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*Research Article*

# Study of $1D$ Strange Charmed Meson Family Using HQET

**Pallavi Gupta and A. Upadhyay**

*School of Physics and Material Science, Thapar University, Patiala, Punjab 147004, India*

Correspondence should be addressed to A. Upadhyay; alka.iisc@gmail.com

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Recently LHCb predicted spin 1 and spin 3 states $D_{s1}^*(2860)$ and $D_{s3}^*(2860)$ which are studied through their strong decays and are assigned to fit the $1^3D_1$ and $1^3D_3$ states in the charm spectroscopy. In this paper, using the heavy quark effective theory, we state that assigning $D_{s1}^*(2860)$ as the mixing of $1^3D_1 - 2^3S_1$ states is rather a better justification to its observed experimental values than a pure state. We study its decay modes variation with hadronic coupling constant $g_{xh}$ and the mixing angle $\theta$. We appoint spin 3 state $D_{s3}^*(2860)$ as the missing $1D$ $3^- J^P$ state and also study its decay channel behavior with coupling constant $g_{yh}$. To appreciate the above results, we check the variation of decay modes for their spin partners states, that is, $1D_2$ and $1D_2'$, with their masses and strong coupling constant, that is, $g_{xh}$ and $g_{yh}$. Our calculation using HQET approach gives mixing angle of the $1^3D_1 - 2^3S_1$ state for $D_{s1}^*(2860)$ to lie in the range ($-1.6$ radians $\leq \theta \leq -1.2$ radians). Our calculation for coupling constant values gives $g_{xh}$ to lie within value range of 0.17–0.20 and $g_{yh}$ to be 0.40. We expect from experiments to observe this mixing angle to verify our results.

## 1. Introduction

Over the last decade many new heavy-light mesons ($Q\bar{q}$) have been observed by various experimental collaborations. The state $D_{sJ}^*(2860)$ was first observed by the BaBar Collaboration in $D_{sJ}(2860) \rightarrow D^0K^+, D^+K^0$ with mass $M = 2856.6 \pm 1.5$ MeV and width $\Gamma = 48 \pm 7$ MeV [1]. It was supposed to have natural parity states, that is, $0^+$, $1^-$, $2^+$, $3^-$, and so forth. But the assignment of $D_{sJ}(2860)$ as the $0^+$ state was ruled out after the observation of $D_{sJ}(2860) \rightarrow D^*K$ [2]. Along with the $D^*K$ channel [2] BaBar also gives the ratio $R$ measured as $R = \text{Br}(D_{sJ}^*(2860) \rightarrow D^*K)/\text{Br}(D_{sJ}^*(2860) \rightarrow DK) = 1.10 \pm 0.15 \pm 0.19$. Along with this $D_{sJ}^*(2860)$, BaBar Collaboration also observed $D_{s1}^*(2710)$ state in the $DK$ invariant mass spectrum with mass $= 2688 \pm 4 \pm 3$ MeV and decay width $= 112 \pm 736$ [1]. In [2], BaBar Collaboration reported the branching ratio for this $D_{s1}^*(2710)$ state as $R = \text{Br}(D_{s1}^*(2710) \rightarrow D^*K)/\text{Br}(D_{s1}^*(2710) \rightarrow DK) = 0.91 \pm 0.13 \pm 0.12$. The $D_{sJ}(2860)$ and $D_{s1}^*(2710)$ state had went through extensive discussions by various theoretical models, to find a place in strange charm spectrum. Various discussions suggest $D_{s1}^*(2710)$ to be suitable as $1^3D_3$ state or as a radial excitation of $S$-wave, that is, $2^3S_1$ state. Zhang et al. have assigned $D_{sJ}^*(2860)$ as $2^3P_0$ or $1^3D_3$ states using the $^3P_0$ model [3], Colangelo et al. assign $D_{sJ}^*(2860)$ to be $1^3D_3$ state using the heavy meson effective theory [4, 5], and Li et al. favor $D_{sJ}^*(2860)$ as the $2^3P_0$ or $1^3D_3$ state using Regge phenomenology [6]. All these different approaches calculated different value of the $R$ ratio $R = (\text{Br}(D_{sJ}^*(2860) \rightarrow D^*K)/\text{Br}(D_{sJ}^*(2860) \rightarrow DK))$. Heavy quark effective theory predicts $R$ to be $\approx 0.39$ [4], while $^3P_0$ model calculated it to be $R = 0.59$; both of the predicted values of $R$ are far from the experimental value $R = 1.10$. All these references favored $D_{sJ}^*(2860)$ as $1^3D_{s3}$ state due to observed narrow decay width, at the cost of mismatch of $R$ with experiments.

Recently LHCb Collaboration predicted a new resonance around 2.86 GeV in the $\overline{D}^0K^-$ invariant mass spectrum from decay channel $B_s^0 \rightarrow \overline{D}^0K^-\Pi^+$, containing the mixture of spin 1 and spin 3 states components corresponding to $D_{s1}^*(2860)$ and $D_{s3}^*(2860)$ [7, 8] where the mass and width parameters are

$$M\left(D_{s1}^*(2860)\right) = 2859 \pm 12 \pm 6 \pm 23\,\text{MeV},$$

$$\Gamma\left(D_{s1}^*(2860)\right) = 159 \pm 23 \pm 27 \pm 72\,\text{MeV},$$

$$M\left(D_{s3}^{*}(2860)\right) = 2860.5 \pm 2.6 \pm 2.5 \pm 6.0\,\text{MeV},$$
$$\Gamma\left(D_{s3}^{*}(2860)\right) = 53 \pm 7 \pm 4 \pm 6\,\text{MeV}. \tag{1}$$

Here the first error is statistical error, the second is the experimental systematic effects, and the last one is due to model variations. Thus LHCb observed two $D_s^*(2860)$ states with spin 1 and spin 3. From the previous study it can be speculated that it is the spin 3 resonance of $D_s^*(2860)$ that belongs to $1^3D_{s3}$ state, with a narrow width $\Gamma = 53\,\text{MeV}$. Theoretically, $R$ value can be matched with the experimental value, considering its contribution coming from the spin 1 state of $D_{s1}^*(2860)$ resonance. Comparing with earlier theoretical mass predictions, LHCb spin 1 resonance $D_{s1}^*(2860)$ can be assumed to fit in $1^3D_{s1}$ state of $1D$ family or can be a mixture of $1^3D_1$ and $2^3S_1$ states since both these states have the same orbital angular momentum. Assigning $D_{s1}^*$ as a mixing state of $1^3D_1 - 2^3S_1$ may give a better justification than assigning it as a pure state, because the $R$ value calculated by taking $D_{s1}^*(2860)$ to be a mixed state of $1^3D_1 - 2^3S_1$ now depends on the mixing angle of these mixed states. By choosing suitable mixing angle ($\theta$), the calculated $R$ value can be better justified with the experimental value. Li and Ma assign $D_{sJ}(2860)$ to be mixing state of $1^3D_1 - 2^3S_1$ and $D_{s1}^*(2700)$ to be its orthogonal partner [9] and obtained $R = 0.8$, nearly close to the experimental value. Zhong and Zhao by chiral quark model [10, 11] studied the $D_{sJ}^*$ state as the $1^3D_3$ state with some $1^3D_2 - 1^1D_2$ mixing. Wang [12] tried to reproduce the experimental value $R = 1.10$ with some suitable hadronic coupling constants, by including chiral symmetry breaking corrections in heavy quark effective theory. Besides these studies, Vijande et al. also assign $D_{sJ}(2860)$ to be the multiquark exotic state as $c\bar{s} - cn\bar{s}\,\bar{n}$ [13]. Godfrey and Jardine by adopting the pseudoscalar emission decay model [14] and Song et al. by adopting QPC model [15] studied $D_{sJ}^*$ as $1^3D_1 - 2^3S_1$. Various predictions are made to study the mixing effects in $D_{sJ}^*$ state [16–19].

In Particle Data Group [20] $1S$ and $1P$ strange charmed states are nicely described, but information for other states is still missing. The strange meson states with their $J^P$ states predicted by various theoretical model are gathered in Table 1. From the mass predicted by various theoretical models, that is, from second, third, and fourth columns of Table 1, $D_{s1}^*(2860)$ and $D_{s3}^*(2860)$ can be fitted as spin 1 and spin 3 state of $1D$ family.

In this paper, the four states of $1D$ stranged charm meson family are analyzed by studying their decay widths and branching ratios. For this heavy quark effective theory is used and the importance of mixing of the two states is surveyed. In the past years, HQET has been successful in assigning suitable $J^P$ states to the observed $D$ and $B$ mesons using their decays widths in terms of coupling constants. We use HQET approach to study spin 1 resonance of LHCb $D_{s1}^*(2860)$, by assigning it to be the first member of $1D$ stranged charm meson family. Properties of this state are examined in two ways, firstly by considering it as a pure spin 1 state of $1^3D_1$ and secondly by assuming this state as a mixture of $1^3D_1$ and $2^3S_1$ states. LHCb predicted spin 3 state $D_{s3}^*(2860)$ is studied by assigning it the $1^3D_3$ position in $1D$ strange charm mesons. To complete the $1D$ family, we also try to study the behavior of their spin partners, that is, $1D_2$ and $1D_{2'}$, which are still missing experimentally.

Table 1: Theoretically predicted masses.

| $J^P(^{2s+1}L_j)$ | GI [21] (MeV) | PE [22] (MeV) | EFG [23] (MeV) | Experimental [20] (MeV) |
|---|---|---|---|---|
| $0^-\ (^1S_0)$ | 1979 | 1965 | 1969 | 1968 |
| $1^-\ (^3S_1)$ | 2129 | 2113 | 2111 | 2112 |
| $0^+\ (^3P_0)$ | 2484 | 2487 | 2509 | 2318 |
| $1^+\ (^1P_1)$ | 2459 | 2535 | 2536 | 2460 |
| $1^+\ (^3P_1)$ | 2556 | 2605 | 2574 | 2536 |
| $2^+\ (^3P_2)$ | 2592 | 2581 | 2571 | 2573 |
| $1^-\ (^3D_1)$ | 2899 | 2913 | 2913 | 2859 |
| $2^-\ (^1D_2)$ | 2900 | 2900 | 2931 | — |
| $2^-\ (^3D_2')$ | 2926 | 2953 | 2961 | — |
| $3^-\ (^3D_3)$ | 2917 | 2925 | 2871 | 2860 |

The paper is divided into the following sections. Section 2 describes the heavy quark effective theory formalism used for the strong decays. Section 3 discusses the members of $1D$ family. In this section, all the four states with their decay modes in terms of their couplings are described in different subsections. To appreciate the experimental value of $R$, various mixing effects in terms of mixing angle theta are studied. We finally conclude our results in Section 4.

## 2. Framework

In the heavy quark limit $m_Q \gg \Lambda_{QCD} \gg m_q$, $Q\bar{q}$ system can be effectively studied using heavy quark effective theory. According to this theory, heavy quark acts like static color source with spin $s_Q$, which, due to heavy flavor symmetry, interacts only with the light degree of freedom having spin $s_l$ through the exchange of soft gluons. This picture can be compared with that of hydrogen atom [24]. The basic idea is that, in a $Q\bar{q}$ system, heavy quark plays the role of a nucleus and the light quark plays the role of an electron. This $Q\bar{q}$ system can be categorized in doublets in relation to the total conserved angular momentum, that is, $s_l = s_{\bar{q}} + L$, where $s_{\bar{q}}$ and $L$ are the spin and orbital angular momentum of the light antiquark, respectively. For $L = 0$ ($S$-wave), the doublet is represented by $(D, D^*)$ with $J_{s_l}^P = (0^-, 1^-)_{1/2}$, in which for $L = 1$ ($P$-wave), there are two doublets represented by $(D_0^*, D_1)$ and $(D_1', D_2^*)$ with $J_{s_l}^P = (0^+, 1^+)_{1/2}$ and $(1^+, 2^+)_{3/2}$, respectively. Two doublets of $L = 2$ ($D$-wave) are represented by $(D_1^*, D_2)$ and $(D_2', D_3^*)$ belonging to $J_{s_l}^P = (1^-, 2^-)_{3/2}$ and $(2^-, 3^-)_{5/2}$, respectively. These doublets are described by the effective superfields $H_a$, $S_a$, $T_a$, $X_a$, $Y_a$ [25, 26], where field $H_a$ describes the $(D, D^*)$ doublet, that is, $S$-wave, and $S_a$ and $T_a$ fields represent the $P$-wave doublets $(0^+, 1^+)_{1/2}$ and

$(1^+, 2^+)_{3/2}$, respectively. $D$-wave doublets are represented by the $X_a$ and $Y_a$ fields. These fields are as follows:

$$H_a = \frac{1+\not{v}}{2}\left\{P^*_{a\mu}\gamma^\mu - P_a\gamma_5\right\},$$
$$S_a = \frac{1+\not{v}}{2}\left\{P^\mu_{1a}\gamma_\mu\gamma_5 - P^*_{0a}\right\},$$
$$T^\mu_a = \frac{1+\not{v}}{2}\left\{P^{*\mu\nu}_{2a}\gamma_\nu - P_{1a\nu}\sqrt{\frac{3}{2}}\gamma_5\left[g^{\mu\nu} - \frac{\gamma^\nu\left(\gamma^\mu - v^\mu\right)}{3}\right]\right\},$$
$$X^\mu_a = \frac{1+\not{v}}{2}\left\{P^{\mu\nu}_{2a}\gamma_5\gamma_\nu - P^*_{1a\nu}\sqrt{\frac{3}{2}}\left[g^{\mu\nu} - \frac{\gamma^\nu\left(\gamma^\mu + \gamma^\mu\right)}{3}\right]\right\}, \quad (2)$$
$$Y^{\mu\nu}_a = \frac{1+\not{v}}{2}\left\{P^{*\mu\nu\sigma}_{3a}\gamma_\sigma - P^{\alpha\beta}_{2a}\sqrt{\frac{5}{3}}\gamma_5\left[g^\mu_\alpha g^\nu_\beta - \frac{g^\nu_\beta\gamma_\alpha\left(\gamma^\mu - v^\mu\right)}{5} - \frac{g^\mu_\alpha\gamma_\beta\left(\gamma^\nu - v^\nu\right)}{5}\right]\right\}.$$

The light pseudoscalar mesons are described by the fields $\xi = \exp^{iM/f_\pi}$. The pion octet is introduced by the vector and axial combinations $V^\mu = (1/2)\xi\partial^\mu\xi^\dagger + \xi^\dagger\partial^\mu\xi$ and $A^\mu = (1/2)\xi\partial^\mu\xi^\dagger - \xi^\dagger\partial^\mu\xi$. We choose $f_\pi = 130$ MeV. Here, all traces are taken over Dirac spinor indices, light quark $SU(3)_V$ flavor indices $a = u, d, s$, and heavy quark flavor indices $Q = c, b$ [25, 26]. The Dirac structure of chiral Lagrangian has been replaced by velocity vector $v$. At the leading approximation, the heavy meson chiral Lagrangian terms $L_{HH}$, $L_{SH}$, $L_{TH}$, $L_{XH}$, $L_{XS}$, $L_{XT}$, $L_{YH}$, $L_{YS}$, $L_{YT}$ for the two-body strong decays to light pseudoscalar mesons can be written as follows:

$$L_{HH} = g_{hh}\mathrm{Tr}\left\{\overline{H}_a H_b\gamma_\mu\gamma_5 A^\mu_{ba}\right\},$$
$$L_{SH} = g_{sh}\mathrm{Tr}\left\{\overline{H}_a S_b\gamma_\mu\gamma_5 A^\mu_{ba}\right\} + \text{h.c.},$$
$$L_{TH} = \frac{g_{th}}{\Lambda}\mathrm{Tr}\left\{\overline{H}_a T^\mu_b\left(iD_\mu\not{A} + i\not{D}A_\mu\right)_{ba}\gamma_5\right\} + \text{h.c.},$$
$$L_{XH} = \frac{g_{xh}}{\Lambda}\mathrm{Tr}\left\{\overline{H}_a X^\mu_b\left(iD_\mu\not{A} + i\not{D}A_\mu\right)_{ba}\gamma_5\right\} + \text{h.c.},$$
$$L_{XS} = \frac{g_{xs}}{\Lambda}\mathrm{Tr}\left\{\overline{S}_a X^\mu_b\left(iD_\mu\not{A} + i\not{D}A_\mu\right)_{ba}\gamma_5\right\} + \text{h.c.},$$
$$L_{XT} = \frac{1}{\Lambda^2}\mathrm{Tr}\left\{\overline{T}^\mu_a X^\nu_b\left[k^T_1\left\{D_\mu, D_\nu\right\}A_\lambda + k^T_2\left(D_\mu D_\lambda A_\nu + D_\nu D_\lambda A_\mu\right)\right]_{ba}\gamma^\lambda\gamma_5\right\} + \text{h.c.},$$
$$L_{YH} = \frac{1}{\Lambda^2}\mathrm{Tr}\left\{\overline{H}_a Y^{\mu\nu}_b\left[k^H_1\left\{D_\mu, D_\nu\right\}A_\lambda + k^H_2\left(D_\mu D_\lambda A_\nu + D_\nu D_\lambda A_\mu\right)\right]_{ba}\gamma^\lambda\gamma_5\right\} + \text{h.c.},$$
$$L_{YS} = \frac{1}{\Lambda^2}\mathrm{Tr}\left\{\overline{S}_a Y^{\mu\nu}_b\left[k^S_1\left\{D_\mu, D_\nu\right\}A_\lambda + k^S_2\left(D_\mu D_\lambda A_\nu + D_\nu D_\lambda A_\mu\right)\right]_{ba}\gamma^\lambda\gamma_5\right\} + \text{h.c.},$$
$$L_{YT} = \frac{g_{yt}}{\Lambda}\mathrm{Tr}\left\{\overline{T}_{a\mu} X^{\mu\nu}_b\left(iD_\nu\not{A} + i\not{D}A_\nu\right)_{ba}\gamma_5\right\} + \text{h.c.} \quad (3)$$

From the chiral Lagrangian terms $L_{HH}$, $L_{SH}$, $L_{TH}$, $L_{XH}$, $L_{YH}$, the two-body strong decay of $Q\overline{q}$ system to final state light pseudoscalar mesons $M$ $(\pi, \eta, K)$ can be described as

$(1^-, 2^-) \to (0^-, 1^-) + M$

$$\Gamma\left(1^- \longrightarrow 0^-\right) = C_M\frac{4g^2_{xh}}{9\pi f^2_\pi\Lambda^2}\frac{M_f}{M_i}\left[p^3_M\left(m^2_M + p^2_M\right)\right],$$
$$\Gamma\left(1^- \longrightarrow 1^-\right) = C_M\frac{2g^2_{xh}}{9\pi f^2_\pi\Lambda^2}\frac{M_f}{M_i}\left[p^3_M\left(m^2_M + p^2_M\right)\right], \quad (4)$$
$$\Gamma\left(2^- \longrightarrow 1^-\right) = C_M\frac{2g^2_{xh}}{3\pi f^2_\pi\Lambda^2}\frac{M_f}{M_i}\left[p^3_M\left(m^2_M + p^2_M\right)\right].$$

$(2^-, 3^-) \to (0^-, 1^-) + M$

$$\Gamma\left(2^- \longrightarrow 1^-\right) = C_M\frac{4g^2_{yh}}{15\pi f^2_\pi\Lambda^4}\frac{M_f}{M_i}\left[p^7_M\right],$$
$$\Gamma\left(3^- \longrightarrow 0^-\right) = C_M\frac{4g^2_{yh}}{35\pi f^2_\pi\Lambda^4}\frac{M_f}{M_i}\left[p^7_M\right], \quad (5)$$
$$\Gamma\left(3^- \longrightarrow 1^-\right) = C_M\frac{16g^2_{yh}}{105\pi f^2_\pi\Lambda^4}\frac{M_f}{M_i}\left[p^7_M\right].$$

$(1^-, 2^-) \to (0^+, 1^+) + M$

$$\Gamma\left(2^- \longrightarrow 1^+\right) = C_M\frac{2g^2_{xs}}{5\pi f^2_\pi\Lambda^2}\frac{M_f}{M_i}\left[p^5_M\right],$$
$$\Gamma\left(2^- \longrightarrow 0^+\right) = C_M\frac{4g^2_{xs}}{15\pi f^2_\pi\Lambda^2}\frac{M_f}{M_i}\left[p^5_M\right], \quad (6)$$
$$\Gamma\left(1^- \longrightarrow 1^+\right) = C_M\frac{2g^2_{xs}}{3\pi f^2_\pi\Lambda^2}\frac{M_f}{M_i}\left[p^5_M\right].$$

$(2^-, 3^-) \to (0^+, 1^+) + M$

$$\Gamma\left(3^- \longrightarrow 1^+\right) = C_M\frac{4g^2_{ys}}{15\pi f^2_\pi\Lambda^4}\frac{M_f}{M_i}\left[p^5_M\left(m^2_M + p^2_M\right)\right],$$
$$\Gamma\left(2^- \longrightarrow 1^+\right) = C_M\frac{8g^2_{ys}}{75\pi f^2_\pi\Lambda^4}\frac{M_f}{M_i}\left[p^5_M\left(m^2_M + p^2_M\right)\right], \quad (7)$$
$$\Gamma\left(2^- \longrightarrow 0^+\right) = C_M\frac{4g^2_{ys}}{25\pi f^2_\pi\Lambda^4}\frac{M_f}{M_i}\left[p^5_M\left(m^2_M + p^2_M\right)\right].$$

Table 2: Calculated partial and total decay widths of $D_{s1}^*(2860)$ as pure $(1^3D_1)$.

| Theory | $DK$ | $D^*K$ | $D_s\eta$ | $D_s^*\eta$ | Total | $D^*K/DK$ | $D_s\eta/DK$ |
|---|---|---|---|---|---|---|---|
| Our | $2865.45g_{xh}^2$ | $693.135g_{xh}^2$ | $508.189g_{xh}^2$ | $85.70g_{xh}^2$ | $4152.48g_{xh}^2$ | 0.24 | 0.177 |
| Experimental [8] | | | | | 159 | 1.10 | |

In the above expressions of decay width, $M_i$, $M_f$ stand for initial and final meson mass. Hadronic coupling constants $g_{xh}$, $g_{xs}$, $g_{xt} = k_1^T + k_2^T$, $g_{yh} = k_1^H + k_2^H$, $g_{ys} = k_1^S + k_2^S$, and $g_{yt}$ are dependent on the radial quantum number, $\Lambda$ is the chiral symmetry breaking scale = 1 GeV, and $p_M$ and $m_M$ are the final momentum and mass of the emitted light pseudoscalar meson. The coefficients $C_{\pi^\pm}, C_{K^\pm}, C_{K^0}, C_{\overline{K}^0} = 1$, $C_{\pi^0} = 1/2$, and $C_\eta = 2/3$ or $1/6$ [25, 26]. Different values of $C_\eta$ correspond to the initial state being $c\bar{u}$, $c\bar{d}$, or $c\bar{s}$, respectively.

## 3. Numerical Results

OZI allowed two-body strong decays of $1D$ strange charm family are calculated using the heavy quark effective approach as given in Section 2. In the present work, partial and total decay widths of these four $1D$ states are studied and compared with their experimental values. OZI allowed decay channels for $D_{s1}^*(2860)$ and $D_{s3}^*$ states are $DK$, $D^*K$, $D_s\eta$, and $D_s^*\eta$, and for their spin partners $1D_{s2}$ and $1D'_{s2}$ states, they are $D^*K$, $D_s^*\eta$, $D(2400)K$, and $D_s^*(2317)\eta$. For this calculation, initial masses of $D_{s1}^*$ and $D_{s3}^*$ states, as given by the LHCb [7, 8], have been used as input parameters along with 2890 MeV and 2900 MeV for their spin partner states $1D_{s2}$ and $1D'_{s2}$, respectively. Heavy quark effective theory shows that decay widths also depend on the strong hadronic couplings $g_{xh}$, $g_{yh}$, $g_{xs}$, and $g_{ys}$. The theoretical value of the strong coupling constants has been constrained within the range of 0 and 1 [27] though their experimental information is still missing. In the next subsections, we have calculated two of these coupling constants, that is, $g_{xh}$ and $g_{yh}$, using the decay widths and available experimental data.

*3.1. $D_{s1}^*(2860)$.* $D_{s1}^*(2860)$ was first observed by BaBar Collaboration and in 2014 its spin, mass, and decay width were confirmed by LHCb. In this subsection, heavy quark effective theory is adopted to reproduce the experimental data given by these collaborations. Also the coupling constant $g_{xh}$ is determined by assigning $D_{s1}^*(2860)$ state as the $1^-$ member of the $1D$ charm family. Assuming it to be the pure $1D$ $1^-$ state, we calculated the total and partial decay widths of $D_{s1}^*(2860)$ using the decay width formulae given in Section 2 in terms of their hadronic coupling constants. These partial decay widths and ratios are tabulated in Table 2. Along with the partial decay widths, we also studied the ratios such as

$$R = \frac{\mathrm{Br}\left(D_{s1}^*(2860) \longrightarrow D^*K\right)}{\mathrm{Br}\left(D_{s1}^*(2860) \longrightarrow DK\right)}, \qquad R1 = \frac{\mathrm{Br}\left(D_{s1}^*(2860) \longrightarrow D_s\eta\right)}{\mathrm{Br}\left(D_{s1}^*(2860) \longrightarrow DK\right)}. \tag{8}$$

Table 3

| | Reference [15] | Reference [18] | Reference [5] |
|---|---|---|---|
| $\Gamma(D^*K/DK)$ | 0.46–0.70 | 12.5–7.6 | 0.06 |
| $\Gamma(D_s\eta/DK)$ | 0.10–0.14 | 0.30–0.14 | 0.23 |

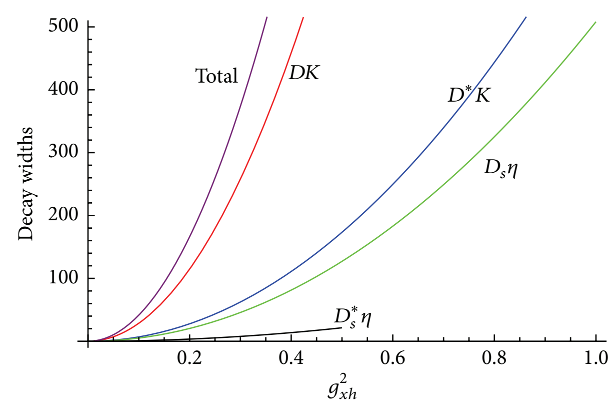


Figure 1: Decay widths of $D_{s1}(2860)$ for $\theta = 0^\circ$.

It can be seen from Table 2 that our calculated $R$ value does not match with the experimental value 1.10. Ratios calculated in Table 2 have also been calculated by various other theoretical models as shown in Table 3.

It can be seen that $R$ value, that is, $\Gamma(D^*K/DK)$ calculated by our HQET approach and by other theoretical approaches [5, 15, 18], does not match with the experimental $R$ value, that is, 1.10. As $R$ is independent of couplings, to justify the experimental value of $R$, we include the mixing of the states. According to this scheme, state $D_{s1}^*(2860)$ is assumed to be the mixture of $2^3S_1$ and $1^3D_1$ states with $D_s(2700)$ to be its orthogonal partner satisfying the relation

$$\begin{pmatrix} D_{s1}(2S) \\ D_{s1}(2860) \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} 2^3S_1 \\ 1^3D_1 \end{pmatrix}, \tag{9}$$

where $\theta$ is the mixing angle between these two mixed states. Effect of variation of total decay width of $D_{s1}(2860)$ state with coupling constant $g_{xh}$ for different mixing angles is shown in Figures 1–4, which shows the variation for some typical values of mixing angle at $\theta = 0^\circ$, $\theta = -30^\circ$ and for $\theta = -60^\circ$ and $\theta = -80^\circ$ where $\theta = 0^\circ$ correspond to nonmixing, that is, pure $1^3D_1$ state.

Figures 1, 2, 3, and 4 show that $DK$ is the main decay channel of $1^3D_1$ state. Apart from $DK$, $D^*K$ and $D_s\eta$ are also important decay channels of $1^3D_1$, whereas the calculated

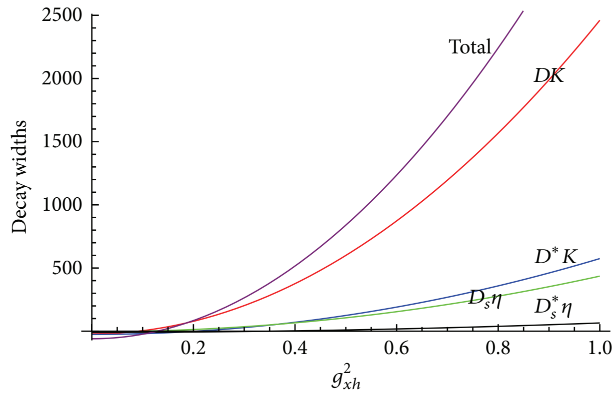


Figure 2: Decay widths of $D_{s1}(2860)$ for $\theta = -30°$.

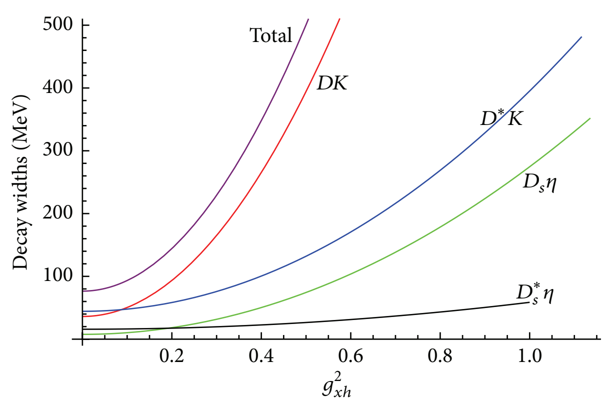


Figure 3: Decay widths of $D_{s1}(2860)$ for $\theta = -60°$.

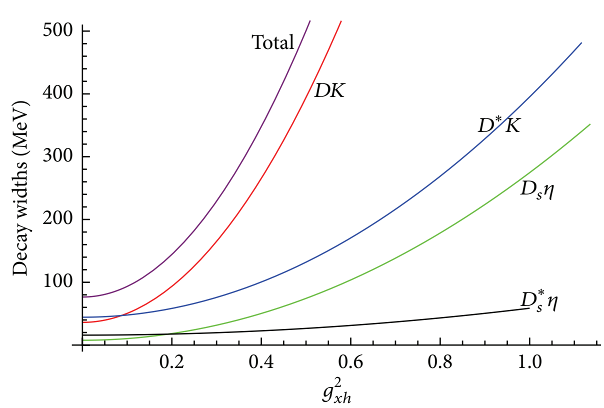


Figure 4: Decay widths of $D_{s1}(2860)$ for $\theta = -80°$.

decay width for $D_s^*\eta$ is found to be small. Dominance of $D_s^*\eta$ decay channel increases with large value of mixing angle. $R$ ratio defined in Section 1 now depends on both the mixing angle and strong coupling constants $g_{xh}$ and $g_{hh}$. Variation of $R$ value with the mixing angle, by fixing $g_{hh} = 0.17$ [25], is shown in Figure 5. This figure shows that $R \simeq 8.5$ corresponding to the mixing angle of range $-1.6 \leq \theta \leq -1.2$

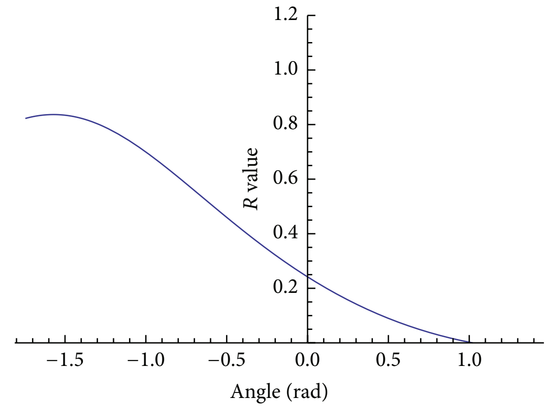


Figure 5: Variation of $R$ value with mixing angle.

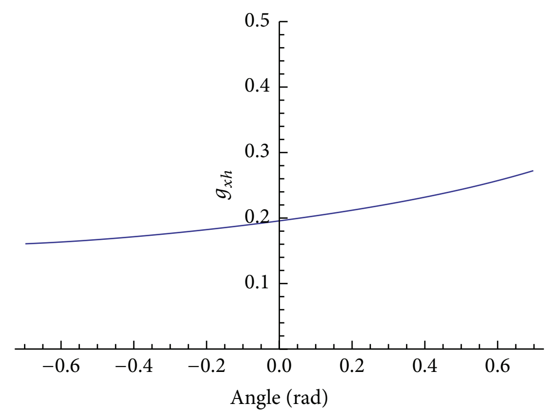


Figure 6: Variation of coupling constant $g_{xh}$ with mixing angle.

radians. This obtained $R$ value is near to the experimental $R$ value, $R = 1.10 \pm 0.15 \pm 0.19$. For this range of mixing angle our hadronic coupling constant comes out to be within $0.17 \leq g_{xh} \leq 0.20$. This variation of $g_{xh}$ hadronic coupling with the mixing angle has been shown in Figure 6.

For these calculated values of mixing angle and coupling constant, partial and total decay widths are again studied. Total width $\Gamma$ comes out to be 159 MeV, which matches very well with the experimental data. Other partial decay widths are listed in Table 4. The calculated $R$ value from our approach is matching well with the experimental observed value $1.10 \pm 0.15 \pm 0.19$.

*3.2. $D_{s3}^*(2860)$.* Considering spin 3 resonance $D_{s3}^*(2860)$ of LHCb, as the $1^3D_{s3}$ state, decay channels and partial decay widths are presented in Table 5. Figure 7 shows the variation of the partial and total decay width with coupling constant $g_{yh}$.

Figure 7 clearly shows that $DK$ is the dominant decay mode of $D_{s3}^*(2860)$. Other important decay channels are $D^*K$, $D_s\eta$ with $D_s^*\eta$ contributing least. Computing it with the experimental value of total decay width $\Gamma = 53$ MeV,

TABLE 4: Calculated partial and total decay widths of $D_{s1}^*(2860)$ as a mixture of $(1^3D_1)$ and $2^3S_1$.

| Theory | $DK$ (MeV) | $D^*K$ (MeV) | $D_s\eta$ (MeV) | $D_s^*\eta$ (MeV) | Total (MeV) | $D^*K/DK$ | $D_s\eta/DK$ |
|---|---|---|---|---|---|---|---|
| Our | 60.61 | 55.26 | 11.85 | 18.66 | 146.39 | 0.91 | 0.19 |
| Experimental | | | | | 159 | 1.10 | |

TABLE 5: Calculated partial and total decay widths of $D_{s3}^*(2860)$ as $1^3D_{s3}$ state in terms of hadronic coupling $g_{yh}$.

| Theory | $DK$ (MeV) | $D^*K$ (MeV) | $D_s\eta$ (MeV) | $D_s^*\eta$ (MeV) | Total (MeV) |
|---|---|---|---|---|---|
| Our | $249.18g_{yh}^2$ | $96.39g_{yh}^2$ | $24.72g_{yh}^2$ | $4.54g_{yh}^2$ | $374.846g_{yh}^2$ |
| Experimental | | | | | 53 |

TABLE 6

| | Reference [5] | Reference [10] | Reference [7] |
|---|---|---|---|
| $\Gamma(D^*K/DK)$ | 0.39 | 0.43 | 0.8 |
| $\Gamma(D_s\eta/DK)$ | 0.13 | 0.11 | 0.05 |

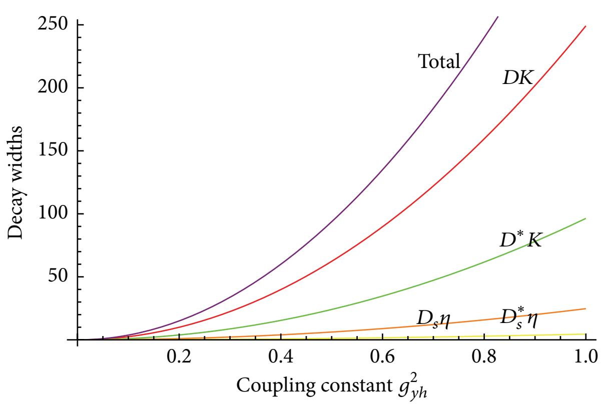


FIGURE 7: The variation of partial decay widths of $D_{s3}^*(2860)$ as the $1^3D_{s3}$ state with hadronic coupling.

coupling constant $g_{yh}$ comes out to be 0.40. These partial decay widths can be used to calculate the ratio $R$:

$$R = \frac{\Gamma\left(D_{s3}^*(2860) \longrightarrow D^*K\right)}{\Gamma\left(D_{s3}^*(2860) \longrightarrow DK\right)} = 0.38$$
$$R1 = \frac{\Gamma\left(D_{s3}^*(2860) \longrightarrow D_s\eta\right)}{\Gamma\left(D_{s3}^*(2860) \longrightarrow DK\right)} = 0.03. \quad (10)$$

These ratios are compared with predictions made by various other theoretical models as shown in Table 6.

*3.3.* $1D_{s2}$ *and* $1D_{s2}'$. $1D_{s2}$ is the spin partner of $D_{s1}^*(2860)$ belonging to $J^P$ as $2_{3/2}^-$ state, and $1D_{s2}'$ state belongs to $J_{s_l}^P$ to $2_{5/2}^-$. Both these states are still unknown in the charm meson spectrum. As shown in Table 1, their masses have been already predicted by various theoretical models [21–23]. Taking their

TABLE 7: Calculated partial and total decay widths of $1D_{s2}$ and $1D_{s2}'$. First section is calculated by taking them as pure states and the second section includes mixing scheme into account.

| Decay channel | As pure states ($\theta = 0^\circ$) | | As mixed states ($\theta = -39^\circ$) | |
|---|---|---|---|---|
| | $1D_2$ (MeV) | $1D_2'$ (MeV) | $1D_2$ (MeV) | $1D_2'$ (MeV) |
| $D^*K$ | 61.37 | 20.51 | 36.02 | 56.95 |
| $D_s^*\eta$ | 16.69 | 2.51 | 11.62 | 13.42 |
| $D(2400)K$ | — | $7.5 \times 10^{-5}$ | — | $1.5 \times 10^{-4}$ |
| $D_s^*(2317)\eta$ | 0.0037 | 0.005 | 0.0014 | 0.01 |
| Total | 78.06 | 21.04 | 47.65 | 70.38 |

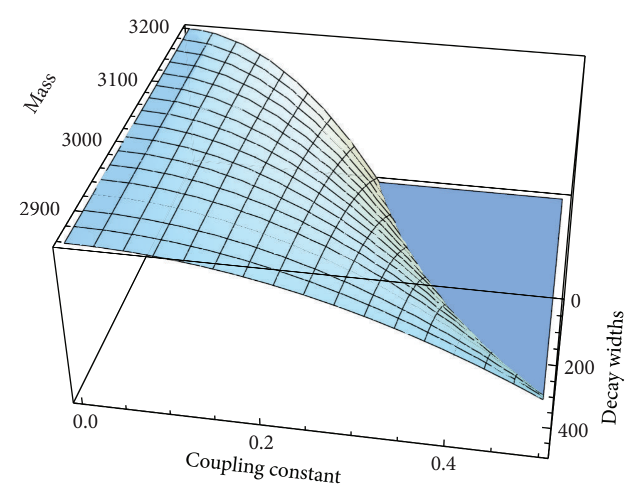


FIGURE 8: Variation of $1D_{s2}$ with its mass and coupling constant $g_{xh}$.

masses to be within the allowed range 2800 MeV to 3000 MeV, variation of their total OZI allowed two-body strong decay width has been plotted with respect to their mass and their corresponding coupling constant, in Figures 8 and 9, respectively.

Using the hadronic couplings obtained in Sections 3.1 and 3.2, $g_{xh} = 0.20$ and $g_{yh} = 0.40$, partial and total decay widths of these states are listed in first column of Table 7. Also,

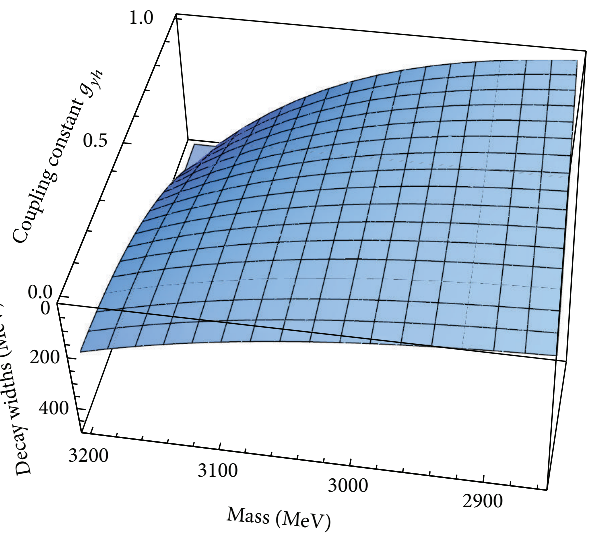


Figure 9: Variation of $1D'_{s2}$ with its mass and coupling constant.

these two states can mix through spin-orbit interaction or by some other mechanism and physically $D'_{s2}$ and $D_{s2}$ can be represented as the linear combination of ${}^3D_2$ and ${}^1D_2$ states as

$$\begin{pmatrix} 1D\left(2^-\right) \\ 1D'\left(2^-\right) \end{pmatrix} = \begin{pmatrix} \cos\theta_{1D} & \sin\theta_{1D} \\ -\sin\theta_{1D} & \cos\theta_{1D} \end{pmatrix} \begin{pmatrix} 1^3D_2 \\ 1^1D_2 \end{pmatrix}, \quad (11)$$

where $\theta_{1D}$ is the mixing angle between the two ${}^3D_2$ and ${}^1D_2$ states. In general the mixing angle between ${}^3L_l$ and ${}^1L_l$ in heavy quark limit is given by $\theta_{1D} = \arctan\sqrt{L/(L+1)}$. For this case the mixing angle corresponds to $L = 2$ and comes out to be $39.2^\circ \sim 39^\circ$. In Table 7, the last column gives partial decay widths by taking this mixing into account.

## 4. Conclusion

Due to advancement in high energy accelerators, large amount of information is available on heavy-light charm and bottom mesons. This information motivates theorists to explore more about these heavy-light mesons. These $D$ and $B$ meson states are studied by observing their decaying behavior, masses, their $J^P$ states, coupling constants, branching ratios, and so forth. Many models like heavy quark effective theory, quark pair creation model, potential models, and so forth, are framed to study these heavy-light mesons. Recently, LHCb predicted spin 1 and spin 3 strange charm mesons. In this paper, we use the heavy quark effective approach to study the recently observed spin 1 and spin 3 strange charm states. This theory treats the heavy quark as static and provides Lagrangian and decay widths formulas to the available states. This theory has adequately studied the previously determined experimental states and successfully allotted their positions in the charm and bottom spectroscopy.

Observation of spin 1 and spin 3 resonances of $D_s^*(2860)$ by LHCb has clearly indicated that there are two different states of $D_{sJ}^*(2860)$. In the last 5 years, various theoretical models [5–7, 10–19], which studied $D_{sJ}^*(2860)$, favored it as $1^3D_3$ state with narrow decay width. From the LHCb data, $D_{s3}^*(2860)$ state with $\Gamma = 53$ MeV can be correlated with this $D_{sJ}^*(2860)$ state. We too studied the decay behavior of $D_{s3}^*(2860)$ assuming it to be in the $1^3D_3$ state and calculated the hadronic coupling constant $g_{yh} = 0.40$. This value can be compared with the one obtained by Wang $g_{yh} = 0.52$ [25].

We also studied the remaining spin 1 observed state by LHCb $D_{s1}^*(2860)$, assuming it to be pure $1^3D_{1s}$ state and to be a mixture of $1^3D_1$ and $2^3S_1$ state. We study its decay channels ($D^*K, DK, D_s^*\eta, D_s\eta$) and $R$ value ($D^*K/DK$) calculated for the pure state ($R = 0.48$) which does not lie within the given experimental data ($R = 1.10$). So we adopted it to be as a mixture of radially excited $2^3S_1$ and orbitally excited $1^3D_1$. Using this interpretation, decay widths and $R$ value depend on mixing angle ($\theta$) and coupling constant $g_{xh}$. We studied the variation of partial widths with coupling constant $g_{xh}$ for some fixed values of mixing angle ($\theta$) which shows $DK$ is the dominant decay channel. In the variation of $R$ value with mixing angle ($\theta$), experimental $R$ value favors the large mixing angle. This large mixing angle implies the predominance of $2^3S_1$ state for $D_{s1}^*(2860)$. We obtained $R = 0.85$ corresponding to the mixing angle $-1.6 \le \theta \le -1.2$. Along with this mixing angle, we constrain the coupling constant $g_{xh}$ to be lying in the range $0.17 \le g_{xh} \le 0.20$. This obtained coupling value is close to the value given by Wang $g_{xh} = 0.19$ in [25].

Using these coupling constants, we also calculated the decay behavior of the spin partners of these states $1D_2$ and $1D'_{2s}$. These states are studied using two ways, first by considering them as pure states and secondly by taking their mixing into account. In both cases $D^*K$ is the dominating decay channel. Decay width for $1D'_{2s}$ as a pure state comes to be small indicating the presence of other decay modes. As we have only considered the decays to pseudoscalar mesons, there may be a possibility that decays to light vectors mesons may also be present for this state.

## Competing Interests

The authors declare that they have no competing interests.

## Acknowledgments

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