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Research Article

Extinction in Two-Species Nonlinear Discrete Competitive System

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We propose a nonlinear discrete system of two species with the effect of toxic substances. By constructing a suitable Lyapunov-type function, we obtain the sufficient conditions which guarantee that one of the components will be driven to extinction while the other will be globally attractive with any positive solution of a discrete equation. Two examples together with their numerical simulations illustrate the feasibility of our main results. The results not only improve but also complement some known results.

1. Introduction

Let Z denote the set of all nonnegative integers. For any bounded sequence $\{f(n)\}$, set $f^u = \sup_{n\in Z} f(n)$ and $f^l = \inf_{n\in Z} f(n)$.

In the real world, there are many types of interactions between two species. Competitive relations are among the most common ecological interactions. As we all know, the competitive system has been established and was accepted by many scientists and now it became the most important means to explain the ecological phenomenon. During the last decade, the study of the dynamic behaviors of competitive system with toxic substance or feedback control have been discussed by many authors; see, for example, [1–16]. However, most of the studies are based on the traditional Lotka-Volterra competitive system [5, 7, 8, 17–20]; seldom did scholars consider the nonlinear case [1–4, 8, 9, 11, 12, 15, 16, 21–25].

In [1], Li and Chen studied the extinction property of the following two species competitive system:

$$\begin{aligned}\dot{x}_1(t) &= x_1(t)\left[r_1(t) - a_1(t)x_1(t) - b_1(t)x_2(t) - c_1(t)x_1(t)x_2(t)\right],\\ \dot{x}_2(t) &= x_2(t)\left[r_2(t) - a_2(t)x_1(t) - b_2(t)x_2(t) - c_2(t)x_1(t)x_2(t)\right],\end{aligned} \tag{1}$$

where $r_i(t), a_i(t), b_i(t), c_i(t)$, $i = 1, 2$, are assumed to be continuous and bounded above and below by positive constants and $x_1(t), x_2(t)$ are population density of species x_1 and x_2 at time t, respectively.

In fact, when the size of the population is relatively small, the discrete time models governed by difference equations are more appropriate than the continuous ones. Therefore, Li and Chen [2] and Guo et al. [3] studied the following discrete Lotka-Volterra competition system:

$$\begin{aligned}x_1(n+1) &= x_1(n)\exp\left[r_1(n) - a_1(n)x_1(n) - b_1(n)x_2(n) - c_1(n)x_1(n)x_2(n)\right],\\ x_2(n+1) &= x_2(n)\exp\left[r_2(n) - a_2(n)x_1(n) - b_2(n)x_2(n) - c_2(n)x_1(n)x_2(n)\right],\end{aligned} \tag{2}$$

where $\{r_i(n)\}$, $\{a_i(n)\}$, $\{b_i(n)\}$, and $\{c_i(n)\}$, $i = 1, 2$, are bounded nonnegative sequences defined on $n \in Z$. In [2], Li and Chen showed that if the coefficients of system (2) satisfy

$$\frac{r_2^u}{r_1^l} < \min\left\{\frac{a_2^l}{a_1^u}, \frac{b_2^l}{b_1^u}, \frac{c_2^l}{c_1^u}\right\}, \tag{H_0}$$

species x_2 will be driven to extinction. In [3], Guo et al. introduced the average growth rate and showed that if the coefficients of system (2) satisfy the following inequality:

$$\frac{M[r_2]}{m[r_1]} < \min\left\{\frac{a_2^l}{a_1^u}, \frac{b_2^l}{b_1^u}, \frac{c_2^l}{c_1^u}\right\}, \qquad (H_0')$$

then the same conclusion holds. Obviously, condition (H_0') is weaker than that of (H_0).

Since conditions (H_0) and (H_0') are all sufficient conditions, one of the interesting problems is whether the results still hold under the weaker condition. Now let us consider the following example.

Example 1. Consider the following system:

$$\begin{aligned} x_1(n+1) &= x_1(n)\exp\left[(0.6+0.3\sin(n)) - (1+0.3\sin(n))x_1(n) - (1+0.5\sin(n))x_2(n) - (1.1+\sin(n))x_1(n)x_2(n)\right], \\ x_2(n+1) &= x_2(n)\exp\left[(0.6+0.3\sin(n)) - (2+0.6\sin(n))x_1(n) - (2+\sin(n))x_2(n) - (2.2+2\sin(n))x_1(n)x_2(n)\right]. \end{aligned} \qquad (3)$$

In this case

$$\begin{aligned} r_1(n) &= 0.6+0.3\sin(n), \\ a_1(n) &= 1+0.3\sin(n), \\ b_1(n) &= 1+0.5\sin(n), \\ c_1(n) &= 1.1+\sin(n), \\ r_2(n) &= 0.6+0.3\sin(n), \\ a_2(n) &= 2+0.6\sin(n), \\ b_2(n) &= 2+\sin(n), \\ c_2(n) &= 2.2+2\sin(n). \end{aligned} \qquad (4)$$

By simple computation, one can see that

$$\frac{a_2^l}{a_1^u} = \frac{1.4}{1.3}, \quad \frac{b_2^l}{b_1^u} = \frac{1}{1.5}, \quad \frac{c_2^l}{c_1^u} = \frac{0.2}{2.1}, \qquad (5)$$

$$\frac{r_2^u}{r_1^l} = \frac{0.9}{0.3} > \frac{0.2}{2.1} = \min\left\{\frac{a_2^l}{a_1^u}, \frac{b_2^l}{b_1^u}, \frac{c_2^l}{c_1^u}\right\}, \qquad (6)$$

$$\frac{M[r_2]}{m[r_1]} = 1 > \frac{0.2}{2.1} = \min\left\{\frac{a_2^l}{a_1^u}, \frac{b_2^l}{b_1^u}, \frac{c_2^l}{c_1^u}\right\}, \qquad (7)$$

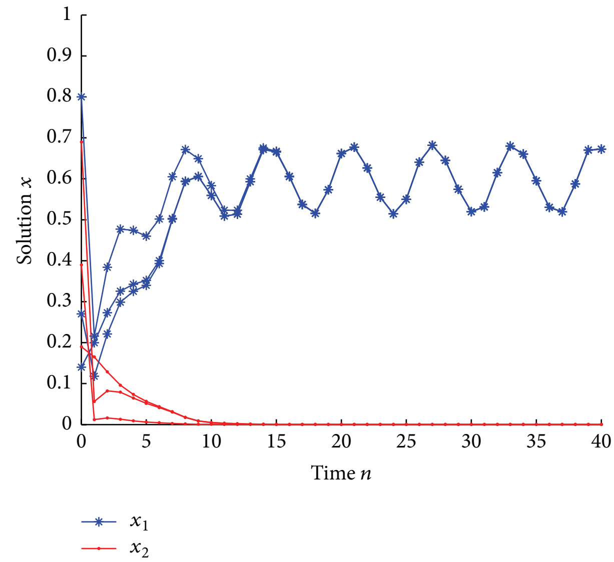

Figure 1: Dynamic behaviors of system (3) with initial values $(x_1(0), x_2(0)) = (0.14, 0.19), (0.27, 0.69)$, and $(0.8, 0.39)$, respectively.

and (6) and (7) show that neither (H_0) nor (H_0') holds; hence one could not draw any conclusion about the dynamic behaviors of the system. However, Figure 1 shows species x_2 will be driven to extinction in this case. This motivates us to revisit the extinction property of system (2).

On the other hand, Gilpin and Ayala [21] conducted experiment on fruit fly dynamics to test the validity of 10 models of competitions. One of the models accounting best for the experimental results is given by

$$\begin{aligned} \dot{x}_1(t) &= r_1 x_1(t)\left(1 - \left(\frac{x_1(t)}{K_1}\right)^{\theta_1} - \alpha_{12}\frac{x_2(t)}{K_2}\right), \\ \dot{x}_2(t) &= r_2 x_2(t)\left(1 - \left(\frac{x_2(t)}{K_2}\right)^{\theta_2} - \alpha_{21}\frac{x_1(t)}{K_1}\right). \end{aligned} \qquad (8)$$

Fan and Wang [22] studied the dynamic behaviors of the following nonautonomous n-species Gilpin-Ayala competitive system:

$$\dot{x}_i(t) = x_i(t)\left[b_i(t) - \sum_{j=1}^{n} a_{ij}(t)\left(x_j\left(t-\tau_{ij}(t)\right)\right)^{\theta_{ij}}\right], \quad i = 1, 2, \ldots, n, \qquad (9)$$

where $b_i(t)$, $\tau_{ij}(t)$, and $a_{ij}(t)$, $i, j = 1, 2, \ldots, n$, are continuous for $0 \le t < +\infty$ and θ_{ij} are positive constants.

Chen et al. [23] studied a discrete n-species Gilpin-Ayala competitive system

$$x_i(k+1) = x_i(k)\exp\left[b_i(k) - \sum_{j=1}^{n} a_{ij}(k)\left(x_j(k)\right)^{\theta_{ij}}\right], \quad i = 1, 2, \ldots, n, \qquad (10)$$

where $b_i(k)$, $a_{ij}(k)$, $i, j = 1, 2, \ldots, n$, are all positive sequences bounded above and below by positive constants. θ_{ij} are positive constants.

Recently, stimulated by the works of [1, 21, 22], Chen et al. [24] proposed the following, a nonautonomous nonlinear competition system:

$$\begin{aligned}\dot{x}_1(t) &= x_1(t)\left[r_1(t) - a_1(t)x_1^{\alpha_1}(t) - b_1(t)x_2^{\alpha_2}(t) - c_1(t)x_1^{\alpha_1}(t)x_2^{\alpha_2}(t)\right],\\ \dot{x}_2(t) &= x_2(t)\left[r_2(t) - a_2(t)x_1^{\alpha_1}(t) - b_2(t)x_2^{\alpha_2}(t) - c_2(t)x_1^{\alpha_1}(t)x_2^{\alpha_2}(t)\right].\end{aligned} \tag{11}$$

Chen et al. showed that if the coefficients of system (10) satisfy

$$\limsup_{t\to+\infty}\frac{r_2(t)}{r_1(t)} < \liminf_{t\to+\infty}\left\{\frac{a_2(t)}{a_1(t)}, \frac{b_2(t)}{b_1(t)}, \frac{c_2(t)}{c_1(t)}\right\}, \tag{12}$$

the second species will be driven to extinction while the first one will stabilize at a certain solution of the system

$$\dot{x}_1(t) = x_1(t)\left[r_1(t) - a_1(t)x_1^{\alpha_1}(t)\right]. \tag{13}$$

Stimulated by the works of [1–3, 21–24], we propose the following a nonlinear discrete two species competition system:

$$\begin{aligned}x_1(n+1) &= x_1(n)\exp\left[r_1(n) - a_1(n)x_1^{\alpha_1}(n) - b_1(n)x_2^{\alpha_2}(n) - c_1(n)x_1^{\alpha_1}(n)x_2^{\alpha_2}(n)\right],\\ x_2(n+1) &= x_2(n)\exp\left[r_2(n) - a_2(n)x_1^{\alpha_1}(n) - b_2(n)x_2^{\alpha_2}(n) - c_2(n)x_1^{\alpha_1}(n)x_2^{\alpha_2}(n)\right].\end{aligned} \tag{14}$$

We introduce the following assumptions:

(H_1) $\{r_i(n)\}$ are bounded sequence defined on Z; $\{a_i(n)\}$, $\{b_i(n)\}$, and $\{c_i(n)\}$, $i = 1, 2$, are bounded nonnegative sequences defined on Z; α_i, $i = 1, 2$, are positive constants.

(H_2) There exists positive integer ω such that for each $i = 1, 2$

$$\liminf_{n\to\infty}\sum_{s=n}^{n+\omega-1} r_i(s) > 0. \tag{15}$$

From the point of view of biology, we assume that $x_i(0) > 0$, $i = 1, 2$; then system (14) has a positive solution $(x_1(n), x_2(n))$ passing through $(x_1(0), x_2(0))$.

The aim of this paper is, by developing the analysis technique of Li and Chen [2], Chen et al. [4, 24], and Xu et al. [6], to study the extinction property of system (14).

The organization of this paper is as follows. In Section 2, sufficient conditions for the permanence of system (14) are obtained. In Section 3, we study the extinction of species x_2. In Section 4, we study the global stability of species x_1 when species x_2 is eventual extinction. Examples are presented in Section 5 to show the feasibility of our main results.

2. Permanence

Lemma 2 (see [26]). *Assume that $\{x(n)\}$ satisfy $x(n) > 0$ and*

$$x(n+1) \le x(n)\exp\{a(n) - b(n)x(n)\}, \quad n \in N, \tag{16}$$

where $a(n)$ and $b(n)$ are nonnegative sequences bounded above and below by positive constants. Then

$$\limsup_{n\to\infty} x(n) \le \frac{\exp\{a^u - 1\}}{b^l}. \tag{17}$$

Lemma 3 (see [26]). *Assume that $\{x(n)\}$ satisfies*

$$x(n+1) \ge x(n)\exp\{a(n) - b(n)x(n)\}, \quad n \ge N_0, \tag{18}$$

$\limsup_{n\to\infty} x(n) \le x^*$, *and $x(N_0) > 0$, where $a(n)$ and $b(n)$ are nonnegative sequences bounded above and below by positive constants and $N_0 \in N$. Then*

$$\liminf_{n\to\infty} x(n) \ge \min\left\{\frac{a^l}{b^u}\exp\left\{a^l - b^u x^*\right\}, \frac{a^l}{b^u}\right\}. \tag{19}$$

Lemma 4. *Assume that $r_1^l > 0$; every positive solution $(x_1(n), x_2(n))$ of system (14) satisfies*

$$\limsup_{n\to\infty} x_1(n) \le M_1, \tag{20}$$

where $M_1 = (\exp\{\alpha_1 r_1^u - 1\}/\alpha_1 a_1^l)^{1/\alpha_1}$.

Proof. By the first equation of system (14), we have

$$x_1(n) \le x_1(n)\exp\left\{r_1(n) - a_1(n)x_1^{\alpha_1}(n)\right\}. \tag{21}$$

Suppose $y_1(n) = x_1^{\alpha_1}(n)$; then $x_1(n) = [y_1(n)]^{1/\alpha_1}$.

From (21), we have

$$\begin{aligned}&\left[y_1(n+1)\right]^{1/\alpha_1}\\ &\quad\le \left[y_1(n)\right]^{1/\alpha_1}\exp\left\{r_1(n) - a_1(n)y_1(n)\right\}.\end{aligned} \tag{22}$$

That is,

$$y_1(n+1) \le y_1(n)\exp\left\{\alpha_1 r_1(n) - \alpha_1 a_1(n)y_1(n)\right\}. \tag{23}$$

Applying Lemma 2 such that

$$\limsup_{t\to+\infty} y_1(n) \le \frac{\exp\{\alpha_1 r_1^u - 1\}}{\alpha_1 a_1^l}, \tag{24}$$

hence

$$\limsup_{t\to+\infty} x_1(n) \le \left(\frac{\exp\{\alpha_1 r_1^u - 1\}}{\alpha_1 a_1^l}\right)^{1/\alpha_1} \overset{\text{def}}{=} M_1. \tag{25}$$

□

Lemma 5. *Assume that $r_2^l > 0$; every positive solution $(x_1(n), x_2(n))$ of system (14) satisfies*

$$\limsup_{n\to\infty} x_2(n) \le M_2, \tag{26}$$

where $M_2 = (\exp\{\alpha_2 r_2^u - 1\}/\alpha_2 b_2^l)^{1/\alpha_2}$.

Proof. The proof of Lemma 5 is similar to that of Lemma 4, so we omit the detail here. □

Lemma 6. *Assume that*

(H_3) $E_1 > 0$

holds; every positive solution $(x_1(n), x_2(n))$ *of system (14) satisfies*

$$\liminf_{n\to\infty} x_1(n) \geq m_1, \quad (27)$$

where $m_1 = (E_1/E_2)^{1/\alpha_1}\exp\{E_1 - E_2 M_1^{\alpha_1}\}$, $E_1 = r_1^l - b_1^u M_2^{\alpha_2}$, *and* $E_2 = a_1^u + c_1^u M_2^{\alpha_2}$.

Proof. In view of (26), for each $\varepsilon > 0$, there exists a $N_1 > 0$ such that

$$x_2(n) \leq M_2 + \varepsilon, \quad \forall n \geq N_1. \quad (28)$$

By the first equation of system (14), we have

$$\begin{aligned} x_1(n+1) &\geq x_1(n)\exp\left\{r_1(n) - a_1(n)x_1^{\alpha_1}(n)\right. \\ &\quad \left. - b_1(n)(M_2+\varepsilon)^{\alpha_2} - c_1(n)x_1^{\alpha_1}(n)(M_2+\varepsilon)^{\alpha_2}\right\} \\ &= x_1(n)\exp\left\{r_1(n) - b_1(n)(M_2+\varepsilon)^{\alpha_2}\right. \\ &\quad \left. - \left(a_1(n) + c_1(n)(M_2+\varepsilon)^{\alpha_2}\right)x_1^{\alpha_1}(n)\right\}. \end{aligned} \quad (29)$$

Suppose $y_1(n) = x_1^{\alpha_1}(n)$; then $x_1(n) = [y_1(n)]^{1/\alpha_1}$.

From (28), we have

$$\begin{aligned} \left[y_1(n+1)\right]^{1/\alpha_1} &\geq \left[y_1(n)\right]^{1/\alpha_1}\exp\left\{r_1(n)\right. \\ &\quad - b_1(n)(M_2+\varepsilon)^{\alpha_2} \\ &\quad \left. - \left(a_1(n) + c_1(n)(M_2+\varepsilon)^{\alpha_2}\right)y_1(n)\right\}. \end{aligned} \quad (30)$$

That is,

$$\begin{aligned} y_1(n+1) &\geq y_1(n)\exp\left\{\alpha_1 r_1(n) - \alpha_1 b_1(n)(M_2+\varepsilon)^{\alpha_2}\right. \\ &\quad \left. - \left(\alpha_1 a_1(n) + \alpha_1 c_1(n)(M_2+\varepsilon)^{\alpha_2}\right)y_1(n)\right\} \\ &\geq y_1(n)\exp\left\{\alpha_1 r_1^l - \alpha_1 b_1^u(M_2+\varepsilon)^{\alpha_2}\right. \\ &\quad \left. - \left(\alpha_1 a_1^u + \alpha_1 c_1^u(M_2+\varepsilon)^{\alpha_2}\right)y_1(n)\right\} \\ &\stackrel{\text{def}}{=} y_1(n)\exp\left\{\alpha_1 E_{1\varepsilon} - \alpha_1 E_{2\varepsilon} y_1(n)\right\}, \end{aligned} \quad (31)$$

where $E_{1\varepsilon} = r_1^l - b_1^u(M_2+\varepsilon)^{\alpha_2}$ and $E_{2\varepsilon} = a_1^u + c_1^u(M_2+\varepsilon)^{\alpha_2}$.

Applying Lemma 3 such that

$$\begin{aligned} &\liminf_{n\to+\infty} y_1(n) \\ &\quad \geq \min\left\{\frac{E_1}{E_2}\exp\left\{\alpha_1 E_1 - \alpha_1 E_2 M_1^{\alpha_1}\right\}, \frac{E_1}{E_2}\right\}, \end{aligned} \quad (32)$$

where $E_1 = r_1^l - b_1^u M_2^{\alpha_2}$ and $E_2 = a_1^u + c_1^u M_2^{\alpha_2}$.

Hence

$$\begin{aligned} &\liminf_{n\to+\infty} x_1(n) \\ &\quad \geq \min\left\{\left(\frac{E_1}{E_2}\right)^{1/\alpha_1}\exp\left\{E_1 - E_2 M_1^{\alpha_1}\right\}, \left(\frac{E_1}{E_2}\right)^{1/\alpha_1}\right\}. \end{aligned} \quad (33)$$

Note that

$$\begin{aligned} M_1 &= \left(\frac{\exp\left\{\alpha_1 r_1^u - 1\right\}}{\alpha_1 a_1^l}\right)^{1/\alpha_1} \geq \left(\frac{\alpha_1 r_1^u}{\alpha_1 a_1^l}\right)^{1/\alpha_1} \\ &= \left(\frac{r_1^u}{a_1^l}\right)^{1/\alpha_1}. \end{aligned} \quad (34)$$

Thus

$$r_1^l - a_1^u M_1^{\alpha_1} \leq 0. \quad (35)$$

And so

$$E_1 - E_2 M_1^{\alpha_1} \leq 0. \quad (36)$$

Hence

$$\liminf_{n\to+\infty} x_1(n) \geq \left(\frac{E_1}{E_2}\right)^{1/\alpha_1}\exp\left\{E_1 - E_2 M_1^{\alpha_1}\right\} \stackrel{\text{def}}{=} m_1. \quad (37)$$

□

Lemma 7. *Assume that*

(H_4) $F_1 > 0$

holds; every positive solution $(x_1(n), x_2(n))$ *of system (14) satisfies*

$$\liminf_{n\to\infty} x_2(n) \geq m_2, \quad (38)$$

where $m_2 = (F_1/F_2)^{1/\alpha_2}\exp\{F_1 - F_2 M_2^{\alpha_2}\}$, $F_1 = r_2^l - a_2^u M_1^{\alpha_1}$, *and* $F_2 = b_2^u + c_2^u M_1^{\alpha_1}$.

Proof. The proof of Lemma 7 is similar to that of Lemma 6, so we omit the detail here. □

Lemma 8. *Assume that* (H_1)–(H_4) *hold; then system (14) is permanent. That is, for every solution* $(x_1(n), x_2(n))$ *of system (14), one has*

$$m_i \leq \liminf_{n\to+\infty} x_i(n) \leq \limsup_{n\to+\infty} x_i(n) \leq M_i, \quad i = 1, 2. \quad (39)$$

3. Extinction

Theorem 9. *Assume that* (H_1), (H_2) *hold; assume further that*

(H_5) $\limsup_{n\to\infty}(\sum_{s=n}^{n+\omega-1} r_2(s)/\sum_{s=n}^{n+\omega-1} r_1(s)) < \liminf_{n\to\infty}\{a_2(n)/a_1(n), b_2(n)/b_1(n), c_2(n)/c_1(n) \mid n \in Z\}$.

Let $(x_1(n), x_2(n))$ *be any positive solution of system (14); then* $x_2(n) \to 0$ *as* $n \to +\infty$.

Proof. By Lemma 2 we know that there exists $n_1 > 0$ such that

$$x_1(n) \le 2M_1, \quad n \ge n_1. \tag{40}$$

By (H_2), there exist positive constants η_0 and $n_2 > n_1$ such that

$$\sum_{s=n}^{n+\omega-1} r_i(s) \ge \eta_0, \quad n \ge n_2. \tag{41}$$

By (H_5), we can choose positive constants α, β, and ε such that

$$\begin{aligned}\limsup_{n\to\infty}\frac{\sum_{s=n}^{n+\omega-1} r_2(s)}{\sum_{s=n}^{n+\omega-1} r_1(s)} &< \frac{\alpha}{\beta}-\varepsilon < \frac{\alpha}{\beta} \\ &< \liminf_{n\to\infty}\left\{\frac{a_2(n)}{a_1(n)}, \frac{b_2(n)}{b_1(n)}, \frac{c_2(n)}{c_1(n)}\right\}.\end{aligned} \tag{42}$$

Thus, there exists a $n_3 > n_2 > 0$, such that for all $n \ge n_3$

$$\sum_{s=n}^{n+\omega-1}\left(\beta r_2(s)-\alpha r_1(s)\right) < -\varepsilon\beta\sum_{s=n}^{n+\omega-1} r_1(s) < -\varepsilon\beta\eta_0, \tag{43}$$

$$\begin{aligned}&\alpha a_1(n)-\beta a_2(n) < 0,\\ &\alpha b_1(n)-\beta b_2(n) < 0,\\ &\alpha c_1(n)-\beta c_2(n) < 0.\end{aligned} \tag{44}$$

Let $V(n) = x_1^{-\alpha}(n)x_2^{\beta}(n)$; then

$$\begin{aligned}\frac{V(n+1)}{V(n)} &= \left[\frac{x_1(n+1)}{x_1(n)}\right]^{-\alpha}\left[\frac{x_2(n+1)}{x_2(n)}\right]^{\beta}\\ &= \exp\{-\alpha r_1(n)+\alpha a_1(n)x_1^{\alpha_1}(n)+\alpha b_1(n)x_2^{\alpha_2}(n)\\ &\quad+\alpha c_1(n)x_1^{\alpha_1}(n)x_2^{\alpha_2}(n)+\beta r_2(n)-\beta a_2(n)x_1^{\alpha_1}(n)\\ &\quad-\beta b_2(n)x_2^{\alpha_2}(n)-\beta c_2(n)x_1^{\alpha_1}(n)x_2^{\alpha_2}(n)\}\\ &= \exp\{\left(\beta r_2(n)-\alpha r_1(n)\right)\\ &\quad+\left(\alpha a_1(n)-\beta a_2(n)\right)x_1^{\alpha_1}(n)\\ &\quad+\left(\alpha b_1(n)-\beta b_2(n)\right)x_2^{\alpha_2}(n)\\ &\quad+\left(\alpha c_1(n)-\beta c_2(n)\right)x_1^{\alpha_1}(n)x_2^{\alpha_2}(n)\}.\end{aligned} \tag{45}$$

From (43) and (44), it follows that

$$V(n+1) \le V(n)\exp\left(\beta r_2(n)-\alpha r_1(n)\right), \quad n \ge n_3. \tag{46}$$

For any $n \ge n_3$, we choose an integer $m \ge 0$ such that $n \in [n_3+m\omega, n_3+(m+1)\omega)$. Integrating (46) from n_3 to $n-1$, from (43), we have

$$\begin{aligned}V(n) &\le V(n_3)\exp\left\{\sum_{s=n_3}^{n-1}\left(\beta r_2(s)-\alpha r_1(s)\right)\right\}\\ &= V(n_3)\exp\left(\sum_{s=n_3}^{n_3+m\omega-1}+\sum_{s=n_3+m\omega-1}^{n-1}\right)\\ &\quad\cdot\left(\beta r_2(s)-\alpha r_1(s)\right)\\ &\le V(n_3)\exp\{-\varepsilon\beta\eta_0 m + A_1\}\\ &< V(n_3)\exp\left\{-\varepsilon\beta\eta_0\left(\frac{n-n_3}{\omega}-1\right)+A_1\right\}\\ &= V(n_3)\exp\left\{-\frac{\varepsilon\beta\eta_0 n}{\omega}+A_1^*\right\},\end{aligned} \tag{47}$$

where $A_1^* = \varepsilon\beta\eta_0 n_3/\omega + \varepsilon\beta\eta_0 + A_1$ and $A_1 = \sup_{n\in Z}|\beta r_2(n) - \alpha r_1(n)|\omega$.

(47) implies that

$$\begin{aligned}x_2(n) &< \left[x_1^{-\alpha}(n_3)x_2^{\beta}(n_3)(2M_1)^{\alpha}\exp\{A_1^*\}\right]^{1/\beta}\\ &\quad\cdot\exp\left\{-\frac{\varepsilon\eta_0 n}{\omega}\right\},\end{aligned} \tag{48}$$

for all $n \ge n_3$. Hence, $x_2(n) \to 0$ exponentially as $n \to +\infty$. □

4. Global Stability

In Section 3, we prove that species x_2 will be driven to extinction if the conditions (H_1), (H_2), and (H_5) hold. Now we investigate the stability property of species x_1 under the same conditions.

Before we state the main result of this section, we first introduce some lemmas.

Lemma 10. *Assume that (H_1), (H_2), (H_5), and $r_1^l > 0$ hold; let $(x_1(n), x_2(n))$ be any positive solution of system (14); then*

$$m_1' \le \liminf_{n\to\infty} x_1(n) \le \limsup_{n\to\infty} x_1(n) \le M_1, \tag{49}$$

where $m_1' = (r_1^l/a_1^u)^{1/\alpha_1}\exp\{r_1^l - a_1^u M_1^{\alpha_1}\}$.

Proof. Under the assumption conditions (H_1), (H_2), and (H_5), it follows from Theorem 9 that

$$\lim_{n\to\infty} x_2(n) = 0. \tag{50}$$

From Lemma 4, we have

$$\limsup_{n\to\infty} x_1(n) \le M_1. \tag{51}$$

By Lemma 10, it is enough to show that

$$\liminf_{n\to\infty} x_1(n) \ge m_1'. \tag{52}$$

In view of (50) and (51), for each $\varepsilon > 0$, there exists an integer $n_0 \in Z$ such that

$$x_1(n) \le M_1 + \varepsilon, \quad x_2(n) \le \varepsilon, \quad \forall n \ge n_0. \tag{53}$$

We consider the following two cases.

Case 1. We assume that there exists an $l_0 \ge n_0$ such that $x_1(l_0+1) \le x_1(l_0)$. Note that for $n \ge l_0$

$$\begin{aligned} x_1(n+1) &= x_1(n)\exp\left[r_1(n) - a_1(n)x_1^{\alpha_1}(n) - b_1(n)x_2^{\alpha_2}(n) - c_1(n)x_1^{\alpha_1}(n)x_2^{\alpha_2}(n)\right] \\ &\ge x_1(n)\exp\left[r_1^l - a_1^u x_1^{\alpha_1}(n) - b_1^u\varepsilon^{\alpha_2} - c_1^u(M_1+\varepsilon)^{\alpha_1}\varepsilon^{\alpha_2}\right]. \end{aligned} \tag{54}$$

In particular, with $n = l_0$, we obtain

$$r_1^l - a_1^u x_1^{\alpha_1}(l_0) - b_1^u\varepsilon^{\alpha_2} - c_1^u(M_1+\varepsilon)^{\alpha_1}\varepsilon^{\alpha_2} \le 0, \tag{55}$$

which implies that

$$x_1(l_0) \ge \left(\frac{r_1^l - \left(b_1^u + c_1^u(M_1+\varepsilon)^{\alpha_1}\right)\varepsilon^{\alpha_2}}{a_1^u}\right)^{1/\alpha_1}. \tag{56}$$

From (54) and (56), it follows that

$$\begin{aligned} x_1(l_0+1) &\ge \left(\frac{r_1^l - \left(b_1^u + c_1^u(M_1+\varepsilon)^{\alpha_1}\right)\varepsilon^{\alpha_2}}{a_1^u}\right)^{1/\alpha_1} \cdot \exp\left[r_1^l - a_1^u x_1^{\alpha_1}(l_0) - b_1^u\varepsilon^{\alpha_2} - c_1^u(M_1+\varepsilon)^{\alpha_1}\varepsilon^{\alpha_2}\right] \\ &\ge \left(\frac{r_1^l - \left(b_1^u + c_1^u(M_1+\varepsilon)^{\alpha_1}\right)\varepsilon^{\alpha_2}}{a_1^u}\right)^{1/\alpha_1} \exp\left[r_1^l - a_1^u(M_1+\varepsilon)^{\alpha_1} - b_1^u\varepsilon^{\alpha_2} - c_1^u(M_1+\varepsilon)^{\alpha_1}\varepsilon^{\alpha_2}\right]. \end{aligned} \tag{57}$$

Let

$$x_{1\varepsilon} = \left(\frac{r_1^l - \left(b_1^u + c_1^u(M_1+\varepsilon)^{\alpha_1}\right)\varepsilon^{\alpha_2}}{a_1^u}\right)^{1/\alpha_1} \exp\left[r_1^l - a_1^u(M_1+\varepsilon)^{\alpha_1} - b_1^u\varepsilon^{\alpha_2} - c_1^u(M_1+\varepsilon)^{\alpha_1}\varepsilon^{\alpha_2}\right]. \tag{58}$$

Note that

$$M_1 = \left(\frac{\exp\{\alpha_1 r_1^u - 1\}}{\alpha_1 a_1^l}\right)^{1/\alpha_1} \ge \left(\frac{\alpha_1 r_1^u}{\alpha_1 a_1^l}\right)^{1/\alpha_1} \ge \left(\frac{r_1^u}{a_1^l}\right)^{1/\alpha_1}, \tag{59}$$

and thus $r_1^l - a_1^u M_1^{\alpha_1} \le 0$; also, for arbitrary ε,

$$r_1^l - a_1^u(M_1+\varepsilon)^{\alpha_1} - b_1^u\varepsilon^{\alpha_2} - c_1^u(M_1+\varepsilon)^{\alpha_1}\varepsilon^{\alpha_2} \le 0, \tag{60}$$

or

$$\left(\frac{r_1^l - \left(b_1^u + c_1^u(M_1+\varepsilon)^{\alpha_1}\right)\varepsilon^{\alpha_2}}{a_1^u}\right)^{1/\alpha_1} \ge x_{1\varepsilon}. \tag{61}$$

We claim that

$$x_1(n) \ge x_{1\varepsilon}, \quad \forall n \ge l_0. \tag{62}$$

By way of contradiction, assume that there exists a $p_0 > l_0$ such that $x_1(p_0) < x_{1\varepsilon}$. Then $p_0 \ge l_0 + 2$. Let $\widetilde{p}_0 \ge l_0 + 2$ be the smallest integer such that $x_1(\widetilde{p}_0) < x_{1\varepsilon}$. Then $x_1(\widetilde{p}_0 - 1) > x_1(\widetilde{p}_0)$. The above argument produces that $x_1(\widetilde{p}_0) \ge x_{1\varepsilon}$, a contradiction. This proves the claim.

Case 2. We assume that $x_1(n+1) > x_1(n)$ for $n \ge n_0$; then $\lim_{n\to\infty} x_1(n) = \underline{x}_1$. We claim that

$$\underline{x}_1 \ge \left(\frac{r_1^l - \left(b_1^u + c_1^u(M_1+\varepsilon)^{\alpha_1}\right)\varepsilon^{\alpha_2}}{a_1^u}\right)^{1/\alpha_1}. \tag{63}$$

By way of contradiction, assume that

$$\underline{x}_1 < \left(\frac{r_1^l - \left(b_1^u + c_1^u(M_1+\varepsilon)^{\alpha_1}\right)\varepsilon^{\alpha_2}}{a_1^u}\right)^{1/\alpha_1}. \tag{64}$$

Taking limit in the first equation in system (14) gives

$$\lim_{n\to\infty}\left(r_1(n) - a_1(n)x_1^{\alpha_1}(n) - b_1(n)x_2^{\alpha_2}(n) - c_1(n)x_1^{\alpha_1}(n)x_2^{\alpha_2}(n)\right) = 0, \tag{65}$$

which is a contradiction since

$$\begin{aligned} &\lim_{n\to\infty}\left(r_1(n) - a_1(n)x_1^{\alpha_1}(n) - b_1(n)x_2^{\alpha_2}(n) - c_1(n)x_1^{\alpha_1}(n)x_2^{\alpha_2}(n)\right) \ge r_1^l - a_1^u\underline{x}_1^{\alpha_1} - b_1^u\varepsilon^{\alpha_2} \\ &\quad - c_1^u(M_1+\varepsilon)^{\alpha_1}\varepsilon^{\alpha_2} > 0. \end{aligned} \tag{66}$$

This proves the claim; then we have

$$\liminf_{n\to\infty} x_1(n) = \lim_{n\to\infty} x_1(n) = \underline{x}_1 \ge x_{1\varepsilon}. \tag{67}$$

Combining Case 1 and Case 2, we see that

$$\liminf_{n\to\infty} x_1(n) \ge x_{1\varepsilon}. \tag{68}$$

Setting $\varepsilon \to 0$, note that

$$\lim_{\varepsilon\to 0} x_{1\varepsilon} = \left(\frac{r_1^l}{a_1^u}\right)^{1/\alpha_1} \exp\left\{r_1^l - a_1^u M_1^{\alpha_1}\right\} \overset{\text{def}}{=} m_1'. \tag{69}$$

Now, we can easily see that (52) holds. This completes the proof of Lemma 10. □

We consider a discrete equation

$$x(n+1) = x(n)\exp\left(r_1(n) - a_1(n)x^{\alpha_1}(n)\right), \quad n \in N, \tag{70}$$

where $\{r_1(n)\}$ and $\{a_1(n)\}$ are bounded nonnegative sequences; similarly to the proof of Lemma 10, we can obtain the following lemma.

Lemma 11. *For any positive solution $\{x(n)\}$ of (70), one has*

$$m \le \liminf_{n\to\infty} x(n) \le \limsup_{n\to\infty} x(n) \le M, \tag{71}$$

where $M = (\exp\{\alpha_1 r_1^u - 1\}/\alpha_1 a_1^l)^{1/\alpha_1}$ *and* $m = (r_1^l/a_1^u)^{1/\alpha_1}\exp\{r_1^l - a_1^u M^{\alpha_1}\}$.

Now, we state the main result of this section.

Theorem 12. *Assume that (H_1), (H_2), (H_5), and $r_1^l > 0$ hold; assume further that*

$$\frac{a_1^u}{a_1^l}\exp\left(\alpha_1 r_1^u - 1\right) < 2; \tag{H_6}$$

then for any positive solution $(x_1(n), x_2(n))$ of system (14) and any positive solution $\{x(n)\}$ of system (70), one has

$$\lim_{n\to\infty}\left(x_1(n) - x(n)\right) = 0, \qquad \lim_{n\to\infty} x_2(n) = 0. \tag{72}$$

Proof. Since (H_1), (H_2), and (H_5) hold, it follows from Theorem 12 that

$$\lim_{n\to\infty} x_2(n) = 0. \tag{73}$$

To prove $\lim_{n\to\infty}(x_1(n) - x(n)) = 0$, let

$$x_1(n) = x(n)\exp\left(y(n)\right). \tag{74}$$

It follows from the first equation of system (14) and (74) that

$$\begin{aligned} y(n+1) = {} & y(n) - a_1(n)x^{\alpha_1}(n)\left(\exp\left(\alpha_1 y(n)\right) - 1\right) \\ & - b_1(n)x_2^{\alpha_2}(n) - c_1(n)x_1^{\alpha_1}(n)x_2^{\alpha_2}(n). \end{aligned} \tag{75}$$

Using the Mean Value Theorem, we get

$$\exp\left(\alpha_1 y(n)\right) - 1 = \alpha_1\exp\left(\theta(n)\right)y(n), \quad \theta(n) \in \left(0, \alpha_1 y(n)\right). \tag{76}$$

Then the first equation of system (14) is equivalent to

$$\begin{aligned} y(n+1) = {} & \left(1 - \alpha_1 a_1(n)x^{\alpha_1}(n)\exp\left(\theta(n)\right)\right)y(n) \\ & - \left(b_1(n) + c_1(n)x_1^{\alpha_1}(n)\right)x_2^{\alpha_2}(n), \end{aligned} \tag{77}$$

where $\theta(n) \in (0, \alpha_1 y(n))$.

To complete the proof, it suffices to show that

$$\lim_{n\to\infty} y(n) = 0. \tag{78}$$

We first assume that

$$\lambda = \max\left\{\left|1 - \alpha_1 a_1^u M_1^{\alpha_1}\right|, \left|1 - \alpha_1 a_1^l m^{\alpha_1}\right|\right\} < 1, \tag{79}$$

and then we can choose positive constant $\varepsilon > 0$ small enough such that

$$\lambda_\varepsilon = \max\left\{\left|1 - \alpha_1 a_1^u\left(M_1 + \varepsilon\right)^{\alpha_1}\right|, \left|1 - \alpha_1 a_1^l\left(m - \varepsilon\right)^{\alpha_1}\right|\right\} < 1. \tag{80}$$

For above ε, according to Lemmas 10 and 11 and (73), there exists an integer $n_0 \in Z$ such that

$$\begin{aligned} & m - \varepsilon \le x(n), \\ & x_1(n) \le M_1 + \varepsilon, \\ & x_2(n) \le \varepsilon, \end{aligned} \quad \text{for } n \ge n_0. \tag{81}$$

It follows from (81) that

$$b_1(n) + c_1(n)x_1^{\alpha_1}(n) \le b_1^u + c_1^u\left(M_1 + \varepsilon\right)^{\alpha_1} \stackrel{\text{def}}{=} M_\varepsilon, \quad \text{for } n \ge n_0. \tag{82}$$

Note that $\theta(n) \in (0, \alpha_1 y(n))$ implies that $\alpha_1 x^{\alpha_1}(n)\exp(\theta(n))$ lies between $\alpha_1 x^{\alpha_1}(n)$ and $\alpha_1 x_1^{\alpha_1}(n)$. From (77) and (80)–(82), we get

$$\begin{aligned} \left|y(n+1)\right| & \le \max\left\{\left|1 - \alpha_1 a_1^u\left(M_1 + \varepsilon\right)^{\alpha_1}\right|, \left|1 - \alpha_1 a_1^l\left(m - \varepsilon\right)^{\alpha_1}\right|\right\} \\ & \cdot \left|y(n)\right| + \left(b_1^u + c_1^u\left(M_1 + \varepsilon\right)^{\alpha_1}\right)\varepsilon^{\alpha_2} = \lambda_\varepsilon\left|y(n)\right| \\ & + M_\varepsilon\varepsilon^{\alpha_2}, \quad \text{for } n \ge n_0. \end{aligned} \tag{83}$$

This implies that

$$\left|y(n)\right| \le \lambda_\varepsilon^{n-n_0}\left|y(n_0)\right| + \frac{1 - \lambda_\varepsilon^{n-n_0}}{1 - \lambda_\varepsilon}M_\varepsilon\varepsilon^{\alpha_2}, \quad \text{for } n \ge n_0. \tag{84}$$

Since $\lambda_\varepsilon < 1$ and ε is arbitrary small, we obtain $\lim_{n\to\infty} y(n) = 0$; it means that (78) holds when $\lambda < 1$.

Note that

$$1 - \alpha_1 a_1^u M_1^{\alpha_1} \le 1 - \alpha_1 a_1^l m^{\alpha_1} < 1, \tag{85}$$

and thus, $\lambda < 1$ is equivalent to

$$1 - \alpha_1 a_1^u M_1^{\alpha_1} > -1, \tag{86}$$

or

$$\alpha_1 a_1^u M_1^{\alpha_1} = \frac{a_1^u}{a_1^l}\exp\left(\alpha_1 r_1^u - 1\right) < 2. \tag{87}$$

Now, we can conclude that (78) is satisfied as (H_6) holds, and so $\lim_{n\to\infty}(x_1(n) - x(n)) = 0$. This completes the proof of Theorem 12. □

As a direct corollary of Theorems 9 and 12, for system (2), we have the following result.

Corollary 13. *Assume that* (H_1), (H_2), (H_5), *and* $r_1^l > 0$ *hold; assume further that*

$$\frac{a_1^u}{a_1^l}\exp\left(r_1^u - 1\right) < 2; \tag{88}$$

then for any positive solution $(x_1(n), x_2(n))$ *of system (2) and any positive solution* $\{x(n)\}$ *of*

$$x_1(n+1) = x_1(n)\exp\left\{r_1(n) - a_1(n)x_1(n)\right\}, \tag{89}$$

one has

$$\begin{gathered}\lim_{n\to\infty}\left(x_1(n) - x(n)\right) = 0,\\ \lim_{n\to\infty} x_2(n) = 0.\end{gathered} \tag{90}$$

5. Examples

The following examples show the feasibility of our main results.

Example 14. Now let us consider Example 1; in this case, one can easily check that

$$\begin{gathered}\liminf_{n\to\infty}\sum_{s=n}^{n+\omega-1} r_i(s) > 0,\\ \limsup_{n\to\infty}\frac{\sum_{s=n}^{n+\omega-1} r_2(s)}{\sum_{s=n}^{n+\omega-1} r_1(s)} = 1,\\ \frac{a_2(n)}{a_1(n)} = \frac{b_2(n)}{b_1(n)} = \frac{c_2(n)}{c_1(n)} = 2;\end{gathered} \tag{91}$$

hence

$$\begin{aligned}\limsup_{n\to\infty}\frac{\sum_{s=n}^{n+\omega-1} r_2(s)}{\sum_{s=n}^{n+\omega-1} r_1(s)} &< 2\\ &= \liminf_{n\to\infty}\left\{\frac{a_2(n)}{a_1(n)}, \frac{b_2(n)}{b_1(n)}, \frac{c_2(n)}{c_1(n)}\right\}.\end{aligned} \tag{92}$$

Also

$$\frac{a_1^u}{a_1^l}\exp\left(\alpha_1 r_1^u - 1\right) = \frac{1.3}{0.7}\exp(0.9 - 1) \approx 1.680 < 2. \tag{93}$$

Equations (91)–(93) show that all the conditions of Corollary 13 hold; then species x_2 will be driven to extinction while species x_1 will be globally attractive with any positive solution of the following discrete equation:

$$\begin{aligned}x_1(n+1) &= x_1(n)\\ &\cdot \exp\left[0.6 + 0.3\sin(n) - (1 + 0.3\sin(n))\,x_1(n)\right].\end{aligned} \tag{94}$$

Example 15. Consider the following system:

$$\begin{aligned}x_1(n+1) = x_1(n)\exp\Big[&0.6 - (1.5 + 0.2\sin(n))\,x_1^2(n)\\ &- (1 + 0.5\sin(n))\,x_2^{1/2}(n)\\ &- (1.1 + \sin(n))\,x_1^2(n)\,x_2^{1/2}(n)\Big],\\ x_2(n+1) = x_2(n)\exp\Big[&0.3 - (1.4 + 0.5\sin(n))\,x_1^2(n)\\ &- (1.3 + 0.5\sin(n))\,x_2^{1/2}(n) - 1.2x_1^2(n)\,x_2^{1/2}(n)\Big].\end{aligned} \tag{95}$$

In this case, corresponding to system (2), $r_1(n) = 0.6$, $r_2(n) = 0.3$, $a_1(n) = 1.5 + 0.2\sin(n)$, $b_1(n) = 1 + 0.5\sin(n)$, $c_1(n) = 1.1 + \sin(n)$, $a_2(n) = 1.4 + 0.5\sin(n)$, $b_2(n) = 1.3 + 0.5\sin(n)$, $c_2(n) = 1.2$, $\alpha_1 = 2$, and $\alpha_2 = 1/2$. By simple computation, one can see that

$$\begin{gathered}\liminf_{n\to\infty}\sum_{s=n}^{n+\omega-1} r_i(s) > 0,\\ \limsup_{n\to\infty}\frac{\sum_{s=n}^{n+\omega-1} r_2(s)}{\sum_{s=n}^{n+\omega-1} r_1(s)} = \frac{1}{2},\end{gathered} \tag{96}$$

$$\frac{a_2(n)}{a_1(n)} \geq \frac{0.9}{1.7},\quad \frac{b_2(n)}{b_1(n)} \geq \frac{0.8}{1.5},\quad \frac{c_2(n)}{c_1(n)} \geq \frac{1.2}{2.1}, \tag{97}$$

$$\begin{aligned}\limsup_{n\to\infty}\frac{\sum_{s=n}^{n+\omega-1} r_2(s)}{\sum_{s=n}^{n+\omega-1} r_1(s)} &< \frac{0.9}{1.7}\\ &\leq \liminf_{n\to\infty}\left\{\frac{a_2(n)}{a_1(n)}, \frac{b_2(n)}{b_1(n)}, \frac{c_2(n)}{c_1(n)}\right\},\end{aligned} \tag{98}$$

$$\begin{aligned}\frac{a_1^u}{a_1^l}\exp\left(\alpha_1 r_1^u - 1\right) &\leq \frac{1.7}{1.3}\exp(2\times 0.6 - 1) \approx 1.5972\\ &< 2.\end{aligned} \tag{99}$$

Equations (97) and (99) show that all the conditions of Theorem 12 hold; thus species x_2 is driven to extinction while species x_1 is asymptotic to any positive solution of

$$\begin{aligned}x(n+1)&\\ &= x(n)\exp\left(0.6 - (1.5 + 0.2\sin(n))\,x^2(n)\right).\end{aligned} \tag{100}$$

Figure 2 shows the dynamic behaviors of system (95).

6. Conclusion

In this paper, we consider a nonlinear discrete two species competition system with the effect of toxic substances. In

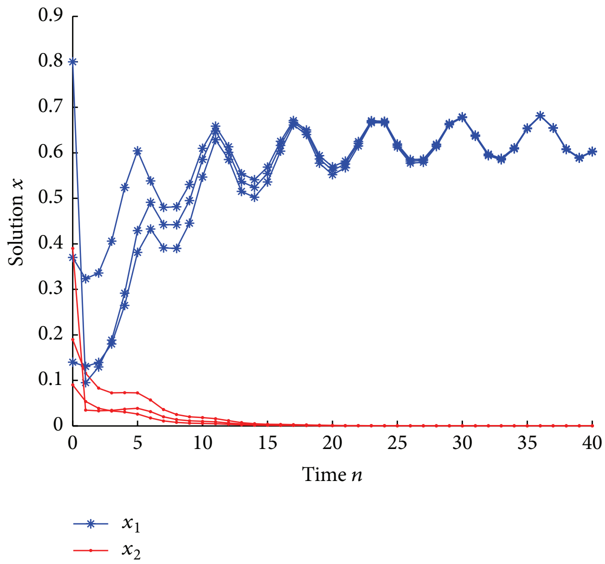

Figure 2: Dynamic behaviors of system (95) with initial values $(x_1(0), x_2(0)) = (0.14, 0.19)$, $(0.27, 0.69)$, and $(0.8, 0.39)$, respectively.

Theorem 9, by constructing a suitable Lyapunov-type function, we obtain a set of sufficient conditions which ensure species x_2 will be driven to extinction. Our results improve and generalize Theorem 2.1 of [2] and Theorem 1.1 of [3].

Competing Interests

The authors declare that there are no competing interests.

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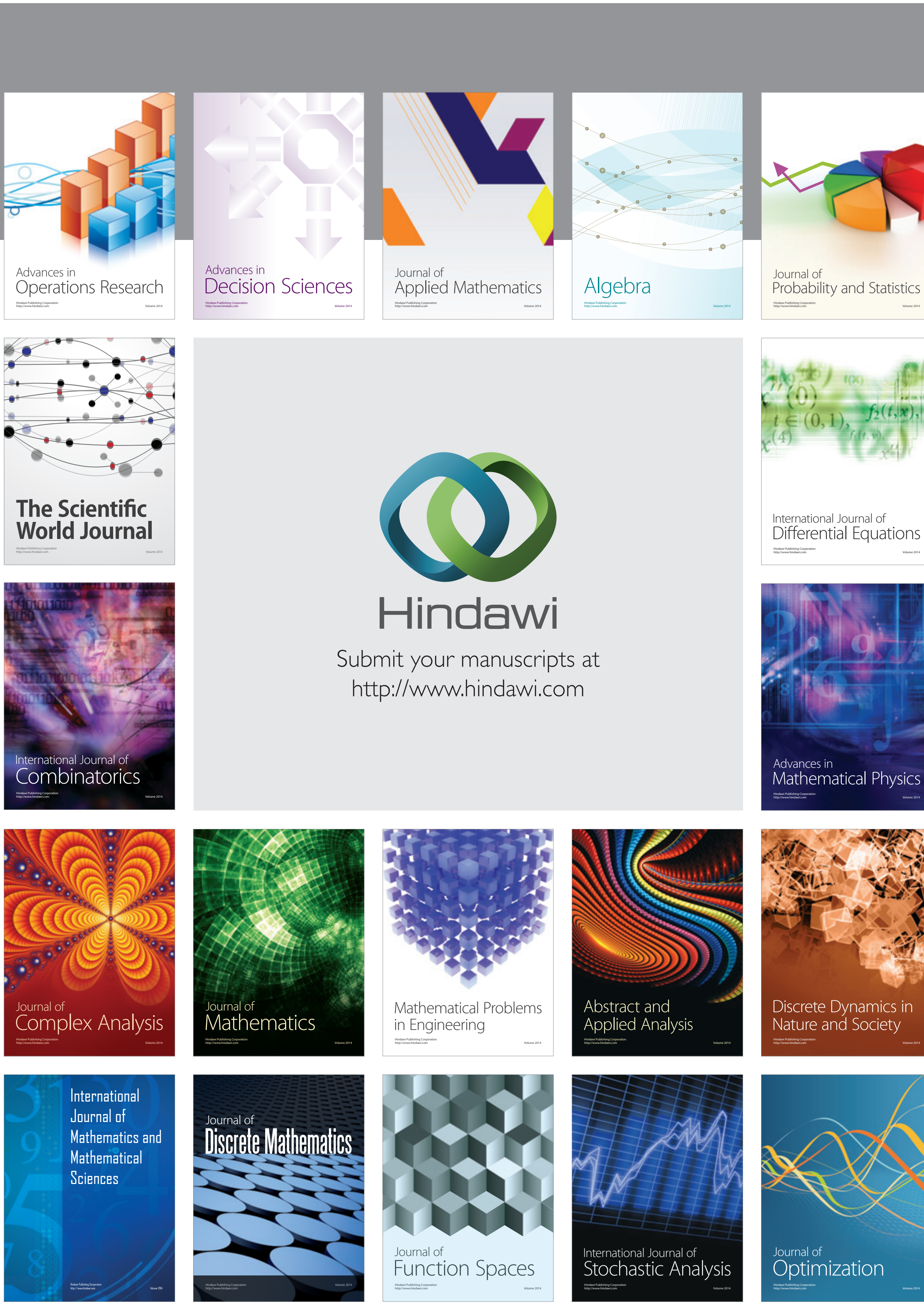
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Mathematical Problems in Engineering
Abstract and Applied Analysis
Discrete Dynamics in Nature and Society
International Journal of Mathematics and Mathematical Sciences
Journal of Discrete Mathematics
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Journal of Optimization