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*Research Article*

# Availability Equivalence Analysis of a Repairable Multistate Parallel-Series System with Different Performance Rates

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This paper extends the concept of availability equivalence from general binary system to discrete multistate system with different performance rates. It considers a repairable discrete multistate parallel-series system with different performance rates. The system availability is defined as the ability of the system to satisfy consumer demand. The universal generating function technique is adopted to derive the availability of both original and improved systems according to factor method and standby redundancy method. Two types of availability equivalence factors of the system are analyzed. A numerical example is presented to illustrate the theoretical results obtained in this paper.

## 1. Introduction

The reliability and availability of system depend on the system structure as well as on the reliability and availability of its components. Their values can be increased by different improvement methods, for example, using more reliable components and adding redundant components to the system. Sometimes these measures can be equivalent as they will have the same effect on the reliability and availability of the system.

The equivalence concept of different system designs with respect to a reliability characteristic was first introduced in [1]. Råde [2, 3], Sarhan [4–7], and Montaser and Sarhan [8] applied such concept to discuss various systems with exponential distribution in the case of no repairs. Xia and Zhang [9] investigated the reliability equivalence of a parallel system with Gamma life time distribution. Mustafa [10] studied the reliability equivalence of some systems with mixture Weibull distribution. Pogány et al. [11] derived reliability equivalence factors of composite system with Gamma-Weibull distribution. Shawky et al. [12] analyzed the reliability equivalence problem of a parallel system with exponentiated exponential distribution.

For repairable systems, Hu et al. [13] analyzed the availability equivalence of different designs for a repairable series-parallel system with identical components in each subsystem. Sarhan and Mustafa [14] investigated the availability equivalence factors of a general repairable series-parallel system and assumed that the system components are repairable and independent but not identical. Recently, Mustafa and Sarhan [15] studied the availability equivalence factors for a general repairable parallel-series system.

In the real world, many systems are designed to perform their intended tasks in a given environment. Multistate system is one type of these systems. The multistate system widely exists in industrial engineering [16, 17], for example, power generation system, computing systems, and transportation systems. However, in the previous literatures for reliability/availability equivalence design, the reported works mainly focus on the issues of the systems with two possible states (completely working and totally failed). Comparable work on repairable multistate systems with availability equivalence design is rarely found in the literature. This motivates us to develop the availability equivalence design of multistate system.

In this paper, we consider a repairable discrete multistate parallel-series system with different performance rates. Assume that the performance rate of each series subsystem is the minimum of the performance rates of all of its components and the performance rate of the system is the sum of the performance rates of all series subsystems. The purpose of this paper is to accomplish two objectives. The first one is to derive the availability of the original and improved systems according to different improvement methods by using universal generating function (UGF) technique [18]. The second one is to analyze the availability equivalence factors of the system.

The structure of this paper is organized as follows. The availability of the original repairable multistate parallel-series system is provided in Section 2. Section 3 derives the availability of the systems improved according to factor method and standby redundancy method. Two types of availability equivalence factors of the system are investigated in Section 4. A numerical example is presented to illustrate the analysis method for availability equivalence factors of the system in Section 5. Finally, conclusion is given in Section 6.

## 2. Availability of Repairable Multistate Parallel-Series System

A repairable discrete multistate parallel-series system is composed of $n$ subsystems connected in parallel, and subsystem $i$ consists of $m_i$ different components connected in series, as depicted in Figure 1. The component $j$ with two states (failed or operating) in the subsystem $i$ is sorted by performance rates $\mathbf{g}_{ij} = \{0, \omega_{ij}\}$, constant failure rate $\lambda_{ij}$, and constant repair rate $\mu_{ij}$, $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, m_i$.

The state probability distributions of the repairable component $j$ in the subsystem $i$ corresponding to performance rates $\mathbf{g}_{ij} = \{0, \omega_{ij}\}$ are

$$P\{w = 0\} = \frac{\lambda_{ij}}{\mu_{ij} + \lambda_{ij}} = \frac{\eta_{ij}}{1 + \eta_{ij}}, \quad P\{w = \omega_{ij}\} = \frac{\mu_{ij}}{\mu_{ij} + \lambda_{ij}} = \frac{1}{1 + \eta_{ij}}, \tag{1}$$

where performance rate $w$ is a random variable, taking value from $\mathbf{g}_{ij} : w \in \mathbf{g}_{ij}$, $\eta_{ij} = \lambda_{ij}/\mu_{ij}$. The UGF of the component $j$ in the subsystem $i$ is defined as follows [16]:

$$u_{ij}(z) = P\{w = 0\} z^0 + P\{w = \omega_{ij}\} z^{\omega_{ij}} = \frac{\eta_{ij}}{1 + \eta_{ij}} z^0 + \frac{1}{1 + \eta_{ij}} z^{\omega_{ij}}. \tag{2}$$

The performance rate of the series subsystem $i$ we consider here is assumed to be equal to the minimum of the performance rates of individual components in the subsystem. According to the following operator $\Theta_s$ [18],

$$\Theta_s(u_1(z), u_2(z)) = \Theta_s\left(\sum_{k_1=1}^{M_1} p_{1k_1} \cdot z^{\omega_{1k_1}}, \sum_{k_2=1}^{M_2} p_{2k_2} \cdot z^{\omega_{2k_2}}\right) = \sum_{k_1=1}^{M_1}\sum_{k_2=1}^{M_2} p_{1k_1} p_{2k_2} \cdot z^{\min\{\omega_{1k_1}, \omega_{2k_2}\}}, \tag{3}$$

where $M_j$ is the number of possible states of the component $j$, $\omega_{jk_j}$ is the performance rate in state $k_j$, and $p_{jk_j}$ is the corresponding state probability, $j = 1, 2$. We can obtain the UGF of the series subsystem $i$:

$$U_i(z) = \Theta_s\left(u_{i1}(z), u_{i2}(z), \ldots, u_{im_i}(z)\right) = \Theta_s\left(\frac{\eta_{i1}}{1 + \eta_{i1}} z^0 + \frac{1}{1 + \eta_{i1}} z^{\omega_{i1}}, \frac{\eta_{i2}}{1 + \eta_{i2}} z^0 + \frac{1}{1 + \eta_{i2}} z^{\omega_{i2}}, \ldots, \frac{\eta_{im_i}}{1 + \eta_{im_i}} z^0 + \frac{1}{1 + \eta_{im_i}} z^{\omega_{im_i}}\right) = \left(1 - \prod_{j=1}^{m_i} \frac{1}{1 + \eta_{ij}}\right) z^0 + \left(\prod_{j=1}^{m_i} \frac{1}{1 + \eta_{ij}}\right) \cdot z^{\min\{\omega_{i1}, \omega_{i2}, \ldots, \omega_{im_i}\}}. \tag{4}$$

The performance rate of the repairable multistate parallel-series system is assumed to be equal to the sum of the performance rates of individual series subsystems. According to the following operator $\Theta_p$ [18],

$$\Theta_p(u_1(z), u_2(z)) = \Theta_p\left(\sum_{k_1=1}^{M_1} p_{1k_1} \cdot z^{\omega_{1k_1}}, \sum_{k_2=1}^{M_2} p_{2k_2} \cdot z^{\omega_{2k_2}}\right) = \sum_{k_1=1}^{M_1}\sum_{k_2=1}^{M_2} p_{1k_1} p_{2k_2} \cdot z^{\omega_{1k_1} + \omega_{2k_2}} = u_1(z) \cdot u_2(z). \tag{5}$$

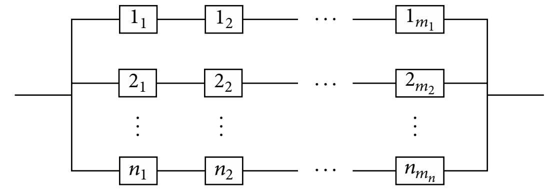


Figure 1: General structure of a repairable multistate parallel-series system.

The UGF of the entire system can be obtained as follows:

$$U(z) = \Theta_p(U_1(z), U_2(z), \ldots, U_n(z)) = \prod_{i=1}^{n} U_i(z)$$
$$= \prod_{i=1}^{n}\left(\left(1 - \prod_{j=1}^{m_i}\frac{1}{1+\eta_{ij}}\right)z^0 + \left(\prod_{j=1}^{m_i}\frac{1}{1+\eta_{ij}}\right)z^{\min\{\omega_{i1},\omega_{i2},\ldots,\omega_{im_i}\}}\right) = \sum_{k=1}^{M} p_k \cdot z^{\omega_k}, \tag{6}$$

where $M$ is the number of possible states of the system, $\omega_k$ is the state performance rate in state $k$, and $p_k = p_k(\boldsymbol{\eta}_1, \boldsymbol{\eta}_2, \ldots, \boldsymbol{\eta}_n)$ is the corresponding state probability, $\boldsymbol{\eta}_i = (\eta_{i1}, \eta_{i2}, \ldots, \eta_{im_i})$, $i = 1, 2, \ldots, n$.

For a given demand performance rate $\omega$, the system availability $A(\omega)$ is

$$A(\omega) = \sum_{k=1}^{M} p_k \cdot 1(\omega_k \geq \omega), \tag{7}$$

where the function $1(\omega_k \geq \omega) = 1$ if $\omega_k \geq \omega$, and $1(\omega_k \geq \omega) = 0$ if $\omega_k < \omega$.

## 3. Availability of Improved Systems

In this section, we present the availability of the improved systems according to factor method and standby redundancy method.

*3.1. The Factor Method.* In the factor method, it is assumed that the system can be improved by reducing failure rates of some components by a factor $\rho$ $(0 < \rho < 1)$ or increasing repair rates of some components by a factor $\sigma$ $(\sigma > 1)$. The two methods will be referred to as reduction and increase methods, respectively.

Let $R_i$ and $S_i$ be the sets of the components for which the failure rates are reduced and the repair rates are increased in the series subsystem $i$, respectively, and $\overline{R_i} = M_i \setminus R_i$ and $\overline{S_i} = M_i \setminus S_i$, where $M_i = \{i_1, i_2, \ldots, i_{m_i}\}$ denotes a set of all components in the subsystem $i$.

For the reduction method, the state probability distributions of the component $j$ in the subsystem $i$ after reducing its failure rate $\lambda_{ij}$ by the factor $\rho$ corresponding to performance rates $\mathbf{g}_{ij} = \{0, \omega_{ij}\}$ are

$$P\{w = 0\} = \frac{\rho\lambda_{ij}}{\mu_{ij} + \rho\lambda_{ij}} = \frac{\rho\eta_{ij}}{1+\rho\eta_{ij}},$$
$$P\{w = \omega_{ij}\} = \frac{\mu_{ij}}{\mu_{ij} + \rho\lambda_{ij}} = \frac{1}{1+\rho\eta_{ij}}, \quad j \in R_i, \tag{8}$$

where $\eta_{ij} = \lambda_{ij}/\mu_{ij}$.

For increase method, the state probability distributions of the component $j$ in the subsystem $i$ after increasing its repair rate $\mu_{ij}$ by the factor $\sigma$ corresponding to performance rates $\mathbf{g}_{ij} = \{0, \omega_{ij}\}$ are

$$P\{w = 0\} = \frac{\lambda_{ij}}{\sigma\mu_{ij} + \lambda_{ij}} = \frac{\eta_{ij}}{\sigma+\eta_{ij}},$$
$$P\{w = \omega_{ij}\} = \frac{\sigma\mu_{ij}}{\sigma\mu_{ij} + \lambda_{ij}} = \frac{\sigma}{\sigma+\eta_{ij}}, \quad j \in S_i, \tag{9}$$

where $\eta_{ij} = \lambda_{ij}/\mu_{ij}$. The UGFs of the component $j$ with reduced failure rate or increased repair rate are

$$u_{ijr}(z) = \frac{\rho\eta_{ij}}{1+\rho\eta_{ij}}z^0 + \frac{1}{1+\rho\eta_{ij}}z^{\omega_{ij}}, \quad j \in R_i,$$
$$u_{ijs}(z) = \frac{\eta_{ij}}{\sigma+\eta_{ij}}z^0 + \frac{\sigma}{\sigma+\eta_{ij}}z^{\omega_{ij}}, \quad j \in S_i. \tag{10}$$

The UGFs of the rest of the components in the subsystem $i$ are still determined by (2).

According to the operator $\Theta_s$, the UGFs of the subsystem $i$ improved by the reduction method or the increase method can be obtained as follows:

$$U_{ir}(z) = \Theta_s\left(\overbrace{u_{ik_1r}(z), u_{ik_2r}(z), \ldots, u_{ik_{r_i}r}(z)}^{r_i}, \overbrace{u_{ik_{r_{i+1}}}(z), u_{ik_{r_{i+2}}}(z), \ldots, u_{ik_{m_i}}(z)}^{(m_i - r_i)}\right),$$
$$U_{is}(z) = \Theta_s\left(\overbrace{u_{il_1s}(z), u_{il_2s}(z), \ldots, u_{il_{s_i}s}(z)}^{s_i}, \overbrace{u_{il_{s_{i+1}}}(z), u_{il_{s_{i+2}}}(z), \ldots, u_{il_{m_i}}(z)}^{(m_i - s_i)}\right), \tag{11}$$

where $k_1, k_2, \ldots, k_{r_i} \in R_i$, $k_{r_i+1}, k_{r_i+2}, \ldots, k_{m_i} \in \overline{R_i}$, $l_1, l_2, \ldots, l_{s_i} \in S_i$, and $l_{s_i+1}, l_{s_i+2}, \ldots, l_{m_i} \in \overline{S_i}$. Using (2), (10), and (11), $U_{ir}(z)$ and $U_{is}(z)$ can be written as

$$U_{ir}(z) = \left(1 - \prod_{j\in R_i}\frac{1}{1+\rho\eta_{ij}}\prod_{j\in\overline{R_i}}\frac{1}{1+\eta_{ij}}\right)z^0 + \left(\prod_{j\in R_i}\frac{1}{1+\rho\eta_{ij}}\prod_{j\in\overline{R_i}}\frac{1}{1+\eta_{ij}}\right)z^{\min\{\omega_{i1},\omega_{i2},\ldots,\omega_{im_i}\}},$$

$$U_{is}(z) = \left(1 - \prod_{j\in S_i}\frac{\sigma}{\sigma+\eta_{ij}}\prod_{j\in\overline{S_i}}\frac{1}{1+\eta_{ij}}\right)z^0 + \left(\prod_{j\in S_i}\frac{\sigma}{\sigma+\eta_{ij}}\prod_{j\in\overline{S_i}}\frac{1}{1+\eta_{ij}}\right)z^{\min\{\omega_{i1},\omega_{i2},\dots,\omega_{im_i}\}}. \quad (12)$$

Using the operator $\Theta_p$, the UGFs of the improved system by the reduction method $U_r(z)$ or the increase method $U_s(z)$ can be obtained as follows:

$$\begin{aligned}U_r(z) &= \Theta_p\left(U_{1r}(z), U_{2r}(z), \dots, U_{nr}(z)\right)\\ &= \prod_{i=1}^{n}\left(\left(1 - \prod_{j\in R_i}\frac{1}{1+\rho\eta_{ij}}\prod_{j\in\overline{R_i}}\frac{1}{1+\eta_{ij}}\right)z^0 + \left(\prod_{j\in R_i}\frac{1}{1+\rho\eta_{ij}}\prod_{j\in\overline{R_i}}\frac{1}{1+\eta_{ij}}\right)z^{\min\{\omega_{i1},\omega_{i2},\dots,\omega_{im_i}\}}\right)\\ &= \sum_{k=1}^{M} p_{kr}\cdot z^{\omega_k},\\ U_s(z) &= \Theta_p\left(U_{1s}(z), U_{2s}(z), \dots, U_{ns}(z)\right)\\ &= \prod_{i=1}^{n}\left(\left(1 - \prod_{j\in S_i}\frac{\sigma}{\sigma+\eta_{ij}}\prod_{j\in\overline{S_i}}\frac{1}{1+\eta_{ij}}\right)z^0 + \left(\prod_{j\in S_i}\frac{\sigma}{\sigma+\eta_{ij}}\prod_{j\in\overline{S_i}}\frac{1}{1+\eta_{ij}}\right)z^{\min\{\omega_{i1},\omega_{i2},\dots,\omega_{im_i}\}}\right)\\ &= \sum_{k=1}^{M} p_{ks}\cdot z^{\omega_k},\end{aligned} \quad (13)$$

where $p_{kr} = p_{kr}(\rho, \boldsymbol{\eta}_1, \boldsymbol{\eta}_2, \dots, \boldsymbol{\eta}_n)$ and $p_{ks} = p_{ks}(\sigma, \boldsymbol{\eta}_1, \boldsymbol{\eta}_2, \dots, \boldsymbol{\eta}_n)$ denote the state probabilities of the improved entire system according to the reduction method and the increase method in state $k$ ($k = 1, 2, \dots, M$), respectively, $\boldsymbol{\eta}_i = (\eta_{i1}, \eta_{i2}, \dots, \eta_{im_i})$, $i = 1, 2, \dots, n$.

For a given demand performance rate $\omega$, the availability of the system improved by the reduction method or the increase method can be determined in the following forms:

$$A_{r,\rho}(\omega) = \sum_{k=1}^{M} p_{kr}\cdot 1(\omega_k \geq \omega), \quad (14)$$

$$A_{s,\sigma}(\omega) = \sum_{k=1}^{M} p_{ks}\cdot 1(\omega_k \geq \omega). \quad (15)$$

*3.2. The Standby Redundancy Method.* In reliability theory, standby redundancy is a technique widely used to improve system availability and reliability [19]. In our work, the standby redundancy method contains warm standby method and cold standby method. It is assumed that some components of the system are connected with some warm or cold standby components via perfect switches.

Let $W_i$ and $C_i$ be the sets of the components with warm standby components and cold standby components in the series subsystem $i$, respectively, and $\overline{W_i} = M_i \setminus W_i$ and $\overline{C_i} = M_i \setminus C_i$, where $M_i = \{i_1, i_2, \dots, i_{m_i}\}$ denotes a set of all components in the subsystem $i$.

It is assumed that each warm standby component in the subsystem $i$ has constant standby failure rate $v_{ij}$ and constant repair rate $\mu_{ij}$. According to [20], the state probability distributions of the component $j$ with a warm standby component corresponding to performance rates $\mathbf{g}_{ij} = \{0, \omega_{ij}\}$ are

$$\begin{aligned}P\{w=0\} &= \frac{(1/2)\lambda_{ij}^2 + (1/2)\lambda_{ij}v_{ij}}{\mu_{ij}^2 + \lambda_{ij}\mu_{ij} + v_{ij}\mu_{ij} + (1/2)\lambda_{ij}^2 + (1/2)\lambda_{ij}v_{ij}}\\ &= \frac{(1/2)\eta_{ij}^2 + (1/2)\eta_{ij}\xi_{ij}}{1+\eta_{ij}+\xi_{ij}+(1/2)\eta_{ij}^2+(1/2)\eta_{ij}\xi_{ij}},\\ P\{w=\omega_{ij}\} &= \frac{\mu_{ij}^2 + \lambda_{ij}\mu_{ij} + v_{ij}\mu_{ij}}{\mu_{ij}^2 + \lambda_{ij}\mu_{ij} + v_{ij}\mu_{ij} + (1/2)\lambda_{ij}^2 + (1/2)\lambda_{ij}v_{ij}}\\ &= \frac{1+\eta_{ij}+\xi_{ij}}{1+\eta_{ij}+\xi_{ij}+(1/2)\eta_{ij}^2+(1/2)\eta_{ij}\xi_{ij}},\end{aligned} \quad (16)$$

where $\eta_{ij} = \lambda_{ij}/\mu_{ij}$ and $\xi_{ij} = v_{ij}/\mu_{ij}$, $j \in W_i$.

According to [21], the state probability distributions of the component $j$ with a cold standby component corresponding to performance rates $\mathbf{g}_{ij} = \{0, \omega_{ij}\}$ are

$$\begin{aligned}P\{w=0\} &= \frac{(1/2)\lambda_{ij}^2}{\mu_{ij}^2 + \lambda_{ij}\mu_{ij} + (1/2)\lambda_{ij}^2}\\ &= \frac{(1/2)\eta_{ij}^2}{1+\eta_{ij}+(1/2)\eta_{ij}^2},\\ P\{w=\omega_{ij}\} &= \frac{\mu_{ij}^2 + \lambda_{ij}\mu_{ij}}{\mu_{ij}^2 + \lambda_{ij}\mu_{ij} + (1/2)\lambda_{ij}^2}\\ &= \frac{1+\eta_{ij}}{1+\eta_{ij}+(1/2)\eta_{ij}^2},\end{aligned} \quad (17)$$

where $\eta_{ij} = \lambda_{ij}/\mu_{ij}$, $j \in C_i$.

Let $u_{ijw}(z)$ and $u_{ijc}(z)$ denote the UGFs of the component $j$ with a warm standby component ($j \in W_i$) and a cold standby component ($j \in C_i$), respectively; we have

$$u_{ijw}(z) = \frac{(1/2)\eta_{ij}^2 + (1/2)\eta_{ij}\xi_{ij}}{1+\eta_{ij}+\xi_{ij}+(1/2)\eta_{ij}^2+(1/2)\eta_{ij}\xi_{ij}} z^0 + \frac{1+\eta_{ij}+\xi_{ij}}{1+\eta_{ij}+\xi_{ij}+(1/2)\eta_{ij}^2+(1/2)\eta_{ij}\xi_{ij}} z^{\omega_{ij}},$$
$$u_{ijc}(z) = \frac{(1/2)\eta_{ij}^2}{1+\eta_{ij}+(1/2)\eta_{ij}^2} z^0 + \frac{1+\eta_{ij}}{1+\eta_{ij}+(1/2)\eta_{ij}^2} z^{\omega_{ij}}. \tag{18}$$

The UGFs of the components belonging to the $\overline{W_i}$ or $\overline{C_i}$ are still determined by (2).

According to the operator $\Theta_s$, the UGFs of the subsystem $i$ improved by the warm standby method or the cold standby method can be obtained as follows:

$$U_{iw}(z) = \Theta_s\left(\overbrace{u_{ik_1w}(z), u_{ik_2w}(z), \ldots, u_{ik_{w_i}w}(z)}^{w_i}, \overbrace{u_{ik_{w_{i+1}}}(z), u_{ik_{w_{i+2}}}(z), \ldots, u_{ik_{m_i}}(z)}^{(m_i-w_i)}\right),$$
$$U_{ic}(z) = \Theta_s\left(\overbrace{u_{il_1c}(z), u_{il_2c}(z), \ldots, u_{il_{c_i}c}(z)}^{c_i}, \overbrace{u_{il_{c_{i+1}}}(z), u_{il_{c_{i+2}}}(z), \ldots, u_{il_{m_i}}(z)}^{(m_i-c_i)}\right), \tag{19}$$

where $k_1, k_2, \ldots, k_{w_i} \in W_i$, $k_{w_i+1}, k_{w_i+2}, \ldots, k_{m_i} \in \overline{W_i}$, $l_1, l_2, \ldots, l_{c_i} \in C_i$, and $l_{c_i+1}, l_{c_i+2}, \ldots, l_{m_i} \in \overline{C_i}$. Using (2), (18), and (19), $U_{iw}(z)$ and $U_{ic}(z)$ can be written as

$$U_{iw}(z) = \left(1 - \prod_{j\in W_i}\frac{1+\eta_{ij}+\xi_{ij}}{1+\eta_{ij}+\xi_{ij}+(1/2)\eta_{ij}^2+(1/2)\eta_{ij}\xi_{ij}}\prod_{j\in\overline{W_i}}\frac{1}{1+\eta_{ij}}\right)\cdot z^0 + \left(\prod_{j\in W_i}\frac{1+\eta_{ij}+\xi_{ij}}{1+\eta_{ij}+\xi_{ij}+(1/2)\eta_{ij}^2+(1/2)\eta_{ij}\xi_{ij}}\prod_{j\in\overline{W_i}}\frac{1}{1+\eta_{ij}}\right)\cdot z^{\min\{\omega_{i1},\omega_{i2},\ldots,\omega_{im_i}\}},$$
$$U_{ic}(z) = \left(1 - \prod_{j\in C_i}\frac{1+\eta_{ij}}{1+\eta_{ij}+(1/2)\eta_{ij}^2}\prod_{j\in\overline{C_i}}\frac{1}{1+\eta_{ij}}\right)z^0 + \left(\prod_{j\in C_i}\frac{1+\eta_{ij}}{1+\eta_{ij}+(1/2)\eta_{ij}^2}\prod_{j\in\overline{C_i}}\frac{1}{1+\eta_{ij}}\right)z^{\min\{\omega_{i1},\omega_{i2},\ldots,\omega_{im_i}\}}. \tag{20}$$

Using the operator $\Theta_p$, the UGFs of the improved system by the warm standby method or the cold standby method can be obtained in the following forms:

$$U_w(z) = \Theta_p(U_{1w}(z), U_{2w}(z), \ldots, U_{nw}(z)) = \prod_{i=1}^{n} U_{iw}(z) = \sum_{k=1}^{M} p_{kw}\cdot z^{\omega_k},$$
$$U_c(z) = \Theta_p(U_{1c}(z), U_{2c}(z), \ldots, U_{nc}(z)) = \prod_{i=1}^{n} U_{ic}(z) = \sum_{k=1}^{M} p_{kc}\cdot z^{\omega_k}, \tag{21}$$

where $U_w(z)$ denotes the UGF of the improved system by the warm standby method and $U_c(z)$ denotes the UGF of the improved system by the cold standby method and $p_{kw} = p_{kw}(\boldsymbol{\eta}_1, \boldsymbol{\eta}_2, \ldots, \boldsymbol{\eta}_n; \boldsymbol{\xi}_1, \boldsymbol{\xi}_2, \ldots, \boldsymbol{\xi}_n)$ and $p_{kc} = p_{kc}(\boldsymbol{\eta}_1, \boldsymbol{\eta}_2, \ldots, \boldsymbol{\eta}_n)$ denote the probabilities of the improved entire system by the warm standby method and the cold standby method in state $k$ ($k = 1, 2, \ldots, M$), respectively, $\boldsymbol{\eta}_i = (\eta_{i1}, \eta_{i2}, \ldots, \eta_{im_i})$, $\boldsymbol{\xi}_i = (\xi_{i1}, \xi_{i2}, \ldots, \xi_{im_i})$, $i = 1, 2, \ldots, n$.

For a given demand performance rate $\omega$, the availability of the system improved by the warm standby method or the cold standby method can be determined as follows:

$$A_w(\omega) = \sum_{k=1}^{M} p_{kw}\cdot 1(\omega_k \geq \omega), \tag{22}$$

$$A_c(\omega) = \sum_{k=1}^{M} p_{kc}\cdot 1(\omega_k \geq \omega). \tag{23}$$

## 4. Availability Equivalence Factors

In this section, we analyze the availability equivalence factors of the repairable multistate parallel-series system. The availability equivalence factor is defined as that factor by which the failure rates (the repair rates) of some of the system's components should be reduced (increased) in order to reach equality of the availability of another better system [13]. Two types of availability equivalence factors will be discussed. These types are called availability equivalence reduction and increase factors, respectively, denoted by AERF and AEIF.

*4.1. AERF.* The AERF, say $\rho_d$, $d = w(c)$, for warm (cold) standby method is defined as that factor $\rho$ by which the failure rates of some components of the system should be reduced so that one could obtain an improved system with availability that equals the availability of that system improved by using the warm standby method and cold standby method, respectively.

That is, to obtain the AERF $\rho_d$, $d = w(c)$, we have to solve the following equation:

$$A_{r,\rho}(\omega) = A_d(\omega), \quad d = w, c, \tag{24}$$

with respect to $\rho$. $A_{r,\rho}(\omega)$ and $A_d(\omega)$ $(d = w, c)$ can be obtained by (14), (22), and (23), respectively. Equation (24) never has closed-form solution; we have to use the numerical technique method to obtain $\rho$.

*4.2. AEIF.* The AEIF, say $\sigma_d$, $d = w(c)$, for warm (cold) standby method is defined as that factor $\sigma$ by which the repair rates of some components of the system should be increased so that one could obtain an improved system with availability that equals the availability of that system improved by using the warm standby method and cold standby method, respectively.

That is, to obtain the AEIF $\sigma_d$, $d = w(c)$, we have to solve the following equation:

$$A_{s,\sigma}(\omega) = A_d(\omega), \quad d = w, c, \tag{25}$$

with respect to $\sigma$. $A_{s,\sigma}(\omega)$ and $A_d(\omega)$ $(d = w, c)$ can be obtained by (15), (22), and (23), respectively. As it seems, (25) has no closed-form solution. To find $\sigma$, a numerical technique method can be used to solve the equation.

In the rest of the section, the main steps of computing the availability equivalence factors of the repairable multistate parallel-series system are provided as follows.

*Step 1.* Give fixed $n$ (number of the subsystems) and $m_i$ (number of components in the subsystem $i$), $i = 1, 2, \ldots, n$.

*Step 2.* Based on the available data, determine the parameters $\lambda_{ij}$, $\mu_{ij}$, $v_{ij}$, and $\omega_{ij}$ of the component $j$ in the subsystem $i$, $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, m_i$.

*Step 3.* Calculate the availability of the systems improved by the standby method and the factor method.

*Step 4.* According to the results in Step 3 and (24)-(25), calculate the availability equivalence factors $\rho$ and $\sigma$.

## 5. Numerical Example

A repairable multistate parallel-series system with two subsystems is considered. The component $j$ of the subsystem $i$ $(i = 1, 2)$ has two different performance rates: 0 and $\omega_{ij}$. The parameters $\lambda_{ij}$, $\mu_{ij}$, $v_{ij}$, $\eta_{ij}$, $\xi_{ij}$, and $\omega_{ij}$ for each component are presented in Table 1. Assume that the demand performance rate of the system $\omega$ is 50.

According to (2), (4), (6), and (7), the availability of the original system is 0.7586 when the demand performance rate $\omega = 50$. By using (18) and (20)–(23), we can obtain the availability of the improved systems according to the standby method. Table 2 presents the availability of the improved systems according to the warm and cold standby methods for different components sets.

Table 1: Parameters of components.

| Component $i_j$ | $\lambda_{ij}$ | $\mu_{ij}$ | $v_{ij}$ | $\eta_{ij} = \lambda_{ij}/\mu_{ij}$ | $\xi_{ij} = v_{ij}/\mu_{ij}$ | $\omega_{ij}$ |
|---|---|---|---|---|---|---|
| $1_1$ | 0.014 | 0.2 | 0.004 | 0.07 | 0.02 | 25 |
| $1_2$ | 0.012 | 0.10 | 0.002 | 0.12 | 0.02 | 30 |
| $2_1$ | 0.015 | 0.15 | 0.0045 | 0.1 | 0.03 | 35 |

Table 2: The availability of the improved system according to the warm and cold standby methods.

| $W_i$ | $A_w(50)$ | $C_i$ | $A_c(50)$ |
|---|---|---|---|
| $W_1 = \{1_1\}, W_2 = \phi$ | 0.8094 | $C_1 = \{1_1\}, C_2 = \phi$ | 0.8099 |
| $W_1 = \{1_2\}, W_2 = \phi$ | 0.8435 | $C_1 = \{1_2\}, C_2 = \phi$ | 0.8442 |
| $W_1 = \{1_1, 1_2\}, W_2 = \phi$ | 0.8998 | $C_1 = \{1_1, 1_2\}, C_2 = \phi$ | 0.9012 |
| $W_1 = \phi, W_2 = \{2_1\}$ | 0.8297 | $C_1 = \phi, C_2 = \{2_1\}$ | 0.8307 |
| $W_1 = \{1_1\}, W_2 = \{2_1\}$ | 0.8852 | $C_1 = \{1_1\}, C_2 = \{2_1\}$ | 0.8868 |
| $W_1 = \{1_2\}, W_2 = \{2_1\}$ | 0.9225 | $C_1 = \{1_2\}, C_2 = \{2_1\}$ | 0.9244 |
| $W_1 = \{1_1, 1_2\}, W_2 = \{2_1\}$ | 0.9841 | $C_1 = \{1_1, 1_2\}, C_2 = \{2_1\}$ | 0.9869 |

From the results presented in Table 2, we can see that (1) in terms of one component improved improving component 2 in subsystem 1 according to the warm (cold) standby method gives the highest availability, (2) in terms of two components improved improving component 2 in subsystem 1 and component 1 in subsystem 2 according to the warm (cold) standby method gives the highest availability, and (3) improving all components of the system according to the warm (cold) standby method gives the highest system availability.

Tables 3–6 present the AERF $\rho_d$ $(d = w, c)$ and the AEIF $\sigma_d$ $(d = w, c)$ for different components sets $W_i$, $C_i$, $R_i$, and $S_i$, $i = 1, 2$. The negative values (−Ve) in Tables 3–6 mean that there is no equivalence between the two improved systems: one obtained by the factor method and the other obtained by the standby method.

According to the results presented in Tables 3–6, the following can be seen:

(1) Improving components 1 and 2 in subsystem 1 according to the warm standby method will increase the system availability from 0.7586 to 0.8998; see Table 2. The same increase can be obtained by the factor method (reduction method and increase method).

   (i) *The Reduction Method.* Reducing the failure rates of (1) components 1 and 2 in subsystem 1 by the factor $\rho_w = 0.0543$, (2) component 2 in subsystem 1 and component 1 in subsystem 2 by the factor $\rho_w = 0.1741$, and (3) all components of the system by the factor $\rho_w = 0.3709$, see Table 3.

   (ii) *The Increase Method.* Increasing the repair rates of (1) components 1 and 2 in subsystem 1 by the factor $\sigma_w = 18.4271$, (2) component 2 in subsystem 1 and component 1 in subsystem 2 by the factor $\sigma_w = 5.7424$, and (3) all components of the system by the factor $\sigma_w = 2.6965$, see Table 4.

TABLE 3: The AERF $\rho_w$ for different components sets.

| $W_i$ | $R_i$: $R_1=\{1_1\}$, $R_2=\phi$ | $R_1=\{1_2\}$, $R_2=\phi$ | $R_1=\{1_1,1_2\}$, $R_2=\phi$ | $R_1=\phi$, $R_2=\{2_1\}$ | $R_1=\{1_1\}$, $R_2=\{2_1\}$ | $R_1=\{1_2\}$, $R_2=\{2_1\}$ | $R_1=\{1_1,1_2\}$, $R_2=\{2_1\}$ |
|---|---|---|---|---|---|---|---|
| $W_1=\{1_1\}$, $W_2=\phi$ | 0.0412 | 0.4143 | 0.6307 | 0.3101 | 0.5924 | 0.6780 | 0.7567 |
| $W_1=\{1_2\}$, $W_2=\phi$ | −Ve | 0.0607 | 0.4022 | −Ve | 0.3398 | 0.4784 | 0.6046 |
| $W_1=\{1_1,1_2\}$, $W_2=\phi$ | −Ve | −Ve | 0.0543 | −Ve | −Ve | 0.1741 | 0.3709 |
| $W_1=\phi$, $W_2=\{2_1\}$ | −Ve | 0.2003 | 0.4929 | 0.0579 | 0.4401 | 0.5577 | 0.6651 |
| $W_1=\{1_1\}$, $W_2=\{2_1\}$ | −Ve | −Ve | 0.1412 | −Ve | 0.0511 | 0.2503 | 0.4295 |
| $W_1=\{1_2\}$, $W_2=\{2_1\}$ | −Ve | −Ve | −Ve | −Ve | −Ve | 0.0594 | 0.2821 |
| $W_1=\{1_1,1_2\}$, $W_2=\{2_1\}$ | −Ve | −Ve | −Ve | −Ve | −Ve | −Ve | 0.0554 |

TABLE 4: The AEIF $\sigma_w$ for different components sets.

| $W_i$ | $S_i$: $S_1=\{1_1\}$, $S_2=\phi$ | $S_1=\{1_2\}$, $S_2=\phi$ | $S_1=\{1_1,1_2\}$, $S_2=\phi$ | $S_1=\phi$, $S_2=\{2_1\}$ | $S_1=\{1_1\}$, $S_2=\{2_1\}$ | $S_1=\{1_2\}$, $S_2=\{2_1\}$ | $S_1=\{1_1,1_2\}$, $S_2=\{2_1\}$ |
|---|---|---|---|---|---|---|---|
| $W_1=\{1_1\}$, $W_2=\phi$ | 24.2606 | 2.4134 | 1.5855 | 3.2247 | 1.6881 | 1.4749 | 1.3216 |
| $W_1=\{1_2\}$, $W_2=\phi$ | −Ve | 16.4723 | 2.4865 | −Ve | 2.9433 | 2.0902 | 1.6540 |
| $W_1=\{1_1,1_2\}$, $W_2=\phi$ | −Ve | −Ve | 18.4271 | −Ve | −Ve | 5.7424 | 2.6965 |
| $W_1=\phi$, $W_2=\{2_1\}$ | −Ve | 4.9920 | 2.0287 | 17.2854 | 2.2722 | 1.7930 | 1.5035 |
| $W_1=\{1_1\}$, $W_2=\{2_1\}$ | −Ve | −Ve | 7.0811 | −Ve | 19.5845 | 3.9960 | 2.3281 |
| $W_1=\{1_2\}$, $W_2=\{2_1\}$ | −Ve | −Ve | −Ve | −Ve | −Ve | 16.8271 | 3.5447 |
| $W_1=\{1_1,1_2\}$, $W_2=\{2_1\}$ | −Ve | −Ve | −Ve | −Ve | −Ve | −Ve | 18.0431 |

TABLE 5: The AERF $\rho_c$ for different components sets.

| $C_i$ | $R_i$: $R_1=\{1_1\}$, $R_2=\phi$ | $R_1=\{1_2\}$, $R_2=\phi$ | $R_1=\{1_1,1_2\}$, $R_2=\phi$ | $R_1=\phi$, $R_2=\{2_1\}$ | $R_1=\{1_1\}$, $R_2=\{2_1\}$ | $R_1=\{1_2\}$, $R_2=\{2_1\}$ | $R_1=\{1_1,1_2\}$, $R_2=\{2_1\}$ |
|---|---|---|---|---|---|---|---|
| $C_1=\{1_1\}$, $C_2=\phi$ | 0.0324 | 0.4089 | 0.6273 | 0.3037 | 0.5886 | 0.6750 | 0.7544 |
| $C_1=\{1_2\}$, $C_2=\phi$ | −Ve | 0.0537 | 0.3976 | −Ve | 0.3347 | 0.4745 | 0.6016 |
| $C_1=\{1_1,1_2\}$, $C_2=\phi$ | −Ve | −Ve | 0.0460 | −Ve | −Ve | 0.1669 | 0.3653 |
| $C_1=\phi$, $C_2=\{2_1\}$ | −Ve | 0.1900 | 0.4863 | 0.0457 | 0.4327 | 0.5519 | 0.6607 |
| $C_1=\{1_1\}$, $C_2=\{2_1\}$ | −Ve | −Ve | 0.1316 | −Ve | 0.0404 | 0.2418 | 0.4230 |
| $C_1=\{1_2\}$, $C_2=\{2_1\}$ | −Ve | −Ve | −Ve | −Ve | −Ve | 0.0500 | 0.2748 |
| $C_1=\{1_1,1_2\}$, $C_2=\{2_1\}$ | −Ve | −Ve | −Ve | −Ve | −Ve | −Ve | 0.0456 |

Table 6: The AEIF $\sigma_c$ for different components sets.

| $C_i$ | $S_i$: $S_1 = \{1_1\}$, $S_2 = \phi$ | $S_1 = \{1_2\}$, $S_2 = \phi$ | $S_1 = \{1_1, 1_2\}$, $S_2 = \phi$ | $S_1 = \phi$, $S_2 = \{2_1\}$ | $S_1 = \{1_1\}$, $S_2 = \{2_1\}$ | $S_1 = \{1_2\}$, $S_2 = \{2_1\}$ | $S_1 = \{1_1, 1_2\}$, $S_2 = \{2_1\}$ |
|---|---|---|---|---|---|---|---|
| $C_1 = \{1_1\}$, $C_2 = \phi$ | 30.8888 | 2.4453 | 1.5942 | 3.2923 | 1.6990 | 1.4815 | 1.3256 |
| $C_1 = \{1_2\}$, $C_2 = \phi$ | −Ve | 18.6054 | 2.5149 | −Ve | 2.9875 | 2.1076 | 1.6623 |
| $C_1 = \{1_1, 1_2\}$, $C_2 = \phi$ | −Ve | −Ve | 21.7186 | −Ve | −Ve | 5.9901 | 2.7375 |
| $C_1 = \phi$, $C_2 = \{2_1\}$ | −Ve | 5.2618 | 2.0565 | 21.8605 | 2.3109 | 1.8119 | 1.5136 |
| $C_1 = \{1_1\}$, $C_2 = \{2_1\}$ | −Ve | −Ve | 7.5997 | −Ve | 24.7552 | 4.1353 | 2.3638 |
| $C_1 = \{1_2\}$, $C_2 = \{2_1\}$ | −Ve | −Ve | −Ve | −Ve | −Ve | 19.9924 | 3.6388 |
| $C_1 = \{1_1, 1_2\}$, $C_2 = \{2_1\}$ | −Ve | −Ve | −Ve | −Ve | −Ve | −Ve | 21.9416 |

(2) Improving components 1 and 2 in subsystem 1 according to the cold standby method will increase the system availability from 0.7586 to 0.9012; see Table 2. The same increase can be obtained by the factor method (reduction method and increase method).

  (i) *The Reduction Method*. Reducing the failure rates of (1) components 1 and 2 in subsystem 1 by the factor $\rho_c = 0.0460$, (2) component 2 in subsystem 1 and component 1 in subsystem 2 by the factor $\rho_c = 0.1669$, and (3) all components of the system by the factor $\rho_c = 0.3653$; see Table 5.

  (ii) *The Increase Method*. Increasing the repair rates of (1) components 1 and 2 in subsystem 1 by the factor $\sigma_c = 21.7186$, (2) component 2 in subsystem 1 and component 1 in subsystem 2 by the factor $\sigma_c = 5.9901$, and (3) all components of the system by the factor $\sigma_c = 2.7375$, see Table 6.

## 6. Conclusion

In this paper the extension of availability equivalence concept is considered for a repairable discrete multistate parallel-series system with different performance rates. The system availability is defined as the ability of the system to satisfy consumer demand. The quality of the system availability can be improved by using the factor method and the standby method of the system improvements. The UGF technique is used to analyze the availability of both original and improved systems according to different methods. Two types of availability equivalence factors of the system are derived. The results can be used to analyze the equivalence between different designs of improving methods. In future research, we will be concerned with the development of availability equivalence design for repairable multistate system with nonconstant failure rates, repair rates, and multiple failure modes.

## Competing Interests

The authors declare that they have no competing interests.

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