Hindawi Publishing Corporation
Mathematical Problems in Engineering
Volume 2011, Article ID 978612, 15 pages
doi:10.1155/2011/978612

Research Article

Generalized Synchronization between Two Complex Dynamical Networks with Time-Varying Delay and Nonlinear Coupling

Qiuxiang Bian[1, 2] and Hongxing Yao[1]

[1] *Faculty of Science, Jiangsu University, Jiangsu, Zhenjiang, 212013, China*
[2] *Faculty of Mathematics and Physics, Jiangsu University of Science and Technology, Jiangsu, Zhenjiang, 212003, China*

Correspondence should be addressed to Qiuxiang Bian, bianqx_1@163.com

Received 30 August 2010; Revised 26 March 2011; Accepted 17 May 2011

Academic Editor: E. E. N. Macau

The generalized synchronization between two complex networks with nonlinear coupling and time-varying delay is investigated in this paper. The novel adaptive schemes of constructing controller response network are proposed to realize generalized synchronization with the drive network to a given mapping. Two specific examples show and verify the effectiveness of the proposed method.

1. Introduction

Over the past decade, complex networks have gained a lot of attention in various fields, such as sociology, biology, physical sciences, mathematics, and engineering [1–5]. A complex network is a large number of interconnected nodes, in which each node represents a unit (or element) with certain dynamical system and edge represents the relationship or connection between two units (or elements). Synchronization is one of the most important dynamical properties of dynamical systems, there are different kinds of methods to realize synchronization such as active control [6], feedback control [7], adaptive control [8], impulsive control [9], passive method [10], and so forth. Synchronization of complex networks includes complete synchronization (CS) [11, 12], projective synchronization (PS) [13, 14], phase synchronization [15, 16], generalized synchronization (GS) [17, 18], and so on.

As a sort of synchronous behavior, GS is an extension of CS and PS, so GS is more widespread than CS and PS in nature and in technical applications. GS of chaos system has been widely researched. However, most of theoretical results about synchronization of complex networks focus on CS and PS. Especially, due to the complexity of GS, the theoretical results for GS are lacking, but GS of complex networks is attracting special attention; in [17], the author gives a novel definition of GS on networks and a numerical simulation example. Reference [18] applies the auxiliary-system approach to study paths to globally generalized synchronization in scale-free networks of identical chaotic oscillators.

Recently, GS of drive-response chaos systems is investigated by the nonlinear control theory in [19]. In this letter, we extend this method to investigate GS between two complex networks, and some criterions for GS are deduced.

This letter is organized as follows. In Section 2, the definition of GS between the drive-response complex networks is given and some preliminary knowledge, including three assumptions and one lemma is also introduced. By employing the Lyapunov theory and Barbălat lemma, some schemes are designed to construct response networks to realize GS with respect to the given nonlinear smooth mapping. In Section 3, two numerical examples are given to demonstrate the effectiveness of the proposed method in Section 2. Finally, conclusions are given in Section 4.

2. GS Theorems between Two Complex Networks with Nonlinear Coupling

2.1. Definition and Assumptions

Definition 2.1. Suppose $x_i = (x_{i1}, x_{i2}, \ldots, x_{in})^T \in R^n$, $y_i = (y_{i1}, y_{i2}, \ldots, y_{in})^T \in R^n$, $i = 1, 2, \ldots, N$ are the state variables of the drive network and the response network, respectively. Given the smooth vector function $\Phi : R^n \to R^n$, the drive network and response network are said to achieve GS with respect to Φ. If

$$\lim_{t\to\infty} \|e_i(t)\| = 0, \quad i = 1, 2, \ldots, N, \tag{2.1}$$

where $e_i(t) = x_i(t) - \Phi(y_i(t)),\ i = 1, 2, \ldots, N$, the norm $\|\cdot\|$ of a vector x is defined as $\|x\| = (x^T x)^{1/2}$.

Remark 2.2. If $\Phi(y_i) = y_i$, then GS is CS in [20]. If $\Phi(y_i) = \lambda y_i$, then GS is PS in [13, 14].

In this paper, we consider a general complex dynamical network with time-varying nonlinear coupling and consisting of N nonidentical nodes:

$$\dot{x}_i(t) = f_i(x_i(t)) + \sum_{j=1}^{N} c_{ij} h\big(x_j(t - \tau(t))\big), \quad i = 1, 2, \ldots, N, \tag{2.2}$$

where $x_i = (x_{i1}, x_{i2}, \ldots, x_{in})^T \in R^n,\ i = 1, 2, \ldots, N$ are the state variables of the drive network, $f_i : R^n \to R^n$, $h : R^n \to R^n$ are smooth nonlinear vector functions, and $\tau(t)$ is time-varying delay. $C = (c_{ij})_{N\times N}$ is unknown or uncertain coupling matrix; if there is a connection between node i and node $j\ (j \neq i)$, then $c_{ij} \neq 0$, otherwise $c_{ij} = 0\ (i \neq j)$, and the diagonal elements of C are defined by

$$c_{ii} = -\sum_{\substack{j=1 \\ j\neq i}}^{N} c_{ij}. \tag{2.3}$$

It should be noted that the complex dynamical network model (2.2) is quite general. If $f_i = f$, $i = 1, 2, \ldots, l$; $g_i = g,\ i = l+1,\ l+2, \ldots, N$, then we can get the following complex dynamical network:

$$\begin{aligned}\dot{x}_i(t) &= f(x_i(t)) + \sum_{j=1}^{N} c_{ij} h\left(x_j(t-\tau(t))\right), \quad i = 1, \ldots, l, \\ \dot{x}_i(t) &= g(x_i(t)) + \sum_{j=1}^{N} c_{ij} h\left(x_j(t-\tau(t))\right), \quad i = l+1, \ldots, N.\end{aligned} \tag{2.4}$$

On the other hand, if $h(x_i) = Ax_i$, with $A = (a_{ij})_{N\times N}$ being an inner-coupling constant matrix of the network, then the complex network model (2.2) degenerates into the model of linearly and diffusively coupled network with coupling delays:

$$\dot{x}_i(t) = f_i(x_i(t)) + \sum_{j=1}^{N} c_{ij} A x_j(t-\tau(t)), \quad i = 1, 2, \ldots, N. \tag{2.5}$$

Let

$$D\Phi(y_i) = \begin{pmatrix} \dfrac{\partial\phi_1(y_i)}{\partial y_{i1}} & \dfrac{\partial\phi_1(y_i)}{\partial y_{i2}} & \cdots & \dfrac{\partial\phi_1(y_i)}{\partial y_{in}} \\ \dfrac{\partial\phi_2(y_i)}{\partial y_{i1}} & \dfrac{\partial\phi_2(y_i)}{\partial y_{i2}} & \cdots & \dfrac{\partial\phi_2(y_i)}{\partial y_{in}} \\ \cdots & \cdots & \cdots & \cdots \\ \dfrac{\partial\phi_n(y_i)}{\partial y_{i1}} & \dfrac{\partial\phi_n(y_i)}{\partial y_{i2}} & \cdots & \dfrac{\partial\phi_n(y_i)}{\partial y_{in}} \end{pmatrix} \tag{2.6}$$

be the Jacobian matrix of the mapping $\Phi(y_i) = (\phi_1(y_i), \phi_2(y_i), \ldots, \phi_n(y_i))^T$, $\phi_j(y_i) \in R$, $i = 1, 2, \ldots, N$, $j = 1, 2, \ldots, n$. When matrix $D\Phi(y_i(t))$ is reversible, we can give the following controller response network:

$$\dot{y}_i(t) = D\Phi(y_i(t))^{-1}\left[f_i(\Phi(y_i(t))) + \sum_{j=1}^{N} \widehat{c}_{ij} h\left(\Phi\left(y_j(t-\tau(t))\right)\right)\right] + u_i, \quad i = 1, 2, \ldots, N, \tag{2.7}$$

where $y_i = (y_{i1}, y_{i2}, \ldots, y_{in})^T \in R^n$, $i = 1, 2, \ldots, N$ are the state variables of the response network, $u_i \in R^n$, $i = 1, 2, \ldots, N$ are nonlinear controllers to be designed, and $\widehat{C} = (\widehat{c}_{ij})_{N\times N}$ is the estimate of the unknown coupling matrix $C = (c_{ij})_{N\times N}$.

Let $e_i(t) = x_i(t) - \Phi(y_i(t))$, with the aid of (2.2) and (2.7), the following error network can be obtained:

$$\begin{aligned}\dot{e}_i(t) &= \dot{x}_i(t) - \left[D\Phi(y_i(t))\right]\dot{y}_i(t) \\ &= f_i(x_i(t)) - f_i(\Phi(y_i(t))) + \sum_{j=1}^{N} c_{ij} h\left(x_j(t-\tau(t))\right) - \sum_{j=1}^{N} \widehat{c}_{ij} h\left(\Phi\left(y_j(t-\tau(t))\right)\right) - D\Phi(y_i(t)) u_i \\ &= f_i(x_i(t)) - f_i(\Phi(y_i(t))) + \sum_{j=1}^{N} c_{ij} H\left(e_j(t-\tau(t))\right) - \sum_{j=1}^{N} \widetilde{c}_{ij} h\left(\Phi\left(y_j(t-\tau(t))\right)\right) - D\Phi(y_i(t)) u_i,\end{aligned} \tag{2.8}$$

where

$$H(e_j(t)) = h(x_j(t)) - h(\Phi(y_j(t))), \tag{2.9}$$

$$\widetilde{c}_{ij} = \widehat{c}_{ij} - c_{ij}. \tag{2.10}$$

The following conditions are needed for the solutions of (2.8) to achieve the objective (2.1).

Assumption 1. (A1) Time delay $\tau(t)$ is a differential function with $0 \le \tau(t) \le h$, $\dot{\tau}(t) \le \mu < 1$, where h and μ are positive constants. Obviously, this assumption holds for constant $\tau(t)$.

Assumption 2. (A2) Suppose that $f_i(\cdot)$ is bounded and there exists a nonnegative constant α such that

$$\| f_i(x_i(t)) - f_i(\Phi(y_i(t))) \| \le \alpha \|e_i(t)\|, \quad i = 1, 2, \ldots, N. \tag{2.11}$$

Assumption 3. (A3) Suppose that $h(\cdot)$ is bounded and there exists a nonnegative constant β such that

$$\| h(x_i(t)) - h(\Phi(y_i(t))) \| \le \beta \|e_i(t)\|, \quad i = 1, 2, \ldots, N. \tag{2.12}$$

Remark 2.3. The condition (2.12) is reasonable due to [21, 22]. For example, the Hopfield neural network [23] is described by

$$\frac{\mathrm{d}x_i(t)}{\mathrm{d}t} = -\frac{x_i(t)}{R_i} + \sum_{j=1}^{N} w_{ij} f_j(x_j(t - \tau_{ij}(t))) + I_i, \quad i = 1, 2, \ldots, N. \tag{2.13}$$

Take $f_j(x_j) = (\pi/2)\arctan((\pi/2)\lambda x_j)$, where λ is positive constant. It is obvious that $f_j(\cdot)$ satisfies Assumption 3.

Lemma 2.4. *For any vectors $X, Y \in R^n$, the following inequality holds*

$$2X^T Y \le X^T X + Y^T Y. \tag{2.14}$$

Next section, we will give some sufficient conditions of complex dynamical networks (2.2) and (2.7) obtaining GS.

2.2. Main Results

Theorem 2.5. *Suppose that (A1)–(A3) hold. Using the following controller:*

$$u_i = D\Phi(y_i(t))^{-1} d_i e_i(t) \tag{2.15}$$

and the update laws

$$\dot{d}_i = k_i e_i^T(t) e_i(t), \tag{2.16}$$

$$\dot{\tilde{c}}_{ij} = \delta_{ij} e_i^T(t) h\big(\Phi\big(y_j(t-\tau(t))\big)\big), \tag{2.17}$$

where $i, j = 1, 2, \ldots, N$, d_i *is feedback strength, and* $\delta_{ij} > 0$, $k_i > 0$ *are arbitrary constants, then the complex dynamical networks* (2.2) *and* (2.7) *will achieve GS with respect to* Φ.

Proof. Select a Lyapunov-Krasovskii functional candidate as

$$\begin{aligned} V(t, e(t)) = {} & \sum_{i=1}^{N} e_i^T(t) e_i(t) + \sum_{i=1}^{N}\sum_{j=1}^{N} \frac{1}{\delta_{ij}} \tilde{c}_{ij}^2 + \sum_{i=1}^{N} \frac{1}{k_i}(d_i - d)^2 \\ & + \frac{N\beta^2}{1-\mu} \int_{t-\tau(t)}^{t} \sum_{i=1}^{N} e_i^T(\xi) e_i(\xi) \mathrm{d}\xi, \end{aligned} \tag{2.18}$$

where $e(t) = (e_1^T(t), e_2^T(t), \ldots, e_N^T(t))^T$ and d is a positive constant to be determined.

The time derivative of V along the solution of the error system (2.8) is

$$\begin{aligned} \frac{\mathrm{d}V}{\mathrm{d}t} = {} & \sum_{i=1}^{N} 2 e_i^T(t) \dot{e}_i(t) + \sum_{i=1}^{N}\sum_{j=1}^{N} \frac{2}{\delta_{ij}} \tilde{c}_{ij} \dot{\tilde{c}}_{ij} + \sum_{i=1}^{N} \frac{2}{k_i}(d_i - d)\dot{d}_i + \frac{N\beta^2}{1-\mu} \sum_{i=1}^{N} e_i^T(t) e_i(t) \\ & - \frac{1-\dot{\tau}(t)}{1-\mu} N\beta^2 \sum_{i=1}^{N} e_i^T(t-\tau(t)) e_i(t-\tau(t)) \\ = {} & \sum_{i=1}^{N} 2 e_i^T(t) \Bigg[f_i(x_i(t)) - f_i\big(\Phi\big(y_i(t)\big)\big) + \sum_{j=1}^{N} c_{ij} H\big(e_j(t-\tau(t))\big) \\ & \qquad - \sum_{j=1}^{N} \tilde{c}_{ij} h\big(\Phi\big(y_j(t-\tau(t))\big)\big) - D\Phi\big(y_i(t)\big) u_i(t) \Bigg] \\ & + \sum_{i=1}^{N}\sum_{j=1}^{N} \frac{2}{\delta_{ij}} \tilde{c}_{ij} \dot{\tilde{c}}_{ij} + \sum_{i=1}^{N} \frac{2}{k_i}(d_i - d)\dot{d}_i \\ & + \frac{N\beta^2}{1-\mu} \sum_{i=1}^{N} e_i^T(t) e_i(t) - \frac{1-\dot{\tau}(t)}{1-\mu} N\beta^2 \sum_{i=1}^{N} e_i^T(t-\tau(t)) e_i(t-\tau(t)). \end{aligned} \tag{2.19}$$

Substituting the controller (2.15) and update laws (2.16)-(2.17) into (2.19) and considering Assumption 2, we obtain

$$\begin{aligned} \frac{\mathrm{d}V}{\mathrm{d}t} \le {} & \sum_{i=1}^{N} \left(2\alpha - 2d + \frac{N\beta^2}{1-\mu} \right) e_i^T(t) e_i(t) + 2 \sum_{i=1}^{N}\sum_{j=1}^{N} e_i^T(t) c_{ij} H\big(e_j(t-\tau(t))\big) \\ & - \frac{1-\dot{\tau}(t)}{1-\mu} N\beta^2 \sum_{i=1}^{N} e_i^T(t-\tau(t)) e_i(t-\tau(t)). \end{aligned} \tag{2.20}$$

By Lemma 2.4 and considering Assumptions 1 and 3, we have

$$\begin{aligned}
&\frac{1-\dot{\tau}(t)}{1-\mu} \geq 1, \\
&2\sum_{i=1}^{N}\sum_{j=1}^{N} e_i^T(t) c_{ij} H\left(e_j(t-\tau(t))\right) \\
&\qquad \leq \sum_{i=1}^{N}\sum_{j=1}^{N} c_{ij}^2 e_i^T(t) e_i(t) + \sum_{i=1}^{N}\sum_{j=1}^{N} H^T\left(e_j(t-\tau(t))\right) H\left(e_j(t-\tau(t))\right) \\
&\qquad \leq N \max_{1\leq i,j\leq N}\left\{c_{ij}^2\right\} \sum_{i=1}^{N} e_i^T(t) e_i(t) + N\beta^2 \sum_{j=1}^{N} e_j^T(t-\tau(t)) e_j(t-\tau(t)),
\end{aligned} \tag{2.21}$$

then

$$\begin{aligned}
\frac{dV}{dt} &\leq \sum_{i=1}^{N}\left(2\alpha - 2d + \frac{N\beta^2}{1-\mu} + N \max_{1\leq i,j\leq N}\left\{c_{ij}^2\right\}\right) e_i^T(t) e_i(t) \\
&= -\left(2d - 2\alpha - \frac{N\beta^2}{1-\mu} - N \max_{1\leq i,j\leq N}\left\{c_{ij}^2\right\}\right) e^T(t) e(t).
\end{aligned} \tag{2.22}$$

Note that we can choose constant d to make $dV/dt \leq -e^T(t)e(t) \leq 0$, thus V is nonincreasing in $t \geq 0$. One has V is bounded since $0 \leq V(t, e(t)) \leq V(0, e(0))$, so $\lim_{t\to+\infty} V(t, e(t))$ exists and

$$\lim_{t\to+\infty}\int_0^t e^T(s)e(s)ds \leq -\lim_{t\to+\infty}\int_0^t \frac{dV}{ds}ds = V(0, e(0)) - \lim_{t\to+\infty} V(t, e(t)). \tag{2.23}$$

From (2.18), we have $0 \leq e^T(t)e(t) \leq V(t, e(t))$, so $e^T(t)e(t)$ is bounded. According to error system (2.8), $(d/dt)e^T(t)e(t) = 2e^T(t)\dot{e}(t)$ is bounded for $t \geq 0$ due to the boundedness of $f_i(\cdot)$ and $h(\cdot)$. From the above, we can see that $e(t) \in L^2 \cap L^\infty$ and $\dot{e}(t) \in L^\infty$. By using another form of Barbălat lemma [24], one has $\lim_{t\to+\infty} e^T(t)e(t) = 0$, so $\lim_{t\to+\infty} e(t) = 0$ and the complex dynamical networks (2.2) and (2.7) can obtain generalized synchronization under the controller (2.15) and update laws (2.16)-(2.17). This completes the proof. □

Remark 2.6. If $\lim_{t\to+\infty}\dot{e}(t)$ exists,then we can obtain $\lim_{t\to+\infty}\dot{e}(t) = 0$ for $\lim_{t\to+\infty} e(t) = 0$. According to error system (2.8), we have $\lim_{t\to+\infty}\sum_{j=1}^{N}\tilde{c}_{ij} h(\Phi(y_j(t-\tau(t)))) = 0$. When $\{h(\Phi(y_j(t-\tau(t))))\}_{j=1}^{N}$ are linearly independent on the orbit $\{y_j(t-\tau(t))\}_{j=1}^{N}$ of synchronization manifold, $\lim_{t\to+\infty}\tilde{c}_{ij} = 0$. We can get $\lim_{t\to+\infty}\hat{c}_{ij} = c_{ij}$, $i, j = 1, 2, \ldots, N$; that is, the uncertain coupling matrix C can be successfully estimated using the update laws (2.17).

In a special case $\Phi(y_i) = \lambda\, y_i$ (λ is nonzero constant), based on Theorem 2.5, we can construct the following response network

$$\dot{y}_i(t) = \frac{1}{\lambda}\left[f_i(\lambda y_i(t)) + \sum_{j=1}^{N}\hat{c}_{ij} h(\lambda y_j(t-\tau(t)))\right] + u_i, \quad i = 1, 2, \ldots, N. \tag{2.24}$$

Corollary 2.7. *Suppose that (A1)–(A3) hold. Using the controller*

$$u_i = \frac{1}{\lambda} d_i e_i(t) \tag{2.25}$$

and the update laws

$$\begin{aligned} \dot{d}_i &= k_i e_i^T(t) e_i(t), \\ \dot{\hat{c}}_{ij} &= \delta_{ij} e_i^T(t) h\left(\lambda y_j(t-\tau(t))\right), \end{aligned} \tag{2.26}$$

where $i, j = 1, 2, \ldots, N, d_i$ *is feedback strength, and* $\delta_{ij} > 0,\ k_i > 0$ *are arbitrary constants, then the complex dynamical networks* (2.2) *and* (2.24) *will obtain PS.*

To networks (2.4), according to Theorem 2.5, one can construct the following response network:

$$\begin{aligned} \dot{y}_i(t) &= D\Phi\left(y_i(t)\right)^{-1}\left[f\left(\Phi\left(y_i(t)\right)\right) + \sum_{j=1}^{N} \hat{c}_{ij} h\left(\Phi\left(y_j(t-\tau(t))\right)\right)\right] + u_i, \quad i = 1, \ldots, l, \\ \dot{y}_i(t) &= D\Phi\left(y_i(t)\right)^{-1}\left[g\left(\Phi\left(y_i(t)\right)\right) + \sum_{j=1}^{N} \hat{c}_{ij} h\left(\Phi\left(y_j(t-\tau(t))\right)\right)\right] + u_i, \quad i = l+1, \ldots, N \end{aligned} \tag{2.27}$$

and get the following corollary:

Corollary 2.8. *Suppose that (A1)–(A3) hold. Using the controller*

$$u_i = D\Phi\left(y_i(t)\right)^{-1} d_i e_i(t) \tag{2.28}$$

and the update laws

$$\begin{aligned} \dot{d}_i &= k_i e_i^T(t) e_i(t), \\ \dot{\hat{c}}_{ij} &= \delta_{ij} e_i^T(t) h\left(\Phi\left(y_j(t-\tau(t))\right)\right), \end{aligned} \tag{2.29}$$

where $i, j = 1, 2, \ldots, N, d_i$ *is feedback strength,* $\delta_{ij} > 0,\ k_i > 0$ *are arbitrary constants, then the complex dynamical network networks* (2.4) *and* (2.27) *will achieve GS with respect to* Φ*.*

If coupling function $h(x_i) = Ax_i$; that is, the network is linearly coupled, then the complex network (2.2) degenerates into (2.5). Note that $\|Ae_i(t)\| \le \|A\| \cdot \|e_i(t)\|,\ i = 1, 2, \ldots, N$ hold. We construct the following response network:

$$\dot{y}_i(t) = D\Phi\left(y_i(t)\right)^{-1}\left[f_i\left(\Phi\left(y_i(t)\right)\right) + \sum_{j=1}^{N} \hat{c}_{ij} A\Phi\left(y_j(t-\tau(t))\right)\right] + u_i, \quad i = 1, 2, \ldots, N. \tag{2.30}$$

Corollary 2.9. *Suppose that (A1) and (A2) hold. Using the controller*

$$u_i = D\Phi\big(y_i(t)\big)^{-1} d_i e_i(t) \tag{2.31}$$

and the update laws

$$\begin{aligned}\dot{d}_i &= k_i e_i^T(t) e_i(t),\\ \dot{\tilde{c}}_{ij} &= \delta_{ij} e_i^T(t) A\Phi\big(y_j(t-\tau(t))\big),\end{aligned} \tag{2.32}$$

where $i, j = 1, 2, \ldots, N$, d_i *is feedback strength, and* $\delta_{ij} > 0$, $k_i > 0$ *are arbitrary constants, then the complex dynamical networks* (2.5) *and* (2.30) *will obtain GS.*

Using different control, we can obtain the following theorem.

Theorem 2.10. *Suppose that (A1) and (A3) hold. Using the following controller:*

$$u_i = D\Phi\big(y_i(t)\big)^{-1}\big[d_i e_i(t) + f_i(x_i(t)) - f_i\big(\Phi\big(y_i(t)\big)\big)\big], \tag{2.33}$$

and the update laws

$$\dot{d}_i = k_i e_i^T(t) e_i(t), \tag{2.34}$$

$$\dot{\tilde{c}}_{ij} = \delta_{ij} e_i^T(t) h\big(\Phi\big(y_j(t-\tau(t))\big)\big), \tag{2.35}$$

where $i, j = 1, 2, \ldots, N$, d_i *is feedback strength, and* $\delta_{ij} > 0$, $k_i > 0$ *are arbitrary constants, then the complex dynamical networks* (2.2) *and* (2.7) *will achieve GS with respect to* Φ.

Proof. Select the same Lyapunov-Krasovskii function as Theorem 2.5, then

$$\begin{aligned}\frac{dV}{dt} &= 2\sum_{i=1}^{N} e_i^T(t)\Bigg[f_i(x_i(t)) - f_i\big(\Phi\big(y_i(t)\big)\big) + \sum_{j=1}^{N} c_{ij} H\left(e_j(t-\tau(t))\right) - \sum_{j=1}^{N}\tilde{c}_{ij} h\big(\Phi\big(y_j(t-\tau(t))\big)\big)\\ &\qquad - D\Phi\big(y_i(t)\big)u_i(t)\Bigg] + \sum_{i=1}^{N}\sum_{j=1}^{N}\frac{2}{\delta_{ij}}\tilde{c}_{ij}\dot{\tilde{c}}_{ij} + \sum_{i=1}^{N}\frac{2}{k_i}(d_i - d)\dot{d}_i\\ &\quad + \frac{N\beta^2}{1-\mu}\sum_{i=1}^{N} e_i^T(t)e_i(t) - \frac{1-\dot{\tau}(t)}{1-\mu}N\beta^2\sum_{i=1}^{N} e_i^T(t-\tau(t))e_i(t-\tau(t))\\ &\le \sum_{i=1}^{N}\left(-2d + \frac{N\beta^2}{1-\mu}\right)e_i^T(t)e_i(t)\\ &\quad + \sum_{i=1}^{N}\sum_{j=1}^{N} e_i^T(t)c_{ij}H\big(e_j(t-\tau(t))\big) - \frac{1-\dot{\tau}(t)}{1-\mu}N\beta^2\sum_{i=1}^{N} e_i^T(t-\tau(t))e_i(t-\tau(t)).\end{aligned} \tag{2.36}$$

The rest of the proof is similar to Theorem 2.5 and omitted here. This completes the proof. □

Remark 2.11. According to Remark 2.6, when $\{h(\Phi(y_j(t-\tau(t))))\}_{j=1}^N$ are linearly independent on the orbit $\{y_j(t-\tau(t))\}_{j=1}^N$ of synchronization manifold, we can get $\lim_{t\to+\infty}\widehat{c}_{ij} = c_{ij},\ i,j = 1,2,\ldots,N$; that is, the uncertain coupling matrix C can be successfully estimated using the updating laws (2.35).

Remark 2.12. Based on Theorem 2.10, we can get corollaries corresponding to Corollaries 2.7–2.9.

3. Illustrative Numerical Examples

In this section, two groups of drive and response networks are concretely presented to demonstrate the effectiveness of the proposed method in the previous section.

It is well known that the unified chaotic system [25] is described by

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{pmatrix} -(25\beta+10)(x_1-x_2) \\ -x_1x_3 + (28-35\beta)x_1 + (29\beta-1)x_2 \\ x_1x_2 - \dfrac{(\beta+8)}{3}x_3 \end{pmatrix} \tag{3.1}$$

$$= \begin{pmatrix} -10(x_1-x_2) \\ -x_1x_3 + 28x_1 - x_2 \\ x_1x_2 - \dfrac{8}{3}x_3 \end{pmatrix} + \begin{pmatrix} -25(x_1-x_2) \\ -35x_1 + 29x_2 \\ -\dfrac{1}{3}x_3 \end{pmatrix}\beta = F(x) + G(x)\beta, \tag{3.2}$$

which is chaotic if $\beta \in [0,1]$. Obviously, system (3.2) is the original Lorenz system for $\beta = 0$ while system (3.2) belongs to the original Chen system for $\beta = 1$. In fact, system (3.2) bridges the gap between the Lorenz system and Chen system.

The unified new chaotic system [26] can be described as

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{pmatrix} ax_1 - x_2x_3 \\ bx_2 + x_1x_3 \\ cx_3 + \dfrac{1}{3}x_1x_2 \end{pmatrix} = g(x). \tag{3.3}$$

It is chaotic when $a = 5.0,\ b = -10.0$, and $c = -3.8$.

In the following, we will take these two chaotic systems as node dynamics to validate the effectiveness of Theorems 2.5 and 2.10. To do that, we first verify that function $f(x) = F(x) + G(x)\beta\ (\beta \in [0,1])$ satisfies Assumption 2.

Since the attractor is confined to a bounded region, there exists a constant $M > 0$, satisfying for all $y = (y_1, y_2, y_3)$, $z = (z_1, z_2, z_3) \in R^3$, $\|y\| \le M$, $\|z\| \le M$; therefore,

$$\begin{aligned}
\|f(y) - f(z)\|^2 &= (25\beta + 10)^2 [(y_2 - y_1) - (z_2 - z_1)]^2 + [z_1 z_3 - y_1 y_3 + (28 - 35\beta)(y_1 - z_1) \\
&\quad + (29\beta - 1)(y_2 - z_2)]^2 + \left[y_1 y_2 - z_1 z_2 - \frac{(\beta + 8)}{3}(y_3 - z_3)\right]^2 \\
&= (25\beta + 10)^2 [(y_2 - z_2) - (y_1 - z_1)]^2 \\
&\quad + [z_1(z_3 - y_3) + (-y_3 + 28 - 35\beta)(y_1 - z_1) + (29\beta - 1)(y_2 - z_2)]^2 \\
&\quad + \left[y_1(y_2 - z_2) + z_2(y_1 - z_1) - \frac{(\beta + 8)}{3}(y_3 - z_3)\right]^2 \\
&\le 35^2 \left[2(y_2 - z_2)^2 + 2(y_1 - z_1)^2\right] + 3M^2(y_3 - z_3)^2 + 6\left(28^2 + M^2\right)(y_1 - z_1)^2 \\
&\quad + 3 \times 28^2 (y_2 - z_2)^2 + 3M^2(y_2 - z_2)^2 + 3M^2(y_1 - z_1)^2 + 9(y_3 - z_3)^2 \\
&\le \left(2 \times 35^2 + 6 \times 28^2 + 9M^2\right)\|y - z\|^2.
\end{aligned} \tag{3.4}$$

Thus, function $f(x) = F(x) + G(x)\beta(\beta \in [0, 1])$ satisfies Assumption 2. By the same process, we can obtain that function $g(x)$ satisfies Assumption 2, too.

Example 3.1. In this subsection, we consider a weighted complex dynamical network with coupling delay consisting of 3 Lorenz systems and 2 new chaotic systems (3.3). The entire networked system is given as

$$\begin{aligned}
\dot{x}_i(t) &= F(x_i(t)) + \sum_{j=1}^{5} c_{ij} h(x_j(t - \tau(t))), \quad i = 1, 2, 3, \\
\dot{x}_i(t) &= g(x_i(t)) + \sum_{j=1}^{5} c_{ij} h(x_j(t - \tau(t))), \quad i = 4, 5,
\end{aligned} \tag{3.5}$$

where $x_i(t) = (x_{i1}(t), x_{i2}(t), x_{i3}(t))^T$, $i = 1, 2, \ldots, 5$. $\tau(t) = 0.1$, the weight configuration matrix

$$C = (c_{ij})_{5\times 5} = \begin{pmatrix} -5 & 1 & 3 & 1 & 0 \\ 1 & -2 & 1 & 0 & 0 \\ 3 & 1 & -4 & 0 & 0 \\ 1 & 0 & 0 & -2 & 1 \\ 0 & 0 & 0 & 1 & -1 \end{pmatrix}. \tag{3.6}$$

The coupling functions are $h(x_j(t)) = (\sin(x_{j1}(t)), \arctan(x_{j2}(t)), \arctan(x_{j3}(t)))^T$, $j = 1, 2, \ldots, 5$.

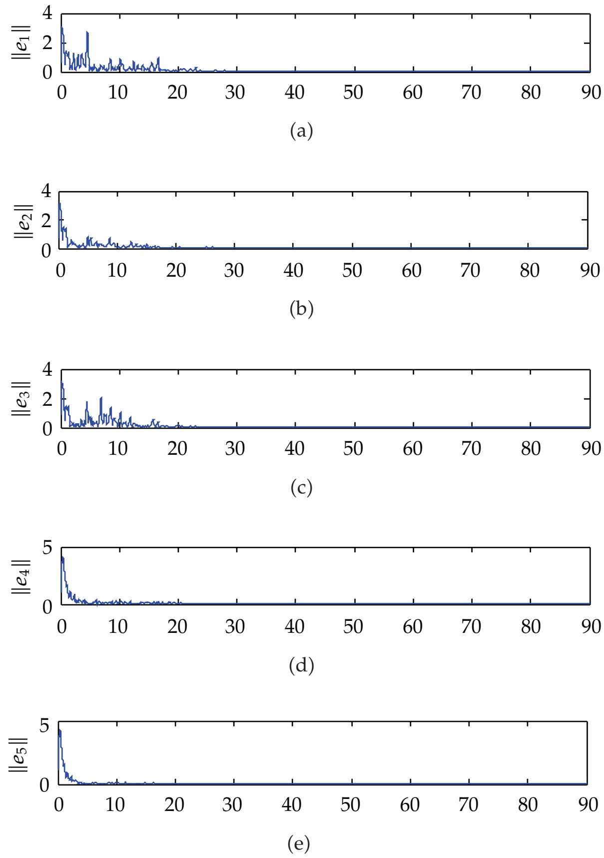

Figure 1: GS errors of model (3.5) and (3.7) with respect to $\Phi(y) = (y_1 + y_2, 2y_2, 2y_3)^T$.

Let $\Phi(y_i) = (y_{i1} + y_{i2},\ 2y_{i2},\ 2y_{i3})^T$, then $D\Phi(y_i) = \left(\begin{smallmatrix} 1 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{smallmatrix}\right)$, $i = 1, 2, \ldots, 5$.

Since (A1)–(A3) hold, therefore, according to Theorem 2.5, we can use the following response network:

$$\dot{y}_i(t) = D\Phi(y_i(t))^{-1}\left[F(\Phi(y_i(t))) + \sum_{j=1}^{N}\hat{c}_{ij}h(\Phi(y_j(t-\tau(t))))\right] + u_i, \quad i = 1, 2, 3,$$
$$\dot{y}_i(t) = D\Phi(y_i(t))^{-1}\left[g(\Phi(y_i(t))) + \sum_{j=1}^{N}\hat{c}_{ij}h(\Phi(y_j(t-\tau(t))))\right] + u_i, \quad i = 4, 5. \tag{3.7}$$

The controller and update laws are given by (2.15)–(2.17). The initial values are given as follows: $\hat{c}_{ij}(0) = 3$, $\delta_{ij} = 1$, $d_i(0) = 1$, $k_i = 1$, $x_i(0) = (12 + i \times 0.1, 15 + i \times 0.1, 30 + i \times 0.15)^T$, $\hat{x}_i(0) = (5 + i \times 0.1,\ 7.5 + i \times 0.1,\ 15 + i \times 0.1)$, $i,\ j = 1, 2, \ldots, 5$. Figure 1 shows GS errors $\|e_i(t)\|$, $i = 1, 2, \ldots, 5$. One can see that all nodes' errors converge to zero. Some elements of matrix $\widehat{C}$ are displayed in Figure 2. The numerical results show that this adaptive scheme is effective and we can get $\lim_{t\to\infty}\hat{c}_{ij} = c_{ij}$, $i,\ j = 1, 2, \ldots, 5$.

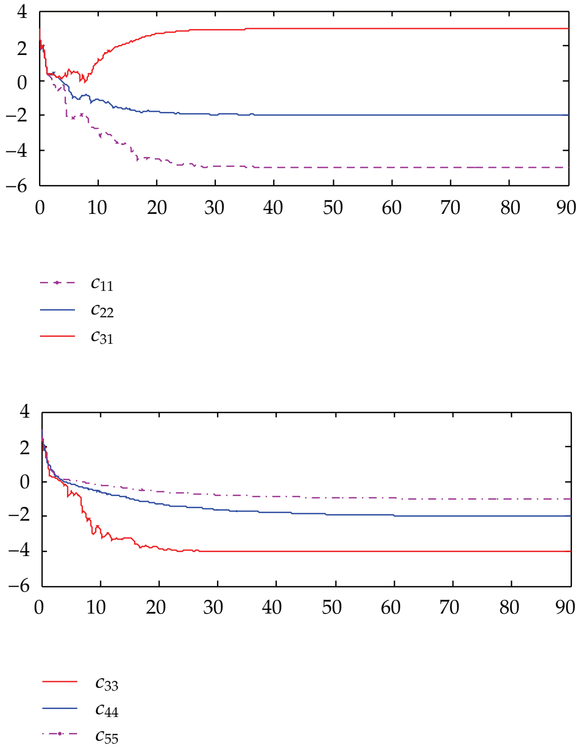

Figure 2: Estimation of topology of model (3.5).

Example 3.2. In the following simulation, we choose a weighted complex dynamical network with coupling delay consisting of 5 unified chaotic systems. The entire networked system is given as

$$\dot{x}_i(t) = f_i(x_i(t)) + \sum_{j=1}^{5} c_{ij} h\big(x_j(t-\tau(t))\big), \quad i = 1,2,\ldots,5, \tag{3.8}$$

where $x_i(t) = (x_{i1}(t), x_{i2}(t), x_{i3}(t))^T$, $f_i(x) = F(x)\beta_i + G(x)$, $\beta_i = 0.1 \times (i-1)$, $i = 1,2,\ldots,5$. We assume $\tau(t) = 0.3, h(x_j(t)) = (\arctan(x_{j1}(t)), \arctan(x_{j2}(t)), \arctan(x_{j3}(t)))^T$, $j = 1,2, \ldots,5$. C is the same as that in model (3.5).

Let $\Phi(y_i) = (y_{i1} + y_{i2},\ 2y_{i2},\ y_{i3}^3 + y_{i3})^T$, $D\Phi(y_i) = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3y_{i3}^2+1 \end{pmatrix}$, $i = 1,2,\ldots,5$.

According to Theorem 2.10, the response network is given by

$$\dot{y}_i(t) = D\Phi\big(y_i(t)\big)^{-1}\left[f_i\big(\Phi\big(y_i(t)\big)\big) + \sum_{j=1}^{N} \hat{c}_{ij} h\big(\Phi\big(y_j(t-\tau(t))\big)\big)\right] + u_i, \quad i = 1,2,\ldots,5. \tag{3.9}$$

The controller and update laws are given by (2.33)–(2.35). The initial values are given as follows: $\hat{c}_{ij}(0) = 6, \delta_{ij} = 1,\ d_i(0) = 1,\ k_i = 1,\ x_i(0) = (12,15,30)^T,\ \hat{x}_i(0) = (5,7.5,3)^T,\ i,j = 1,2,\ldots,5$. Figures 3 and 4 show the GS errors $\|e_i(t)\|$, $i = 1,2,\ldots,5$ and some elements of

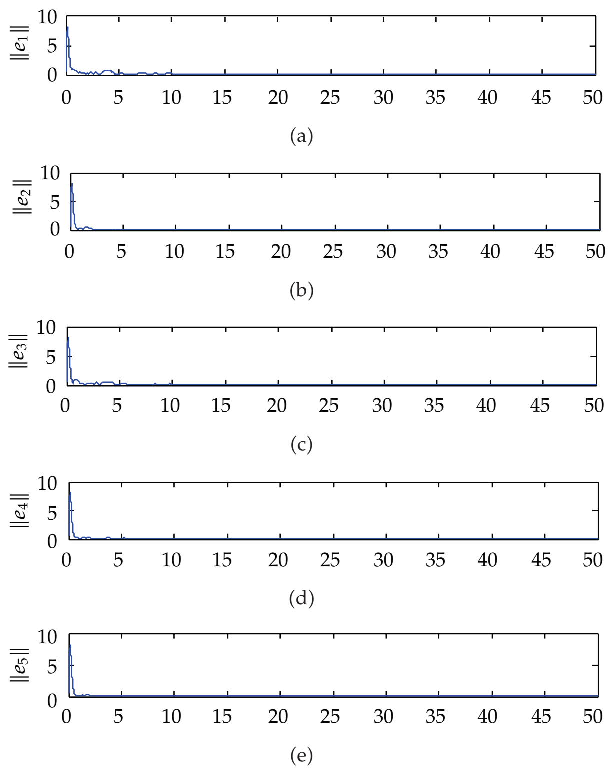

Figure 3: GS errors of model (3.8) and (3.9) with respect to $\Phi(y) = (y_1 + y_2, 2y_2, y_3^3 + y_3)^T$.

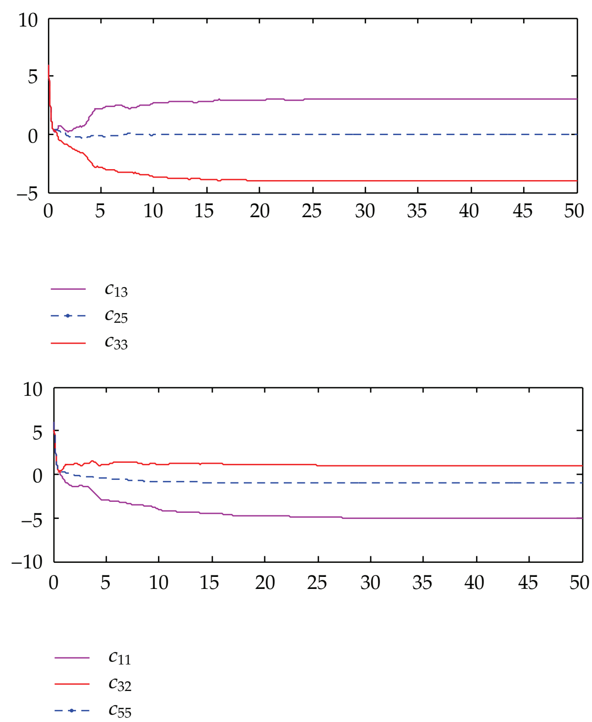

Figure 4: Estimation of topology of model (3.8).

matrix $\widehat{C}$, respectively. The results illustrate that this scheme is effective and we can get $\lim_{t\to+\infty}\widehat{c}_{ij} = c_{ij}$, $i, j = 1, 2, \ldots, 5$.

4. Conclusion

In this paper, we have investigated GS between two complex networks with different node dynamics and proposed some new GS schemes via nonlinear control using Lyapunov theory and Barbălat lemma. Our results generalize CS of complex dynamical network with linear coupling and without delay in [20] to GS of complex dynamical network with nonidentical nodes and time-varying delay nonlinear coupling. Numerical examples are provided to verify the effectiveness of the theoretical results. This work extends the study of GS.

Acknowledgment

This research is supported by the National Natural Science Foundation of China under Grant no. 70871056.

References

[1] D. J. Watts and S. H. Strogatz, "Collective dynamics of "small-world" networks," *Nature*, vol. 393, no. 6684, pp. 440–442, 1998.
[2] A. L. Barabási, R. Albert, and H. Jeong, "Mean-field theory for scale-free random network," *Physica A*, vol. 272, no. 1-2, pp. 173–187, 1999.
[3] S. H. Strogatz, "Exploring complex networks," *Nature*, vol. 410, no. 6825, pp. 268–276, 2001.
[4] L. Y. Xiang, Z. Q. Chen, Z. X. Liu, and Z. Z. Yuane, "Research on the modelling, analysis and control of complex dynamical networks," *Progress in Natural Science*, vol. 16, no. 13, pp. 1543–1551, 2006.
[5] T. Zhou, Z. Q. Fu, and B. H. Wang, "Epidemic dynamics on complex networks," *Progress in Natural Science*, vol. 16, no. 5, pp. 452–457, 2006.
[6] M. C. Ho, Y. C. Hung, and C. H. Chou, "Phase and anti-phase synchronization of two chaotic systems by using active control," *Physics Letters A*, vol. 296, no. 1, pp. 43–48, 2002.
[7] H. H. Chen, G. J. Sheu, Y. L. Lin, and C. S. Chen, "Chaos synchronization between two different chaotic systems via nonlinear feedback control," *Nonlinear Analysis: Theory, Methods & Applications*, vol. 70, no. 12, pp. 4393–4401, 2009.
[8] J. Zhou, J. A. Lu, and J. H. Lü, "Adaptive synchronization of an uncertain complex dynamical network," *IEEE Transactions on Automatic Control*, vol. 51, no. 4, pp. 652–656, 2006.
[9] S. M. Cai, J. Zhou, L. Xiang, and Z. R. Liu, "Robust impulsive synchronization of complex delayed dynamical networks," *Physics Letters A*, vol. 372, no. 30, pp. 4990–4995, 2008.
[10] X. R. Chen and C. X. Liu, "Passive control on a unified chaotic system," *Nonlinear Analysis: Real World Applications*, vol. 11, no. 2, pp. 683–687, 2010.
[11] L. M. Pecora and T. L. Carroll, "Master stability functions for synchronized coupled systems," *Physical Review Letters*, vol. 80, no. 10, pp. 2109–2112, 1998.
[12] A. Arenas, A. Díaz-Guilera, and C. J. Pérez-Vicente, "Synchronization reveals topological scales in complex networks," *Physical Review Letters*, vol. 96, no. 11, Article ID 114102, 4 pages, 2006.
[13] M. Sun, C. Y. Zeng, and L. X. Tian, "Projective synchronization in drive-response dynamical networks of partially linear systems with time-varying coupling delay," *Physics Letters A*, vol. 372, no. 46, pp. 6904–6908, 2008.
[14] S. Zheng, Q. S. Bi, and G. L. Cai, "Adaptive projective synchronization in complex networks with time-varying coupling delay," *Physics Letters A*, vol. 373, no. 17, pp. 1553–1559, 2009.
[15] X. Q. Yu, Q. S. Ren, J. L. Hou, and J. Y. Zhao, "The chaotic phase synchronization in adaptively coupled-delayed complex networks," *Physics Letters A*, vol. 373, no. 14, pp. 1276–1282, 2009.
[16] X. Li, "Phase synchronization in complex networks with decayed long-range interactions," *Physica D*, vol. 223, no. 2, pp. 242–247, 2006.
[17] X. L. Xu, Z. Q. Chen, G. Y. Si, X. F. Hu, and P. Luo, "A novel definition of generalized synchronization on networks and a numerical simulation example," *Computers & Mathematics with Applications*, vol. 56, no. 11, pp. 2789–2794, 2008.

[18] Y. C. Hung, Y. T. Huang, M. C. Ho, and C. K. Hu, "Paths to globally generalized synchronization in scale-free networks," *Physical Review E*, vol. 77, no. 1, Article ID 016202, 8 pages, 2008.
[19] J. F. Li, N. Li, Y. P. Liu, and Y. Gan, "Linear and nonlinear generalized synchronization of a class of chaotic systems by using a single driving variable," *Acta Physica Sinica*, vol. 58, no. 2, pp. 779–784, 2009.
[20] J. Zhou and J. A. Lu, "Topology identification of weighted complex dynamical networks," *Physica A*, vol. 386, no. 1, pp. 481–491, 2007.
[21] X. W. Liu and T. P. Chen, "Exponential synchronization of nonlinear coupled dynamical networks with a delayed coupling," *Physica A*, vol. 381, no. 15, pp. 82–92, 2007.
[22] G. M. He and J. Y. Yang, "Adaptive synchronization in nonlinearly coupled dynamical networks," *Chaos, Solitons and Fractals*, vol. 38, no. 5, pp. 1254–1259, 2008.
[23] J. G. Peng, H. Qiao, and Z. B. Xu, "A new approach to stability of neural networks with time-varying delays," *Neural Networks*, vol. 15, no. 1, pp. 95–103, 2002.
[24] G. Tao, "A simple alternative to the Barbălat lemma," *IEEE Transactions on Automatic Control*, vol. 42, no. 5, p. 698, 1997.
[25] J. H. Lü, G. R. Chen, D. Z. Cheng, and S. Celikovsky, "Bridge the gap between the Lorenz system and the Chen system," *International Journal of Bifurcation and Chaos*, vol. 12, no. 12, pp. 2917–2926, 2002.
[26] H. K. Chen and C. I. Lee, "Anti-control of chaos in rigid body motion," *Chaos, Solitons and Fractals*, vol. 21, no. 4, pp. 957–965, 2004.