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Research Article

Chaos Analysis and Control of Relative Rotation System with Mathieu-Duffing Oscillator

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Chaos analysis and control of relative rotation nonlinear dynamic system with Mathieu-Duffing oscillator are investigated. By using Lagrange equation, the dynamics equation of relative rotation system has been established. Melnikov's method is applied to predict the chaotic behavior of this system. Moreover, the chaotic dynamical behavior can be controlled by adding the Gaussian white noise to the proposed system for the sake of changing chaos state into stable state. Through numerical calculation, the Poincaré map analysis and phase portraits are carried out to confirm main results.

1. Introduction

Chaos, as a kind of physical phenomenon, exists widely in various nonlinear dynamic systems [1–5]. The main ideal of chaos control approach is that one can use given unstable periodic orbits or external excitations to force chaotic system into stable system [6–9]. There are many ways to be used to suppress chaos, such as parametric driven [10–12] and time delayed approach [13, 14], and the Gaussian white noise as random phase control is an interesting one. The control of chaos we discuss in this paper is based on this case.

The theory of mechanics of relative rotation system was established by Carmeli in 1986 [15]; bifurcation and chaos of this system got rapid development in variety of areas. Shi et al. [16] studied the chaotic behavior and its control for a class of nonlinear dynamics equation of relative rotation system. Liu et al. [17] found stability and bifurcation for a coupled nonlinear relative rotation system with multitime delay feedbacks. Therefore, this system plays an important role in nonlinear dynamic systems.

Stochastic forces or random noise are the most frequently control strategies to be used in suppressing chaos. Ramesh and Narayanan [18] investigated the robustness in presence of uniform noise and found that the system would lose control while noise intensity was raided to a threshold level. Liu et al. [19] explored the effect of bounded noise on chaotic motion of Duffing oscillator under parametric excitation. Wu et al. [20] studied stochastic chaos and its control by the top Lyapunov exponent. Yin et al. [21] proved the peculiar solitary waves are more likely to be chaos by using the Melnikov theory and found that the system can be well controlled when the frequency of the perturbation surpasses the peculiar perturbation frequency with fixed parameters of the unperturbed system. Noise, as random phase, has been used in studying the control of chaos in the paper. So as a kind of effective method of chaos control, whether in theory or in the practical application, the noise has some of the research significance.

In this paper, we explore chaos analysis and control in relative rotation system with Mathieu-Duffing oscillator. The paper is organized as follows. The dynamics equation of relative rotation system is established in Section 2. In Section 3, the prediction of chaotic motion is given by Melnikov's method. In Section 4, the Gaussian white noise can increase or decrease the threshold values of chaos, so as to realize the control of chaos.

2. Dynamics Equation of Relative Rotation System with Mathieu-Duffing Oscillator

Mathieu-Duffing system is a class of typical vibration system, and researching its nonlinear dynamic characteristics shows to be extremely essential. Considering Mathieu-Duffing oscillator expression form as follows [22, 23]

$$f(t) = N_1(t)x + N_2(t)x^3, \quad (1)$$

where

$$N_1(t) = cK(t),$$
$$N_2(t) = bK(t), \quad (2)$$
$$K(t) = k_0^2 + k_1\cos(\omega_1 t)$$

$N_1(t)$, $N_2(t)$ are linear torsional stiffness of the system. For two quality relative rotation systems of Mathieu-Duffing oscillator, the kinetic energy is

$$E = \frac{1}{2}J_1\dot{\theta}_1^2 + \frac{1}{2}J_2\dot{\theta}_2^2, \quad (3)$$

where J_1, J_2 are moment of inertia and θ_i, $\dot{\theta}_i$ $(i = 1,2)$ are rotational angle and speed of rotational angle, respectively. The potential energy of system is

$$U = \frac{1}{2}N_1(t)\theta^2 + \frac{1}{2}N_2(t)\theta^4. \quad (4)$$

Generalized forces are

$$Q_1 = F_1^1\frac{\partial\theta_1}{\partial q_1} + F_2^2\frac{\partial\theta_2}{\partial q_1},$$
$$Q_2 = F_1^1\frac{\partial\theta_1}{\partial q_2} + F_2^2\frac{\partial\theta_2}{\partial q_2}, \quad (5)$$

where

$$F_1^1 = T_1 - A(\dot{\theta}_1 - \dot{\theta}_2),$$
$$F_2^2 = T_2 - A(\dot{\theta}_2 - \dot{\theta}_1), \quad (6)$$

where T_1, T_2 are generalized external force, q_1, q_2 are generalized coordinates, and A is linear damping coefficient. The Lagrange equation is

$$\frac{d}{dt}\frac{\partial E}{\partial \dot{q}_j} - \frac{\partial E}{\partial q_j} + \frac{\partial U}{\partial q_j}. \quad (7)$$

Substituting (1)–(5) into Lagrange equation (7), we have

$$J_1\ddot{\theta}_1 + cK(t)(\theta_1 - \theta_2) + bK(t)(\theta_1 - \theta_2)^3 + A(\dot{\theta}_1 - \dot{\theta}_2) = T_1, \quad (8)$$

$$J_2\ddot{\theta}_2 + cK(t)(\theta_2 - \theta_1) + bK(t)(\theta_2 - \theta_1)^3 + A(\dot{\theta}_2 - \dot{\theta}_1) = T_2, \quad (9)$$

where $\ddot{\theta}_1, \ddot{\theta}_2$ are acceleration of rotational angle of moment of inertia. Combining (8) and (9), we have

$$(\ddot{\theta}_1 - \ddot{\theta}_2) + \frac{J_1 + J_2}{J_1J_2}cK(t)(\theta_1 - \theta_2) + \frac{J_1 + J_2}{J_1J_2}bK(t)(\theta_1 - \theta_2)^3 + \frac{J_1 + J_2}{J_1J_2}A(\dot{\theta}_1 - \dot{\theta}_2) = \frac{J_2T_1 - J_1T_2}{J_1J_2}. \quad (10)$$

Let

$$x = \theta_1 - \theta_2,$$
$$\dot{x} = \dot{\theta}_1 - \dot{\theta}_2,$$
$$\ddot{x} = \ddot{\theta}_1 - \dot{\theta}_2,$$
$$\frac{J_2T_1 - J_1T_2}{J_1J_2} = F\cos(\omega_2 t), \quad (11)$$
$$a = \frac{J_1 + J_2}{J_1J_2},$$
$$\lambda = \frac{J_1 + J_2}{J_1J_2}A.$$

Hence, the above equation turns to

$$\ddot{x} + a(k_0^2 + k_1\cos(\omega_1 t))(cx + bx^3) + \lambda\dot{x} = F\cos(\omega_2 t). \quad (12)$$

Equation (12) is dynamics equation of relative rotation system with Mathieu-Duffing oscillator.

3. Melnikov's Method Analysis

Melnikov's method [24–26] is an effective approach to predict chaotic behavior, its basic idea mainly makes the dynamics system into a Poincaré map system, and we obtain chaotic properties by studying conditions of the mapping system whether there exist homoclinic orbits or heteroclinic orbits.

Equation (12) can be transformed into first-order equation

$$\dot{x}_1 = x_2,$$
$$\dot{x}_2 = -ack_0^2x_1 - abk_0^2x_1^3 - ak_1\cos(\omega_1 t)(cx_1 + bx_1^3) - \lambda x_2 + F\cos(\omega_2 t). \quad (13)$$

Small parameter ε is added to the nonlinear term of (13) and it may be written as

$$\dot{x}_1 = x_2,$$
$$\dot{x}_2 = -ack_0^2x_1 - abk_0^2x_1^3 - \varepsilon(ak_1\cos(\omega_1 t)(cx_1 + bx_1^3) + \lambda x_2 - F\cos(\omega_2 t)). \quad (14)$$

When $\varepsilon = 0$, the above equation turns into

$$\dot{x}_1 = x_2,$$
$$\dot{x}_2 = -ack_0^2x_1 - abk_0^2x_1^3. \quad (15)$$

Equation (15) is a unperturbed Hamilton system and its Hamiltonian is

$$H(x_1, x_2) = \frac{1}{2}x_2^2 + \frac{1}{2}ack_0^2x_1^2 + \frac{1}{4}abk_0^2x_1^4. \quad (16)$$

There exist heteroclinic orbits and they satisfy

$$\dot{x}_1 = x_2,$$
$$H\left(\pm\sqrt{-\frac{c}{b}}, 0\right) = \frac{1}{2}x_2^2 + \frac{1}{2}ack_0^2x_1^2 + \frac{1}{4}abk_0^2x_1^4. \quad (17)$$

Then the parametric equations of two heteroclinic orbits are

$$x_1(t) = \pm\sqrt{-\frac{c}{b}}\tanh\left(\frac{\sqrt{2ac}}{a}k_0t\right),$$
$$x_2(t) = \pm\frac{ck_0}{2}\sqrt{-\frac{2a}{b}}\frac{1}{\cosh^2}\left(\frac{\sqrt{2ac}}{a}k_0t\right). \quad (18)$$

Melnikov function can be given by

$$\begin{aligned} M(t_0) &= \int_{-\infty}^{+\infty} x_2(t) \\ &\cdot \left[-ak_1\cos(\omega_1(t+t_0))\left(cx_1(t)+bx_1(t)^3\right)\right. \\ &\left.-\lambda x_2(t) + F\cos(\omega_2(t+t_0))\right]dt \\ &= \pm ack_1\int_{-\infty}^{+\infty} x_1(t)x_2(t)\cos(\omega_1(t+t_0))\,dt \\ &\pm abk_1\int_{-\infty}^{+\infty} x_1(t)^3x_2(t)\cos(\omega_1(t+t_0))\,dt \\ &-\lambda\int_{-\infty}^{+\infty} x_2(t)^2\,dt \pm F\int_{-\infty}^{+\infty} x_2(t)\cos(\omega_2(t+t_0))\,dt, \end{aligned} \quad (19)$$

where

$$\begin{aligned} &\int_{-\infty}^{+\infty} x_1(t)x_2(t)\cos(\omega_1(t+t_0))\,dt \\ &\quad= \int_{-\infty}^{+\infty} x_1(t)x_2(t)\sin(\omega_1t)\,dt\sin(\omega_1t_0), \\ &\int_{-\infty}^{+\infty} x_1(t)^3x_2(t)\cos(\omega_1(t+t_0))\,dt \\ &\quad= \int_{-\infty}^{+\infty} x_1(t)^3x_2(t)\sin(\omega_1t)\,dt\sin(\omega_1t_0), \\ &\int_{-\infty}^{+\infty} x_2(t)\cos(\omega_2(t+t_0))\,dt \\ &\quad= \int_{-\infty}^{+\infty} x_2(t)\cos(\omega_2t)\,dt\cos(\omega_2t_0). \end{aligned} \quad (20)$$

Let

$$\begin{aligned} Z_1 &= \int_{-\infty}^{+\infty} x_2(t)^2\,dt, \\ Z_2 &= \int_{-\infty}^{+\infty} x_1(t)x_2(t)\sin(\omega_1t)\,dt, \\ Z_3 &= \int_{-\infty}^{+\infty} x_1(t)^3x_2(t)\sin(\omega_1t)\,dt, \\ Z_4 &= \int_{-\infty}^{+\infty} x_2(t)\cos(\omega_2t)\,dt. \end{aligned} \quad (21)$$

Substituting $Z_1 - Z_4$ into Melnikov function, we have

$$\begin{aligned} M(t_0) = -\lambda Z_1 &\pm (ack_1Z_2 + abk_1Z_3)\sin(\omega_1t_0) \\ &\pm FZ_4\cos(\omega_2t). \end{aligned} \quad (22)$$

When $\omega_1 = \omega_2 = \sqrt{ack_0^2}$, (22) can be written as

$$\begin{aligned} &M(t_0) \\ &= -\lambda Z_1 \\ &\quad\pm\sqrt{(ack_1Z_2+abk_1Z_3)^2+(FZ_4)^2}\sin(\omega_1t_0+\varphi), \end{aligned} \quad (23)$$

where

$$\cos\varphi = \frac{ack_1Z_2+abk_1Z_3}{\sqrt{(ack_1Z_2+abk_1Z_3)^2+(FZ_4)^2}},$$
$$\sin\varphi = \frac{FZ_4}{\sqrt{(ack_1Z_2+abk_1Z_3)^2+(FZ_4)^2}}. \quad (24)$$

Because of $M(t_0) = 0$, we have

$$\sin(\omega_1t_0+\varphi) = \pm\frac{\lambda Z_1}{\sqrt{(ack_1Z_2+abk_1Z_3)^2+(FZ_4)^2}}. \quad (25)$$

Due to $|\sin(\omega_1t_0+\varphi)| \le 1$, then

$$\begin{aligned} &\left|\frac{\lambda Z_1}{\sqrt{(ack_1Z_2+abk_1Z_3)^2+(FZ_4)^2}}\right| \le 1, \\ &\frac{\partial M(t_0)}{\partial t_0} = \pm\omega_1\sqrt{(ack_1Z_2+abk_1Z_3)^2+(FZ_4)^2} \\ &\quad\cdot\cos(\omega_1t_0+\varphi) \ne 0. \end{aligned} \quad (26)$$

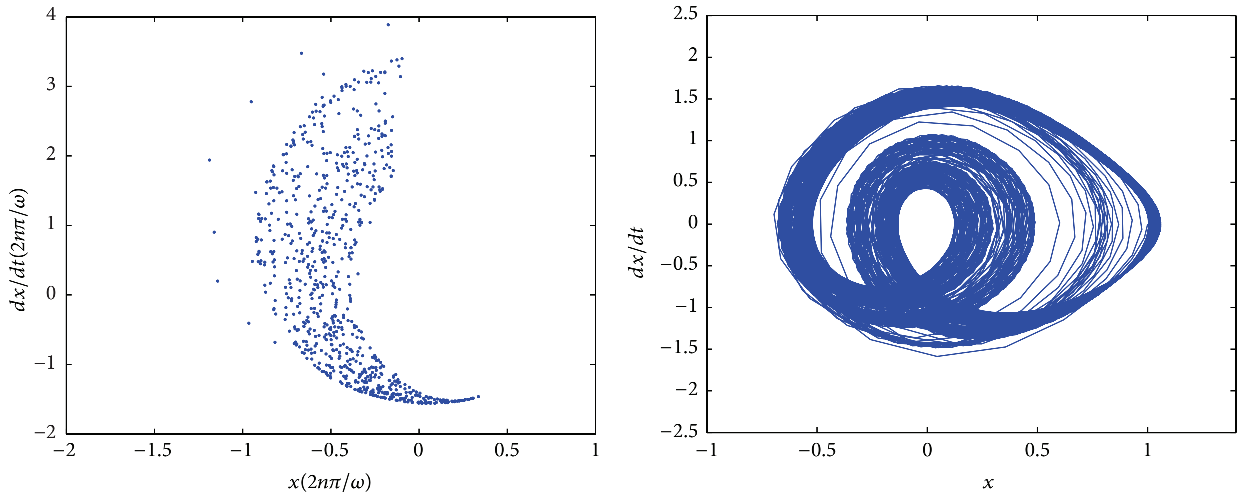
That is, $\cos(\omega_1t_0+\varphi) \ne 0$, so $\sin(\omega_1t_0+\varphi) \ne 0$; we can know

$$\left|\frac{\lambda Z_1}{\sqrt{(ack_1Z_2+abk_1Z_3)^2+(FZ_4)^2}}\right| < 1. \quad (27)$$

There exists a sufficient small ε, making $M(t_0) = 0$ and $\partial M(t_0)/\partial t_0 \ne 0$; therefore, the system produces chaotic behavior.

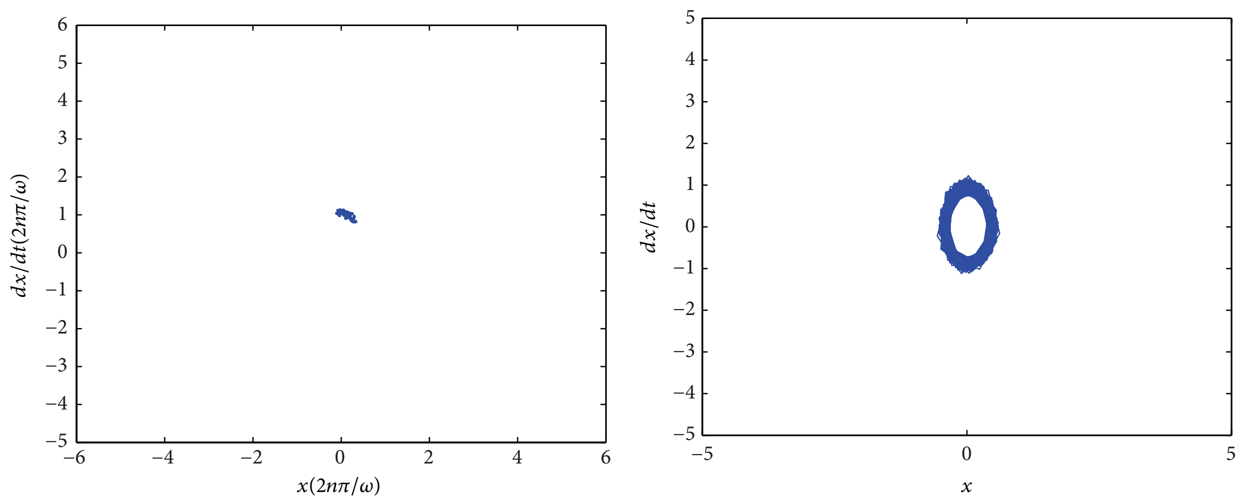
For system (13), taking parameters $ack_0^2 = 4$, $abk_0^2 = -4$, $ack_1 = 1$, $abk_1 = -1$, $\lambda = 0.5$, $F = 1.42$, and $\omega_1 = \omega_2 = 2$ with the initial conditions $x(0) = 1$, $\dot{x}(0) = 0$, the Poincaré map and phase portraits were plotted by using the numerical simulation in Figure 1.

From Figure 1, we can see that the Poincaré map has chaotic attractors, phase portraits show nonoverlapping and are disorganized, and these can prove that relative rotation system with Mathieu-Duffing oscillator is chaos.



(a) (b)

FIGURE 1: (a) Poincaré map; (b) phase portraits.



(a) (b)

FIGURE 2: (a) Poincaré map with $\sigma = 0.4$; (b) phase portraits with $\sigma = 0.4$.

4. Chaos Control of Relative Rotation System with Mathieu-Duffing Oscillator Using Noise

Adding Gaussian white noise, (13) can be written as

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= -ack_0^2 x_1 - abk_0^2 x_1^3 - ak_1 \cos(\omega_1 t)\left(cx_1 + bx_1^3\right) \\ &\quad - \lambda x_2 + F\cos(\omega_2 t + \sigma\xi(t)), \end{aligned} \tag{28}$$

where $\xi(t)$ denotes standard Gaussian white noise; it satisfies

$$\begin{aligned} E\xi(t) &= 0, \\ E\xi(t)\xi(t+\tau) &= \zeta(\tau), \end{aligned} \tag{29}$$

where $\zeta(\tau)$ is Dirac-Delta function and σ is the intensity of noise.

Taking the same parameters and initial conditions as Section 3. Equation (28) can be written:

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= -4x_1 + 4x_1^3 - \cos(2t)\,x_1 + \cos(2t)\,x_1^3 - 0.5x_2 \\ &\quad + 1.42\cos(2t + \sigma\xi(t)). \end{aligned} \tag{30}$$

Making the Poincaré map as

$$\begin{aligned} &\Sigma \longrightarrow \Sigma, \\ &\Sigma\left\{(x(t), \dot{x}(t)) \mid t = 0, \frac{2\pi}{\omega_2}, \frac{4\pi}{\omega_2}, \dots\right\} \subseteq \mathbb{R}^2. \end{aligned} \tag{31}$$

Under the given initial conditions, the solution of differential equation (30) is solved by the fourth-order Runge-Kutta method and the solution is plotted for every $T = 2\pi/\omega_2$. After deleting the first 100 dots, we plot the Poincaré map by using surplus 200 iteration dots, when the intensity of noise $\sigma = 0.4$; it is shown in Figure 2(a).

The Poincaré map appears to be a stable stochastic attractor, which means that the system is stable. To further verify the obtained results, we plotted the phase portraits. Taking the same intensity of noise, the phase portraits are shown in Figure 2(b). We can see from Figure 2(b) that the phase portraits turned into a regular annulus. It is proved that chaotic behavior was suppressed, and the system changed chaos state into stable state.

5. Concluding Remarks

We investigate the chaos analysis and control of relative rotation system with Mathieu-Duffing oscillator in this paper. By analysis of Melnikov's method, we confirm chaotic behavior of this system under the given parameters and initial conditions. Adding the Gaussian white noise to phase of system, we plot the Poincaré map and phase portraits under the intensity of noise $\sigma = 0.4$. We get that the Poincaré map changed chaotic attractors into a stable attractor, and phase portraits also displayed regular state, so the relative rotation system with Mathieu-Duffing oscillator eventually becomes stable.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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