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Research Article

Flow Simulation of Suspension Bridge Cable Based on Lattice-Boltzmann Method

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Suspension bridge is a kind of bridge which uses cables as the main bearing structure. Suspension bridge has the characteristics of saving materials and weak stiffness. With the increase of the span of suspension bridge, wind induced vibration has resulted in injury of several suspension bridges, which leads to a significant loss. Thus, it is imperative to study the wind vibration mechanism of cables. As for this problem, this paper based on motion theory of mesoscopic particles performs flow simulation of cables by LBM which is different from traditional computing method of fluid mechanics. By calculating the distribution function of the distribution on the grid of uniform flow field, the macroscopic motion law of the flow field around cables can be obtained, which can provide reference for wind resistant design of suspension.

1. Introduction

Suspension bridges use cables as the main bearing structure of the superstructure and the cable support tower is anchored on both banks and ends of the bridge. The suspension bridge which was developed from the rope bridge initially appeared in the early nineteenth century. Compared with other kinds of bridge structures, the suspension bridge can be used to span a relatively long distance with fewer materials. Therefore, the suspension bridge is especially suitable for large span highway bridges. Allowing for this consideration, many large span bridges were built based on this structure at present. The suspension bridge is composed of cable, tower, anchorage, and bridge deck system and the cables are used as the main bearing component. Suspension bridge [1] has its own unique nature: it can be built relatively high, allowing boats to pass and therefore it can be built where the water is deep or the current is swift. Suspension is a cable associating the main cable with beam carrying loads. Each point in the suspension can only withstand tension and the tension is along the tangential direction of the suspension. Suspension is relatively soft and the stiffness is small. Thus, it is easy to generate fierce deflection and vibration under wind load [2]. The performance analysis of the suspension cable under wind load is necessary.

Presently, with the development of the theory of aerodynamics and computing technology, however, numerical simulation has become an important tool for wind resistance of bridges. But apparently the blunt body dynamic method is somehow imprecise and imperfect. For complicated blunt body dynamic flow problems in structural wind engineering, it is still difficult to solve Navier-Stokes equations. Even the simplified model, such as eddy viscosity model, cannot effectively reveal the intrinsic physical mechanism of the flow around the blunt body [3]. What is more, the CFD method based on the Navier-Stokes equation has a low computational efficiency and the progress of the efficiency is limited in spite of the rapid development of computer technology. Therefore, numerical simulation is carried out in this paper by LBM based on the mesoscopic particle velocity distribution function. This method can obtain smaller vortexing behavior under the same discrete rate. In addition, the evolution of the distribution function is completed on the local grid, which is suitable for large scale parallel computing and therefore LBM has higher computational efficiency, accuracy, and stability [4].

2. The Lattice Boltzmann Method

The Lattice Boltzmann method [5] (LBM) is one of the most important achievements in recent 20 years of computational fluid mechanics and different from the traditional numerical methods [6] for fluid calculation and modeling method describing the movement of the molecule. LBM regards the fluid as discrete system composed of a large number of mesoscopic particles. According to the movement characteristics of the particle, a simplified LB equation is established to calculate the evolution of particle distribution function.

The Lattice Boltzmann equation [7] (LBE) is linear, but actually its nonlinearity is embedded in the left side of the LBE. In LBM, the nonlinear convection term in the macroscopic method is replaced by the linear transfer process, which is similar to the method of solving the compressible flow characteristics. LBM can be easily implemented on a parallel processing computer as a result of the fact that the collision and streaming processes are local. LBE is an integropartial differential equation, so one of the difficulties in solving LBE is the complexity of the collision integral. In order to simplify the solving process, a collision function model with a simple operator instead of collision was proposed. LBE is a special discrete form of Boltzmann-BGK equation including discrete velocity, discrete time, and space discretization. Discrete time and space can be linked by discrete velocity of particles, which makes LBM have effective parallel computing ability and effective ability to deal with complex boundaries.

The Lattice Boltzmann evolution equation of the single relaxation time is

$$f_i\left(x + c_i\Delta t, t + \Delta t\right) - f_i\left(x, t\right) = -\frac{1}{\tau}\left[f_i\left(x, t\right) - f_i^{\text{eq}}\left(x, t\right)\right]. \quad (1)$$

In the formula, τ, c_i, f_i, and f_i^{eq} are the relaxation factor, discrete velocity, particle distribution function, and equilibrium distribution function, respectively.

In 1992, researchers including Yuehong Qian proposed the model DnQb [8]. n and b refer to space dimension and discrete velocity, respectively, and the equilibrium distribution function is

$$f_i^{\text{eq}} = \omega_i\rho\left[1 + \frac{\left(c_i \cdot u^{\text{eq}}\right)}{c_s^2} + \frac{\left(c_i \cdot u^{\text{eq}}\right)^2}{2c_s^4} - \frac{\left(u^{\text{eq}} \cdot u^{\text{eq}}\right)}{2c_s^2}\right]. \quad (2)$$

In the formula, u^{eq} is the macroscopic velocity. And c_s is sound velocity of the grid and ω_i is weight coefficient. The two parameters decide that the model of lattices depends on the selecting of c_i in discrete velocity model.

In this paper, we adopted D2Q9 model (shown as Figure 1):

$$c_i = \begin{cases} (0,0)\,c, & i = 0 \\ \left[\cos\frac{(i-1)\pi}{2}, \sin\frac{(i-1)\pi}{2}\right]c, & i = 1,2,3,4 \\ \sqrt{2}\left[\cos\frac{(2i-9)\pi}{4}, \sin\frac{(2i-9)\pi}{4}\right]c, & i = 5,6,7,8. \end{cases} \quad (3)$$

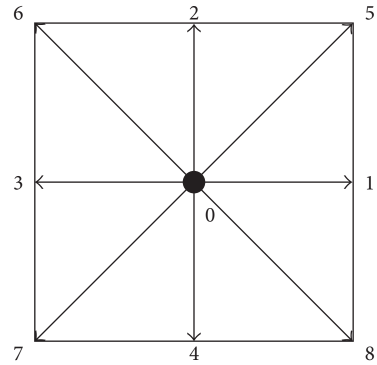

Figure 1: D2Q9 model.

Weight coefficient and sound velocity of lattices are as follows:

$$\omega_i = \begin{cases} \frac{4}{9}, & i = 0 \\ \frac{1}{9}, & i = 1,2,3,4 \\ \frac{1}{36}, & i = 5,6,7,8, \end{cases} \qquad c_s^2 = \frac{1}{3}c^2. \quad (4)$$

In the formula, $c = \Delta x/\Delta t$ is velocity of grid. Δx and Δt are grid length and time step, respectively.

The macroscopic velocity and momentum of fluid are

$$\rho = \sum_i f_i, \qquad \rho u = \sum_i c_i f_i. \quad (5)$$

The relationship between the fluid viscosity coefficient and the relaxation factor of the model is

$$\nu = \left(\tau - \frac{1}{2}\right)c_s^2\Delta t. \quad (6)$$

Generally, the calculation process of LBE is as follows:

(1) Initialize the distribution function:

$$f_i\left(x, 0\right) \quad \left(i = 1, 2, \ldots, b\right). \quad (7)$$

(2) Perform collision at t:

$$f_i'\left(x, t\right) = f_i\left(x, t\right) + \frac{1}{\tau}\left(x, t\right), \quad i = 1, 2, \ldots, b. \quad (8)$$

(3) Perform migration:

$$f_i\left(x + c_i\Delta t, t + \Delta t\right) = f_i'\left(x, t\right), \quad i = 1, 2, \ldots, b. \quad (9)$$

(4) Calculate macroscopic thermodynamic quantities:

$$\rho(x, t + \Delta t) = \sum_i f_i(x, t + \Delta t),$$
$$\rho u(x, t + \Delta t) = \sum_i c_i f_i(x, t + \Delta t). \quad (10)$$

(5) Repeat steps (2)–(4) until the terminal conditions are met.

In LBM, the accurate simulation of boundary conditions [9–11] is an important and crucial problem, because the boundary conditions are not easy to determine, which needs to confirm the distribution function of the boundary. At present, the types of boundary of LBM can be divided into heuristic schemes, dynamic schemes, and interpolation/extrapolation schemes. According to the type of boundary condition, it also can be divided into the velocity boundary and the pressure boundary. Additionally, there are some special artificial boundaries, such as the entrance, exit, infinity, and symmetry.

The rebound format is usually used for simulating the boundary condition of stationary solid or obstacle flow. It is mainly refers to the fact that the projectile of solid boundary will rebound to fluid field. We can easily know that $f_5 = f_7$, $f_2 = f_4$, $f_6 = f_8$, and f_7, f_4, and f_8 can be obtained from the streaming process.

In practical application, the velocity component is usually known. Thus, a method is proposed by Zou and He to calculate the 3 unknown equilibrium distribution functions under equilibrium conditions:

$$\rho = \frac{1}{1-u}\left[f_0 + f_2 + f_4 + 2(f_3 + f_6 + f_7)\right],$$
$$f_1 = f_3 + \frac{2}{3}\rho u,$$
$$f_5 = f_7 - \frac{1}{2}(f_2 - f_4) + \frac{1}{6}\rho u + \frac{1}{2}\rho u, \quad (11)$$
$$f_8 = f_6 + \frac{1}{2}(f_2 - f_4) + \frac{1}{6}\rho u - \frac{1}{2}\rho u.$$

Sometimes, the velocity of the exit is unknown. In these situations, we need to use extrapolation method to get the unknown distribution functions. For example,

$$f_{3,n} = 2 \cdot f_{3,n-1} - f_{3,n-2},$$
$$f_{6,n} = 2 \cdot f_{6,n-1} - f_{6,n-2}, \quad (12)$$
$$f_{7,n} = 2 \cdot f_{7,n-1} - f_{7,n-2}.$$

Through the research done before, we can know that, in traditional CFD, the convection term is nonlinear. What is more, for incompressible flow problems, the pressure term is implicit. In LBM, the integral differential equation, however, is transformed into a linear differential equation. Additionally, the distribution function is used to describe the motion of the fluid, which makes us not have to construct Poisson equation. But this BGK model is only valid for small Reynolds number. We have to further research the model for high Reynolds number.

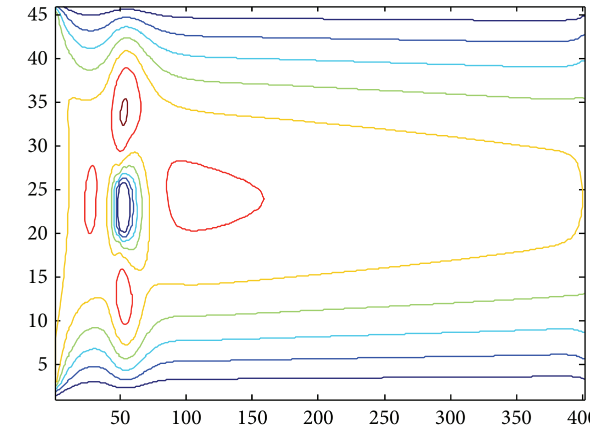

Figure 2: Streamline, Re = 20.

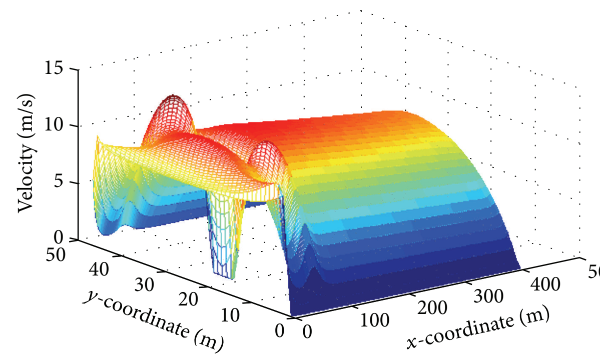

Figure 3: 3D graph, Re = 20.

3. Flow Simulation of Cables

Because cables of suspension bridges have a large dynamic response under wind load [12] and the theory of LBM suitable for blunt body flow [13], this paper simulates flow around cables by LBM in order to verify the feasibility and accuracy of LBM. We choose a rectangle with 400 meters' length and 45 meters' width as the computational domain. We suppose the wind velocity is 5 m/s. We chose the standard rebounded scheme as the surface free slip boundary and dealt with the flow inlet velocity boundary and the exit constant pressure boundary by the nonequilibrium rebounded scheme. As follows, we selected cables with different quantity to simulate the flow.

3.1. Single Flow Simulation

3.1.1. Re = 20. The barrier is a rectangular cylinder [14] of 5 m ∗ 5 m placed 50 meters away from the entrance. When the Reynolds number is 20, we obtain the streamline and three-dimensional graph by use of the MATLAB after 40000 steps of the calculation, as shown in Figures 2 and 3.

3.1.2. Re = 100. The barrier is a rectangular cylinder of 5 m ∗ 5 m placed 50 meters away from the entrance. When the Reynolds number is 100, we obtain the streamline and

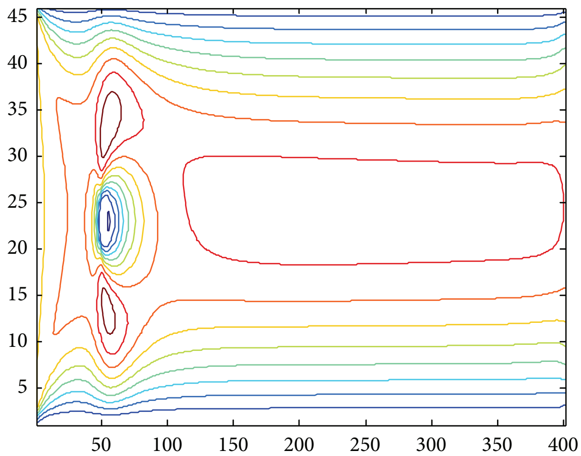

Figure 4: Streamline, Re = 100.

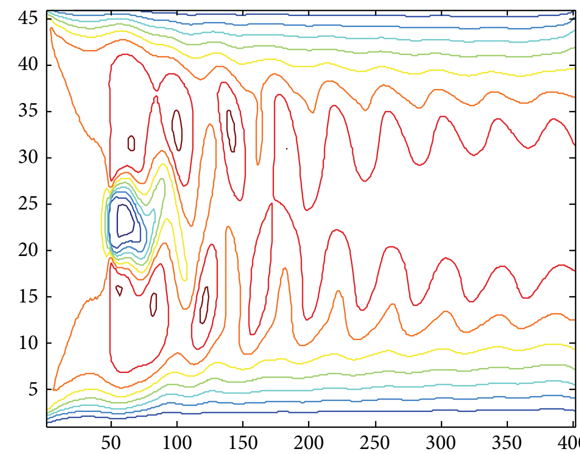

Figure 6: Streamline, Re = 1000.

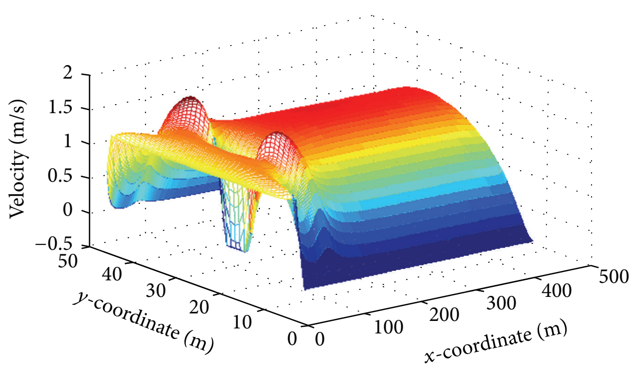

Figure 5: 3D graph, Re = 100.

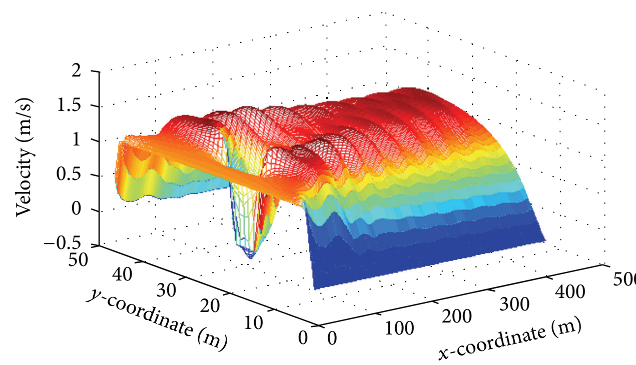

Figure 7: 3D graph, Re = 1000.

three-dimensional graph by use of the MATLAB after 40000 steps of the calculation, as shown in Figures 4 and 5.

3.1.3. Re = 1000. The barrier is a rectangular cylinder of 5 m ∗ 5 m placed 50 meters away from the entrance. When the Reynolds number is 1000, we obtain the streamline and three-dimensional graph by use of the MATLAB after 40000 steps of the calculation, as shown in Figures 6 and 7.

We can see from the flow around the wake that the variation of the flow field is obvious, and it will gradually tend to be stable. When the Reynolds number is 20, the flow field in front of the square cylinder surface has the maximum vorticity, while the vorticity of other regions is small. When the Reynolds number increases to 100, the flow field around the square cylinder becomes unstable, which is no longer a laminar flow. When the Reynolds number increases to 1000, the flow field after a long distance around the column is still in a state of volatility. By the comparison of Figures 2–7, we can see that, with the increase of Reynolds number [15, 16], steady flow changes to unsteady flow gradually.

3.2. Flow Simulation of Two Circular Cylinders in Tandem

3.2.1. Re = 20. The barriers are two rectangular cylinders of 5 m ∗ 5 m placed 50 meters away from the entrance. When the Reynolds number is 20, we obtain the streamline and three-dimensional graph by use of the MATLAB after 40000 steps of the calculation, as shown in Figures 8 and 9.

3.2.2. Re = 100. The barriers are two rectangular cylinders of 5 m ∗ 5 m placed 50 meters away from the entrance. When the Reynolds number is 100, we obtain the streamline and three-dimensional graph by use of the MATLAB after 40000 steps of the calculation, as shown in Figures 10 and 11.

We can see from the flow around the wake that the variation of the flow field is obvious, and it will gradually tend to be stable. When the Reynolds number is 20, the streamline has been closed at 400 m, while when the Reynolds number is 100, the streamline may be closed in a long distance. By comparison of Figures 8–11, we can see that, with the increase of Reynolds number, steady flow changes to unsteady flow gradually.

After single and double flow simulation, we can conclude that when the Reynolds number is small, flow field will become stable quickly. With the increase of Reynolds number, steady flow changes to unsteady flow gradually, which proves that LBM is not fit for numerical simulation with high Reynolds number. Therefore, we should carry out numerical simulation with small Reynolds number.

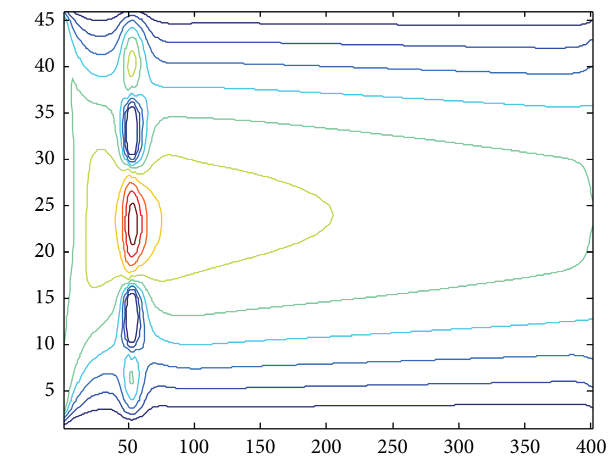

Figure 8: Streamline, Re = 20.

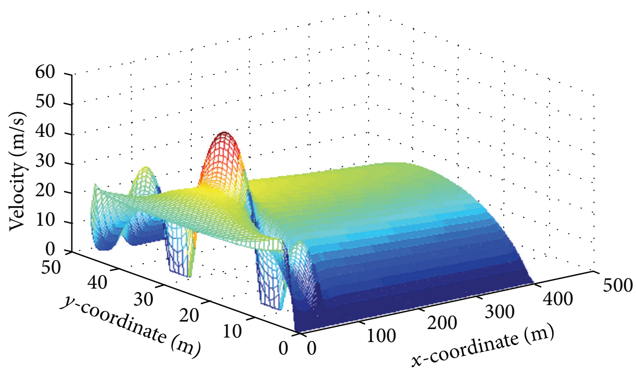

Figure 9: 3D graph, Re = 20.

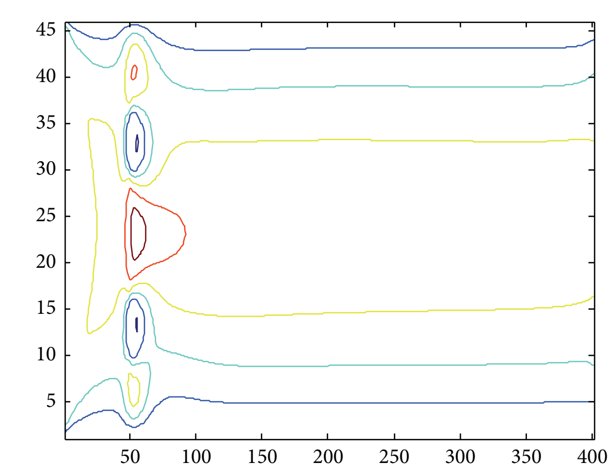

Figure 10: Streamline, Re = 100.

3.3. Flow around Three Cylinders in Tandem. The barriers are three rectangular cylinders of 5 m ∗ 5 m placed 50 meters away from the entrance. When the Reynolds number is 20, we obtain the streamline and three-dimensional graph by use of the MATLAB after 40000 steps of the calculation, as shown in Figures 12 and 13.

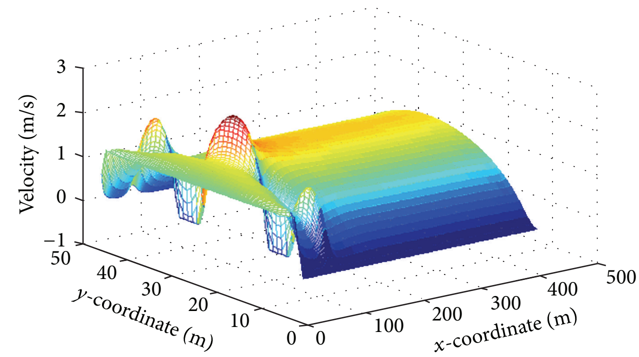

Figure 11: 3D graph, Re = 100.

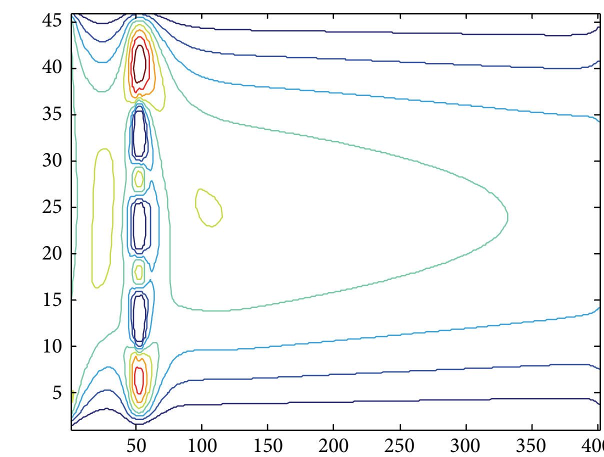

Figure 12: Streamline, Re = 20.

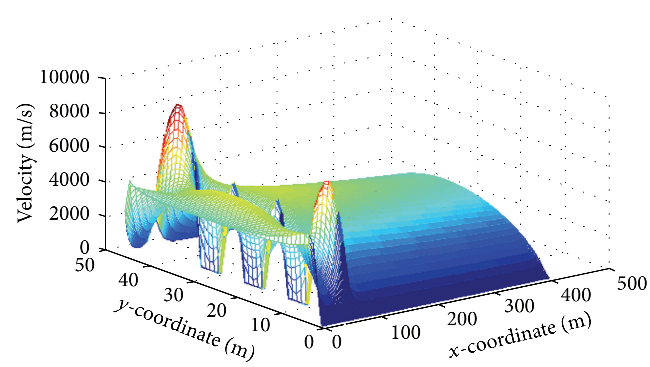

Figure 13: 3D graph, Re = 20.

We can see from the flow around the wake that the variation of the flow field is obvious, and it will gradually tend to be stable. The flow field at both sides of the side column varies greatly, but the flow field between the two columns varies little. By comparison of Figures 2 and 3, Figures 8 and 9, and Figures 12 and 13, we can see that the less the obstacles, the easier for flow field tending to be stable.

3.4. Flow Simulation of Double Column Parallel. The barriers are two rectangular cylinders of 5 m ∗ 5 m placed 50 meters

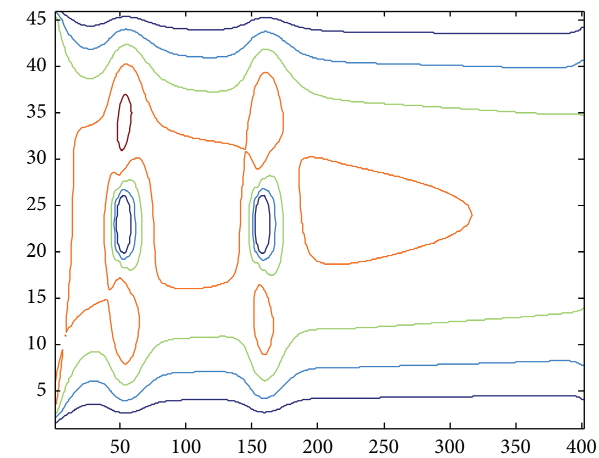

FIGURE 14: Streamline, Re = 20.

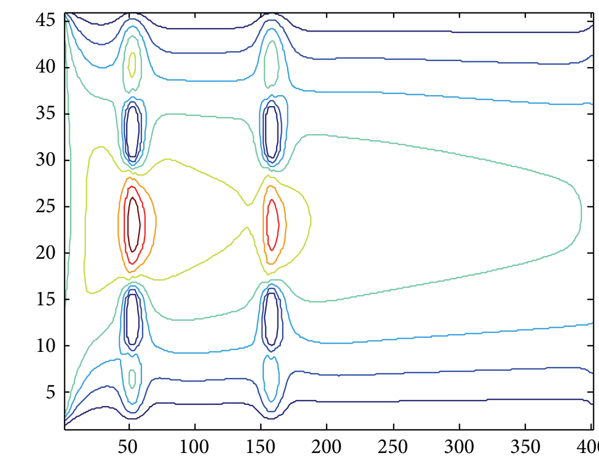

FIGURE 16: Streamline, Re = 20.

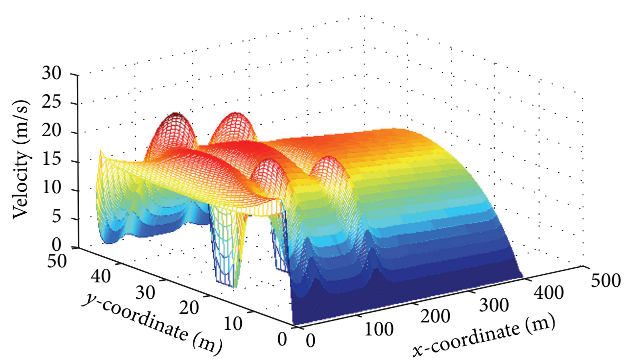

FIGURE 15: 3D graph, Re = 20.

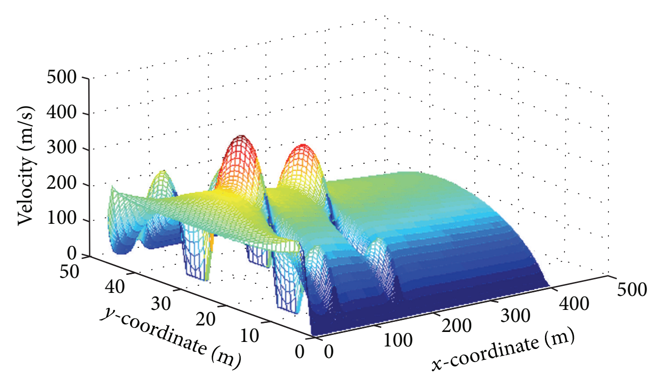

FIGURE 17: 3D graph, Re = 20.

and 155 meters away from the entrance, respectively. When the Reynolds number is 20, we obtain the streamline and three-dimensional graph by use of the MATLAB after 40000 steps of the calculation, as shown in Figures 14 and 15.

We can see from the flow around the wake that the variation of the flow field is obvious, and it will gradually tend to be stable. The flow field at both sides of the side column varies greatly, but the flow field between the two columns varies little. By comparison of Figures 8 and 9 and Figures 14 and 15, we can see that the less the obstacles, the easier for flow field tending to be stable.

3.5. Flow Simulation of Four Column Parallel. The barriers are four rectangular cylinders of 5 m $*$ 5 m placed 50 meters and 155 meters away from the entrance, respectively. When the Reynolds number is 20, we obtain the streamline and three-dimensional graph by use of the MATLAB after 40000 steps of the calculation, as shown in Figures 16 and 17.

We can see from the flow around the wake that the variation of the flow field is obvious, and it will gradually tend to be stable. By comparison of Figures 14 and 15 and Figures 16 and 17, we can see that the less the obstacles, the easier for flow field tending to be stable.

4. Conclusion

Because the accuracy of existing numerical simulation method is low, this paper puts forward a numerical simulation with high accuracy and good stability based on LBE. We compile the calculation program by use of MATLAB to simulate stationary flow of single, double, or multiple columns, and we obtain the streamlines and three-dimension graph which can be used to study the characteristics of cables around the flow. Through the calculation, we draw the conclusions: when the Reynolds number is high, steady flow changes to unsteady flow resulting in the fact that LBM is not suitable for numerical simulation with high Reynolds number, while when the Reynolds number is low, the flow is steady flow, and the variation of the flow field around cables is obvious. The flow field tends to be stable after a while, which is in accordance with practical theory. Thus, LBM is suitable for flow simulation around cables with small Reynolds number.

Competing Interests

The authors declare that they have no competing interests.

Acknowledgments

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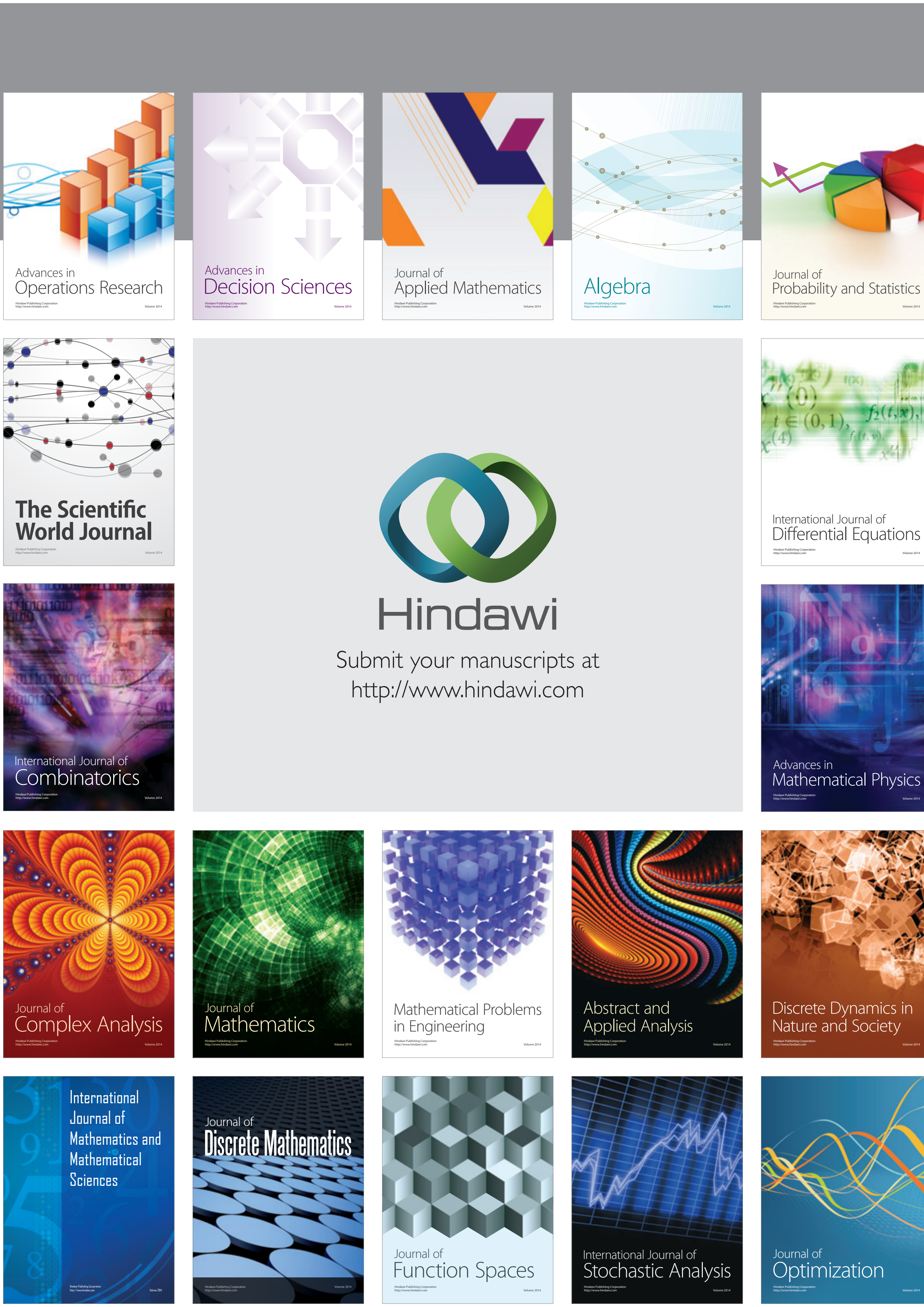
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