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Research Article

Conditions of Perfect Imaging in Negative Refraction Materials with Gain

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Light propagation is analyzed in a negative refraction material (NRM) with gain achieved by pumping. An inherent spatial "walk-off" between the directions of phase propagation and energy transfer is known to exist in lossy NRMs. Here, the analysis is extended to the case where the NRM acts as an active material under various pumping conditions. It is shown that the condition for perfect imaging is only possible for specific wavelengths under special excitation conditions. Under excessive gain, the optical imaging can no longer be perfect.

1. Introduction

Negative refraction is known to offer a wide range of potential applications [1–4]. However, losses, which are an inherent feature of the negative refraction, present a major impediment to the performance of NRMs [5–9]. To overcome these problems, NRMs with gain were proposed to compensate the losses, even to turn the materials into amplified systems. Nevertheless, it is often stated that the gain will destroy the negative refraction due to causality considerations [10], although the statement was disputed by a theory demonstrating that negative refraction may be preserved in a limited spectral region [11, 12].

Common methods to introduce gain in NRMs include optical parametric amplification (OPA) [13] and externally pumped gain materials [14–18]. Optical imaging needs to collect both propagating and evanescent waves. However, only within a limited range may the wave vectors receive gain from OPA because of the strict phase-matching condition, the application of OPA to achieve perfect imaging in NRMs is not possible.

In this paper, we demonstrate that, under the action of the pumping gain, lossless and amplified light propagation may occur in a special spectral window of the NRM. The propagation behavior is shown to be closely related to the dispersion and pumping configuration. Propagation in NRMs is also examined in different pumping configurations.

1.1. Spatial "Walk-Off" in Lossy NRMs. Light incidents from free space onto a homogeneous, isotropic, lossy NRM, of permittivity $\varepsilon_r(\omega) = \varepsilon' + i\varepsilon''$ and permeability $\mu_r(\omega) = \mu' + i\mu''$, were studied in detail [8]. The complex effective refractive index is then defined as $n^2(\omega) = \varepsilon_r(\omega)\mu_r(\omega)$ or $n(\omega) = n' + in''$. In free space, the incident wave vector $\vec{k}$ is real, while in the lossy NRM, the wave vector $\vec{q} = q_x\hat{x} + (q_z' + iq_z'')\hat{z}$ is complex. At a given optical frequency ω, this implies that $\vec{q}^{\,2} = n^2(\omega)k^2$ for both the propagating wave ($|q_x| < |n(\omega)k|$) and the evanescent one ($|q_x| > |n(\omega)k|$).

To analyze light propagation in the NRM, the phase and group velocities are expressed as $\vec{v}_p = (\omega/|\vec{q}|^2)\vec{q}$ and $\vec{v}_g = \nabla\omega(\vec{q}) = ((A - iB)/(A^2 + B^2))\vec{q}$, where $A = d[\mathrm{Re}(n^2(\omega))k^2]/d\omega$ and $B = d[\mathrm{Im}(n^2(\omega))k^2]/d\omega$ are determined by the NRM dispersion. The energy propagation is approximately determined by the group velocity under the assumption of low losses [19–21]. The Poynting vector can also be used to define the energy propagation.

For complex vectors $\vec{v}_p$ and $\vec{v}_g$, the direction of the phase propagation and energy transfer in the wave packet are determined by their real parts [22]:

$$\vec{v}_p = \frac{\omega}{|\vec{q}|^2}(q_x\hat{x} + q_z'\hat{z}), \tag{1}$$

$$\vec{v}_g = \frac{A}{A^2+B^2}(q_x\hat{x} + q_z'\hat{z}) + \frac{B}{A^2+B^2}q_z''\hat{z}. \tag{2}$$

In an ideal NRM, where the refractive index is negative without losses, the phase velocity and group velocity are strictly antiparallel [1, 19]. However, (1) and (2) show that the group velocity is no longer antiparallel because of the contribution of the last term in (2). This *spatial "walk-off,"* that is, the noncollinearity between the phase propagation and the energy transfer, becomes obvious in a homogenous, isotropic, lossy NRM. The angle between the phase velocity and the group velocity is

$$\delta = \theta_p + \theta_g, \tag{3}$$

with the "walk-off" angle defined as $180^\circ - \delta$.

The propagation behavior is discussed here for both propagating and evanescent waves. The dispersive curve is described as the Lorentz model, with $\varepsilon_{r\,\mathrm{host}}(\omega) = 1+\omega_p^2/(\omega_p^2 - \omega^2 - i\gamma_1\omega)$, $\mu_{r\,\mathrm{host}}(\omega) = 1 + \omega_p^2/(\omega_p^2 - \omega^2 - i\gamma_2\omega)$, and $\omega_p = 100\times10^{12}\,\mathrm{s}^{-1}$, $\gamma_1 = 3\times10^{12}\,\mathrm{s}^{-1}$, $\gamma_2 = 5\times10^{12}\,\mathrm{s}^{-1}$ in Figure 1(a) for the real and imaginary parts of the refractive index. For a typical propagating wave with $|q_x| < |q|$, the size of the "walk-off" is numerically simulated as shown in Figure 1(b). The analysis of the "walk-off" can be extended to evanescent waves with $|q_x| > |q|$, where it is found that the "walk-off" dramatically increases with $|q_x|$, also shown in Figure 1(b).

For perfect focusing, the "walk-off" appearing at different $|q_x|$ should be suppressed. It was shown that this goal can be achieved by a pumping gain scheme [18]. Here, we show, in a pumped four-level model of signal amplification, that the realization of perfect imaging is possible only for a specific wavelength under strict pumping condition.

2. NRMs with Pumping Gain

Four-level systems represent conventional gain media. The intensity of light in a chosen spectral interval can be amplified in NRMs by introducing an extra term in the electrical field susceptibility [14, 17, 23]. It is assumed here that the gain medium is pumped in the linear regime, and no gain saturation arises. Accordingly, the population of the ground level N_1 (which can be considered as the population of the gain medium) is much larger than in the other three levels as per the usual pumping condition. With the definition of polarization $\vec{P} = \chi_{\mathrm{ex}}\varepsilon_0\vec{E}$, the permittivity, with the extra term in the susceptibility, is given by

$$\begin{aligned}\varepsilon_r(\omega) &= \varepsilon_{r\mathrm{host}} + \chi_{\mathrm{ex}} \\ &= 1 + \frac{\omega_p^2}{\omega_p^2 - \omega^2 - i\gamma_1\omega} - \frac{\sigma\Gamma_{\mathrm{pump}}N_1\tau_{32}/\varepsilon_0}{\omega_x^2 - \omega^2 - i\gamma_{\mathrm{ex}}\omega}.\end{aligned} \tag{4}$$

As shown below, the last term of (4) is crucial to the performance of the gain-compensated NRMs. The components of the effective refractive index, n' and n'', are shown in Figure 2.

Whereas the optical losses in the NRM can be effectively compensated by pumping, as shown in Figure 2(b), an amplification of the input signal is achievable; the elimination of the "walk-off" depends on both the real and imaginary parts of the refractive index. The ideal case is the one with $n' = -1$ and $n'' = 0$, giving rise to perfect imaging [1, 6]. However, as shown in Figure 2, this condition holds only at the optimal frequency where $\omega = 121.28\times10^{12}\,\mathrm{s}^{-1}$ under appropriate pumping conditions.

Optical imaging with resolution above or below the diffraction limit depends on the system's ability to recover the wavevector's component for either propagating or evanescent waves. Figure 3(b) shows the size of the "walk-off" for different $|q_x|$ with the appropriate pumping rate of $0.21\times10^9\,\mathrm{s}^{-1}$. The propagating waves correspond to $|q_x| < |q|$, while the evanescent ones correspond to $|q_x| > |q|$. After introducing the pumping gain, a red shift is observed in the n' curve. At the optimal frequency ($\omega = 121.28\times10^{12}\,\mathrm{s}^{-1}$) where $n' = -1$, the angles between the group and phase velocities are strictly antiparallel for all $|q_x|$. Because of the antiparallel directions of the energy transfer and phase propagation, the spatial "walk-off" is suppressed, so that the ability of directional transmission (for the propagating waves) and perfect focusing (for evanescent waves in the near-field) will be preserved. Thus, the pumping can effectively cancel the losses only in a limited spectral region, under appropriate pumping conditions. This conclusion is in agreement with those reported in [11, 12].

By contrast, the propagation in an active NRM under excessive pumping exhibits a peculiar behavior. Figure 3(c) shows the "walk-off" angles at different $|q_x|$ for excessive pumping rate (here $\Gamma_{\mathrm{pump}} = 0.48\times10^9\,\mathrm{s}^{-1}$) at the frequency where $n' = -1$ (here $\omega = 121.44\times10^{12}\,\mathrm{s}^{-1}$). The δ angles are then larger than 180°, indicating that the "walk-off" reappears, with the respective angles $180^\circ + \delta$. The "walk-off" becomes more significant at larger $|q_x|$. It also shows that δ increases dramatically with the increase of $|q_x|$ for the evanescent wave with $|q_x| > |q|$. Hence, the perfect focus for the near-field component is impossible under excessive pumping. Notice that because of the red shift in $n'(\omega)$, the "walk-off" is suppressed at the frequency of $121.20\times10^{12}\,\mathrm{s}^{-1}$, where $n'(\omega) \approx -1.3$ (the red arrow in Figure 3(c)). However, perfect lensing requires $n'(\omega) = -1$ [1, 6], hence the perfect focus cannot be obtained under excessive pumping.

In order to achieve perfect focusing, the pumping rates should be reduced and the pumping central frequencies should be blue-shifted, as shown in Figure 3(c). Light with all values of $|q_x|$ can then perfectly focus through the slab.

3. Conclusions

To conclude, we have analyzed the effect of gain on the negative refraction in NRMs. In a lossy NRM, even though it is isotropic and homogeneous, the group and phase velocities

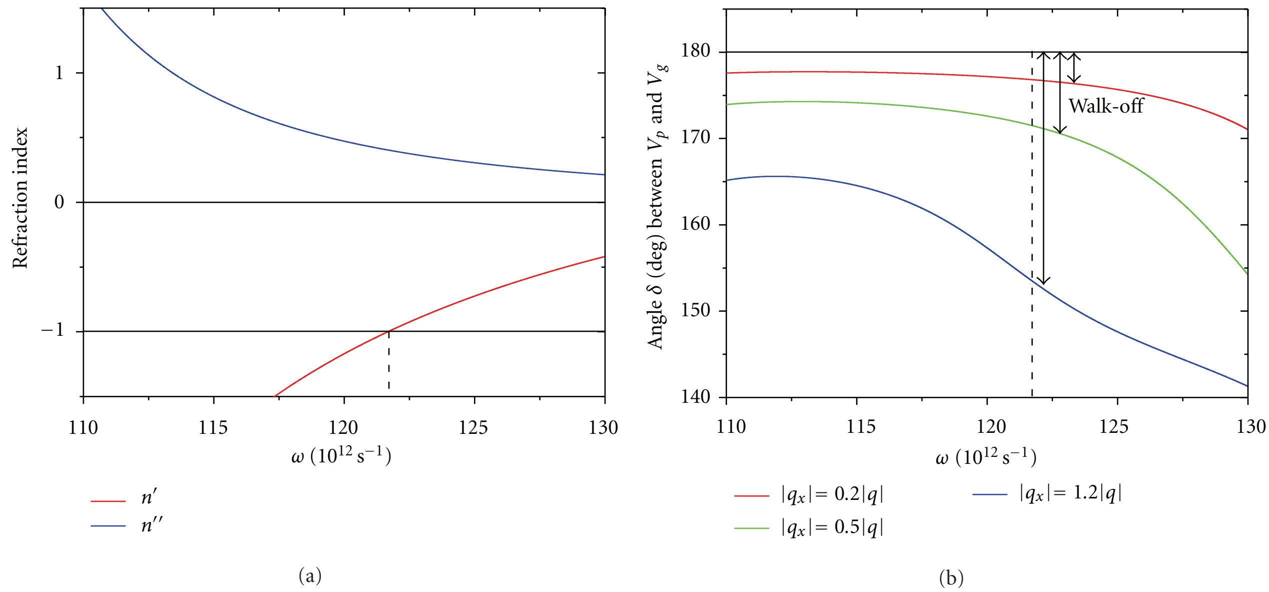

Figure 1: (a) The refractive index of the NRM slab. The red and blue lines show the real and imaginary parts of the refractive index $n(\omega)$, respectively. The frequency is $\omega = 121.67 \times 10^{12}\ \mathrm{s}^{-1}$, where $\varepsilon_r(\omega) \approx \mu_r(\omega) \approx -1$ [15]. (b) $\vec{v_p}$ and $\vec{v_g}$ are not strictly antiparallel ($\delta < 180^\circ$) in the lossy NRM slab, featuring the "walk-off" angle about 3.2° at $|q_x| = 0.2|q|$, 8.5° at $|q_x| = 0.5|q|$, and 26.4° at $|q_x| = 1.2|q|$. The carrier frequency is $\omega = 121.67 \times 10^{12}\ \mathrm{s}^{-1}$.

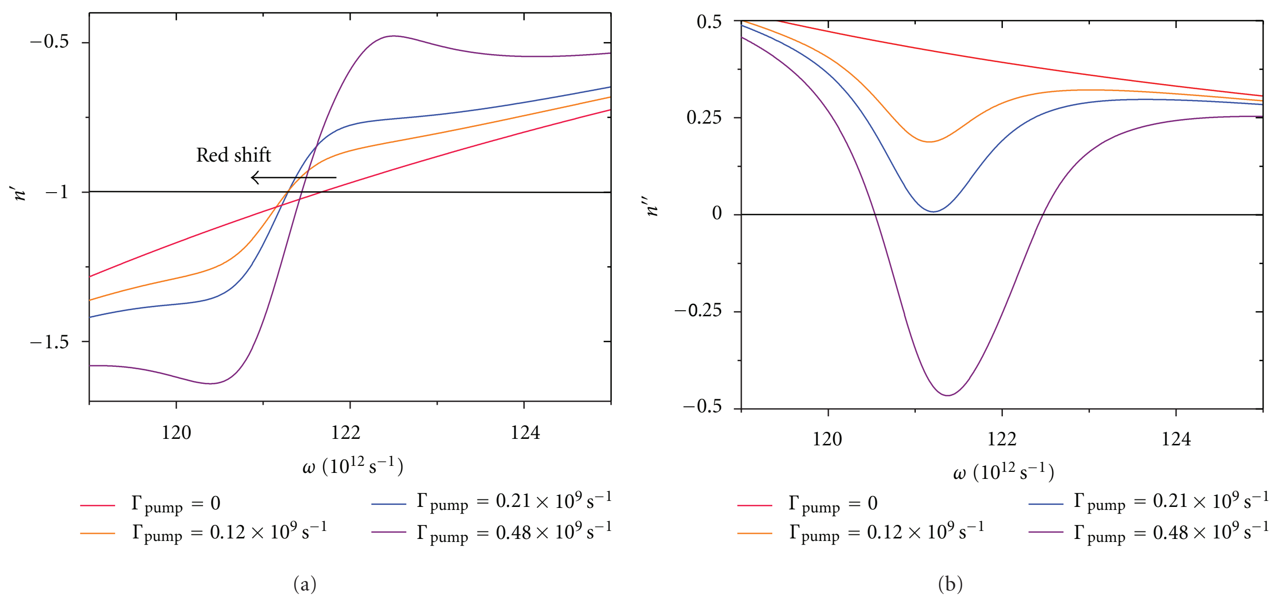

Figure 2: The parameters for the four-level system are chosen as $\sigma = 10^{-4}\ \mathrm{C}^2/\mathrm{kg}$ (C stands for Coulomb), which is the strength of the coupling between the gain material and the host NRM, $\gamma_{\mathrm{ex}} = 0.76 \times 10^{12}\ \mathrm{s}^{-1}$ is related to the linewidth of the gain medium, $\omega_x = 121.6 \times 10^{12}\ \mathrm{s}^{-1}$ is the central pumping frequency, determined by the frequency difference between state 3 and state 2, at which $n\prime(\omega) \approx -1$ is satisfied without the gain (see Figure 1(a)), and the decay time of the gain level 2 is $\tau_{32} = 5 \times 10^{-12}$ s. The value of the occupation density is set to be $N_1 = 5 \times 10^{23}\ \mathrm{m}^{-3}$. The pumping rates Γ_{pump} are assumed to be $0.12 \times 10^9\ \mathrm{s}^{-1}$ for insufficient pumping, $0.21 \times 10^9\ \mathrm{s}^{-1}$ for the appropriate pumping (to be discussed below), and $0.48 \times 10^9\ \mathrm{s}^{-1}$ for the excessive pumping, respectively. (a) The real part of the refractive index for different pumping amplitudes. A red shift of the frequency is observed at $n' = -1$. (b) The imaginary part of the refractive index for different pumping amplitudes. Loss-free windows (intervals of amplification) are revealed by the n'' curves. Increasing the pumping rate may compensate the losses, or even turn the material into an active medium.

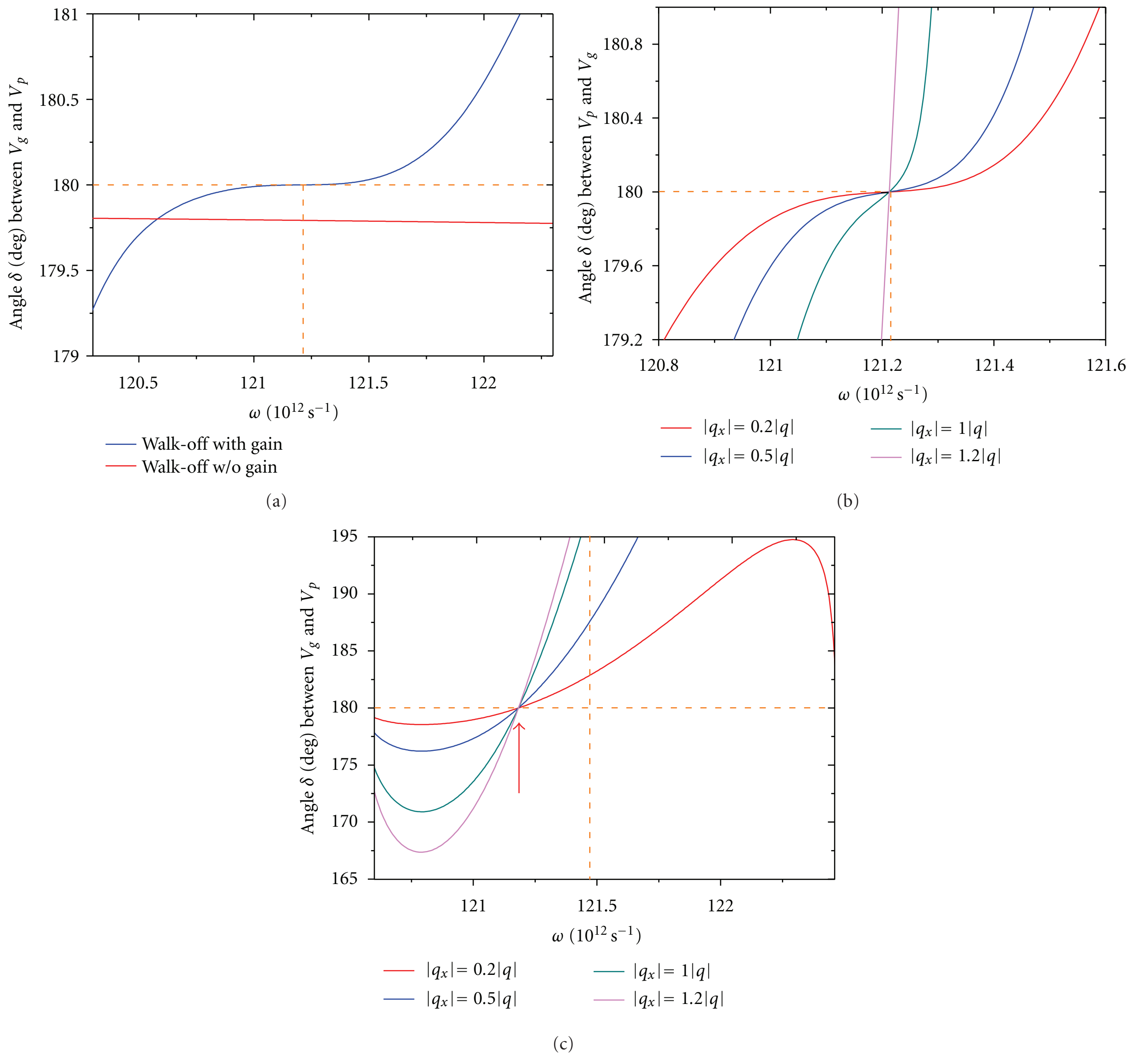

FIGURE 3: The "walk-off" angles as function of frequency ω in an active (pumped) NRM. The dashed line in all the plots shows the frequencies at which $n'(\omega) = -1$. (a) The blue line is the spatial "walk-off" angle with the pumping rate of $\Gamma_{\text{pump}} = 0.21 \times 10^9\ \text{s}^{-1}$ at $|q_x| = 0.01|q|$. Both losses and the "walk-off" within the gain window are precisely canceled. (b) The δ angles obtained at the pumping rate of $\Gamma_{\text{pump}} = 0.21 \times 10^9\ \text{s}^{-1}$. The group velocities and phase velocities are strictly antiparallel in this case ($\delta = 180^\circ$) at different $|q_x|$ when $n'(\omega) = -1$ (at $\omega = 121.28 \times 10^{12}\ \text{s}^{-1}$). (c) The δ angles obtained at the pumping rate of $\Gamma_{\text{pump}} = 0.48 \times 10^9\ \text{s}^{-1}$. Even though at this pumping rate the NRM is turned into a light-amplifying medium, the spatial "walk-off" reappears and depends on the value of $|q_x|$ when $n'(\omega) = -1$, which may lead to degradation of the optical performance of an NRM.

are not strictly antiparallel, yielding a spatial "walk-off", which may restrict the applications of NRMs in a variety of fields. By introducing gain, losses can be effectively reduced, and light amplification can be realized within a narrow spectral range. Appropriately setting the gain to strictly cancel the losses, the "walk-off" for both propagating and evanescent waves can be effectively eliminated for all values of $|q_x|$, leading to an ideal NRM. However, for excessively pumped NRMs, the spatial "walk-off" reappears. Thus, the use of optical pumping to realize perfect imaging is restricted to a very narrow spectral region, under precisely defined pumping conditions. An alternative method of overcoming NRM losses without signal distortion may involve self-induced transparency (SIT) solitons, which were predicted in metamaterials [24], in analogy with SIT in other resonantly absorbing structures [25, 26].

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References

[1] J. B. Pendry, "Negative refraction makes a perfect lens," *Physical Review Letters*, vol. 85, no. 18, pp. 3966–3969, 2000.

[2] R. A. Shelby, D. R. Smith, and S. Schultz, "Experimental verification of a negative index of refraction," *Science*, vol. 292, no. 5514, pp. 77–79, 2001.

[3] K. L. Tsakmakidis, A. D. Boardman, and O. Hess, "'Trapped rainbow' storage of light in metamaterials," *Nature*, vol. 450, no. 7168, pp. 397–401, 2007.

[4] H. G. Chen, C. T. Chan, and P. Sheng, "Transformation optics and metamaterials," *Nature Materials*, vol. 9, no. 5, pp. 387–396, 2010.

[5] R. W. Ziolkowski and E. Heyman, "Wave propagation in media having negative permittivity and permeability," *Physical Review E*, vol. 64, no. 5, Article ID 056625, 2001.

[6] D. R. Smith, D. Schurig, M. Rosenbluth, S. Schultz, S. A. Ramakrishna, and J. B. Pendry, "Limitations on subdiffraction imaging with a negative refractive index slab," *Applied Physics Letters*, vol. 82, no. 10, pp. 1506–1508, 2003.

[7] A. G. Ramm, "Does negative refraction make a perfect lens?" *Physics Letters A*, vol. 372, no. 43, pp. 6518–6520, 2008.

[8] Y.-J. Jen, A. Lakhtakia, C.-W. Yu, and C.-T. Lin, "Negative refraction in a uniaxial absorbent dielectric material," *European Journal of Physics*, vol. 30, no. 6, pp. 1381–1390, 2009.

[9] W. H. Wee and J. B. Pendry, "Looking beyond the perfect lens," *New Journal of Physics*, vol. 12, Article ID 053018, 2010.

[10] M. I. Stockman, "Criterion for negative refraction with low optical losses from a fundamental principle of causality," *Physical Review Letters*, vol. 98, no. 17, Article ID 177404, 2007.

[11] P. Kinsler and M. W. McCall, "Causality-based criteria for a negative refractive index must be used with care," *Physical Review Letters*, vol. 101, no. 16, Article ID 167401, 2008.

[12] K. J. Webb and L. Thylén, "Perfect-lens-material condition from adjacent absorptive and gain resonances," *Optics Letters*, vol. 33, no. 7, pp. 747–749, 2008.

[13] A. K. Popov and V. M. Shalaev, "Compensating losses in negative-index metamaterials by optical parametric amplification," *Optics Letters*, vol. 31, no. 14, pp. 2169–2171, 2006.

[14] P. P. Orth, J. Evers, and C. H. Keitel, "Lossless negative refraction in an active dense gas of atoms," 2007, http://arxiv.org/abs/0711.0303.

[15] A. Fang, T. Koschny, M. Wegener, and C. M. Soukoulis, "Self-consistent calculation of metamaterials with gain," *Physical Review B*, vol. 79, no. 24, Article ID 241104, 2009.

[16] Y. Sivan, S. Xiao, U. K. Chettiar, A. V. Kildishev, and V. M. Shalaev, "Frequency-domain simulations of a negativeindex material with embedded gain," *Optics Express*, vol. 17, no. 26, pp. 24060–24074, 2009.

[17] S. Wuestner, A. Pusch, K. L. Tsakmakidis, J. M. Hamm, and O. Hess, "Overcoming losses with gain in a negative refractive index metamaterial," *Physical Review Letters*, vol. 105, no. 12, Article ID 127401, 2010.

[18] S. Xiao, V. P. Drachev, A. V. Kildishev et al., "Loss-free and active optical negative-index metamaterials," *Nature*, vol. 466, no. 7307, pp. 735–738, 2010.

[19] M. W. McCall, "What is negative refraction?" *Journal of Modern Optics*, vol. 56, no. 16, pp. 1727–1740, 2009.

[20] V. Gerasik and M. Stastna, "Complex group velocity and energy transport in absorbing media," *Physical Review E*, vol. 81, no. 5, Article ID 056602, 2010.

[21] L. Muschietti and C. T. Dum, "Real group velocity in a medium with dissipation," *Physics of Fluids B*, vol. 5, no. 5, pp. 1383–1397, 1993.

[22] M. Born and E. Wolf, *Principals of Optics*, Cambridge University Press, 7th edition, 1999.

[23] A. A. Govyadinov, V. A. Podolskiy, and M. A. Noginov, "Active metamaterials: sign of refractive index and gain-assisted dispersion management," *Applied Physics Letters*, vol. 91, no. 19, Article ID 191103, 2007.

[24] J. Zeng, J. Zhou, G. Kurizki, and T. Opatrny, "Backward self-induced transparency in metamaterials," *Physical Review A*, vol. 80, no. 6, Article ID 061806, 2009.

[25] M. Blaauboer, B. A. Malomed, and G. Kurizki, "Spatiotemporally localized multidimensional solitons in self-induced transparency media," *Physical Review Letters*, vol. 84, no. 9, pp. 1906–1909, 2000.

[26] T. Opatrný, B. A. Malomed, and G. Kurizki, "Dark and bright solitons in resonantly absorbing gratings," *Physical Review E*, vol. 60, no. 5, pp. 6137–6149, 1999.