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Research Article

H_∞ Gain-Scheduled Control for LPV Stochastic Systems

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A robust control problem for discrete-time uncertain stochastic systems is discussed via gain-scheduled control scheme subject to H_∞ attenuation performance. Applying Linear Parameter Varying (LPV) modeling approach and stochastic difference equation, the uncertain stochastic systems can be described by combining time-varying weighting function and linear systems with multiplicative noise terms. Due to the consideration of stochastic behavior, the stability in the sense of mean square is applied for the system. Furthermore, two kinds of Lyapunov functions are employed to derive their corresponding sufficient conditions to solve the stabilization problems of this paper. In order to use convex optimization algorithm, the derived conditions are converted into Linear Matrix Inequality (LMI) form. Via solving those conditions, the gain-scheduled controller can be established such that the robust asymptotical stability and H_∞ performance of the disturbed uncertain stochastic system can be achieved in the sense of mean square. Finally, two numerical examples are applied to demonstrate the effectiveness and applicability of the proposed design method.

1. Introduction

In control problems, accurate parameters of dynamic system are always important premised assumption. Unfortunately, the accurate parameters are hardly to be obtained in practical applications due to modeling errors and natural perturbations. For this reason, robust control schemes [1–12] were proposed to guarantee stability of dynamic system with admissible uncertainties. Through [1–3], the uncertainty of system is described by norm bounded time-varying function. On the other hand, based on LPV modeling approach [4–6], the uncertain systems can be interpreted by combining several subsystems and chosen time-varying weighting function. Referring to [4–6, 9], LPV system can be established to completely represent the uncertain systems by using LPV modeling approach. Furthermore, Lyapunov stability theory has been widely applied for stability analysis and synthesis of LPV systems. In the Lyapunov stability theory, the choice of Lyapunov function to present the system energy is an important issue that will influence conservatism of the derived stability criterion. Generally, Parameter Independent Lyapunov Function (PILF) [4, 7] and Parameter Dependent Lyapunov Function (PDLF) [10, 11] are applied to propose their stability criteria for LPV systems. In this paper, both PILF and PDLF are, respectively, applied to derive the corresponding stability criterion for the considered LPV system.

Referring to the literature [13, 14], gain-scheduled design scheme provides powerful tool to deal with stabilization problems of the LPV systems. Moreover, based on the gain-scheduled design scheme, robustness of LPV systems can be increased due to a gain-table designed by numerous operation points. Besides, it is well known that external disturbance often causes poor control performance and unstable source of controlled systems. Therefore, H_∞ gain-scheduled controller design methods have been proposed by [8, 14–16] to constrain the effect of external disturbance on LPV systems. With H_∞ control theory, the performance index can cope with the worst case as the effect of external disturbance.

Practically, stochastic behavior of dynamic systems often appears around operating environment. Due to unmeasurable and unpredictable property, stability of stochastic system is difficult to be analyzed and discussed. Referring to [17, 18], the stochastic behavior is considered as external disturbance or unknown perturbation. On the other hand, stochastic difference equation has been proposed by [19] to formulate the stochastic behavior into multiplicative noise term expressed by multiplication of states and noises. Via the multiplicative noise term, the stochastic behavior of system is more representative and understandable than that described by disturbance or perturbation. Therefore, many efforts [20–27] have been developed to discuss stability analysis and synthesis of stochastic systems. Referring to [26, 27], the uncertainty is described by specific norm bounded time-varying function that limits the description of uncertain stochastic system. In order to extend stability criterion to uncertain stochastic systems, the LPV stochastic system is proposed and considered in this paper.

To the best of our knowledge, there have been less works on discussing robust stabilization problems of the LPV stochastic systems subject to H_∞ performance. The main purpose of this paper is thus to develop the gain-scheduled controller design methods for the LPV stochastic systems. According to the consideration of stochastic behavior, the robust stability criterion proposed in this paper is more general than the one in [4, 8, 14]. And both PILF and PDLF are applied to derive their corresponding sufficient conditions that are converted into the LMI form. Via solving those conditions, the feasible solutions can be obtained to establish the corresponding gain-scheduled controller to guarantee the asymptotical stability and H_∞ performance of the LPV stochastic system in the sense of mean square. For discussing the conservatism of proposed design methods, a numerical example is proposed to find the minimum performance index of the derived conditions. In addition, a ship autopilot servosystem is proposed to show the effectiveness and applicability of the proposed design methods.

The paper is organized as follows. In Section 2, disturbed discrete-time LPV stochastic systems and its stabilization problems are described. The gain-scheduled H_∞ controller design method is proposed in Section 3. And less conservative stability criterion is proposed in Section 4. Finally, two numerical examples are employed to demonstrate effectiveness and application of the proposed design methods in Section 5. Some conclusions are stated in Section 6.

2. Systems Description and Problem Formulation

In this section, the following discrete-time disturbed uncertain stochastic system is proposed:

$$\begin{aligned} x(t+1) &= \mathbf{A}(\alpha(t))x(t) + \mathbf{B}(\alpha(t))u(t) + \mathbf{E}(\alpha(t)) \\ &\quad \cdot w(t) \\ &\quad + \left(\overline{\mathbf{A}}(\alpha(t))x(t) + \overline{\mathbf{B}}(\alpha(t))u(t) + \overline{\mathbf{E}}(\alpha(t))w(t)\right) \\ &\quad \cdot \beta(t), \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control input vector, $w(t) \in \mathbb{R}^p$ is the exogenous disturbance input, and $\beta(t)$ is a discrete type scalar Brownian motion satisfying the independent increment property [19]; that is, $E\{x(t)\beta(t)\} = 0$. And the covariance of $\beta(t)$ can be assumed as $E\{\beta^T(t)\beta(t)\} = \tau^2$, where the τ is intensity level of the motion. $E\{\cdot\}$ denotes the expected value of $\cdot$. $\mathbf{A}(\alpha(t)) \in \mathbb{R}^{n\times n}$, $\mathbf{B}(\alpha(t)) \in \mathbb{R}^{n\times m}$, $\mathbf{E}(\alpha(t)) \in \mathbb{R}^{n\times p}$, $\overline{\mathbf{A}}(\alpha(t)) \in \mathbb{R}^{n\times n}$, $\overline{\mathbf{B}}(\alpha(t)) \in \mathbb{R}^{n\times m}$, and $\overline{\mathbf{E}}(\alpha(t)) \in \mathbb{R}^{n\times p}$ which are matrices depending on time-varying parameters vector $\alpha(t) = [\alpha_1(t)\ \ \alpha_2(t)\ \ \cdots\ \ \alpha_r(t)]$. Referring to [12], the time-varying parameter $\alpha(t)$ can be expressed as a convex combination. Thus, the matrices of system (1) depending on the $\alpha(t)$ can be reconstructed by the following equation:

$$\begin{aligned} &\begin{bmatrix} \mathbf{A}(\alpha(t)) & \mathbf{B}(\alpha(t)) & \mathbf{E}(\alpha(t)) \\ \overline{\mathbf{A}}(\alpha(t)) & \overline{\mathbf{B}}(\alpha(t)) & \overline{\mathbf{E}}(\alpha(t)) \end{bmatrix} \\ &\quad = \sum_{i=1}^{N} \vartheta_i(t) \begin{bmatrix} \mathbf{A}_i & \mathbf{B}_i & \mathbf{E}_i \\ \overline{\mathbf{A}}_i & \overline{\mathbf{B}}_i & \overline{\mathbf{E}}_i \end{bmatrix}, \end{aligned} \tag{2}$$

where $N = 2^r$ and $\vartheta_i(t)$ is measurable at each time instant. Moreover, $\vartheta_i(t)$ satisfies $\sum_{i=1}^{N} \vartheta_i(t) = 1$ and $0 \le \vartheta_i(t) \le 1$. The $\mathbf{A}_i$, $\mathbf{B}_i$, $\mathbf{E}_i$, $\overline{\mathbf{A}}_i$, $\overline{\mathbf{B}}_i$, and $\overline{\mathbf{E}}_i$ are constant matrices with appropriate dimensions. Based on (2), system (1) can be rewritten as follows:

$$\begin{aligned} x(t+1) = \sum_{i=1}^{N} \vartheta_i(t) \Big(&\mathbf{A}_i x(t) + \mathbf{B}_i u(t) + \mathbf{E}_i w(t) \\ &+ \left(\overline{\mathbf{A}}_i x(t) + \overline{\mathbf{B}}_i u(t) + \overline{\mathbf{E}}_i w(t)\right)\beta(t)\Big). \end{aligned} \tag{3}$$

Based on the LPV modeling approach and stochastic difference equation, the LPV stochastic system (3) is structured to substitute the uncertain stochastic system (1) to develop gain-scheduled controller design methods. Moreover, the design methods are proposed to satisfy the H_∞ performance such as

$$E\left\{\sum_{0}^{t_f} x^T(t)\,\mathbf{S}x(t)\right\} < E\left\{\eta^2 \sum_{0}^{t_f} w^T(t)\,w(t)\right\} \tag{4}$$

for $w(t) \neq 0$ and $x(0) = 0$, in which t_f is the terminal time of control, η is a prescribed value which denotes the worst case effect of $w(t)$ on $x(t)$, and $\mathbf{S}$ is a positive definite weighting matrix. Besides, in case such as $w(t) = 0$, the robust stability of (3) is an important issue. The concerned stability of (3) is thus provided as the following definition by the sense of mean square [20, 23].

Definition 1. For LPV stochastic system (3) with zero external disturbance $w(t) = 0$, the solution with admissible robust uncertainties is asymptotically mean square stable if $E\{x(t)\}$ and state correlation matrix $E\{x^T(t)x(t)\}$ are converged to zero as $t \to \infty$.

In next section, both PILF and PDLF are applied to derive their corresponding sufficient condition into LMI problem for applying the convex optimization algorithm [28, 29]. Through solving the condition, the gain-scheduled controller can be established to achieve robust asymptotical stability and H_∞ performance of (3) in the sense of mean square.

3. Stability Criterion for Disturbed LPV Stochastic Systems

In this section, the gain-scheduled control scheme [14] is employed to discuss the stabilization problem of (3). Thus, the following gain-scheduled controller is proposed:

$$u(t) = -\mathbf{F}(\alpha(t))\,x(t) \tag{5a}$$

or

$$u(t) = \sum_{j=1}^{N}\vartheta_j(t)\left(-\mathbf{F}_j x(t)\right). \tag{5b}$$

Substituting (5a)-(5b) into (1), the following closed-loop system can be inferred:

$$\begin{aligned} x(t+1) &= (\mathbf{A}(\alpha(t)) - \mathbf{B}(\alpha(t))\mathbf{F}(\alpha(t)))\,x(t) \\ &\quad + \mathbf{E}(\alpha(t))\,w(t) + \left(\left(\overline{\mathbf{A}}(\alpha(t)) - \overline{\mathbf{B}}(\alpha(t))\mathbf{F}(\alpha(t))\right)\right. \\ &\quad \left.\cdot x(t) + \overline{\mathbf{E}}(\alpha(t))\,w(t)\right)\beta(t) = \mathbf{R}(\alpha(t))\,x(t) \\ &\quad + \mathbf{E}(\alpha(t))\,w(t) + \left(\overline{\mathbf{R}}(\alpha(t))\,x(t) + \overline{\mathbf{E}}(\alpha(t))\,w(t)\right) \\ &\quad \cdot \beta(t) = \sum_{i=1}^{N}\vartheta_i(t)\left(\left(\mathbf{A}_i - \mathbf{B}_i\sum_{j=1}^{N}\vartheta_j(t)\,\mathbf{F}_j\right)x(t)\right. \\ &\quad + \mathbf{E}_i w(t) \\ &\quad \left.+\left(\left(\overline{\mathbf{A}}_i - \overline{\mathbf{B}}_i\sum_{j=1}^{N}\vartheta_j(t)\,\mathbf{F}_j\right)x(t) + \overline{\mathbf{E}}_i w(t)\right)\beta(t)\right) \\ &= \sum_{i=1}^{N}\sum_{j=1}^{N}\vartheta_i(t)\,\vartheta_j(t)\left(\mathbf{R}_{ij}x(t) + \mathbf{E}_i w(t) + \left(\overline{\mathbf{R}}_{ij}x(t)\right.\right. \\ &\quad \left.\left. + \overline{\mathbf{E}}_i w(t)\right)\beta(t)\right), \end{aligned} \tag{6}$$

where $\mathbf{R}(\alpha(t)) = \mathbf{A}(\alpha(t)) - \mathbf{B}(\alpha(t))\mathbf{F}(\alpha(t))$, $\overline{\mathbf{R}}(\alpha(t)) = \overline{\mathbf{A}}(\alpha(t)) - \overline{\mathbf{B}}(\alpha(t))\mathbf{F}(\alpha(t))$, $\mathbf{R}_{ij} = \mathbf{A}_i - \mathbf{B}_i\mathbf{F}_j$, and $\overline{\mathbf{R}}_{ij} = \overline{\mathbf{A}}_i - \overline{\mathbf{B}}_i\mathbf{F}_j$. For closed-loop system (6), the following sufficient condition is derived via the PILF.

Theorem 2. *With given positive scalars τ and η, if there exist gains $\mathbf{F}_j$, positive definite matrices $\mathbf{P}$ and $\mathbf{S}$, and value $\eta > 0$ satisfying the following inequality, the robust asymptotical stability and H_∞ performance of the closed-loop system (6) are achieved in the sense of mean square:*

$$\begin{bmatrix} -\mathbf{Q} & * & * & * & * \\ 0 & -\eta^2\mathbf{I} & * & * & * \\ \mathbf{A}_i\mathbf{Q} - \mathbf{B}_i\mathbf{Y}_j & \mathbf{E}_i & -\mathbf{Q} & * & * \\ \tau\left(\overline{\mathbf{A}}_i\mathbf{Q} - \overline{\mathbf{B}}_i\mathbf{Y}_j\right) & \tau\left(\overline{\mathbf{E}}_i\right) & 0 & -\mathbf{Q} & * \\ \mathbf{Q} & 0 & 0 & 0 & -\mathbf{U} \end{bmatrix} < 0, \quad \text{for } i, j = 1, 2, \ldots, N, \tag{7}$$

where $\mathbf{Q} = \mathbf{P}^{-1}$, $\mathbf{Y}_j = \mathbf{F}_j\mathbf{Q}$, $\mathbf{U} = \mathbf{S}^{-1}$, the $*$ denotes the transposed elements of the symmetric position, and $\mathbf{I}$ denotes identity matrix.

Proof. Choosing a Lyapunov function as $V(x(t)) = x^T(t)\mathbf{P}x(t)$, one can obtain first forward difference of the $V(x(t))$

$$\begin{aligned} \Delta V(x(t)) &= V(x(t+1)) - V(x(t)) = \left(\mathbf{R}(\alpha(t))\,x(t)\right. \\ &\quad + \mathbf{E}(\alpha(t))\,w(t) + \left(\overline{\mathbf{R}}(\alpha(t))\,x(t) + \overline{\mathbf{E}}(\alpha(t))\,w(t)\right) \\ &\quad \left.\cdot \beta(t)\right)^T \mathbf{P}\left(\mathbf{R}(\alpha(t))\,x(t) + \mathbf{E}(\alpha(t))\,w(t)\right. \\ &\quad \left.+\left(\overline{\mathbf{R}}(\alpha(t))\,x(t) + \mathbf{E}(\alpha(t))\,w(t)\right)\beta(t)\right) - x^T(t) \\ &\quad \cdot \mathbf{P}x(t) = \sum_{i=1}^{N}\sum_{j=1}^{N}\vartheta_i(t)\,\vartheta_j(t) \\ &\quad \cdot\left(\left(\mathbf{R}_{ij}x(t) + \mathbf{E}_i w(t) + \left(\overline{\mathbf{R}}_{ij}x(t) + \overline{\mathbf{E}}_i w(t)\right)\beta(t)\right)^T\right. \\ &\quad \left.\cdot \mathbf{P}\left(\mathbf{R}_{ij}x(t) + \mathbf{E}_i w(t) + \left(\overline{\mathbf{R}}_{ij}x(t) + \overline{\mathbf{E}}_i w(t)\right)\beta(t)\right)\right) \\ &\quad - x^T(t)\,\mathbf{P}x(t), \end{aligned} \tag{8}$$

where $\mathbf{R}(\alpha(t))$, $\overline{\mathbf{R}}(\alpha(t))$, $\mathbf{R}_{ij}$, and $\overline{\mathbf{R}}_{ij}$ are defined in (6). Taking expectation of (8), the following equation can be obtained with the independent increment property of Brownian motion; that is, $E\{x(t)\beta(t)\} = 0$ and $E\{\beta(t)\beta(t)\} = \tau^2$. Consider

$$\begin{aligned} E\{\Delta V(x(t))\} &= E\left\{\sum_{i=1}^{N}\sum_{j=1}^{N}\vartheta_i(t)\,\vartheta_j(t)\right. \\ &\quad \cdot\left(x^T(t)\left(\mathbf{R}_{ij}^T\mathbf{P}\mathbf{R}_{ij} + \tau^2\overline{\mathbf{R}}_{ij}^T\mathbf{P}\overline{\mathbf{R}}_{ij} - \mathbf{P}\right)x(t)\right. \\ &\quad + w^T(t)\left(\mathbf{E}_i^T\mathbf{P}\mathbf{R}_{ij} + \tau^2\overline{\mathbf{E}}_i^T\mathbf{P}\overline{\mathbf{R}}_{ij}\right)x(t) \\ &\quad + x^T(t)\left(\mathbf{R}_{ij}^T\mathbf{P}\mathbf{E}_i + \tau^2\overline{\mathbf{R}}_{ij}^T\mathbf{P}\overline{\mathbf{E}}_i\right)w(t) \\ &\quad \left.\left.+ w^T(t)\left(\mathbf{E}_i^T\mathbf{P}\mathbf{E}_i + \tau^2\overline{\mathbf{E}}_i^T\mathbf{P}\overline{\mathbf{E}}_i\right)w(t)\right)\right\} = E\{\Psi\}, \end{aligned} \tag{9}$$

where

$$\begin{aligned} \Psi &= \sum_{i=1}^{N}\sum_{j=1}^{N}\vartheta_i(t)\,\vartheta_j(t)\begin{bmatrix} x(t) \\ w(t) \end{bmatrix}^T \\ &\quad \cdot\begin{bmatrix} \mathbf{R}_{ij}^T\mathbf{P}\mathbf{R}_{ij} + \tau^2\overline{\mathbf{R}}_{ij}^T\mathbf{P}\overline{\mathbf{R}}_{ij} - \mathbf{P} & * \\ \mathbf{E}_i^T\mathbf{P}\mathbf{R}_{ij} + \tau^2\overline{\mathbf{E}}_i^T\mathbf{P}\overline{\mathbf{R}}_{ij} & \mathbf{E}_i^T\mathbf{P}\mathbf{E}_i + \tau^2\overline{\mathbf{E}}_i^T\mathbf{P}\overline{\mathbf{E}}_i \end{bmatrix}\begin{bmatrix} x(t) \\ w(t) \end{bmatrix}. \end{aligned} \tag{10}$$

Let us define the following performance function:

$$J_D = \sum_{0}^{t_f}\left(x^T(t)\,\mathbf{S}x(t) - \eta^2 w^T(t)\,w(t)\right). \tag{11}$$

Then, one has the following relations with zero initial conditions:

$$J_D = E\left\{\sum_{0}^{t_f}\left(x^T(t)\mathbf{S}x(t) - \eta^2 w^T(t)w(t)\right) + \sum_{0}^{t_f}(\Delta V(x(t))) - V(x(t))\right\} \le E\left\{\sum_{0}^{t_f}\left(x^T(t)\mathbf{S}x(t) - \eta^2 w^T(t)w(t) + \Delta V(x(t))\right)\right\} = E\left\{\sum_{0}^{t_f}\left(x^T(t)\mathbf{S}x(t) - \eta^2 w^T(t)w(t) + \Psi\right)\right\} = E\left\{\sum_{0}^{t_f}L(x,w,t)\right\}. \quad (12)$$

According to (9), one has

$$L(x,w,t) = E\left\{\sum_{i=1}^{N}\sum_{j=1}^{N}\vartheta_i(t)\vartheta_j(t)\begin{bmatrix}x(t)\\w(t)\end{bmatrix}^T\Lambda\begin{bmatrix}x(t)\\w(t)\end{bmatrix}\right\}, \quad (13)$$

where

$$\Lambda = \begin{bmatrix}\mathbf{R}_{ij}^T\mathbf{P}\mathbf{R}_{ij} + \tau^2\overline{\mathbf{R}}_{ij}^T\mathbf{P}\overline{\mathbf{R}}_{ij} - \mathbf{P} + \mathbf{S} & * \\ \mathbf{E}_i^T\mathbf{P}\mathbf{R}_{ij} + \tau^2\overline{\mathbf{E}}_i^T\mathbf{P}\overline{\mathbf{R}}_{ij} & \mathbf{E}_i^T\mathbf{P}\mathbf{E}_i + \tau^2\overline{\mathbf{E}}_i^T\mathbf{P}\overline{\mathbf{E}}_i - \eta^2\mathbf{I}\end{bmatrix}. \quad (14)$$

Applying Schur complement [28], the following inequality can be obtained from (7):

$$\left[\begin{array}{c|c}\left(\mathbf{A}_i\mathbf{Q} - \mathbf{B}_i\mathbf{Y}_j\right)^T\mathbf{Q}^{-1}\left(\mathbf{A}_i\mathbf{Q} - \mathbf{B}_i\mathbf{Y}_j\right) + \tau^2\left(\overline{\mathbf{A}}_i\mathbf{Q} - \overline{\mathbf{B}}_i\mathbf{Y}_j\right)^T\mathbf{Q}^{-1}\left(\overline{\mathbf{A}}_i\mathbf{Q} - \overline{\mathbf{B}}_i\mathbf{Y}_j\right) - \mathbf{Q} + \mathbf{Q}\mathbf{U}^{-1}\mathbf{Q} & * \\ \hline \mathbf{E}_i^T\mathbf{Q}^{-1}\left(\mathbf{A}_i\mathbf{Q} - \mathbf{B}_i\mathbf{Y}_j\right) + \tau^2\overline{\mathbf{E}}_i^T\mathbf{Q}^{-1}\left(\overline{\mathbf{A}}_i\mathbf{Q} - \overline{\mathbf{B}}_i\mathbf{Y}_j\right) & \mathbf{E}_i^T\mathbf{Q}^{-1}\mathbf{E}_i + \tau^2\overline{\mathbf{E}}_i^T\mathbf{Q}^{-1}\overline{\mathbf{E}}_i - \eta^2\mathbf{I}\end{array}\right] < 0. \quad (15)$$

Due to definitions as $\mathbf{Q} = \mathbf{P}^{-1}$, $\mathbf{Y}_j = \mathbf{F}_j\mathbf{Q}$, and $\mathbf{U} = \mathbf{S}^{-1}$, inequality (15) can be rewritten as follows:

$$\begin{bmatrix}\mathbf{P}^{-1}\mathbf{R}_{ij}^T\mathbf{P}\mathbf{R}_{ij}\mathbf{P}^{-1} + \tau^2\mathbf{P}^{-1}\overline{\mathbf{R}}_{ij}^T\mathbf{P}\overline{\mathbf{R}}_{ij}\mathbf{P}^{-1} - \mathbf{P}^{-1} + \mathbf{P}^{-1}\mathbf{S}\mathbf{P}^{-1} & * \\ \mathbf{E}_i^T\mathbf{P}\mathbf{R}_{ij}\mathbf{P}^{-1} + \tau^2\overline{\mathbf{E}}_i^T\mathbf{P}\overline{\mathbf{R}}_{ij}\mathbf{P}^{-1} & \mathbf{E}_i^T\mathbf{P}\mathbf{E}_i + \tau^2\overline{\mathbf{E}}_i^T\mathbf{P}\overline{\mathbf{E}}_i - \eta^2\mathbf{I}\end{bmatrix} < 0. \quad (16)$$

Multiplying both sides of (16) with diag$\{\mathbf{P},\mathbf{I}\}$, where the diag$\{\cdot,\cdot\}$ denotes a block-diagonal matrix with element $\cdot$, one can obtain the following inequality:

$$\begin{bmatrix}\mathbf{R}_{ij}^T\mathbf{P}\mathbf{R}_{ij} + \tau^2\overline{\mathbf{R}}_{ij}^T\mathbf{P}\overline{\mathbf{R}}_{ij} - \mathbf{P} + \mathbf{S} & * \\ \mathbf{E}_i^T\mathbf{P}\mathbf{R}_{ij} + \tau^2\overline{\mathbf{E}}_i^T\mathbf{P}\overline{\mathbf{R}}_{ij} & \mathbf{E}_i^T\mathbf{P}\mathbf{E}_i + \tau^2\overline{\mathbf{E}}_i^T\mathbf{P}\overline{\mathbf{E}}_i - \eta^2\mathbf{I}\end{bmatrix} < 0. \quad (17)$$

Obviously, the left-hand side of inequality (17) is equal to Λ in (13). Thus, $\Lambda < 0$ is found if condition (7) holds. And $L(x,w,t) < 0$ can be obtained from (13) with $\Lambda < 0$. According to $L(x,w,t) < 0$, the following inequalities can be inferred from (12) as follows:

$$J_D < 0 \quad (18)$$

or

$$E\left\{\sum_{0}^{t_f}x^T(t)\mathbf{S}x(t)\right\} < E\left\{\eta^2\sum_{0}^{t_f}w^T(t)w(t)\right\}. \quad (19)$$

Because (19) is equivalent to (4), it is easy to show that the closed-loop system (6) with controller (5a)-(5b) satisfies H_∞ performance when the condition (7) holds. Next, the asymptotical stability is necessary to be proven. By assuming $w(t) = 0$, the following inequality can be found from $L(x,w,t) < 0$ if the condition in Theorem 2 holds:

$$E\left\{\Psi + x^T(t)\mathbf{S}x(t)\right\} < 0 \quad (20a)$$

or

$$E\{\Psi\} < E\left\{-x^T(t)\mathbf{S}x(t)\right\}. \quad (20b)$$

According to $\mathbf{S} > 0$, one can find $E\{\Psi\} < 0$. From (9), $E\{\Psi\} < 0$ implies $E\{\Delta V(x(t))\} < 0$. According to Definition 1, the asymptotical stability of the closed-loop system (6) can be achieved via controller (5a)-(5b) in the sense of mean square due to $E\{\Delta V(x(t))\} < 0$. Thus, the proof of this theorem is complete. □

Based on the PILF, the sufficient conditions are derived in Theorem 2. Via finding the feasible solutions, the controller (5a)-(5b) is designed to guarantee the asymptotical stability and H_∞ performance of the closed-loop system (6) in the sense of mean square. However, Theorem 2 processes conservatism in finding a common matrix $\mathbf{P}$ to satisfy sufficient condition (7) for $i, j = 1, 2, \ldots, N$. For this reason, the less conservative sufficient conditions than the ones in Theorem 2 are proposed in the next section.

4. Relaxed Stability Criterion for Disturbed LPV Stochastic Systems

Referring to [10, 11], the PDLF is proposed to derive relaxed stability criterion for LPV systems. The reason for reducing conservatism in solving stabilization problem of the system (1) is that the PDLF consists of state and multiple positive definite matrices. Based on the PDLF, a relaxed stability criterion for system (1) is proposed in this section. Besides, arbitrary matrices $\mathbf{G}_i$ are introduced to reduce conservatism of the proposed stability criterion in this section. Thus, the following gain-scheduled controller is proposed:

$$u(t) = -\mathbf{F}(\alpha(t))\mathbf{G}^{-1}(\alpha(t))x(t), \tag{21a}$$

or

$$u(t) = -\left(\sum_{j=1}^{N}\vartheta_j(t)\mathbf{F}_j\right)\left(\sum_{j=1}^{N}\vartheta_j(t)\mathbf{G}_j\right)^{-1}x(t). \tag{21b}$$

Remark 3. According to the arbitrary matrices $\mathbf{G}_i$, the freedom of searching feasible solutions of Theorem 4 is increased. Moreover, the sufficient conditions of Theorem 4 can be converted into extended LMI form by using the arbitrary matrices $\mathbf{G}_i$. Referring to [16], the extended form possesses less conservatism than standard LMI form as in (7). Thus, the structure of (21a)-(21b) is applied to the proposed relaxed gain-scheduled controller design method for disturbed uncertain stochastic systems (1).

Substituting (21a)-(21b) into system (1), the corresponding closed-loop system can be represented as follows:

$$\begin{aligned}
x(t+1) &= \mathbf{X}(\alpha(t))x(t) + \mathbf{E}(\alpha(t))w(t) \\
&\quad + \left(\overline{\mathbf{X}}(\alpha(t))x(t) + \overline{\mathbf{E}}(\alpha(t))w(t)\right)\beta(t) \\
&= \sum_{i=1}^{N}\sum_{j=1}^{N}\vartheta_i(t)\vartheta_j(t) \\
&\quad \cdot\left(\mathbf{X}_{ij}x(t) + \mathbf{E}_iw(t) + \left(\overline{\mathbf{X}}_{ij}x(t) + \overline{\mathbf{E}}_iw(t)\right)\beta(t)\right),
\end{aligned} \tag{22}$$

where

$$\begin{aligned}
\mathbf{X}(\alpha(t)) &= \mathbf{A}(\alpha(t))x(t) \\
&\quad - \mathbf{B}(\alpha(t))\mathbf{F}(\alpha(t))\mathbf{G}^{-1}(\alpha(t)), \\
\overline{\mathbf{X}}(\alpha(t)) &= \overline{\mathbf{A}}(\alpha(t))x(t) \\
&\quad - \overline{\mathbf{B}}(\alpha(t))\mathbf{F}(\alpha(t))\mathbf{G}^{-1}(\alpha(t)), \\
\mathbf{X}_{ij} &= \mathbf{A}_i - \mathbf{B}_i\mathbf{F}_j\left(\sum_{j=1}^{N}\vartheta_j(t)\mathbf{G}_j\right)^{-1}, \\
\overline{\mathbf{X}}_{ij} &= \overline{\mathbf{A}}_i - \overline{\mathbf{B}}_i\mathbf{F}_j\left(\sum_{j=1}^{N}\vartheta_j(t)\mathbf{G}_j\right)^{-1}.
\end{aligned} \tag{23}$$

For stability problem of closed-loop system (22), the sufficient conditions are derived by H_∞ performance definition and PDLF.

Theorem 4. *With given positive scalars τ and η, if there exist feedback gains $\mathbf{F}_i$, positive definite matrices $\mathbf{P}_i$ and $\mathbf{S}$, and arbitrary matrices $\mathbf{G}_i$ to satisfy the following conditions, then the asymptotical stability and H_∞ performance of the closed-loop system (22) are guaranteed in the sense of mean square. Consider*

$$\begin{bmatrix}
\mathbf{Q}_i - \mathbf{G}_i^T - \mathbf{G}_i & * & * & * & * \\
0 & -\eta^2\mathbf{I} & * & * & * \\
\mathbf{A}_i\mathbf{G}_j - \mathbf{B}_i\mathbf{F}_j & \mathbf{E}_i & -\mathbf{Q}_k & * & * \\
\tau\left(\overline{\mathbf{A}}_i\mathbf{G}_j - \overline{\mathbf{B}}_i\mathbf{F}_j\right) & \tau\overline{\mathbf{E}}_i & 0 & -\mathbf{Q}_k & * \\
\mathbf{G}_i & 0 & 0 & 0 & -\mathbf{U}
\end{bmatrix} < 0, \quad \text{for } i, j, k = 1, 2, \ldots, N, \tag{24}$$

where $\mathbf{Q}_k = \mathbf{P}_k^{-1}$ and $\mathbf{U} = \mathbf{S}^{-1}$.

Proof. Choosing a Lyapunov function as $V(x(t)) = x^T(t)\mathbf{P}(\alpha(t))x(t)$, the first forward difference of the $V(x(t))$ can be obtained, such as

$$\begin{aligned}
\Delta V(x(t)) &= V(x(t+1)) - V(x(t)) \\
&= \left(\mathbf{X}(\alpha(t))x(t) + \mathbf{E}(\alpha(t))w(t)\right. \\
&\quad \left. + \left(\overline{\mathbf{X}}(\alpha(t))x(t) + \overline{\mathbf{E}}(\alpha(t))w(t)\right)\beta(t)\right)^T \\
&\quad \cdot\mathbf{P}(\alpha(t+1))\left(\mathbf{X}(\alpha(t))x(t) + \mathbf{E}(\alpha(t))w(t)\right. \\
&\quad \left. + \left(\overline{\mathbf{X}}(\alpha(t))x(t) + \overline{\mathbf{E}}(\alpha(t))w(t)\right)\beta(t)\right) - x^T(t) \\
&\quad \cdot\mathbf{P}(\alpha(t))x(t).
\end{aligned} \tag{25}$$

In this paper, $\mathbf{P}(\alpha(t+1))$ is defined by the following equation:

$$\mathbf{P}(\alpha(t+1)) = \sum_{j=1}^{N}\vartheta_j(t+1)\mathbf{P}_j = \left(\sum_{k=1}^{N}\varepsilon_k(t)\mathbf{P}_k\right), \tag{26}$$

where $\varepsilon(t)$ is the time-varying parameter satisfying $\sum_{k=1}^{N}\varepsilon_k(t) = 1$ and $0 \le \varepsilon_k(t) \le 1$. Due to (26), (25) can be rewritten as in the following equation:

$$\begin{aligned}\Delta V(x(t)) &= \Big(\mathbf{X}(\alpha(t))x(t) + \mathbf{E}(\alpha(t))w(t) \\ &\quad + \big(\overline{\mathbf{X}}(\alpha(t))x(t) + \overline{\mathbf{E}}(\alpha(t))w(t)\big)\beta(t)\Big)^T \mathbf{P}(\varepsilon(t)) \\ &\quad \cdot \Big(\mathbf{X}(\alpha(t))x(t) \\ &\quad + \mathbf{E}(\alpha(t))w(t) \\ &\quad + \big(\overline{\mathbf{X}}(\alpha(t))x(t) + \overline{\mathbf{E}}(\alpha(t))w(t)\big)\beta(t)\Big) - x^T(t) \\ &\quad \cdot \mathbf{P}(\alpha(t))x(t).\end{aligned} \tag{27}$$

Taking expectation of (27), the following equation can be found with the independent increment property of Brownian motion:

$$\begin{aligned}E\{\Delta V(x(t))\} &= E\Big\{x^T(t) \\ &\quad \cdot \Big(\mathbf{X}^T(\alpha(t))\mathbf{P}(\varepsilon(t))\mathbf{X}(\alpha(t)) \\ &\quad + \tau^2\overline{\mathbf{X}}^T(\alpha(t))\mathbf{P}(\varepsilon(t))\overline{\mathbf{X}}(\alpha(t))\Big)x(t) + w^T(t) \\ &\quad \cdot \Big(\mathbf{E}^T(\alpha(t))\mathbf{P}(\varepsilon(t))\mathbf{X}(\alpha(t)) \\ &\quad + \tau^2\overline{\mathbf{E}}^T(\alpha(t))\mathbf{P}(\varepsilon(t))\overline{\mathbf{X}}(\alpha(t))\Big)x(t) + x^T(t) \\ &\quad \cdot \Big(\mathbf{X}^T(\alpha(t))\mathbf{P}(\varepsilon(t))\mathbf{E}(\alpha(t)) \\ &\quad + \tau^2\overline{\mathbf{X}}^T(\alpha(t))\mathbf{P}(\varepsilon(t))\overline{\mathbf{E}}(\alpha(t))\Big)w(t) + w^T(t) \\ &\quad \cdot \Big(\mathbf{E}^T(\alpha(t))\mathbf{P}(\varepsilon(t))\mathbf{E}(\alpha(t)) \\ &\quad + \tau^2\overline{\mathbf{E}}^T(\alpha(t))\mathbf{P}(\varepsilon(t))\overline{\mathbf{E}}(\alpha(t))\Big)w(t) - x^T(t) \\ &\quad \cdot \mathbf{P}(\alpha(t))x(t)\Big\}.\end{aligned} \tag{28}$$

Applying the cost function (11), one can find the following relations:

$$\begin{aligned}J_D &= E\Big\{\sum_0^{t_f}\big(x^T(t)\mathbf{S}x(t) - \eta^2 w^T(t)w(t)\big) \\ &\quad + \sum_0^{t_f}\Delta V(x(t)) - V(x(t_f))\Big\} \\ &\le E\Big\{\sum_0^{t_f}\big(x^T(t)\mathbf{S}x(t) - \eta^2 w^T(t)w(t) \\ &\quad + \Delta V(x(t))\big)\Big\} = E\Big\{\sum_0^{t_f}\Phi(x,w,t)\Big\}.\end{aligned} \tag{29}$$

According to (28), one has

$$\Phi(x,w,t) = \begin{bmatrix} x(t) \\ w(t)\end{bmatrix}^T \Xi \begin{bmatrix} x(t) \\ w(t)\end{bmatrix}, \tag{30}$$

where

$$\Xi = \left[\begin{array}{c|c} \mathbf{X}^T(\alpha(t))\mathbf{P}(\varepsilon(t))\mathbf{X}(\alpha(t)) + \tau^2\overline{\mathbf{X}}^T(\alpha(t))\mathbf{P}(\varepsilon(t))\overline{\mathbf{X}}(\alpha(t)) - \mathbf{P}(\alpha(t)) + \mathbf{S} & * \\ \hline \mathbf{E}^T(\alpha(t))\mathbf{P}(\varepsilon(t))\mathbf{X}(\alpha(t)) + \tau^2\overline{\mathbf{E}}^T(\alpha(t))\mathbf{P}(\varepsilon(t))\overline{\mathbf{X}}(\alpha(t)) & \mathbf{E}^T(\alpha(t))\mathbf{P}(\varepsilon(t))\mathbf{E}(\alpha(t)) + \tau^2\overline{\mathbf{E}}^T(\alpha(t))\mathbf{P}(\varepsilon(t))\overline{\mathbf{E}}(\alpha(t)) - \eta^2\mathbf{I}\end{array}\right]. \tag{31}$$

Applying the Schur complement, one has the following inequality from (24):

$$\left[\begin{array}{c|c} \mathbf{Q}_i - \mathbf{G}_i^T - \mathbf{G}_i + \mathbf{G}_i^T\mathbf{U}^{-1}\mathbf{G}_i + (\mathbf{A}_i\mathbf{G}_j - \mathbf{B}_i\mathbf{F}_j)^T\mathbf{Q}_k^{-1}(\mathbf{A}_i\mathbf{G}_j - \mathbf{B}_i\mathbf{F}_j) + \tau^2(\overline{\mathbf{A}}_i\mathbf{G}_j - \overline{\mathbf{B}}_i\mathbf{F}_j)^T\mathbf{Q}_k^{-1}(\overline{\mathbf{A}}_i\mathbf{G}_j - \overline{\mathbf{B}}_i\mathbf{F}_j) & * \\ \hline \mathbf{E}_i^T\mathbf{Q}_k^{-1}(\mathbf{A}_i\mathbf{G}_j - \mathbf{B}_i\mathbf{F}_j) + \tau^2\overline{\mathbf{E}}_i^T\mathbf{Q}_k^{-1}(\overline{\mathbf{A}}_i\mathbf{G}_j - \overline{\mathbf{B}}_i\mathbf{F}_j) & \mathbf{E}_i^T\mathbf{Q}_k^{-1}\mathbf{E}_i + \tau^2\overline{\mathbf{E}}_i^T\mathbf{Q}_k^{-1}\overline{\mathbf{E}}_i - \eta^2\mathbf{I}\end{array}\right] < 0. \tag{32}$$

According to the fact that $\mathbf{P}_i^{-1} - \mathbf{G}_i^T - \mathbf{G}_i \ge -\mathbf{G}_i^T\mathbf{P}_i\mathbf{G}_i$, the following inequality holds from (32) with definition $\mathbf{Q}_k = \mathbf{P}_k^{-1}$ and $\mathbf{U} = \mathbf{S}^{-1}$

$$\left[\begin{array}{c|c} -\mathbf{G}_i^T\mathbf{P}_i\mathbf{G}_i + \mathbf{G}_i^T\mathbf{S}\mathbf{G}_i + (\mathbf{A}_i\mathbf{G}_j - \mathbf{B}_i\mathbf{F}_j)^T\mathbf{P}_k(\mathbf{A}_i\mathbf{G}_j - \mathbf{B}_i\mathbf{F}_j) + \tau^2(\overline{\mathbf{A}}_i\mathbf{G}_j - \overline{\mathbf{B}}_i\mathbf{F}_j)^T\mathbf{P}_k(\overline{\mathbf{A}}_i\mathbf{G}_j - \overline{\mathbf{B}}_i\mathbf{F}_j) & * \\ \hline \mathbf{E}_i^T\mathbf{P}_k(\mathbf{A}_i\mathbf{G}_j - \mathbf{B}_i\mathbf{F}_j) + \tau^2\overline{\mathbf{E}}_i^T\mathbf{P}_k(\overline{\mathbf{A}}_i\mathbf{G}_j - \overline{\mathbf{B}}_i\mathbf{F}_j) & \mathbf{E}_i^T\mathbf{P}_k\mathbf{E}_i + \tau^2\overline{\mathbf{E}}_i^T\mathbf{P}_k\overline{\mathbf{E}}_i - \eta^2\mathbf{I}\end{array}\right] < 0. \tag{33}$$

Since $\vartheta_i \geq 0$ and $\sum_{i=1}^{N} \vartheta_i = 1$, the following inequality can be inferred from (33):

$$\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{N}\vartheta_i(t)\vartheta_j(t)\varepsilon_k(t) \cdot \left[\begin{array}{c|c} -\mathbf{G}_i^T\mathbf{P}_i\mathbf{G}_i + \mathbf{G}_i^T\mathbf{S}\mathbf{G}_i + \left(\mathbf{A}_i\mathbf{G}_j - \mathbf{B}_i\mathbf{F}_j\right)^T\mathbf{P}_k\left(\mathbf{A}_i\mathbf{G}_j - \mathbf{B}_i\mathbf{F}_j\right) + \tau^2\left(\overline{\mathbf{A}}_i\mathbf{G}_j - \overline{\mathbf{B}}_i\mathbf{F}_j\right)^T\mathbf{P}_k\left(\overline{\mathbf{A}}_i\mathbf{G}_j - \overline{\mathbf{B}}_i\mathbf{F}_j\right) & * \\ \hline \mathbf{E}_i^T\mathbf{P}_k\left(\mathbf{A}_i\mathbf{G}_j - \mathbf{B}_i\mathbf{F}_j\right) + \tau^2\overline{\mathbf{E}}_i^T\mathbf{P}_k\left(\overline{\mathbf{A}}_i\mathbf{G}_j - \overline{\mathbf{B}}_i\mathbf{F}_j\right) & \mathbf{E}_i^T\mathbf{P}_k\mathbf{E}_i + \tau^2\overline{\mathbf{E}}_i^T\mathbf{P}_k\overline{\mathbf{E}}_i - \eta^2\mathbf{I} \end{array}\right] < 0. \quad (34)$$

And inequality (34) can be rewritten as follows:

$$\left[\begin{array}{c|c} \begin{array}{c} -\mathbf{G}^T(\alpha(t))\mathbf{P}(\alpha(t))\mathbf{G}(\alpha(t)) + \mathbf{G}^T(\alpha(t))\mathbf{S}\mathbf{G}(\alpha(t)) \\ + (\mathbf{A}(\alpha(t))\mathbf{G}(\alpha(t)) - \mathbf{B}(\alpha(t))\mathbf{F}(\alpha(t)))^T\mathbf{P}(\varepsilon(t)) \\ \cdot(\mathbf{A}(\alpha(t))\mathbf{G}(\alpha(t)) - \mathbf{B}(\alpha(t))\mathbf{F}(\alpha(t))) \\ +\tau^2\left(\overline{\mathbf{A}}(\alpha(t))\mathbf{G}(\alpha(t)) - \overline{\mathbf{B}}(\alpha(t))\mathbf{F}(\alpha(t))\right)^T\mathbf{P}(\varepsilon(t)) \\ \cdot\left(\overline{\mathbf{A}}(\alpha(t))\mathbf{G}(\alpha(t)) - \overline{\mathbf{B}}(\alpha(t))\mathbf{F}(\alpha(t))\right) \end{array} & * \\ \hline \begin{array}{c} \mathbf{E}^T(\alpha(t))\mathbf{P}(\varepsilon(t))(\mathbf{A}(\alpha(t))\mathbf{G}(\alpha(t)) - \mathbf{B}(\alpha(t))\mathbf{F}(\alpha(t))) \\ +\tau^2\overline{\mathbf{E}}^T(\alpha(t))\mathbf{P}(\varepsilon(t))\left(\overline{\mathbf{A}}(\alpha(t))\mathbf{G}(\alpha(t)) - \overline{\mathbf{B}}(\alpha(t))\mathbf{F}(\alpha(t))\right) \end{array} & \begin{array}{c} \mathbf{E}^T(\alpha(t))\mathbf{P}(\varepsilon(t))\mathbf{E}(\alpha(t)) \\ +\tau^2\overline{\mathbf{E}}^T(\alpha(t))\mathbf{P}(\varepsilon(t))\overline{\mathbf{E}}(\alpha(t)) - \eta^2\mathbf{I} \end{array} \end{array}\right] < 0. \quad (35)$$

Before and after multiplying (35) by $\text{diag}\{\mathbf{G}^{-T}(\alpha(t)), \mathbf{I}\}$ and $\text{diag}\{\mathbf{G}^{-1}(\alpha(t)), \mathbf{I}\}$, one has

$$\left[\begin{array}{c|c} \mathbf{X}^T(\alpha(t))\mathbf{P}(\varepsilon(t))\mathbf{X}(\alpha(t)) + \tau^2\overline{\mathbf{X}}^T(\alpha(t))\mathbf{P}(\varepsilon(t))\overline{\mathbf{X}}(\alpha(t)) - \mathbf{P}(\alpha(t)) + \mathbf{S} & * \\ \hline \mathbf{E}^T(\alpha(t))\mathbf{P}(\varepsilon(t))\mathbf{X}(\alpha(t)) + \tau^2\overline{\mathbf{E}}^T(\alpha(t))\mathbf{P}(\varepsilon(t))\overline{\mathbf{X}}(\alpha(t)) & \mathbf{E}^T(\alpha(t))\mathbf{P}(\varepsilon(t))\mathbf{E}(\alpha(t)) + \tau^2\overline{\mathbf{E}}^T(\alpha(t))\mathbf{P}(\varepsilon(t))\overline{\mathbf{E}}(\alpha(t)) - \eta^2\mathbf{I} \end{array}\right] < 0. \quad (36)$$

Obviously, if condition (24) holds, then (36) can be obtained. And $\Xi < 0$ can be also found from (31) due to (36). According to $\Xi < 0$, $\Phi(x, w, t) < 0$ can be inferred from (30). Due to $\Phi(x, w, t) < 0$ and (29), the following inequalities can be obtained:

$$J_D < 0 \quad (37)$$

or

$$E\left\{\sum_{0}^{t_f} x^T(t)\mathbf{S}x(t)\right\} < E\left\{\eta^2\sum_{0}^{t_f} w^T(t)w(t)\right\}. \quad (38)$$

Because (38) is equivalent to (4), it is easy to show that the closed-loop system (22) driven by (21a)-(21b) satisfies H_∞ performance for all nonzero external disturbances. Next, the asymptotical stability of the closed-loop system (22) is proven. If the condition of this theorem is satisfied, then $\Phi(x, w, t) < 0$ is held. By assuming $w(t) = 0$, the following inequality can be found from (29):

$$E\left\{\Delta V(x(t)) + x^T(t)\mathbf{S}x(t)\right\} < 0 \quad (39)$$

or

$$E\{\Delta V(x(t))\} < E\left\{-x^T(t)\mathbf{S}x(t)\right\}. \quad (40)$$

According to $\mathbf{S} > 0$, one can deduce that $E\{\Delta V(x(t))\} < 0$. And then the closed-loop system (22) is asymptotically stable in the sense of mean square according to $E\{\Delta V(x(t))\} < 0$ and Definition 1. The proof of this theorem is complete. □

In this section, the sufficient conditions are derived by PDLF for discussing the stabilization problems of the closed-loop system (22). Through the several positive definite matrices and arbitrary matrices $\mathbf{G}_i$, the conservatism of Theorem 4 can be reduced in finding the feasible solutions of conditions (24). In the following section, two numerical examples are proposed to demonstrate the effectiveness and application of the proposed design method.

5. Simulation Results

In this section, two numerical examples are proposed. The first example is employed to discuss the conservatism of the proposed design methods. Another example is to discuss the stabilization problem of disturbed ship autopilot servosystem with multiplicative noise to show the application of the proposed design methods. Moreover, the design method of [14] is employed to compare with the proposed design methods of this paper.

Example 5. Consider the following disturbed stochastic LPV system:

$$x(t+1) = \mathbf{A}(\alpha(t))\,x(t) + \mathbf{B}(\alpha(t))\,u(t) + \mathbf{E}(\alpha(t)) \cdot w(t) + \left(\overline{\mathbf{A}}(\alpha(t))\,x(t) + \overline{\mathbf{B}}(\alpha(t))\,u(t) + \overline{\mathbf{E}}(\alpha(t))\,w(t)\right)\beta(t) = \sum_{i=1}^{2}\vartheta_i(t)\left(\mathbf{A}_i x(t) + \mathbf{B}_i u(t) + \mathbf{E}_i w(t) + \left(\overline{\mathbf{A}}_i x(t) + \overline{\mathbf{B}}_i u(t) + \overline{\mathbf{E}}_i w(t)\right)\beta(t)\right), \quad (41)$$

where

$$\mathbf{A}_1 = \begin{bmatrix} 2 & -0.1 \\ 0.5 & 1.65 \end{bmatrix}, \quad \mathbf{A}_2 = \begin{bmatrix} 2 & -0.1 \\ 0.5 & 0.35 \end{bmatrix}, \quad \mathbf{B}_1 = \begin{bmatrix} 1 \\ -0.95 \end{bmatrix}, \quad \mathbf{B}_2 = \begin{bmatrix} 1 \\ 0.35 \end{bmatrix},$$

$$\mathbf{E}_1 = \begin{bmatrix} 0.1 \\ 0 \end{bmatrix}, \quad \mathbf{E}_2 = \begin{bmatrix} 0.2 \\ 0 \end{bmatrix}, \quad \overline{\mathbf{A}}_1 = \begin{bmatrix} 0.03 & 0 \\ 0.004 & 0.0165 \end{bmatrix}, \quad \overline{\mathbf{A}}_2 = \begin{bmatrix} 0.03 & 0 \\ 0.004 & 0.0035 \end{bmatrix}, \quad \overline{\mathbf{B}}_1 = \begin{bmatrix} 0.01 \\ -0.0075 \end{bmatrix},$$

$$\overline{\mathbf{B}}_2 = \begin{bmatrix} 0.01 \\ 0.0055 \end{bmatrix}, \quad \overline{\mathbf{E}}_1 = \begin{bmatrix} 0.001 \\ 0 \end{bmatrix}, \quad \overline{\mathbf{E}}_2 = \begin{bmatrix} 0.002 \\ 0 \end{bmatrix}, \quad \vartheta_1(t) = |\sin(t)|, \quad \vartheta_2(t) = 1 - |\sin(t)|. \quad (42)$$

In this numerical example, the intensity level is given as $\tau = 1$. For discussing the conservatism of Theorems 2 and 4, the positive definite matrix $\mathbf{S}$ is determined as identity matrix to find the minimum available value of η. Applying the convex optimization algorithm [29], the minimum available value of η for the sufficient condition of the theorems is shown in Table 1. From Table 1, the minimum available value of η to satisfy Theorem 2 is 1.5166. In case such as $\eta = 1.5166$, the following feasible solutions of condition (7) can be obtained:

$$\mathbf{P} = \begin{bmatrix} 57.1827 & 34.8967 \\ 34.8967 & 22.9765 \end{bmatrix}, \quad \mathbf{F}_1 = \begin{bmatrix} 4.2194 & 1.4732 \end{bmatrix}, \quad \mathbf{F}_2 = \begin{bmatrix} 1.8602 & 0.0691 \end{bmatrix}. \quad (43)$$

Based on (43), the gain-scheduled controller can be designed such as

$$u(t) = -\sum_{j=1}^{2}\vartheta_j(t)\,\mathbf{F}_j x(t). \quad (44)$$

Applying (44), the responses of (41) are stated in Figure 1 with initial condition $x(t) = \begin{bmatrix} 5 & -3 \end{bmatrix}^T$. And the external disturbance $w(t)$ is chosen as zero-mean white noise with unit variance. For checking satisfaction of (4), the following ratio is obtained via using the simulation results:

$$\frac{E\left\{\sum_0^{t_f=5} x^T(t)\,\mathbf{S}x(t)\right\}}{E\left\{\sum_0^{t_f=5} w^T(t)\,w(t)\right\}} = 1.696. \quad (45)$$

Obviously, the ratio in (45) is smaller than the obtained value $\eta^2 = 2.3$ with $\eta = 1.5166$. From Figure 1 and (45), system (41) driven by (44) is robust asymptotically stable with attenuation η in the sense of mean square.

Besides, from Table 1, the minimum available value of η for satisfying Theorem 4 is 1.4. In the case such as $\eta = 1.4$,

Table 1: Comparing results for Theorems 2 and 4.

η	$\cdots$	1.5166	1.5133	1.4	$\cdots$
Theorem 2	Feasible	Feasible	Infeasible	Infeasible	Infeasible
Theorem 4	Feasible	Feasible	Feasible	Feasible	Infeasible

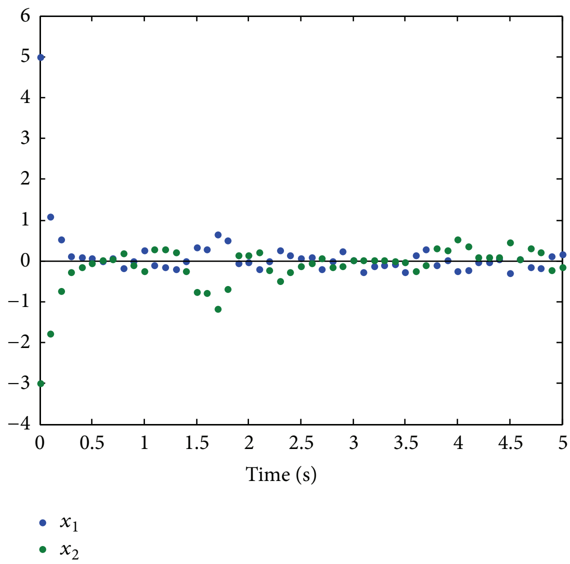

Figure 1: Responses of Example 5 with controller (44).

the feasible solutions of conditions (24) can be obtained such as

$$\mathbf{P}_1 = \begin{bmatrix} 48.989 & 30.9426 \\ 30.9426 & 21.0296 \end{bmatrix},$$
$$\mathbf{P}_2 = \begin{bmatrix} 12.7423 & 7.8087 \\ 7.8087 & 6.5371 \end{bmatrix},$$
$$\mathbf{G}_1 = \begin{bmatrix} 0.2637 & -0.3887 \\ -0.4104 & 0.6518 \end{bmatrix}, \tag{46}$$
$$\mathbf{G}_2 = \begin{bmatrix} 0.2650 & -0.3890 \\ -0.3259 & 0.5924 \end{bmatrix},$$
$$\mathbf{F}_1 = \begin{bmatrix} 0.5621 & -0.7445 \end{bmatrix},$$
$$\mathbf{F}_2 = \begin{bmatrix} 0.4703 & -0.6790 \end{bmatrix}.$$

With the above feasible solutions, gain-scheduled controller (21a)-(21b) is established as follows:

$$u(t) = -\left(\sum_{j=1}^{2} \vartheta_j(t)\,\mathbf{F}_j\right)\left(\sum_{j=1}^{2} \vartheta_j(t)\,\mathbf{G}_j\right)^{-1} x(t). \tag{47}$$

Based on controller (47), the responses of (41) are stated in Figure 2 with the same initial condition and $w(t)$ of the above case. Based on the simulation results, the following ratio value can be obtained:

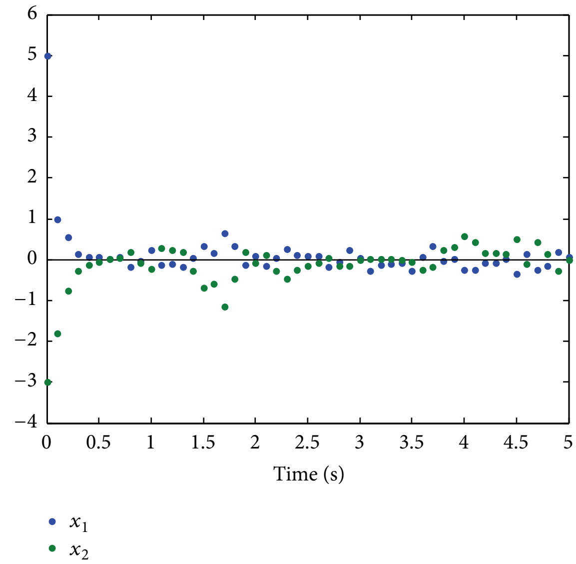

Figure 2: Responses of Example 5 with controller (47).

$$\frac{E\left\{\sum_0^{t_f=5} x^T(t)\,\mathbf{S}x(t)\right\}}{E\left\{\sum_0^{t_f=5} w^T(t)\,w(t)\right\}} = 1.675. \tag{48}$$

Obviously, the value of (48) is smaller than $\eta^2 = 1.96$ with $\eta = 1.4$. From Figure 2 and (48), the asymptotical stability and H_∞ performance of system (41) can be achieved via the controller (46).

From the simulation results of this example, the proposed design methods are useful tools to design gain-scheduled controller for stabilizing the LPV stochastic system (41). Besides, from Table 1, it is obvious to show that the minimum available value of Theorem 2 is bigger than the one of Theorem 4. Thus, the sufficient conditions of Theorem 4 are less conservative than the one in Theorem 2 for discussing stability issue of LPV systems.

Example 6. In this example, the ship autopilot servosystem is applied to show applicability of the proposed controller design methods. Referring to [30], the discretization differential equation of ship motion is proposed. Considering the practical operations, the parameter T_1 in the system is assumed as time-varying parameter $T_1(t)$ in this section. According to $T_1(t)$, the ship autopilot system belongs to uncertain system. Moreover, a multiplicative noise term is added to describe the stochastic behavior of the system. And an external disturbance is added to simulate random force from outside. Thus, the disturbed ship autopilot servosystem with multiplicative noise is considered as follows:

$$x_1(t+1) = x_1(t) + x_2(t) \times \Delta t, \tag{49a}$$

$$x_2(t+1) = x_2(t) + x_3(t) \times \Delta t + 0.0002\left(1 + 0.1\beta(t)\right) w(t), \tag{49b}$$

$$x_3(t+1) = \frac{-K\times\Delta t}{T_1(t)T_2}x_2(t) + \left(\frac{-(T_1(t)+T_2)\times\Delta t}{T_1(t)T_2}+1\right)x_3(t) + \frac{K(T_E-T_3)\times\Delta t}{T_1(t)T_2T_E}x_4(t) + \frac{KT_3\times\Delta t}{T_1(t)T_2T_E}u(t), \tag{49c}$$

$$x_4(t+1) = 0.2\beta(t)x_2(t) + 0.1\beta(t)x_3(t) + \left(\frac{-1\times\Delta t}{T_E}+1\right)x_4(t) + \frac{1\times\Delta t}{T_E}(1+0.6\beta(t))u(t), \tag{49d}$$

where $x_1(t)$ represents the difference of the heading angle and desires heading angle of ship; $x_2(t)$ represents the navigational angle velocity; $x_3(t)$ represents the navigational angle acceleration; $x_4(t)$ represents the actual rudder angle of ship; $u(t)$ represents the steering angle; and $w(t)$ is chosen as zero-mean white noise with unit variance. In order to achieve all possible values of variation of the parameter $T_1(t)$, the time-varying range of $T_1(t)$ is determined as follows:

$$T_1(t) \in [36.25 \quad 108.75]. \tag{50}$$

Besides, the constant parameters $T_2 = 8.54$, $T_3 = 17.61$, and $T_E = 2.5$, rudder gain $K = 0.1141$, and sampling time $\Delta t = 0.4$ are given in this section. According to the LPV modeling approach, system (49a)–(49d) can be described as the following disturbed LPV stochastic system:

$$x(t+1) = \mathbf{A}(\alpha(t))x(t) + \mathbf{B}(\alpha(t))u(t) + \mathbf{E}(\alpha(t)) \cdot w(t) + \left(\overline{\mathbf{A}}(\alpha(t))x(t) + \overline{\mathbf{B}}(\alpha(t))u(t) + \overline{\mathbf{E}}(\alpha(t))w(t)\right)\beta(t) = \sum_{i=1}^{2}\vartheta_i(t)\left(\mathbf{A}_i x(t) + \mathbf{B}_i u(t) + \mathbf{E}_i w(t) + \left(\overline{\mathbf{A}}_i x(t) + \overline{\mathbf{B}}_i u(t) + \overline{\mathbf{E}}_i w(t)\right)\beta(t)\right), \tag{51}$$

where

$$\mathbf{A}_1 = \begin{bmatrix} 1 & 0.4 & 0 & 0 \\ 0 & 1 & 0.4 & 0 \\ 0 & -0.000064 & 0.9508 & -0.000296 \\ 0 & 0 & 0 & 0.8521 \end{bmatrix},$$

$$\mathbf{A}_2 = \begin{bmatrix} 1 & 0.4 & 0 & 0 \\ 0 & 1 & 0.4 & 0 \\ 0 & -0.00013 & 0.9438 & -0.00088 \\ 0 & 0 & 0 & 0.8521 \end{bmatrix},$$

$$\mathbf{B}_1 = \begin{bmatrix} 0 \\ 0 \\ 0.0003 \\ 0.1363 \end{bmatrix}, \quad \mathbf{B}_2 = \begin{bmatrix} 0 \\ 0 \\ 0.0010 \\ 0.1363 \end{bmatrix}, \quad \mathbf{E}_1 = \begin{bmatrix} 0 \\ 0.0002 \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{E}_2 = \mathbf{E}_1,$$

$$\overline{\mathbf{A}}_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0.2 & 0.1 & 0 \end{bmatrix}, \quad \overline{\mathbf{A}}_2 = \overline{\mathbf{A}}_1, \quad \overline{\mathbf{B}}_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.01 \end{bmatrix}, \quad \overline{\mathbf{B}}_2 = \overline{\mathbf{B}}_1,$$

$$\overline{\mathbf{E}}_1 = \begin{bmatrix} 0 \\ 0.00002 \\ 0 \\ 0 \end{bmatrix}, \quad \overline{\mathbf{E}}_2 = \overline{\mathbf{E}}_1, \quad \vartheta_1(t) = |\sin(t)|, \quad \vartheta_2(t) = 1 - |\sin(t)|. \tag{52}$$

For (51), the performance index η is given by 0.0032 and the intensity level $\tau = 1$ is given. Employing the convex optimization algorithm, the feasible solutions of Theorem 2 can be obtained as follows:

$$\mathbf{P} = \begin{bmatrix} 0.0033 & 0.0723 & 0.4088 & -0.0004 \\ 0.0723 & 3.1263 & 21.1886 & -0.0308 \\ 0.4088 & 21.1886 & 148.4819 & -0.2240 \\ -0.0004 & -0.0308 & -0.2240 & 0.0006 \end{bmatrix} \times 10^{-3},$$

$$\mathbf{S}=\begin{bmatrix}0.2227 & 0.0452 & -0.0042 & -0.0030\\ 0.0452 & 0.8313 & 0.0099 & 0.0059\\ -0.0042 & 0.0099 & 0.9017 & -0.0012\\ -0.0030 & 0.0059 & -0.0012 & 0.1516\end{bmatrix}\times 10^{-7},$$

$$\mathbf{F}_1=\begin{bmatrix}0.5172 & 23.2092 & 150.4117 & -0.9578\end{bmatrix},$$

$$\mathbf{F}_2=\begin{bmatrix}1.2392 & 57.8210 & 390.6686 & -1.2176\end{bmatrix}. \tag{53}$$

According to the above feasible solutions, the following gain-scheduled controller can be designed:

$$u(t)=-\sum_{j=1}^{2}\vartheta_j(t)\,\mathbf{F}_i x(t). \tag{54}$$

Based on the gain-scheduled controller (54), the responses of system (51) are stated in Figures 3–6 via initial condition $x(0)=[\pi/2\ \ 0\ \ 0\ \ 0]^T$. For checking the achievement of (4), one can find the following values by substituting the simulated responses into the following ratio function:

$$\frac{E\left\{\sum_0^{t_f=100} x^T(t)\,\mathbf{S}x(t)\right\}}{E\left\{\sum_0^{t_f=100} w^T(t)\,w(t)\right\}}=7.199\times 10^{-7}. \tag{55}$$

It is easy to know that the ratio value in (55) is smaller than the given $\eta^2=1\times10^{-6}$ with $\eta=0.001$. Thus, the H_∞ performance of system (49a)–(49d) can be achieved via controller (54). And, from Figures 3–6, one can find that system (49a)–(49d) driven by (54) is asymptotically stable in the sense of mean square.

Besides, applying Theorem 4, the following feasible solutions of condition (24) are obtained:

$$\mathbf{P}_1=\begin{bmatrix}0.0029 & 0.0655 & 0.3755 & -0.0004\\ 0.0655 & 3.1063 & 21.4553 & -0.0321\\ 0.3755 & 21.4553 & 152.8154 & -0.2360\\ -0.0004 & -0.0321 & -0.2360 & 0.0006\end{bmatrix}\times 10^{-4},$$

$$\mathbf{P}_2=\begin{bmatrix}0.0028 & 0.0585 & 0.3261 & -0.0003\\ 0.0585 & 2.7418 & 18.8854 & -0.0292\\ 0.3261 & 18.8854 & 134.7003 & -0.2158\\ -0.0003 & -0.0292 & -0.2158 & 0.0006\end{bmatrix}\times 10^{-4},$$

$$\mathbf{S}=\begin{bmatrix}0.1817 & 0.0219 & -0.0022 & -0.0032\\ 0.0219 & 0.4025 & 0.0026 & 0.0015\\ -0.0022 & 0.0026 & 0.4152 & -0.0003\\ -0.0032 & 0.0015 & -0.0003 & 0.1365\end{bmatrix}\times 10^{-8},$$

$$\mathbf{G}_1=\begin{bmatrix}4.3444 & -0.5847 & 0.0703 & -0.7245\\ -0.5848 & 0.0899 & -0.0111 & 0.0508\\ 0.0703 & -0.0111 & 0.0014 & 0.0012\\ -0.7242 & 0.0506 & 0.0013 & 4.2746\end{bmatrix}\times 10^{7}, \tag{56}$$

$$\mathbf{G}_2=\begin{bmatrix}4.3445 & -0.5847 & 0.0703 & -0.7229\\ -0.5848 & 0.0900 & -0.0111 & 0.0503\\ 0.0703 & -0.0111 & 0.0014 & 0.0020\\ -0.7233 & 0.0503 & 0.0018 & 4.3617\end{bmatrix}\times 10^{7},$$

$$\mathbf{F}_1=\begin{bmatrix}-0.0393 & 0.0594 & -0.0121 & -3.1880\end{bmatrix}\times 10^{7},$$

$$\mathbf{F}_2=\begin{bmatrix}-0.0370 & 0.0585 & -0.0099 & -2.7806\end{bmatrix}\times 10^{7}.$$

And the following gain-scheduled controller can be designed with the feedback gains in (56):

$$u(t)=-\left(\sum_{j=1}^{2}\vartheta_j(t)\,\mathbf{F}_i\right)\left(\sum_{j=1}^{2}\vartheta_j(t)\,\mathbf{G}_i\right)^{-1}x(t). \tag{57}$$

Applying the controller (57), the responses of system (51) are stated in Figures 3–6 via the same initial condition. From the simulation results, the effect of the disturbance on the system driven by (57) can be criticized as follows:

$$\frac{E\left\{\sum_0^{t_f=100} x^T(t)\,\mathbf{S}x(t)\right\}}{E\left\{\sum_0^{t_f=100} w^T(t)\,w(t)\right\}}=6.587\times 10^{-8}. \tag{58}$$

It is easy to know that the ratio value in (58) is smaller than the given $\eta^2=1\times10^{-6}$ with $\eta=0.001$. Thus, the H_∞ performance of system (49a)–(49d) can be achieved via controller (57). And, from Figures 3–6, one can find that system (49a)–(49d) driven by (57) is asymptotically stable in the sense of mean square.

In order to emphasize the advantages of this paper, the design method of [14] is applied to compare with the proposed methods in this paper. Referring to [14], the H_∞ gain-scheduled controller design method was proposed for LPV systems without consideration of stochastic behavior. On the other hand, the same PDLF was used to derive the sufficient condition in Theorem 8 of [14]. Applying the design method of [14], the corresponding controller can be established as follows:

$$u(t)=\sum_{j=1}^{2}\vartheta_j(t)\,\mathbf{K}_i x(t), \tag{59}$$

where $\mathbf{K}_1=[-0.0188\ \ -1.5204\ \ -15.2101\ \ 0.5341]$ and $\mathbf{K}_2=[1.9481\ \ 30.4800\ \ 145.7313\ \ 0.5315]$. With the same initial

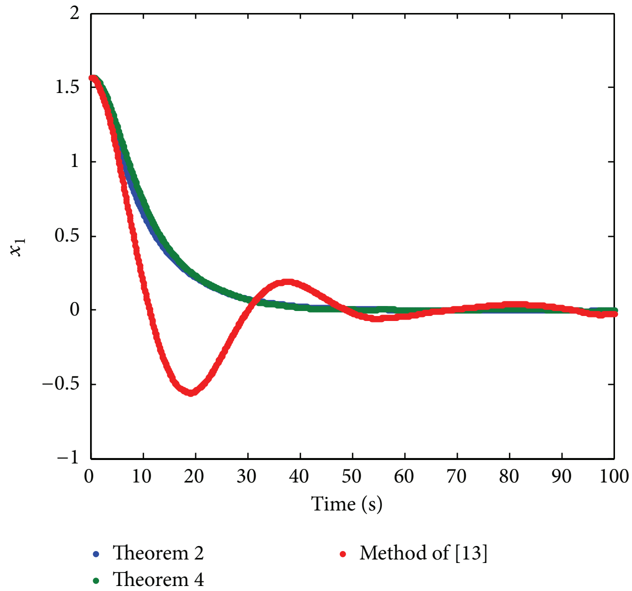

FIGURE 3: Responses for $x_1(t)$ of Example 6.

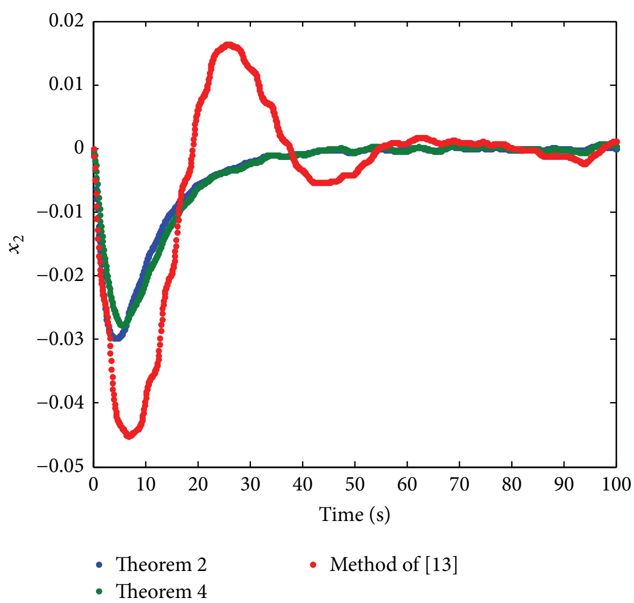

FIGURE 4: Responses for $x_2(t)$ of Example 6.

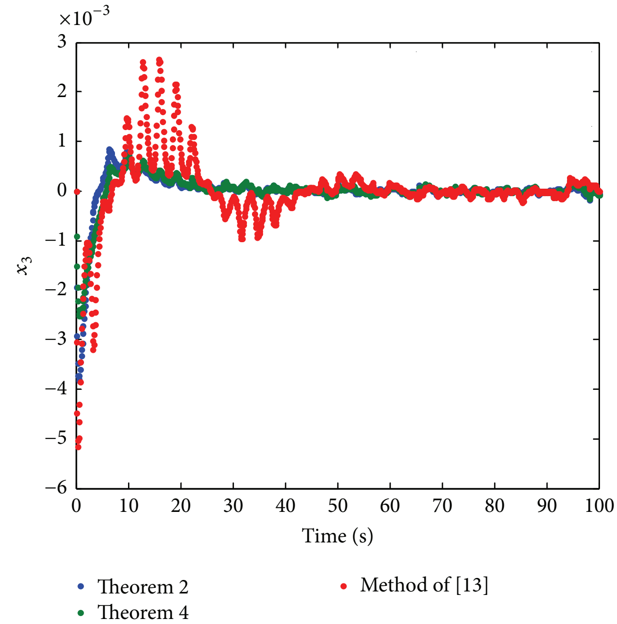

FIGURE 5: Responses for $x_3(t)$ of Example 6.

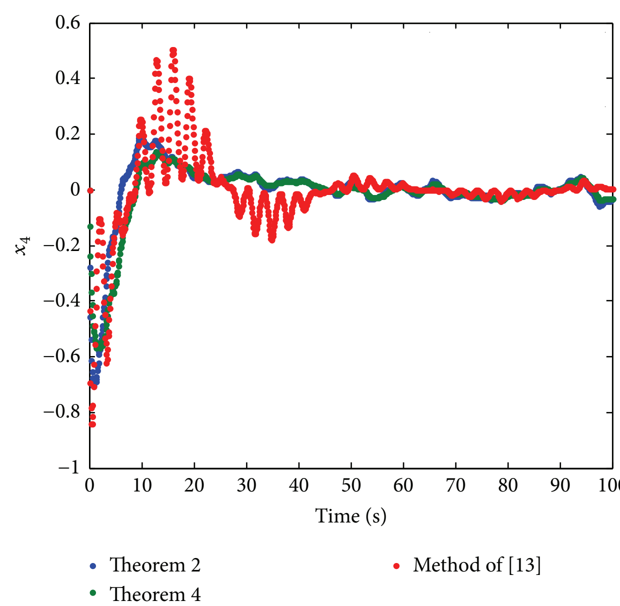

FIGURE 6: Responses for $x_4(t)$ of Example 6.

condition, the responses of (49a)–(49d) driven by (59) are stated in Figures 3–6. From Figures 3–6, one can find that controllers (57) provide better performance in both short term and long term characteristics than others. Besides, the overshoot and setting time of system (49a)–(49d) driven by the controller designed by this paper are smaller than those driven by controller (59). Therefore, the controller designed by [14] provides the worst control performance to stabilize system (49a)–(49d) due to stochastic behavior. Through the simulation results, the proposed design methods provide some improvements for [14] in stabilizing the disturbed uncertain stochastic system (49a)–(49d).

6. Conclusion

The H_∞ gain-scheduled controller design methods have been proposed in this paper for discrete-time disturbed uncertain stochastic systems described by LPV stochastic system. By choosing the Lyapunov functions, the sufficient conditions were derived to establish the corresponding gain-scheduled controller. And the H_∞ attenuation performance has been considered to constrain the effect of external disturbance on the considered systems. Applying the proposed design methods, the simulation results have been proposed to show the effectiveness and applicability of this paper. From the simulation results, the robust asymptotical stability and H_∞ performance of uncertain stochastic systems can achieve the designed gain-scheduled controller in the sense of mean square.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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