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Research Article

Impulsive Synchronization and Adaptive-Impulsive Synchronization of a Novel Financial Hyperchaotic System

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The impulsive synchronization and adaptive-impulsive synchronization of a novel financial hyperchaotic system are investigated. Based on comparing principle for impulsive functional differential equations, several sufficient conditions for impulsive synchronization are derived, and the upper bounds of impulsive interval for stable synchronization are estimated. Furthermore, a nonlinear adaptive-impulsive control scheme is designed to synchronize the financial system using invariant principle of impulsive dynamical systems. Moreover, corresponding numerical simulations are presented to illustrate the effectiveness and feasibility of the proposed methods.

1. Introduction

Since the seminal work of Pecora and Carroll [1], chaos synchronization has been an active topic in nonlinear science, due to its potential applications in secure communication, control theory, telecommunications, biological networks and artificial neural networks, and so forth. So far, many effective approaches have been presented to synchronize chaotic systems such as adaptive control [2, 3], fuzzy control [4], static feedback control [5], variable structure control [6], stochastic control [7], impulsive control [8–10], and others. Impulsive control, as a discontinuous control method, has attracted more interest recently due to its easy implementation in engineering control. In some cases [11, 12], it may be impossible to use synchronization at all times, and to use impulsive control may prove to be more efficient.

The main idea of impulsive synchronization is that the response system received a sequence of synchronizing impulse signals from the drive system only at some discrete time instants. The synchronization velocity is rapid, and it has very strong advantage in practice due to reduced control cost. So, impulsive synchronization has received a great deal of interest from various fields. Yang and Chua [13] presented a theory of impulsive synchronization of two chaotic systems and a promising application of impulsive synchronization of chaotic systems to a secure digital communication scheme. Sun and Zhang [14] investigated the impulsive synchronization of a Chua oscillator, and the experimental results show that the accuracy of the synchronization depends on the period and the width of the impulse. Luo [15] gave the sufficient condition for impulsive synchronization of a new chaotic system. Ma and Wang [16] introduced the impulsive control and synchronization of a new unified hyperchaotic system; the control gains and impulsive intervals are both variable and analyzed by the impulsive synchronization of a class of fractional-order hyperchaotic systems [17]. These works on impulsive synchronization were based on the theory of comparison systems, and it is easy to define the impulsive interval and impulsive control gain.

Recently, some researchers synchronize the chaotic system through combining adaptive control and impulsive control, and they name it adaptive-impulsive control [18–20]. In [18], Li et al. discussed adaptive-impulsive synchronization and parameter identification of a class of chaotic and hyperchaotic systems, and their controllers and identifiers have a limit that the system state variable function independent of parameters must be Lipchitz. Chen and Chang [19] derived an adaptive impulse control with only one restriction criterion to achieve synchronization of nonlinear chaotic systems in the exponential rate of convergence, and they assumed that the system satisfied the local Lipchitz condition. But not every chaotic or hyperchaotic system satisfies the Lipchitz condition; simultaneously the Lipchitz coefficient is hard to estimate. Wan and Sun [20] investigated the nonlinear adaptive-impulsive synchronization of chaotic systems, applied it to quantum cellular neural network (Quantum-CNN), and found adaptive-impulsive controllers more effective than the adaptive control scheme.

Since chaos phenomenon in financial field is founded in 1985, it has huge impacts on Chinese and western economics. There is chaos in economic and financial systems; this means that the system itself has intrinsic instability, and generally it is harmful to systems. So, control and synchronization of the financial chaotic or hyperchaotic system have more significance. Recently, Cai et al. [21] studied the modified function lag projective synchronization of a novel financial hyperchaotic system by continuous adaptive control method. To the best of our knowledge, the impulsive synchronization and adaptive-impulsive synchronization of this novel financial system have not been studied.

Motivated by the aforementioned comments, in the paper, we will discuss impulsive synchronization and adaptive-impulsive synchronization of the novel financial system. Firstly, based on comparing principle, several sufficient conditions for impulsive synchronization are presented, and the upper bounds of impulsive interval for stable synchronization are defined. Furthermore, we will design a nonlinear adaptive-impulsive control scheme to synchronize the financial system using invariant principle of impulsive dynamical systems. Besides, corresponding numerical simulation results are illustrated to verify the effectiveness and feasibility of the theoretical results.

The rest of this paper is organized as follows. In Section 2, some basic theories of impulsive differential equations are called. In Section 3, the novel financial system is given, and its dynamics equations and attractors diagrams are illustrated. In Section 4, several sufficient conditions for impulsive synchronization are introduced, the upper bound of impulsive interval is presented, and numerical simulation results are provided to show the effectiveness of the synchronization criteria. In Section 5, a nonlinear adaptive-impulsive control scheme is constituted to synchronize the financial system, and corresponding numerical simulations are presented to verify the effectiveness of the theoretical results. Finally, the conclusions are drawn in Section 6.

2. Basic Theories of Impulsive Differential Equations

In general, the impulsive differential system is described by

$$\begin{gathered}\dot{\mathbf{x}} = f(t,\mathbf{x}), \quad t \neq \tau_k,\\ \Delta\mathbf{x} = \mathbf{x}(t_k^+) - \mathbf{x}(t_k^-) = U(k,\mathbf{x}), \quad t = \tau_k,\ k = 1,2,\ldots, \\ \mathbf{x}(t_0^+) = \mathbf{x}_0,\end{gathered} \tag{1}$$

where state variable $\mathbf{x} \in R^n$ is left continuous at $t = \tau_k$, discrete set $\{\tau_k\}$ of time instants denotes the time instants at which impulses are sent to the system and satisfies $0 < \tau_1 < \tau_2 < \cdots < \tau_k < \tau_{k+1} < \cdots$, $\tau_k \to \infty$ as $k \to \infty$. $f : R_+ \times R^n \to R^n$ is continuous, and $U(k,\mathbf{x})$ is the state variable at instant τ_k. First, we call the following definitions and theorems [22].

Definition 1. The function $V : R_+ \times R^n \to R_+$ is said to belong to class V_0 if

(1) V is continuous in $(\tau_{k-1},\tau_k]\times R^n$, and, for each $x \in R^n$, $\lim_{(t,y)\to(\tau_k^+,x)} V(t,y) = V(\tau_k^+,x)$ exists for $k = 1,2,\ldots$;

(2) V is locally Lipschitzian in $\mathbf{x}$.

Definition 2. For $(t,\mathbf{x}) \in (\tau_{k-1},\tau_k] \times \mathbf{R}^n$, the right and upper Dini's derivatives of $V \in V_0$ are defined as

$$D^+V(t,\mathbf{x}) \triangleq \limsup_{h\to 0} \frac{1}{h}\left[V(t+h,\mathbf{x}+h\mathbf{f}(t,\mathbf{x})) - V(t,\mathbf{x})\right]. \tag{2}$$

Definition 3 (comparison system). Let $V \in V_0$, and assume that

$$\begin{gathered}D^+V(t,\mathbf{x}) \le g(t,V(t,\mathbf{x})), \quad t \neq \tau_k,\\ V(t,\mathbf{x}+U(k,\mathbf{x})) \le \varphi_k(V(t,\mathbf{x})), \quad t = \tau_k,\end{gathered} \tag{3}$$

where $g : R_+ \times R_+ \to R$ is continuous and $\varphi_k : R_+ \to R_+$ is nondecreasing. Then the system

$$\begin{gathered}\dot{\omega} \le g(t,\omega), \quad t \neq \tau_k,\\ \omega(\tau_k^+) \le \varphi_k(\omega(\tau_k)),\\ \omega(\tau_0^+) = \omega_0 \ge 0\end{gathered} \tag{4}$$

is called the comparison system of (1).

Definition 4. Consider

$$S_\rho = \{\mathbf{x} \in R^n \mid \|\mathbf{x}\| < \rho\}, \tag{5}$$

where $\|\cdot\|$ denotes the Euclidean norm on R^n.

Definition 5. A function α is said to belong to class κ if $\alpha \in C[R_+,R_+]$, $\alpha(0) = 0$, and $\alpha(x)$ is strictly increasing in x. Assume that $f(t,0) = 0$, $U(k,0) = 0$, and $g(t,0) = 0$ for all k.

Theorem 6. *Assume that the following three conditions are satisfied.*

(1) *Consider* $V : R_+ \times S_\rho \to R_+$, $\rho > 0$, $V \in V_0$, $D^+V(t,\mathbf{x}) \le g(t,V(t,\mathbf{x}))$, *and* $t \neq \tau_k$.

(2) *There exists a* $\rho_0 > 0$ *such that* $x \in S_{\rho_0}$ *implies that* $x + U(k,x) \in S_{\rho_0}$ *for all* k, *and* $V(t,\mathbf{x}+U(k,\mathbf{x})) \le \varphi_k(V(t,\mathbf{x}))$, $t = \tau_k$, *and* $x \in S_{\rho_0}$.

(3) *Consider* $\alpha(\|\mathbf{x}\|) \le V(k,\mathbf{x}) \le \beta(\|\mathbf{x}\|)$ *on* $R_+ \times S_\rho$, *where* $\alpha,\beta \in \kappa$.

Then the stability properties of the trivial solution of the comparison system (4) imply the corresponding stability properties of the trivial solution of (1).

Theorem 7 (see [22]). *Let $g(t,\omega) = \dot{\lambda}(t)\omega$, $\lambda \in C^1[R^n, R^n]$, $\varphi_k(\omega) = d_k\omega$, and $\omega \geq 0$ for all k; then the origin of system* (1) *is asymptotically stable if conditions*

$$\lambda(\tau_{k+1}) + \ln(\varepsilon d_k) \leq \lambda(\tau_k), \quad \forall k, \text{ where } \varepsilon > 1, \tag{6}$$

and $\dot{\lambda}(t) \geq 0$ are satisfied.

3. System Descriptions

The novel financial dynamical system [21, 23] is described as follows:

$$\begin{aligned}
\dot{x}_1 &= x_3 + (x_2 - a)x_1 + x_4, \\
\dot{x}_2 &= 1 - bx_2 - x_1^2, \\
\dot{x}_3 &= -x_1 - cx_3, \\
\dot{x}_4 &= -d_1x_1x_2 - d_2x_4,
\end{aligned} \tag{7}$$

where x_1, x_2, x_3, and x_4 are state variables. x_1 denotes the interest rate, x_2 is the investment demand, x_3 is the price exponent, and x_4 is the average profit margin. a, b, c, d_1, and d_2 are system parameters, and when $a = 0.9$, $b = 0.2$, $c = 1.5$, $d_1 = 0.2$, and $d_2 = 0.17$, system (7) is hyperchaotic as displayed in Figures 1(a)–1(d). It is easy to know that the state variables of system (7) are bounded.

4. Impulsive Synchronization of the Financial Hyperchaotic System

Equation (7) is taken as the drive system; then the response system under impulsive control is characterized by

$$\begin{aligned}
\dot{y}_1 &= y_3 + (y_2 - a)y_1 + y_4, \quad t \neq \tau_k, \\
\dot{y}_2 &= 1 - by_2 - y_1^2, \quad t \neq \tau_k, \\
\dot{y}_3 &= -y_1 - cy_3, \quad t \neq \tau_k, \\
\dot{y}_4 &= -d_1y_1y_2 - d_2y_4, \quad t \neq \tau_k, \\
\Delta y_i &= y_i(t_k^+) - y_i(t_k^-) = B_{ik}e_i, \\
i &= 1,2,3,4; \quad t = \tau_k, \quad k = 1,2,\ldots,
\end{aligned} \tag{8}$$

where state variable $y \in R^n$ is left continuous at $t = \tau_k$ and τ_k denotes the moment when impulsive control occurs; discrete set $\{\tau_k\}$ of time instants satisfies $0 < \tau_1 < \tau_2 < \cdots < \tau_k < \tau_{k+1} < \cdots$, $\tau_k \to \infty$ as $k \to \infty$; $B_k = \operatorname{diag}(B_{1k}, B_{2k}, B_{3k}, B_{4k})$ is impulsive control gain constant matrix, and B_{ik} is the impulsive control gain, $i = 1,2,3,4$. $\mathbf{e} = (e_1, e_2, e_3, e_4)^T = (y_1 - x_1, y_2 - x_2, y_3 - x_3, y_4 - x_4)^T$ is the synchronization error.

Subtracting (8) from (7), one obtains the dynamical system of synchronization error:

$$\begin{aligned}
\dot{e}_1 &= e_3 + y_2y_1 - x_2x_1 - ae_1 + e_4, \quad t \neq \tau_k, \\
\dot{e}_2 &= -be_2 - y_1^2 + x_1^2, \quad t \neq \tau_k, \\
\dot{e}_3 &= -e_1 - ce_3, \quad t \neq \tau_k, \\
\dot{e}_4 &= -d_1y_1y_2 + d_1x_1x_2 - d_2e_4, \quad t \neq \tau_k, \\
\Delta e_i &= B_{ik}e_i, \quad i = 1,2,3,4; \ t = \tau_k, \ k = 1,2,\ldots.
\end{aligned} \tag{9}$$

Our aim is to find some conditions on the control gains B_{ik} and the impulsive intervals $(\tau_{k+1} - \tau_k)$ such that the impulsive controlled response system (8) is globally asymptotically synchronous with the drive system (7) for any initial states.

Theorem 8. *Let λ_2 be the largest eigenvalue of $(\mathbf{I}+\mathbf{B}_k^T)(\mathbf{I}+\mathbf{B}_k)$, impulsive control gain matrix $B_k = \operatorname{diag}(B_{1k}, B_{2k}, B_{3k}, B_{4k})$, and the spectral radius of $(\mathbf{I}+\mathbf{B}_k)\rho(\mathbf{I}+\mathbf{B}_k) \leq 1$.*

The conditions are

$$\begin{aligned}
M &= \max\{|1 - d_1y_2|, |x_1|, |d_1x_1|, |x_2|\}, \\
\lambda_1 &= \max\{(2M - a), (M - b), -c, (M - d_2)\}, \\
\lambda_1(\tau_{k+1} - \tau_k) &\leq -\ln(\varepsilon\lambda_2), \quad \varepsilon > 1, \ \lambda_1 \geq 0.
\end{aligned} \tag{10}$$

Proof. Let the Lyapunov function be in the form of

$$V(e) = \frac{1}{2}e^Te. \tag{11}$$

(1) Case I $(t \neq \tau_k)$. The time derivative of (11) along the solution of (9) is

$$\begin{aligned}
\dot{V}(e) &= e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3 + e_4\dot{e}_4 \\
&= e_3e_1 + y_2y_1e_1 - x_2x_1e_1 - ae_1^2 + e_4e_1 - be_2^2 \\
&\quad - e_2y_1^2 + e_2x_1^2 - e_1e_3 - ce_3^2 - dy_1y_2e_4 \\
&\quad + dx_1x_2e_4 - d_2e_4^2 \\
&= -ae_1^2 - be_2^2 - ce_3^2 - d_2e_4^2 \\
&\quad + e_4e_1(1 - dy_2) - e_2e_1x_1 - dx_1e_2e_4 + x_2e_1^2 \\
&\leq -ae_1^2 - be_2^2 - ce_3^2 - d_2e_4^2 + |e_4||e_1||1 - dy_2| \\
&\quad + |e_2||e_1||x_1| + |dx_1||e_2||e_4| + |x_2||e_1^2| \\
&= -ae_1^2 - be_2^2 - ce_3^2 - d_2e_4^2 \\
&\quad + M(|e_4||e_1| + |e_2||e_1| + |e_2||e_4| + |e_1^2|) \\
&\leq (2M - a)e_1^2 + (M - b)e_2^2 \\
&\quad - ce_3^2 + (M - d_2)e_4^2 = \lambda_1V(e).
\end{aligned} \tag{12}$$

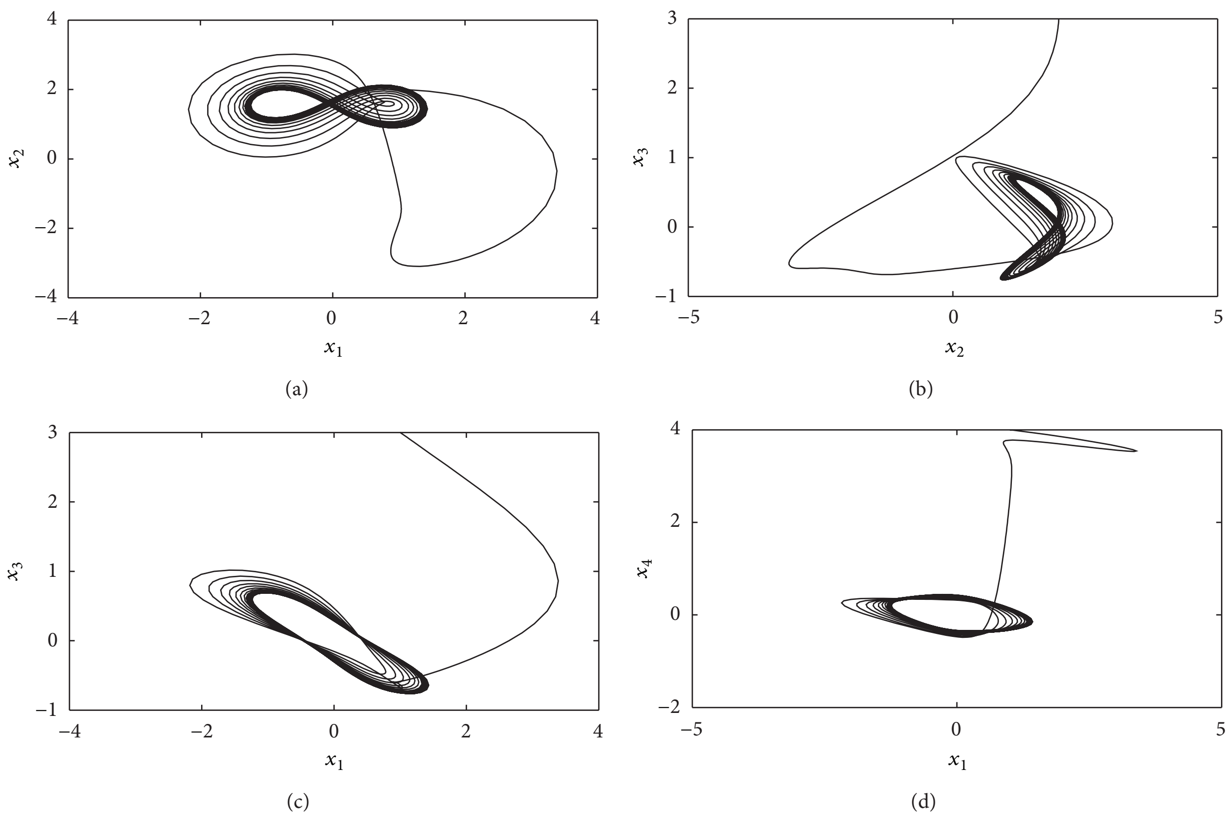

FIGURE 1: Hyperchaotic attractors of the financial hyperchaotic system.

Hence, condition (1) of Theorem 6 is satisfied with $g(t,\omega) = \lambda_1\omega$.

(2) Case II ($t = \tau_k$). Since $\mathbf{I} + \mathbf{B}_k$ is symmetric, by employing Euclidean norm, we have $\rho(\mathbf{I} + \mathbf{B}_k) = \|\mathbf{I} + \mathbf{B}_k\|$. Then for any $\rho_0 > 0$ such that $x \in S_{\rho_0}$, we have $\|\mathbf{x}+U(k,\mathbf{x})\| \le \|\mathbf{I}+\mathbf{B}_k\|\|\mathbf{x}\| = \rho(\mathbf{I} + \mathbf{B}_k)\|\mathbf{x}\| \le \|\mathbf{x}\|$. Then,

$$V(\mathbf{e}) = \frac{1}{2}\mathbf{e}^T(\mathbf{I} + \mathbf{B}_k)^T(\mathbf{I} + \mathbf{B}_k)\mathbf{e} \le \lambda_2 V(\mathbf{e}). \tag{13}$$

So, condition (2) of Theorem 6 is satisfied with $\varphi_k(\omega) = \lambda_2\omega$. Obviously, condition (3) of Theorem 6 is also satisfied. Then, the comparison system is given by

$$\begin{gathered} \dot{\omega} = \lambda_1\omega, \quad t \neq \tau_k, \\ \omega(\tau_k^+) = \lambda_2\omega(\tau_k), \quad \omega(\tau_0^+) = \omega_0 \ge 0. \end{gathered} \tag{14}$$

It follows from Theorem 7 that if

$$\lambda_1(\tau_{k+1}) + \ln(\varepsilon\lambda_2) \le \lambda_1(\tau_k), \quad \forall k, \text{ where } \varepsilon > 1, \tag{15}$$

is satisfied, then the origin of system (9) is asymptotically stable. This completes the proof. □

We assume that the impulses are equidistant and separated by δ; that is, for any $k \in N$, $\tau_{k+1} - \tau_k = \delta$. Then we have the following result.

Corollary 9. *Let the impulses be equidistant and separated by interval δ. Then the origin of system (9) is uniformly asymptotically stable if the following conditions hold:*

$$0 < \delta \le -\frac{\ln(\varepsilon\lambda_2)}{\lambda_1}, \quad \varepsilon > 1. \tag{16}$$

Moreover, an estimate of the upper bound of δ is given as

$$\delta_{\max} = -\frac{\ln(\varepsilon\lambda_2)}{\lambda_1}. \tag{17}$$

Here, impulsive control gain matrix should satisfy $\rho(\mathbf{I}+\mathbf{B}_k) \le 1$.

In order to verify the effectiveness of the impulsive synchronization method, some simulation results are illustrated. For simplicity, we assume $\{\tau_k\}$ is equidistant, and $\tau_{k+1} - \tau_k = \delta > 0$. The parameters are selected as $a = 0.9$, $b = 0.2$, $c = 1.5$, $d_1 = 0.2$, and $d_2 = 0.17$, such that the systems (7) and (8) exhibit hyperchaotic behavior if no control is applied. The initial values of the drive system (7) and the response system (8) are $(x_1(0), x_2(0), x_3(0), x_4(0)) = (-1, -2, -3, -4)$ and $(y_1(0), y_2(0), y_3(0), y_4(0)) = (1, 2, 3, 4)$, respectively. By computing, we can get $M = 4$, $\lambda_1 = 7.1$. We suppose $B_{1k} = B_{2k} = B_{3k} = B_{4k} = B$; then $-2 \le B \le 0$. The estimate of the stable region for different values of B and ε is shown in Figure 2.

We choose impulsive control gain matrix as $\mathbf{B}_k = \operatorname{diag}(-0.8, -0.8, -0.8, -0.8)$, the constant $\varepsilon = 1.5$, then we can

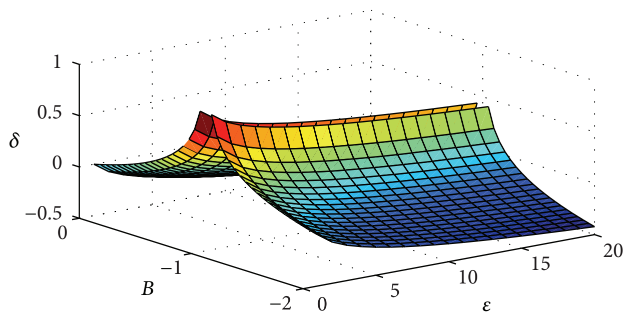

FIGURE 2: The boundaries of the stable region for different values of ε and B.

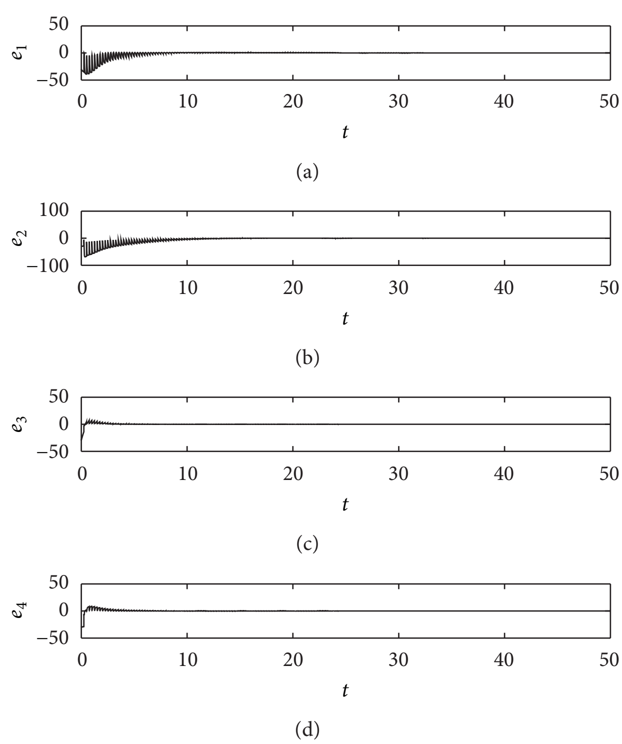

FIGURE 3: The time evolution of impulsive synchronization errors.

get $0 < \delta \leq 0.2607$, and here we select $\delta = 0.25$. Simulation results are displayed in Figures 3 and 4. Figure 3 illustrates that the impulsive synchronization errors $e_i(t)$ $(i = 1, 2, 3, 4)$ asymptotically converge to zero. Figure 4 depicts that the state variables of the drive and response systems reach complete synchronization in a very short period of time.

5. Adaptive-Impulsive Synchronization of the Financial Hyperchaotic System

In this section, we consider complete synchronization of the financial hyperchaotic system under the adaptive-impulsive control.

5.1. Adaptive-Impulsive Synchronization Scheme. The response system under the adaptive-impulsive control is described by the following equation:

$$\begin{aligned}
\dot{y}_1 &= y_3 + (y_2 - a)\, y_1 + y_4 + u_1, \quad t \neq \tau_k, \\
\dot{y}_2 &= 1 - by_2 - y_1^2 + u_2, \quad t \neq \tau_k, \\
\dot{y}_3 &= -y_1 - cy_3 + u_3, \quad t \neq \tau_k, \\
\dot{y}_4 &= -d_1 y_1 y_2 - d_2 y_4 + u_4, \quad t \neq \tau_k, \\
\Delta y_i &= y_i(t^+) - y_i(t^-) = B_{ik} e_i, \\
i &= 1, 2, 3, 4; \quad t = \tau_k, \quad k = 1, 2, \ldots,
\end{aligned} \tag{18}$$

where u_1, u_2, u_3, and u_4 are adaptive controllers to be designed. And $B_k = \operatorname{diag}(B_{1k}, B_{2k}, B_{3k}, B_{4k})$ is impulsive control gain constant matrix, B_{ik} $(i = 1, 2, 3, 4)$ is the impulsive control gain, and $\{u; B_k, \tau_k\}$ is nonlinear adaptive-impulsive controller to be constituted.

We define the error vector $\mathbf{e} = (e_1, e_2, e_3, e_4)^T = (y_1 - x_1, y_2 - x_2, y_3 - x_3, y_4 - x_4)^T$. Our aim is to find the suitable adaptive-impulsive controller for stabilizing the error variables at the origin. To this end, we design the adaptive controllers as follows:

$$\begin{aligned}
u_1 &= x_1 x_2 - y_1 y_2 + m_1 e_1, \\
u_2 &= y_1^2 - x_1^2 + m_2 e_2, \\
u_3 &= m_3 e_3, \\
u_4 &= -d_1 x_1 x_2 + d_2 y_1 y_2 + m_4 e_4, \\
\dot{m}_i &= -\gamma_i e_i^2, \quad i = 1, 2, 3, 4.
\end{aligned} \tag{19}$$

For $i = 1, 2, 3, 4$, $\gamma_i > 0$ are arbitrary constants and m_i are adaptive control gain strengths. The error dynamical system is obtained as follows:

$$\begin{aligned}
\dot{e}_1 &= e_3 - ae_1 + e_4 + m_1 e_1, \quad t \neq \tau_k, \\
\dot{e}_2 &= -be_2 + m_2 e_2, \quad t \neq \tau_k, \\
\dot{e}_3 &= -e_1 - ce_3 + m_3 e_3, \quad t \neq \tau_k, \\
\dot{e}_4 &= -d_2 e_4 + m_4 e_4, \quad t \neq \tau_k, \\
\Delta e_i &= B_{ik} e_i, \quad i = 1, 2, 3, 4; \ t = \tau_k, \ k = 1, 2, \ldots, \\
\dot{m}_i &= -\gamma_i e_i^2, \quad i = 1, 2, 3, 4.
\end{aligned} \tag{20}$$

Theorem 10. *Suppose that $\rho_{ik} = \lambda_{\max}((\mathbf{I}+\mathbf{B}_k^T)(\mathbf{I}+\mathbf{B}_k)) \leq 1$, $i = 1, 2, 3, 4$, $k = 1, 2, \ldots$. Then the drive system* (7) *will be globally asymptotically synchronous with the response system* (18) *by using nonlinear adaptive-impulsive controller $\{u; B_k, \tau_k\}$.*

Proof. Choose a Lyapunov function as

$$V = \frac{1}{2}\mathbf{e}^T\mathbf{e} + \frac{1}{2}\sum_{i=1}^{4}\frac{1}{\gamma_i}(m_i + l)^2, \tag{21}$$

where l is a positive constant.

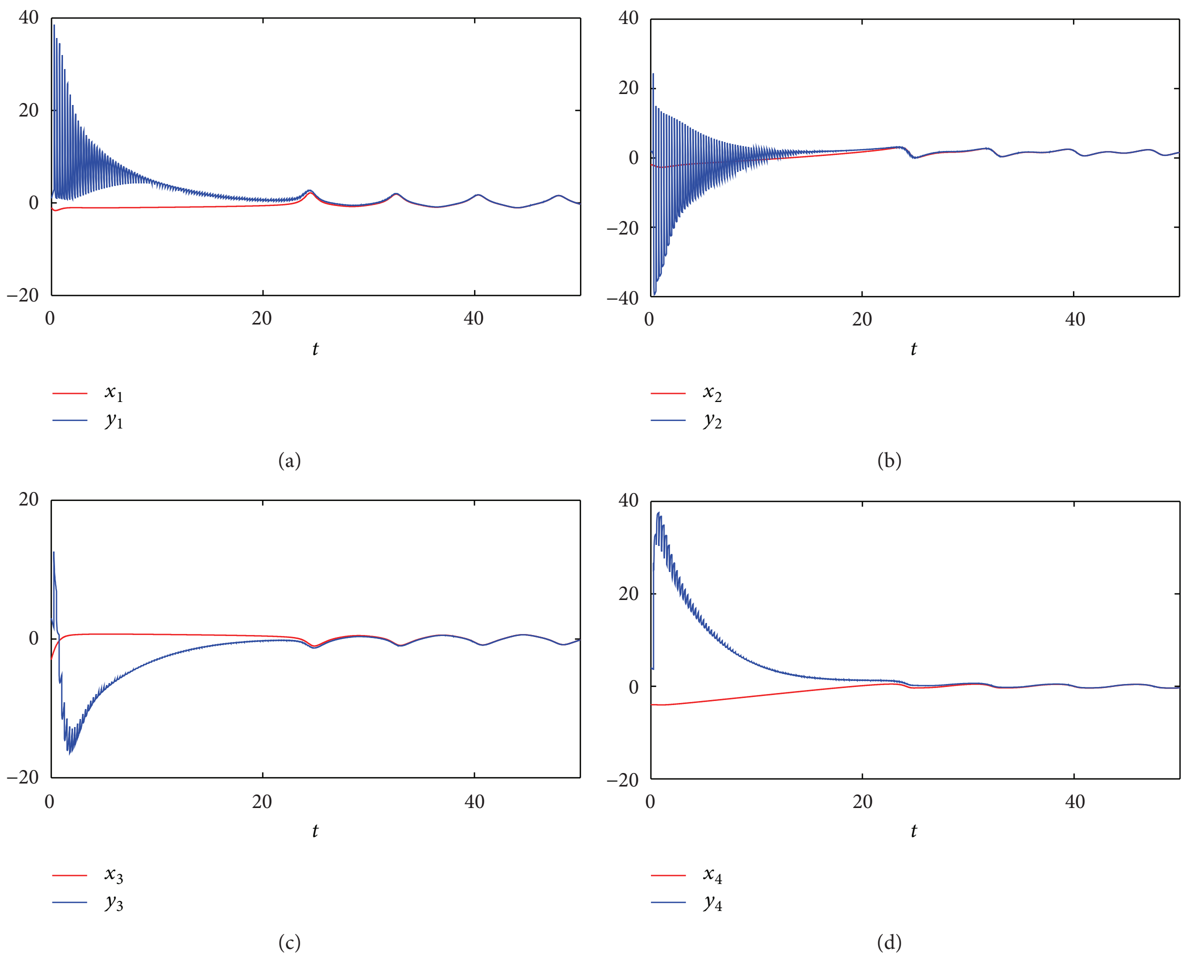

FIGURE 4: The time response of states of the drive system (7) and the response system (8).

In the case $t \neq \tau_k$, the time derivative of (21) along the solution of (20) is

$$\begin{aligned} D^{+}V &= \mathbf{e}^{T}\dot{\mathbf{e}} - \sum_{i=1}^{4}(m_i + l)e_i^2 \\ &= e_1(e_3 - ae_1 + e_4 + m_1e_1) + e_2(-be_2 + m_2e_2) \\ &\quad + e_3(-e_1 - ce_3 + m_3e_3) \\ &\quad + e_4(-ke_4 + m_4e_4) - \sum_{i=1}^{4}(m_i + l)e_i^2 \\ &= -ae_1^2 + e_1e_4 - be_2^2 - ce_3^2 - d_2e_4^2 - \sum_{i=1}^{4} le_i^2 \\ &= \mathbf{e}^{T}\left[\begin{pmatrix} -a & 0 & 0 & \frac{1}{2} \\ 0 & -b & 0 & 0 \\ 0 & 0 & -c & 0 \\ \frac{1}{2} & 0 & 0 & -d_2 \end{pmatrix} - l\mathbf{I}\right]\mathbf{e} = \mathbf{e}^{T}\mathbf{Q}\mathbf{e}, \end{aligned} \tag{22}$$

where $\mathbf{I}$ is the four-dimensional identity matrix. When $l > -0.17$, the matrix $\mathbf{Q}$ is negative definite. Through choosing the proper l, $\dot{V} \leq -\mathbf{e}^{T}\mathbf{e}$ is achieved.

In the case $t = \tau_k$,

$$\begin{aligned} V(\tau_k^{+}) &= \frac{1}{2}e^{T}(\tau_k^{+})e(\tau_k^{+}) + \frac{1}{2}\sum_{i=1}^{4}\frac{1}{\gamma_i}(m_i(\tau_k^{+}) + l)^2 \\ &= \frac{1}{2}\sum_{i=1}^{4}(1 + B_{ik})^2 e_i^2(\tau_k) + \frac{1}{2}\sum_{i=1}^{4}\frac{1}{\gamma_i}(m_i(\tau_k^{+}) + l)^2 \\ &= \frac{1}{2}\rho_{ik}\sum_{i=1}^{4}e_i^2(\tau_k) + \frac{1}{2}\sum_{i=1}^{4}\frac{1}{\gamma_i}(m_i(\tau_k) + l)^2 \\ &< \frac{1}{2}\sum_{i=1}^{4}e_i^2(\tau_k) + \frac{1}{2}\sum_{i=1}^{4}\frac{1}{\gamma_i}(m_i(\tau_k) + l)^2 = V(\tau_k). \end{aligned} \tag{23}$$

Define the set $\mathfrak{R} \triangleq \{\dot{V} = 0, t \neq \tau_k, k = 1, 2, \ldots\} \cup \{V(\tau_k^{+}) = V(\tau_k), k = 1, 2, \ldots\}$, and the set $\Omega = \{(e, m) \in \mathbb{R}^{2n} : e(t) = 0, m = m_0 \in \mathbb{R}^{n}\}$ is the largest invariant set contained in $\mathfrak{R}$ for error dynamical equations (21). According to corollary 5.1 [24], the orbit of the system (20) converges to the set Ω, that

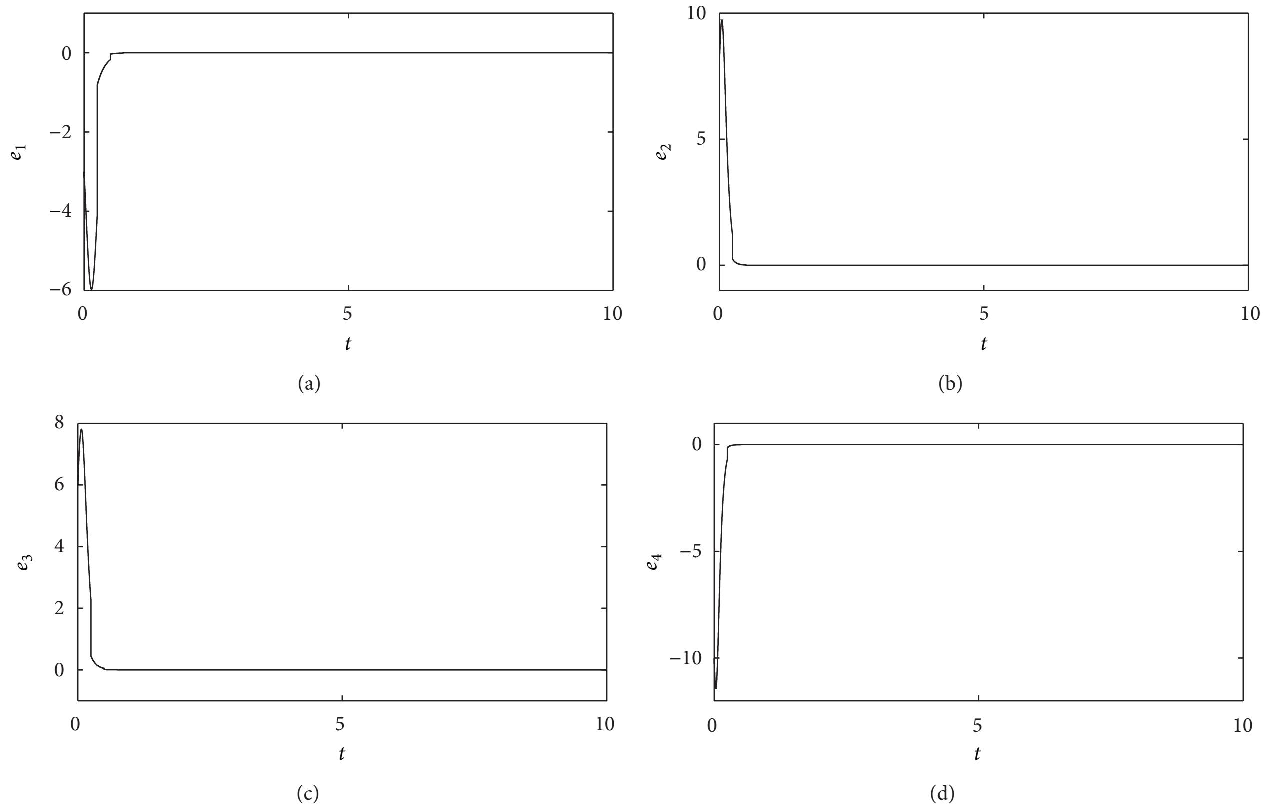

FIGURE 5: The time evolution of the synchronization errors between systems (7) and (18).

is, $e(t) \to 0$ and $m \to m_0$ as $t \to \infty$. That is to say, the drive system (7) and the response system (18) are synchronized. This completes the proof. □

Remark 11. The designed adaptive-impulsive controllers have no specific requirements on impulsive interval. An important research topic of conventional impulsive control is how to get larger impulsive interval [13, 25, 26]. Adaptive-impulsive control scheme which the paper constituted fundamentally eliminates the limit on impulsive interval of conventional impulsive control. When the impulsive gain is fixed, the larger the impulsive interval, the more flexible the impulsive controller, and simultaneously the less the energy used.

Remark 12. The control gain strengths of adaptive controller can be defined adaptively. The adaptive control gain strengths are always fixed [21, 27–29]; sometimes it may be the maximum value, and thus that can give a kind of energy waste. The method of our paper is different from them. The control gain strengths can be automatically adapted to a suitable value depending on constant γ_i and their initial values.

Remark 13. In the adaptive-impulsive control process, impulsive controller wastes less energy than general impulsive control; nevertheless, continuous adaptive controller needs continuous energy. But is the energy wasted by adaptive-impulsive controller less than that of impulsive control? The answer is maybe. So, from the view of energy saving, how to constitute the more effective adaptive-impulsive controller is essential and needs us to investigate in the future.

Remark 14. Adaptive controllers have the merit of simple design, but continuous control needs more energy. Impulsive controllers can save much energy. The new adaptive-impulsive controller which the paper constituted integrates the advantages of adaptive controller and impulsive controller; it is designed simply and wastes less energy. Therefore, the proposed method in this paper can be applied to many fields, such as secure communication and commercial systems.

5.2. Numerical Simulations. Numerical simulations are given in this subsection to verify the effectiveness and feasibility of the theoretical results obtained. We also assume $\{\tau_k\}$ is equidistant, and $\tau_{k+1} - \tau_k = \delta$. We select the parameters as $a = 0.9$, $b = 0.2$, $c = 1.5$, $d_1 = 0.2$, and $d_2 = 0.17$, so that the systems (7) and (18) are hyperchaotic when no control inputs are applied. The initial states of the drive system (7) and the response system (18) are taken as $(x_1(0), x_2(0), x_3(0), x_4(0)) = (-1, -2, -3, -4)$ and $(y_1(0), y_2(0), y_3(0), y_4(0)) = (1, 2, 3, 4)$, respectively. The nonlinear adaptive-impulsive controllers $\{u; B_k, \tau_k\}$ are designed as $\mathbf{B}_k = \operatorname{diag}\{-0.8, -0.8, -0.8, -0.8\}$, $\gamma_1 = \gamma_2 = \gamma_3 = \gamma_4 = -2$, and $\delta = 1$, and the initial values of the control gain strengths are set as $m_1(0) = m_2(0) = m_3(0) = m_4(0) = 8$. The corresponding simulation results are illustrated in Figures 5–7.

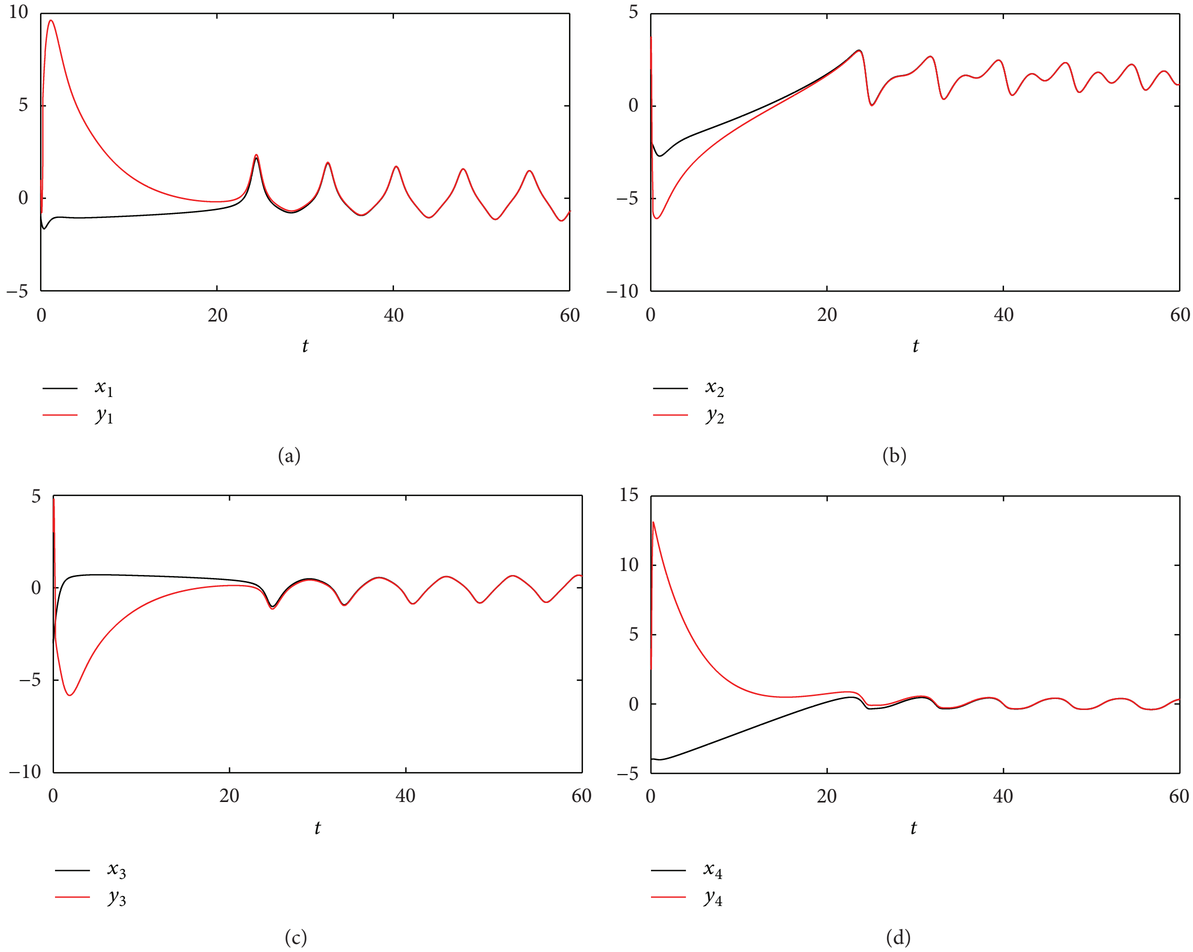

FIGURE 6: The time response of states of the drive system (7) and the response system (18).

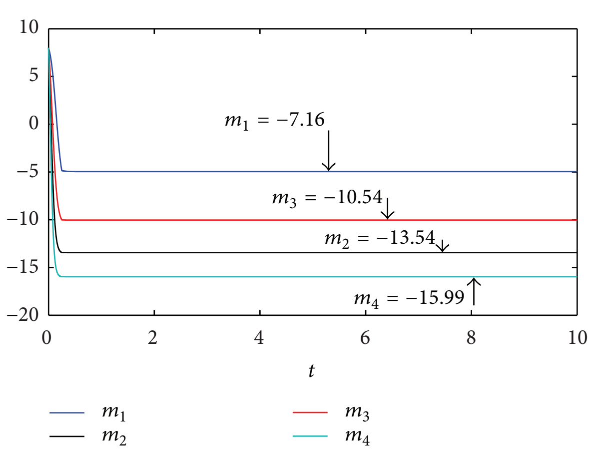

FIGURE 7: The time evolution of the control gain strengths.

Figure 5 shows that the error variables e_1, e_2, e_3, and e_4 tend to zero, respectively. Figure 6 denotes the time response of the drive system (7) and the response system (18). Figure 7 presents the time evolution of the control gain strengths, which displays that they converge to $m_1 = -7.16$, $m_2 = -13.54$, $m_3 = -10.54$, and $m_4 = -15.99$ as $t \rightarrow \infty$. As shown in Figures 5–7, complete synchronization between the drive system (7) and the response system (18) is obtained, and the control gain strengths are estimated adaptively by using the nonlinear adaptive-impulsive controllers $\{u; B_k, \tau_k\}$.

Comparing the above results of impulsive synchronization and adaptive-impulsive synchronization, we can find that the synchronization time using nonlinear adaptive-impulsive control scheme is shorter than that using impulsive control. From this point of view, adaptive-impulsive control is more effective than impulsive control.

6. Conclusions

In this paper, we investigated impulsive synchronization and adaptive-impulsive synchronization of a novel financial hyperchaotic system theoretically and numerically at the first time. We have proposed an impulsive synchronization scheme for the financial system, obtained some synchronization criteria by means of comparing system principle, estimated the upper bounds of impulsive interval for stable synchronization, and provided numerical simulation results to show the effectiveness of the synchronization criteria. Furthermore, an adaptive-impulsive synchronization method for the financial system has been introduced, some synchronization conditions have been given, and corresponding numerical simulations have been presented to verify the effectiveness of the theoretical results. The results are helpful for synchronization development of financial systems and financial markets.

Complete synchronization is achieved for chaotic systems with well-matched parameters. However, parameter mismatch is inevitable in practical implementations of chaos synchronization because of noise or other artificial factors. And very small parameter mismatch might induce loss of perfect synchronization but might reserve quasisynchronization for the given allowable error. So we will investigate the effects of parameter mismatch of synchronization and derive some applicable synchronization criteria in a near future study.

Conflict of Interests

The authors do not have a direct financial relation with any commercial identity mentioned in their paper that might lead to a conflict of interests for any of the authors.

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