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Research Article

Flexible Stock Allocation and Trim Loss Control for Cutting Problem in the Industrial-Use Paper Production

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We consider a one-dimensional cutting stock problem (CSP) in which the stock widths are not used to fulfill the order but kept for use in the future for the industrial-use paper production. We present a new model based on the flexible stock allocation and trim loss control to determine the production quantity. We evaluate our approach using a real data and show that we are able to solve industrial-size problems, while also addressing common cutting considerations such as aggregation of orders, multiple stock widths, and cutting different patterns on the same machine. In addition, we compare our model with others, including trim loss minimization problem (TLMP) and cutting stock problem (CSP). The results show that the proposed model outperforms the other two models regarding total flexibility and trim loss ratio.

1. Introduction

A one-dimensional cutting stock problem (CSP) is one of the famous combinatorial optimization problems, which has many applications in industries, such as paper, wood, textiles, steel, space, ship construction, and logistic transportation [1–6]. Most studies focus on minimizing the trim loss that is the amount of residual pieces of processed stock lengths. A standard one-dimensional cutting stock problem (S1D-CSP) as a kind of the above problems is known as an NP-complete one [7]. Numerous studies have examined how to fulfill orders and optimize production planning. Gilmore and Gomory [8] presented a delayed pattern generation technique for solving a one-dimensional cutting problem using linear programming. Other methods, including pattern-oriented approach, item-oriented approach, mixed approach and exact approach, can be found in [9–27].

In the industrial-use paper industry, the production quantity is usually greater than the customers' order. Using the traditional CSP, the trim loss can be significant. Thus, we need to consider usable leftovers to prevent the trim loss generated after optimization. This issue becomes a one-dimensional CSP with usable leftovers. Yanasse [28] reported that the literature on usable leftovers is scarce and the problem still lacks clear and appropriate definitions. Kos and Duhovnik [29] proposed usable leftover material used in the next cutting plan to reduce trim loss. Related studies can be found in [6, 29–35]. Cherri et al. [31] presented several modifications in some well-known heuristics to solve a one-dimensional CSP with usable leftovers. Poldi and Arenales [32] presented a study with the classical one-dimensional integer CSP, which consists of cutting a set of available stock lengths in order to produce smaller ordered items. Cui and Yang [33] considered a one-dimensional CSP with useful leftover in the cutting plan. Cherri et al. [35] proposed a priority-in-use heuristic approach to solve a one-dimensional CSP with usable leftovers. However, these models cannot be directly used for solving the CSP in the industrial-use production that each reel can only be produced a certain number of rolls depending on its cutting machine. Wang and Liu [36] presented a new decision model for reducing trim loss and inventory in the paper industry.

In this study, we present a new model based on the flexible stock allocation and trim loss control to determine the production quantity. Our proposed model is a flexibility maximization problem (FAP). Under a certain condition of trim loss control, FAP can be confined to cutting stock problem (CSP) or trim loss minimization problem (TLMP).

The remainder of this paper is organized as follows. In Section 2, the definition of problem in the paper industry is presented. A new model is developed in Section 3. In Section 4, some examples illustrate the application of the proposed model. Finally, conclusions are drawn in Section 5.

2. Problem Definition

The production of industrial-use paper starts from raw material to reels and then from reels to the production of rolls as finished goods. The entire operation mode is cyclical production, which is the only method for achieving production efficiency. Therefore, the leftover material is not used in a follow-up production. For the production planning (see Figure 1), the customer's paper requirements are obtained and the marketing demand is predicted. Then, during the combined production-marketing meeting, the number of production days and the production quantity of paper types are determined. The production quantity indicates the N number of reels, and each reel can produce the NR number of rolls that depends on the paper type. It should be noted that the unit of the paper width is millimeter (mm).

To formulate the models of CSP, TLMP and FAP, see the notations section are used.

The main research question is how to improve the stock allocation and trim loss of a CSP with useful leftovers in the paper industry. This problem can be studied for either one- or multidimensional CSPs. In this study, one-dimensional CSP with useful leftovers was used. We first provide two examples to illustrate the differences between CSP, TLMP, and manual adjustment (MA). In practice, MA is used in the industrial-use paper industry in which the CSP and TLMP solutions are candidates as manually selecting as MA solution. The CSP and TLMP are usually solved through column generation [8, 9]. To obtain the solutions of CSP and TLMP, the computer program was written in Lingo 11 Software [37].

The formulation of CSP is defined as follows:

$$\text{(CSP) Minimize} \quad \sum_{r=1}^{t} x_r, \tag{1}$$

s.t.

$$L \geq \sum_{i=1}^{m} a_{ir}\text{ow}_i, \quad \text{(width of reel constraint)} \tag{2}$$

$$\sum_{r=1}^{t} a_{ir}x_r \geq d_i, \quad \text{(demand constraint)} \tag{3}$$

$$\text{UB} \geq L - \sum_{i=1}^{m} a_{ir}\text{ow}_i, \quad \text{(trim loss constraint)}, \tag{4}$$

where a_{ir} and x_r are decision variables and integer variables. Minimizing the total number of patterns is the objective function (1) of the model. Constraint (2) guarantees the cutting stocks regarding the reel width. Constraint (3) guarantees the cutting stocks regarding the demand. In the industrial-use paper production, there exists the maximum trim loss for each cutting, and then constraint (4) guarantees the waste of each roll during the cutting process.

In order to reduce the trim loss, a modified model called TLMP is given as follows:

$$\text{(TLMP) Minimize} \quad \sum_{r=1}^{t} \left(L - \sum_{i=1}^{m} a_{ir}\text{ow}_i \right) x_r, \tag{5}$$

s.t.

$$L \geq \sum_{i=1}^{m} a_{ir}\text{ow}_i, \quad \text{(width of reel constraint)} \tag{6}$$

$$\sum_{r=1}^{t} a_{ir}x_r \geq d_i, \quad \text{(demand constraint)} \tag{7}$$

$$\text{SRQ} = \frac{\sum_{r=1}^{t} x_r}{\text{NR}} \quad \text{(reel set constraint)} \tag{8}$$

$$\text{UB} \geq L - \sum_{i=1}^{m} a_{ir}d_i, \quad \text{(trim loss constraint)}, \tag{9}$$

where a_{ir} and x_r are decision variables and integer variables. Minimizing the total trim loss is the objective function (5) of the model. Constraint (6) guarantees the cutting stocks regarding the reel width. Constraint (7) guarantees the cutting stocks regarding the demand. In industry paper production, the maximum trim loss for each cutting and the limit production volume are considered; then constraint (8) guarantees the number of rolls for each reel, and constraint (9) guarantees the waste of each roll during the cutting process.

For example 1, we assume that the reel width is 10 units, NR is 3, UB is 3, the demand of order widths $\{3, 4\}$ is $\{3, 3\}$, and the stock widths are $\{3, 4, 5\}$. In Figure 2, we provide CSP, TLMP, and MA solutions. The trim loss using CSP is 5 units. And the trim loss using TLMP is zero. We found that the stock width using CSP is obtained as $\{4\}$ and the stock widths using TLMP are $\{3\} * 3$. In order to obtain flexible stock widths, using MA based on CSP and TMLP solutions, the extending stock width is determined as $\{6\}$, and the stock width is obtained as $\{3\}$. Thus, MA can provide more flexible stock width $\{6\}$.

For example 2, we assume that the reel width is 10 units, NR is 3, UB is 3, the demand of order widths $\{3, 5\}$ is $\{1, 1\}$, and the stock widths are $\{3, 4, 5\}$. In Figure 3, we provide CSP, TLMP, and MA solutions. The trim loss using CSP is 2 units. And the trim loss using TLMP is zero. We found that the stock width using CSP is zero, the unused rolls are 2 and 3, and the stock widths using TLMP are $\{3\} * 1$, $\{4\} * 1$, and $\{5\} * 3$. Using MA based on CSP and TMLP solutions, the extending stock widths are determined as $\{10, 10\}$ or $\{7, 10\}$, the stock width is obtained as zero or $\{5\}$, and the trim loss is obtained as 2 units or zero. Thus, MA can provide more flexible stock widths $\{10, 10\}$ or $\{7, 10\}$.

Based on the above discussions, we conclude that the MA approach can provide more flexible stock widths in a one-dimensional CSP with useful leftovers. This motivates

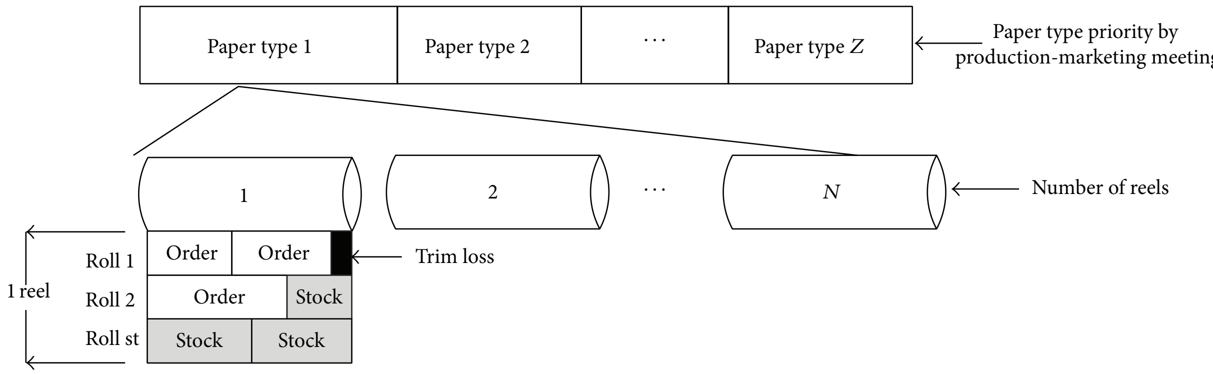

Figure 1: Production planning in the paper industry.

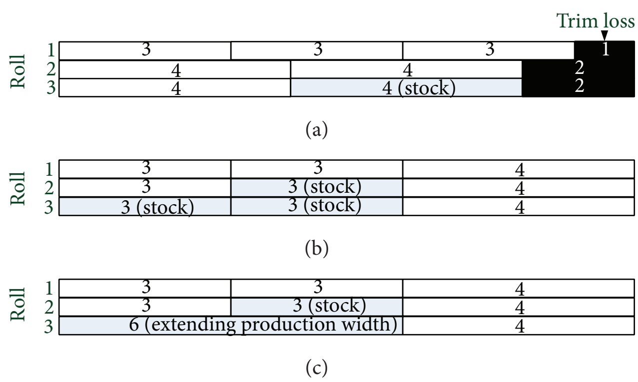

Figure 2: (a) CSP solution, (b) TLMP solution, and (c) MA solution for example 1.

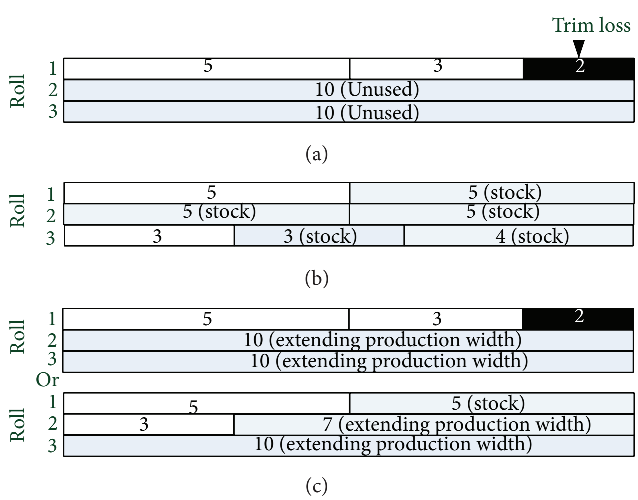

Figure 3: (a) CSP solution, (b) TLMP solution, and (c) MA solution for example 2.

the development of a mathematical formulation for a one-dimensional CSP with useful leftovers in the industrial-use industry.

3. The Proposed Model

To provide more flexible leftovers, we propose the flexible stock allocation approach. It should be noted that the concept of extending stock widths is similar to the usable leftovers [31]. In FAP, we have stock items of width sw_i, $i = \{1, \ldots, m\}$, the reel width is L, each paper width is based on a fixed number called GAP to differentia, and the extending stock width can be obtained from $(\mathrm{sw}_m + \mathrm{GAP})$ to L. Thus, the extending stock item of width is established as

$$\mathrm{ew}_e = \{\mathrm{sw}_m + (e-m) * \mathrm{GAP}\}, \qquad (10)$$

where $e = \{m+1, \ldots, ex\}$, $ex = m + (L - \mathrm{sw}_m)/\mathrm{GAP}$, $(L - \mathrm{sw}_m)/\mathrm{GAP}$ is integer, and m is the number of stock widths. Using (10), we can combine sw_i and ew_e as production width pw_k. This new set of paper widths can provide more flexible stock allocation for cutting plan.

In order to evaluate the benefit of the flexible stock allocation, we define a flexible coefficient f_k for each production width pw_k that is obtained as follows:

$$f_k = \left\{ p \mid \mathrm{pw}_k \geq \sum_{i=1}^{m} \mathrm{pw}_i y_{iz}, \mathrm{UB} \geq \left(\mathrm{pw}_k - \sum_{i=1}^{m} \mathrm{pw}_i y_{iz} \right) \right\}, \qquad (11)$$

where $k = \{1, 2, \ldots, m\}$, $z = \{1, 2, \ldots, p\}$, p is the number of patterns, y_{iz} is the number of widths i in pattern z, and y_{iz} is a positive integer. For instance, we set UB = 999. Since $\mathrm{pw}_1\{1000\}$ has only one combination, the f_1 value is assigned to one. Since $\mathrm{pw}_2\{1100\}$ can be divided into two combinations $\{1000, 1100\}$, the f_2 value is assigned to two. Furthermore, $\mathrm{pw}_{13}\{2200\}$ can be divided into fourteen combinations $\{\{1000, 1000\}, \{1000, 1100\}, \{1000, 1200\}, \{1100, 1100\}, 1300, 1400, 1500, 1600, 1700, 1800, 1900, 2000, 2100, 2200\}$; the f_{13} value is assigned to fourteen. In addition, a flexible coefficient f_k for each extending width is obtained as follows:

$$f_k = f_m + (k - m), \qquad (12)$$

where $k = \{m+1, \ldots, ex\}$ and $e = \{m+1, \ldots, ex\}$.

Table 1: Product information and data for ow_i, d_i, and sw_i.

i	ow_i	d_i	sw_i
1	X	0	1000
2	X	0	1100
3	1200	7	1200
4	1300	22	1300
5	X	0	1400
6	1500	28	1500
7	1600	58	1600
8	1700	47	1700
9	1800	43	1800
10	1900	7	1900
11	2000	20	2000
12	2100	9	2100
13	2200	30	2200
14	2300	7	2300
15	2400	12	2400
16	2500	20	2500

Note: X = not available.

We introduce a coefficient R for controlling the trim loss. Thus, the proposed formulation is as follows:

$$\text{Maximize} \quad \sum_{k=1}^{ex}\left(\sum_{r=1}^{t} a_{kr}x_r - \text{pq}_k\right) \times f_k\text{pw}_k + R\sum_{r=1}^{t}\left(L - \sum_{k=1}^{ex} a_{kr}\text{pw}_k\right)x_r, \tag{13}$$

s.t.

$$\sum_{k=1}^{ex} a_{kr}\text{pw}_k \le L \quad \text{(width of reel constraint)} \tag{14}$$

$$\text{pq}_k \le \sum_{r=1}^{t} a_{kr}x_r \quad \text{(demand constraint)} \tag{15}$$

$$\text{SRQ} = \frac{\left(\sum_{r=1}^{t} x_r\right)}{\text{NR}} \quad \text{(reel set constraint)} \tag{16}$$

$$\text{L} - \sum_{k=1}^{\text{ex}} a_{kr}\text{pw}_k \le \text{UB} \quad \text{(trim loss constraint)}, \tag{17}$$

where a_{kr} and x_r are decision variables and integer variables. Maximizing the total flexibility is the objective function (13) of the model that is the summation of total production coefficient and total trim loss coefficient. The composition of each production coefficient includes nonorder quantity, production width, and flexible coefficient, and the trim loss coefficient includes the coefficient R and the total trim loss. Constraint (14) guarantees the cutting stocks regarding the reel width. Constraint (15) guarantees the cutting stocks regarding the demand. Constraint (16) guarantees the number of rolls for each reel. Constraint (17) guarantees the waste of each roll during the cutting process.

Table 2: Product information and data for pw_k, pq_k, and f_k.

k	pw_k	pq_k	f_k	k	pw_k	pq_k	f_k
1	1000	0	1	17	#2600	0	23
2	1100	0	2	18	#2700	0	24
3	1200	7	3	19	#2800	0	25
4	1300	22	4	20	#2900	0	26
5	1400	0	5	21	#3000	0	27
6	1500	28	6	22	#3100	0	28
7	1600	58	7	23	#3200	0	29
8	1700	47	8	24	#3300	0	30
9	1800	43	9	25	#3400	0	31
10	1900	7	10	26	#3500	0	32
11	2000	20	11	27	#3600	0	33
12	2100	9	12	28	#3700	0	34
13	2200	30	14	29	#3800	0	35
14	2300	7	16	30	#3900	0	36
15	2400	12	19	31	#4000	0	37
16	2500	20	22	32	#4100	0	38
				33	#4200	0	39
				34	#4300	0	40
				35	#4400	0	41
				36	#4500	0	42
				37	#4600	0	43

Note: # = extending stock width.

Table 3: Optimal solutions using FAP and CSP methods.

k	pw_k	pq_k	f_k	CSP		FAP	
				Solution_k	Stock_k	Solution_k	Stock_k
1	1000	0	1	31	31	21	21
2	1100	0	2	0	0	0	0
3	1200	7	3	7	0	12	5
4	1300	22	4	22	0	23	1
5	1400	0	5	0	0	10	10
6	1500	28	6	30	2	28	0
7	1600	58	7	58	0	58	0
8	1700	47	8	47	0	47	0
9	1800	43	9	43	0	43	0
10	1900	7	10	7	0	7	0
11	2000	20	11	20	0	20	0
12	2100	9	12	9	0	9	0
13	2200	30	14	30	0	30	0
14	2300	7	16	7	0	7	0
15	2400	12	19	12	0	12	0
16	2500	20	22	20	0	20	0
21	#3000	0	27	0	0	1	1
37	#4600	0	43	1	1	1	1
	TLR (%)			3.3		1.4	
	TF				86		160

Note: rolls = 135, $R = 0$, and $\text{stock}_k = \text{solution}_k - \text{pq}_k$.

When $R = 0$, the objective function (13) becomes to be a maximize function of $\sum_{k=1}^{ex}(\sum_{r=1}^{t} a_{kr}x_r - \text{pq}_k) f_k\text{pw}_k$; that is,

Table 4: Optimal solutions using FAP and TLMP method.

k	pw_k	pq_k	f_k	TLMP		FAP	
				Solution_k	Stock_k	Solution_k	Stock_k
1	1000	0	1	39	39	37	37
2	1100	0	2	2	2	1	1
3	1200	7	3	9	2	8	1
4	1300	22	4	22	0	23	1
5	1400	0	5	9	9	6	6
6	1500	28	6	28	0	28	0
7	1600	58	7	58	0	58	0
8	1700	47	8	47	0	47	0
9	1800	43	9	43	0	43	0
10	1900	7	10	7	0	7	0
11	2000	20	11	20	0	20	0
12	2100	9	12	9	0	9	0
13	2200	30	14	30	0	30	0
14	2300	7	16	7	0	7	0
15	2400	12	19	12	0	15	3
16	2500	20	22	20	0	20	0
TLR (%)				0.42		0.42	
TF					94		133

Note: rolls = 135, $R = -1000$, and $\text{stock}_k = \text{solution}_k - \text{pq}_k$.

it does not consider trim loss. If the production capacity fails to satisfy (15) during the problem-solving process, a full roll is generated. Subsequently, because the flexibility of $f_e \text{pw}_{ex}$ is greater than any of the leniency and flexibility coefficient combinations, the full roll is substituted by pw_{ex}. In addition, the optimal CSP solution also generates a full roll and the full roll is substituted by pw_{ex}; thus, the FAP results approximate the CSP target function; that is, Minimize $\sum_{r=1}^{t} x_r$. Therefore, the difference between FAP($R = 0$) and the a_{kr} of CSP can be compared.

When $R = -\infty$, $\sum_{k=1}^{ex}(\sum_{r=1}^{t} a_{kr} x_r - \text{pq}_k) f_k \text{pw}_k$ can be neglected and the objective function (13) approaches Maximize $R \sum_{r=1}^{t}(L - \sum_{k=1}^{ex} a_{kr} \text{pw}_k) x_r$. In addition, R approximates the TLMP target function; that is, Minimize $\sum_{r=1}^{t}(L - \sum_{k=1}^{ex} a_{kr} \text{pw}_k) x_r$. Therefore, the difference between FAP($R = -\infty$) and the a_{kr} of TLMP can be compared.

In summary, when $R = 0$, flexible stock becomes the optimal condition and trim loss is maximized. Conversely, when $R = -\infty$, flexible stock becomes the least favorable condition and trim loss is minimized. Therefore, the control of variable R is a flexible stock and trim loss strategy that decision makers adopt during the production process.

4. Illustrative Examples

We consider a real case from an industrial-use paper production and five simulated datasets to illustrate the application of our proposed method. We set the current scheduling quantity as SRQ reels, and each reel can produce NR number of rolls. The cutting machine width limit is L, and the maximum trim loss is UB. These parameters are defined as $\text{NR} = 3$, $L = 4600$ mm, $\text{SRQ} = 45$ reels $= 135$ rolls, and $\text{UB} = 999$ mm.

To obtain the solutions of CSP, TLMP, and FAP, the computer program is divided into the engine and the user interface. The engine interface was written in Lingo 11 Software [37]. The user interface in Visual Basic 5 enables the navigation of data flow from various input sources, such as a common company database and a random number dataset.

4.1. A Real Case from an Industrial-Use Paper Production. According to the FAP model in Section 3, the details are as follows.

Step 1. Define ow_i, d_i, and sw_i, for $i = 1, 2, \ldots, 16$ (see Table 1).

Step 2. Using (10) to obtain the extending widths, since $\text{GAP} = 100$ and $m = 16$, we can obtain that $e = \{17, 18, \ldots, 37\}$ and $\text{ew}_e = \{2600, 2700, \ldots, 4600\}$.

Step 3. Aggregate d_i to pq_k, sw_i to pw_k, and ew_e to pw_k, for $i = 1, 2, \ldots, 16$ and $k = 1, 2, \ldots, 37$.

Step 4. Use (11)-(12) to compute the flexible coefficient f_k for pw_k (see Table 2).

Using FAP to perform optimization, R must be set to 0, thereby allowing FAP results to approximate those of CSP. In this case, we obtained the production capacities of FAP and CSP, stock, trim loss ratio (TLR), and total flexible coefficient (TF), where $\text{TLR} = [\sum_{r=1}^{t}(L - \sum_{k=1}^{ex} a_{kr} \text{pw}_k) x_r] / (L \sum_{r=1}^{t} x_r) \times 100\%$ and $\text{TF} = \sum_{k=1}^{ex} \text{pw}_k f_k$.

The primary reason for comparing CSP was to determine whether FAP effectively reduced TLR and whether the flexible stock of FAP is superior to that of CSP (see Table 3). The TLRs for CSP and FAP were 3.3 and 1.4, respectively; the flexible stocks for CSP, FAP, and extending stock were {{1000, 31}, {1500, 2}, {#4600, 1}},{{1000, 21}, {1200, 5},{1300, 1}, {1400, 10}, {#3000, 1}, {#4600, 1}}, and {{#3000, 1}, {#4600, 1}}, respectively. Thus, the results suggest that FAP outperforms CSP in reducing TLR and that the flexible stock and TF of FAP are superior to that of CSP.

To compare FAP to TLMP, the flexible variable R of FAP was set to -1000, which denotes minimal TLR. In this case, we obtained the production capacities of FAP and TLMP, stock, TLR, and TF(see Table 4).

The primary reason for comparing TLMP was to determine whether the TLR of FAP is similar to that of TLMP or not and whether the flexible stock of FAP is superior to that of TLMP or not (see Table 4). The TLRs for TLMP and FAP were 0.42 and 0.42, respectively; the flexible stocks for TLMP and FAP were {{1000, 39}, {1100, 2}, {1200, 2}, {1400, 9}}, and {{1000, 37},{1100, 1}, {1200, 1}, {1300, 1}, {1400, 6}, {2400, 3}}, respectively. Notably, the flexible stock of FAP {2400, 3} was considerably more lenient. Therefore, based on the results, the TLR of FAP was identical to that of TLMP, and the flexible stock and TF of FAP were superior to those of TLMP.

Moreover, we employed sensitivity analysis to observe the influence that R has on TLR and TF. When $R = 1, 0, -1, \ldots, -\infty$, and R is an integer, the results as shown in Table 5 are obtained.

Table 5: The sensitivity analysis of coefficient R for stock_k using FAP.

k	pw_k	f_k	R						
			1	0	−1 ~ −2	−3 ~ −8	−9 ~ −86	−87 ~ −533	−534 ~ −∞
1	1000	1	11	21	22	28	36	37	37
2	1100	2	0	0	1	1	1	1	1
3	1200	3	5	5	4	1	1	1	1
4	1300	4	1	1	1	1	1	1	1
5	1400	5	10	10	10	10	6	6	6
6	1500	6	0	0	0	0	0	0	0
7	1600	7	0	0	0	0	0	0	0
8	1700	8	0	0	0	0	0	0	0
9	1800	9	0	0	0	0	0	0	0
10	1900	10	0	0	0	0	0	0	0
11	2000	11	0	0	0	0	0	0	0
12	2100	12	0	0	0	0	0	0	0
13	2200	14	0	0	0	0	0	0	0
14	2300	16	0	0	0	0	0	0	0
15	2400	19	0	0	0	0	0	1	3
16	2500	22	0	0	0	0	0	0	0
21	#3000	27	1	1	1	1	1	0	0
37	#4600	43	1	1	1	1	1	1	0
	TLR (%)		4.2	1.4	1.3	0.9	0.52	0.45	0.42
	TF		150	160	160	157	145	138	133

Note: the number of rolls = 135.

Table 6: The range and midpoint of R.

Range of R	0	−1 ~ −2	−3 ~ −8	−9 ~ −86	−87 ~ −533	−534 ~ −∞
Midpoint	0	−1	−5	−47	−310	−1000

Table 7: Information of pq_k for simulated examples.

k	pw_k	f_k	Case				
			1	2	3	4	5
1	1000	1	46	2	6	37	9
2	1100	2	3	15	2	47	41
3	1200	3	33	11	31	5	33
4	1300	4	22	21	22	42	27
5	1400	5	39	12	20	36	1
6	1500	6	46	25	6	31	2
7	1600	7	22	26	21	38	12
8	1700	8	34	21	34	6	41
9	1800	9	30	25	31	32	29
10	1900	10	38	14	44	26	48
11	2000	11	1	20	12	48	22
12	2100	12	23	42	37	42	16
13	2200	14	12	20	48	11	48
14	2300	16	6	31	33	2	19
15	2400	19	27	12	41	28	30
16	2500	22	14	38	41	13	27

Table 8: The results of TF and TLR for CSP, TLMP, and FAP.

Method	Measure	Case				
		1	2	3	4	5
CSP	TF	94	98	10	43	0
	TLR (%)	0	1.1	1.6	0.5	1.8
TLMP	TF	49	19	6	10	9
	TLR (%)	0	0.15	1.2	0.13	1.2
FAP ($R = 0$)	TF	109	115	25	65	26
	TLR (%)	0	1	1.6	0.16	1.4
FAP ($R = -1000$)	TF	109	19	6	20	9
	TLR (%)	0	0.15	1.2	0.13	1.2
Rolls		144	141	183	162	162

According to Table 5, when $R = 1$, TLR increased and $\{1000, 21\}$ changed to $\{1000, 11\}$. This was primarily because trim loss was equivalent to the flexible coefficient of $\{1000\}$, causing the stock capacity of $\{1000\}$ to decrease. Thus, when $0 \leq R < 1$ is defined, we can directly use $R = 0$ for solution identification.

When $R < 0$, we observed that the TF gradually reduced from 160 to 133 and the TLR reduced from 1.4 to 0.42. These results suggest that, when R has a value less than 0, the TF decreases and the TLR declines. Regarding flexible stock, we found that, when R ranged between −1 and −2,

Table 9: Stock information for CSP, TLMP, and FAP.

Case	Method	Solution
1	CSP	{1100, 1}{1500, 1}{#4600, 2}
	TLMP	{1000, 3}{1100, 2}{2200, 3}
	FAP ($R = 0$)	{#2600, 1}{#4600, 2}
	FAP ($R = -1000$)	{#2600, 1}{#4600, 2}
2	CSP	{1000, 1}{2000, 1}{#4600, 2}
	TLMP	{1000, 17}{1100, 1}
	FAP ($R = 0$)	{#3200, 1}{#4600, 2}
	FAP ($R = -1000$)	{1000, 17}{1100, 1}
3	CSP	{1300, 1}{1500, 1}
	TLMP	{1000, 6}
	FAP ($R = 0$)	{#2800, 1}
	FAP ($R = -1000$)	{1000, 6}
4	CSP	{#4600, 1}
	TLMP	{1000, 6}{1300, 1}
	FAP ($R = 0$)	{2500, 1}{#4600, 1}
	FAP ($R = -1000$)	{1000, 4}{1100, 1}{2200, 1}
5	CSP	NA
	TLMP	{1000, 3}{1500, 1}
	FAP ($R = 0$)	{#2900, 1}
	FAP ($R = -1000$)	{1000, 3}{1500, 1}

Note: {paper width, stock quantity}.

the production capacity of {1000} was 22; subsequently, as R decreased to between -3 and -8 and -534 and $-\infty$, the production capacity of {1000} increased to 28 and to 37, respectively. These results suggest that, as R decreases, the allocation of stock gradually coagulates at a lower leniency, negating the effects of extended stock. The decrease in TF from 160 to 133 implies that the degree of permitted flexibility for adjusting stock had already diminished. Therefore, we suggest that R be maintained within a range between $-\infty$ and 0.

Because the trim loss value at each interval of R is a fixed value, we selected the medians of each interval and tabulated them into Table 6, which enabled us to select the desired results. Consequently, the number of medians can be defined by decision makers based on actual conditions.

4.2. Simulated Examples. To verify the superiority of the flexible stock and trim loss produced by using FAP over those produced using CSP and TLMP, we selected 5 Cases for comparison, and randomly obtained the pq_k (where $k = 1, 2, \ldots, 16$), which was achieved by using the RANDBETWEEN function in Microsoft Office Excel 2007. The range of this function was set between $0, 1, 2, \ldots, 50$ (see Table 7). The optimization calculations were then performed for FAP, CSP, and TLMP.

We compared FAP($R = 0$), FAP($R = -1000$), CSP, and TLMP, and the results were tabulated in Table 8. Because using FAP necessitates the consideration of the flexible coefficients, FAP($R = 0$) should effectively reduce TLR when an excessively large CSP's TLR value is produced. Cases 2, 4, and 5 verified that FAP reduced CSP's TLR. FAP($R = -1000$) and TLMP were then examined to determine whether FAP's TLR presented similarities with TLMP's TLR. Consequently, the TLR values observed in all the 5 Cases were consistent.

Subsequently, we endeavored to determine whether FAP could effectively increase the flexibility of stock adjustment (see Table 9). The FAP($R = 0$) for Cases 1 and 3 indicated that the stock leniency demonstrated a merging action. In addition, the extended stock was used in all of the case samples. Furthermore, uncut rolls {#4600} were presented in Cases 1, 2, 4, and 5. Because $R = 0$ is the lowest production capacity model, this model is equivalent to CSP. The FAP($R = -1000$) for Cases 2, 3, and 5 was similar. However, FAP($R = -1000$) presented increased stock adjustment flexibility and extended stock usage in Cases 1 and 4. Thus, FAP($R = 0$) can effectively reduce CSP's TLR and increase stock adjustment flexibility when TLR is at a minimum level. The TLR in FAP($R = -1000$) was equivalent to that of TLMP, which increased stock adjustment flexibility.

A sensitivity analysis was employed to determine the performance of FAP in the 5 Cases and the influence of R on TLR and TF. Consequently, R was set at $1, 0, -1, \ldots, -\infty$, where R was an integer. The results are tabulated in Table 10.

The medians tabulated in Table 6 were used for data reconstruction and the results are presented in Table 11. Subsequently, we collected the R values at each interval for Cases 1, 3, 4, and 5. For Case 2, we were unable to collect the R values at intervals of -55–-79, -80–-124, and -125–-156. Decision makers can determine whether they wish to incorporate the medians at these intervals or not; however, this method of incorporating medians can be used to control the majority of TLR changes.

5. Conclusion

The results of the case study analysis indicate that FAP($R = 0$) was similar to CSP in that both methods could be used to determine the minimal production capacity and the maximal, flexible adjusted stock. Because of the unique production characteristics of industrial-use paper, using the CSP method may produce full rolls and, thus, cannot obtain optimized trim loss problems. Similar to the CSP method, FAP($R = -1000$) generates stock that cannot be flexibly adjusted, despite possessing minimal trim loss. Furthermore, CSP and TLMP failed to control the changes of TLR; therefore, FAP can utilize R to control and maintain TLR in a range between CSP and TLMP'TLR. This approach eliminates the trim loss problem exhibited in CSP and the adjustability problem exhibited in TLMP and allows decision makers to effectively control stock and trim loss according to actual situations.

Future research may consider solving extending stock in stock allocation. In addition, the cost effects during the production process should be addressed.

Notations

i: The index number ($i = 1, 2, \ldots, m$) and m is the number of stock/order widths

sw_i: A stock width with $i = 1, 2, \ldots, m$

ow_i: An order width with $i = 1, 2, \ldots, m$

Table 10: The sensitivity analysis of coefficient R for all cases using FAP.

Case	Measure	R					
		$0\sim\infty$					
1	TLR (%)	0					
	TF	109					
		$0\sim-36$	$-37\sim-54$	$-55\sim-79$	$-80\sim-124$	$-125\sim-156$	$-157\sim\infty$
2	TLR (%)	1	0.94	0.76	0.29	0.22	0.15
	TF	115	112	91	46	39	19
		$0\sim-19$	$-20\sim-\infty$				
3	TLR (%)	1.6	1.2				
	TF	25	6				
		$0\sim-999$	$-1000\sim-\infty$				
4	TLR (%)	0.16	0.13				
	TF	65	20				
		$0\sim-11$	$-12\sim-70$	$-71\sim-\infty$			
5	TLR (%)	1.4	1.3	1.2			
	TF	26	25	9			

Table 11: The results of fixed values of R for all cases using FAP.

Case	Measure	R					
		0	−1	−5	−47	−310	−1000
1	TLR (%)	0	0	0	0	0	0
	TF	109	109	109	109	109	109
2	TLR (%)	1	1	1	0.94	0.15	0.15
	TF	115	115	115	112	19	19
3	TLR (%)	1.6	1.6	1.6	1.2	1.2	1.2
	TF	25	25	25	6	6	6
4	TLR (%)	0.16	0.16	0.16	0.16	0.16	0.13
	TF	65	65	65	65	65	20
5	TLR (%)	1.4	1.4	1.4	1.3	1.2	1.2
	TF	26	26	26	25	9	9

d_i: Demand for ow_i with $i = 1, 2, \ldots, m$
L: Reel width
NR: The number of rolls for a reel
GAP: The difference between two paper widths
ew_e: An extending production width, where e is the index number ($e = m+1, m+2, \ldots, ex$) and $ex = m + (L - sw_m)/\text{GAP}$
pw_k: A production width, where k is the index number ($k = 1, 2, \ldots, ex$)
pq_k: Quantity for the production width, $k = 1, 2, \ldots, ex$
f_k: Flexible coefficient for the production width pw_k, with $k = 1, 2, \ldots, ex$
R: Flexible coefficient for trim loss
SRQ: Production scheduling of reel quantity
UB: Upper bound for trim loss
a_{kr}: The number of widths k in pattern r
x_r: The number of patterns r, where r is the index number ($r = 1, 2, \ldots, t$) and t is the number of patterns.

Conflict of Interests

The authors declare that they have no conflict of interests.

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