin{bmatrix} 0.1742 & 0.1353 \\ 0.1353 & 0.2327 \end{bmatrix}$	$\begin{bmatrix} 0.0607 & 0.03 \\ 0.03 & 0.0878 \end{bmatrix}$	$\begin{bmatrix} 0.0828 & 0.0337 \\ 0.0337 & 0.0883 \end{bmatrix}$
P_s	P_{CI}^*	P_{CI}	$P^{(0)}$	$P^{(1)} = P^{(2)}$
$\begin{bmatrix} 0.1172 & 0.0614 \\ 0.0614 & 0.1554 \end{bmatrix}$	$\begin{bmatrix} 0.2449 & 0.0882 \\ 0.0882 & 0.1573 \end{bmatrix}$	$\begin{bmatrix} 0.1161 & 0.0487 \\ 0.0487 & 0.1199 \end{bmatrix}$	$\begin{bmatrix} 0.0689 & 0.0414 \\ 0.0414 & 0.1106 \end{bmatrix}$	$\begin{bmatrix} 0.0689 & 0.0414 \\ 0.0414 & 0.1106 \end{bmatrix}$
$\overline{P}_s$	$\overline{P}_{\mathrm{CI}}$		$\overline{P}^{(0)}$	$\overline{P}^{(1)} = \overline{P}^{(2)}$
$\begin{bmatrix} 0.0896 & 0.0485 \\ 0.0485 & 0.1235 \end{bmatrix}$	$\begin{bmatrix} 0.0902 & 0.036 \\ 0.036 & 0.0914 \end{bmatrix}$		$\begin{bmatrix} 0.0537 & 0.0304 \\ 0.0304 & 0.0830 \end{bmatrix}$	$\begin{bmatrix} 0.0537 & 0.0304 \\ 0.0304 & 0.0830 \end{bmatrix}$

Proof. The relations (83)–(85) were proved in Theorems 7–12. The relation (86) was proved in [5]. The relation (87) was proved in Remark 11. Taking the trace operations for (83)–(87) yields (88)–(92). The relation (93) was proved in [5]. The proof is completed. □

Remark 14. The trace of the error variance matrix is used to represent the filtering accuracy which is equal to the sum of the filtering error variances for the components of state. The smaller trace means the higher accuracy and the larger trace means the lower accuracy. The accuracy relations (88) and (89) mean that, for any admissible $\overline{Q}$ and $\overline{R}_i$ satisfying (3), the actual accuracy $\operatorname{tr}\overline{P}_\theta$ or $\operatorname{tr}\overline{P}^{(j)}$ of the local or fused filters $\hat{x}_\theta(t \mid t), \theta = 1, \ldots, L, m, d, s, \mathrm{CI}$, $j = 0, 1, 2$, is globally controlled by $\operatorname{tr} P_\theta$ or $\operatorname{tr} P^{(j)}$, and $\operatorname{tr} P_\theta$ or $\operatorname{tr} P^{(j)}$ is independent of all admissible $\overline{Q}$ and $\overline{R}_i$ satisfying (3). Therefore, $\operatorname{tr} P_\theta$ or $\operatorname{tr} P^{(j)}$ is called robust accuracy of $\hat{x}_\theta(t \mid t)$ and is also called global accuracy [24], and $\operatorname{tr}\overline{P}_\theta$ or $\operatorname{tr}\overline{P}^{(j)}$ is called its actual accuracy. Notice that, for different $\overline{Q}$ and $\overline{R}_i$, the corresponding actual accuracies are also different; that is, $\operatorname{tr}\overline{P}_\theta$ or $\operatorname{tr}\overline{P}^{(j)}$ is related to admissible $\overline{Q}$ and $\overline{R}_i$. From (88) and (89), we see that the robust accuracy is the admissible lowest actual accuracy; that is, it is the lowest bound of the actual accuracies. The smaller $\operatorname{tr} P_\theta$ or $\operatorname{tr} P^{(j)}$ means the higher robust accuracy, and the larger $\operatorname{tr} P_\theta$ or $\operatorname{tr} P^{(j)}$ means the lower robust accuracy.

Remark 15. From Theorem 10, we see that P_{CI} with the cross-covariance information is a minimal upper bound of $\overline{P}_{\mathrm{CI}}$, and, from (57) and (58), we have the robust accuracy relation

$$\operatorname{tr}\overline{P}_{\mathrm{CI}} \le \operatorname{tr} P_{\mathrm{CI}} \le \operatorname{tr} P_{\mathrm{CI}}^*. \tag{94}$$

This means that the robust accuracy of the CI fuser is $\operatorname{tr} P_{\mathrm{CI}}$ rather than $\operatorname{tr} P_{\mathrm{CI}}^*$, so that the modified robust accuracy $\operatorname{tr} P_{\mathrm{CI}}$ is higher than the original robust accuracy $\operatorname{tr} P_{\mathrm{CI}}^*$ in [24], and it also develops and extends the ellipsoidal intersection (EI) fuser with the cross-covariance information [33].

6. Simulation Example

Consider a three-sensor tracking system with uncertain noise variances

$$\begin{aligned} x(t+1) &= \Phi x(t) + \Gamma w(t), \\ y_i(t) &= H_i x(t) + v_i(t), \quad i = 1, 2, 3, \\ \Phi &= \begin{bmatrix} 1 & T_0 \\ 0 & 1 \end{bmatrix}, \qquad \Gamma = \begin{bmatrix} 0.5T_0^2 \\ T_0 \end{bmatrix}, \\ H_1 &= \begin{bmatrix} 1 & 0 \end{bmatrix}, \qquad H_2 = I_2, \qquad H_3 = \begin{bmatrix} 1 & 0 \end{bmatrix}, \end{aligned} \tag{95}$$

where $T_0 = 0.25$ is the sampled period, $x(t) = [x_1(t), x_2(t)]^T$ is the state, $x_1(t)$ and $x_2(t)$ are the position and velocity of target at time $tT_0 \cdot w(t)$, and $v_i(t)$ are independent Gaussian white noises with zero mean and unknown actual variances $\overline{Q}$ and $\overline{R}_i$, respectively. Taking the conservative noise variances Q and R_i satisfies $\overline{Q} \le Q$ and $\overline{R}_i \le R_i$. In the simulation, we take $Q = 1$, $R_1 = 0.8$, $R_2 = \operatorname{diag}(8, 0.36)$, $R_3 = 0.64$, $\overline{Q} = 0.8$, $\overline{R}_1 = 0.65$, $\overline{R}_2 = \operatorname{diag}(6, 0.25)$, and $\overline{R}_3 = 0.54$.

Applying the local and fused robust steady-state Kalman filters, the actual and conservative filtering error variances are obtained in Table 1.

Table 1 verifies the accuracy relations (83)–(87), and it is easy to be verified that $P_{\mathrm{CI}} \le P_{\mathrm{CI}}^*$ is satisfied. The traces of the error variances of the local and fused Kalman filters are compared in Table 2, which verify the accuracy relations (88)–(93).

In order to give a geometric interpretation of the matrix accuracy relations, the covariance ellipse is defined as the locus of points $\{x : x^T P^{-1} x = c\}$, where P is the variance matrix and c is a constant. Generally, we select $c = 1$. It has been proved [32] that $P_1 \le P_2$ is equivalent to the covariance ellipse form P_1 by contains that form by P_2.

From Figures 1 and 2, we see that the ellipse of $P^{(j)}$ ($j = 0, 1, 2$) is enclosed in these of P_m, P_d, P_s, and $P_i (i = 1, 2, 3)$ which verifies the matrix accuracy relations (85)–(87). From Figure 3, we see that the ellipses of $\overline{P}^{(j)}$ ($j = 0, 1, 2$) or $\overline{P}_\theta$ ($\theta = 1, 2, 3, m, s, d, \mathrm{CI}$) are enclosed in these of the

Table 2: The accuracy comparison of local and fused robust Kalman filters.

$\mathrm{tr}P_1$	$\mathrm{tr}P_2$	$\mathrm{tr}P_3$	$\mathrm{tr}P_m$	$\mathrm{tr}P_d$	$\mathrm{tr}P_s$	$\mathrm{tr}P_{\mathrm{CI}}^*$, $\mathrm{tr}P_{\mathrm{CI}}$
0.5538	0.5245	0.4952	0.1942	0.2212	0.2725	0.4022, 0.2360
$\mathrm{tr}\overline{P}_1$	$\mathrm{tr}\overline{P}_2$	$\mathrm{tr}\overline{P}_3$	$\mathrm{tr}\overline{P}_m$	$\mathrm{tr}\overline{P}_d$	$\mathrm{tr}\overline{P}_s$	$\mathrm{tr}\overline{P}_{\mathrm{CI}}$
0.4465	0.3815	0.4069	0.1485	0.1711	0.2131	0.1817
$\mathrm{tr}P^{(0)}$	$\mathrm{tr}\overline{P}^{(0)}$	$\mathrm{tr}P^{(1)} = \mathrm{tr}\,P^{(2)}$	$\mathrm{tr}\,\overline{P}^{(1)} = \mathrm{tr}\,\overline{P}^{(2)}$			
0.1795	0.1367	0.1795	0.1367			

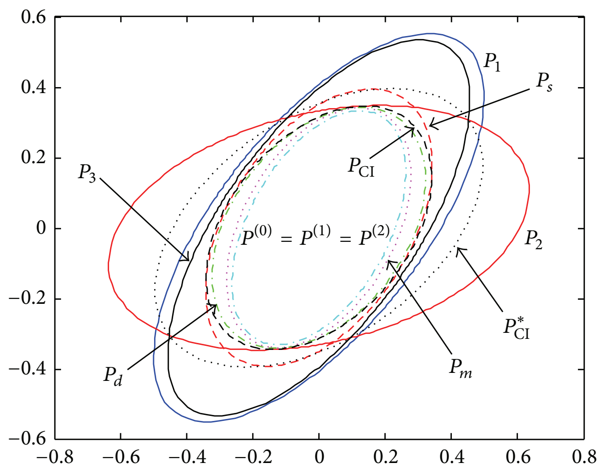

Figure 1: The ellipses of the conservative upper bounds of actual filtering error variances.

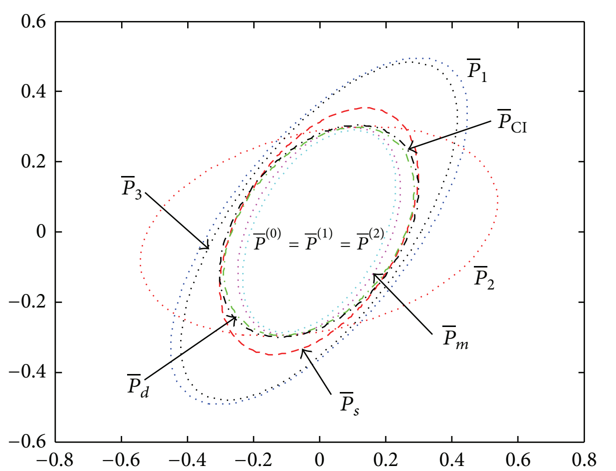

Figure 2: The ellipses of the actual filtering error variances.

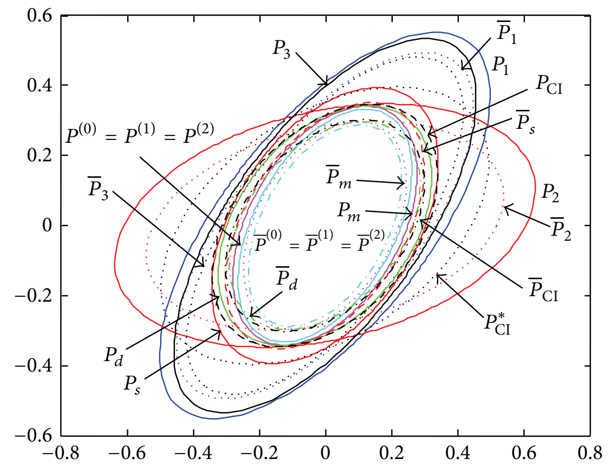

Figure 3: The ellipses of the actual and conservative filtering error variances.

conservative upper bound $P^{(j)}$ or P_θ, respectively, which verify the robustness (83)–(87).

In order to verify the above theoretical accuracy relations, taking $\rho = 200$ Monte-Carlo simulation runs, the mean square error (MSE) value at time t of local or fused Kalman filters is defined as

$$\mathrm{MSE}_\theta(t) = \frac{1}{\rho}\sum_{k=1}^{\rho}\left(x^{(k)}(t) - \hat{x}_\theta^{(k)}(t \mid t)\right)^T \times \left(x^{(k)}(t) - \hat{x}_\theta^{(k)}(t \mid t)\right), \quad \theta = 1, 2, 3, m, s, d, \mathrm{CI},$$

$$\mathrm{MSE}_j(t) = \frac{1}{\rho}\sum_{k=1}^{\rho}\left(x^{(k)}(t) - \hat{x}^{(k)(j)}(t \mid t)\right)^T \times \left(x^{(k)}(t) - \hat{x}^{(k)(j)}(t \mid t)\right), \quad j = 0, 1, 2, \tag{96}$$

where $x^{(k)}(t)$ or $\hat{x}_\theta^{(k)}(t \mid t)$ denotes the kth realization of $x(t)$ or $\hat{x}_\theta(t \mid t)$ and $\hat{x}^{(k)(0)}(t \mid t)$ denotes the kth realization of the centralized fuser. $t = 1, \ldots, t_f$ are the recursive steps, the final step is $t_f = 500$, and the MSE curves of the local and weighted fusion robust Kalman filters are shown in Figure 4, where the straight lines denote the traces of the theoretical error variances, respectively, the curves denote the values of the $\mathrm{MSE}_\theta(t)$ or $\mathrm{MSE}_j(t)$, and $\mathrm{tr}\,P^{(0)} = \mathrm{tr}\,P^{(1)} = \mathrm{tr}\,P^{(2)}$, $\mathrm{tr}\,\overline{P}^{(0)} = \mathrm{tr}\,\overline{P}^{(1)} = \mathrm{tr}\,\overline{P}^{(2)}$.

According to the consistency of the sampled variance, we have

$$\begin{gathered}\mathrm{MSE}_\theta(t) \longrightarrow \mathrm{tr}\,\overline{P}_\theta, \quad \text{as } t \longrightarrow \infty, \\ \rho \longrightarrow \infty, \quad (\theta = 1, 2, 3, m, s, d, \mathrm{CI}), \\ \mathrm{MSE}_j(t) \longrightarrow \mathrm{tr}\,\overline{P}^{(j)}, \quad \text{as } t \longrightarrow \infty, \\ \rho \longrightarrow \infty, \quad (j = 0, 1, 2).\end{gathered} \tag{97}$$

From Figure 4, we see that when t is sufficiently large, the values of $\mathrm{MSE}_i(t)$ $(i = \theta, j)$ are close to the corresponding

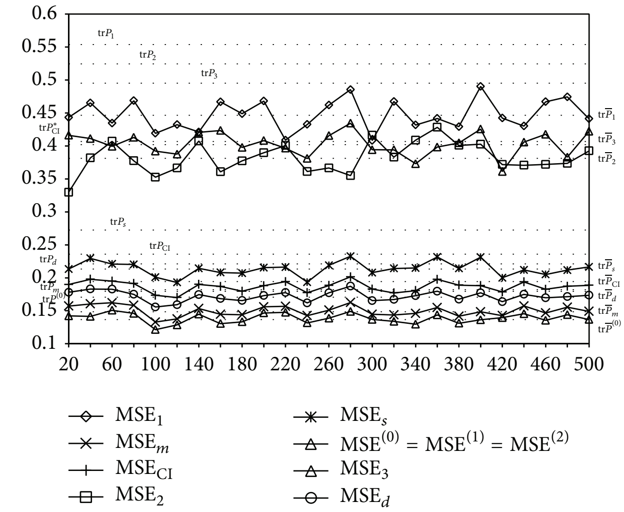

FIGURE 4: The MSE curves for local and fused robust Kalman filters.

theoretical values $\mathrm{tr}\,\overline{P}_i$, which verifies the consistency (97). We also see that $\mathrm{MSE}_i(t) \leq \mathrm{tr}\,P_i$, which verifies the accuracy relations (88) and (89).

7. Conclusions

For the multisensor time-invariant uncertain systems with uncertainties of noise variances, according to the minimax robust estimation principle, based on the worst-case conservative system with conservative upper bound of noise variances, using the ULMV optimal estimation rule and the steady-state Kalman filtering theory, the six robust weighted fusion steady-state Kalman filters have been presented. Their robustness was proved by using the Lyapunov equation method and their robust accuracy relations were proved. Compared with the method and results in [24], the main new contributions are as follows.

(1) Reference [24] presented an indirect design method for obtaining the robust weighted fusion steady-state Kalman filters which are obtained by taking the limit operations for the proposed time-varying robust Kalman fusers. This paper presented a simple direct design method based on the steady-state Kalman filtering theory, which can directly obtain the same steady-state results in [24], and avoided finding the time-varying robust Kalman fusers.

(2) A modified CI fusion method has been presented. A minimal upper bound of actual CI fusion error variances was presented based on the cross-covariance information. It reduces the conservativeness of the original upper bound without the cross-covariance information, and it improves and increases the robust accuracy of the CI fuser as shown in Remark 15. The ellipsoidal intersection (EI) fuser [33] with the cross-covariance information was developed and extended.

(3) Two robust weighted measurement fusion algorithms and the robust centralized fusion algorithm have been presented in a unified framework, their equivalence was proved, and they have the highest robust accuracy than the above other fusers. Only one weighted measurement fusion algorithm was presented in [24].

This paper is limited to the multisensor systems with uncertain noise variances; the extension of the proposed results to the multisensor systems with both the uncertain model parameters and noise variances is in the investigation.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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