

## Research Article

# Decentralized Control for the Interconnected Time-Delay Singular/Nonlinear Subsystems with Closed-Loop Decoupling Property

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This paper presents the decentralized trackers using the observer-based suboptimal method for the interconnected time-delay singular/nonlinear subsystems with closed-loop decoupling property. The observer-based suboptimal method is used to guarantee the high-performance trajectory tracker for two different subsystems. Then, due to the high gain that resulted from the decentralized tracker, the closed-loop system will have the decoupling property. An illustrative example is given to demonstrate the effectiveness of the proposed control structure.

#### 1. Introduction

The singular system model is a natural presentation of dynamic systems, such as power systems [1] and large-scale systems [2, 3]. In general, an interconnection of state variable subsystems is conveniently described as a singular system, even though an overall state space representation may not even exist. Over the past decades, much attention has been focused on the decentralized control [4–6] for time-delay singular systems. In [7], the problem of decentralized stabilization has been discussed for nonlinear singular large-scale time-delay control systems with impulsive solutions. The  $H_{\infty}$  control for singular systems with state delay has been presented in [8]. And the decentralized output feedback control problem [9] is considered for a class of large-scale systems with unknown time-varying delays.

In the recent years, a large number of control systems are characterized by interconnected large-scale subsystems, and many practical examples have been applied to decentralized control systems. The decentralized control of interconnected large-scale systems has commonly appeared in our modern technologies, such as transportation systems, power systems, and communication systems [10–12]. However, a survey of the literature indicates that the singular system issue has seldom been studied in such systems. Many research [13– 16] results concerning the singular/nonlinear system have successfully solved lots of complex problems. For the above reasons, we will discuss the decentralized control of the interconnected large-scale time-delay singular subsystem and nonlinear subsystem.

In this paper, we consider the time-delay effect. In practical applications, the time-delay effect [17–19] may result in an unexpected and unsatisfactory system performance, even including the serious instability, if it is ignored in the design of control systems. In order to overcome this problem, the controller design method [20, 21] is necessary to be further explored in this paper. Sequentially, the decentralized tracker with the high-gain property will make the closed-loop system own the decoupling property.

This paper is organized as follows. Section 2 describes the problem of interest. Section 3 presents the observer-based suboptimal digital tracker. Section 4 presents the simulation results of interconnected time-delay singular/nonlinear subsystems. Finally, Section 5 draws conclusions.



FIGURE 1: The schematic design methodology for the interconnected time-delay singular/nonlinear system.

#### 2. System and Problem Description

Consider the time-delay system consisting of two interconnected MIMO subsystems shown as

S1: 
$$E\dot{x}(t) = Ax_1(t) + A_1x_1(t - \tau_{s1}) + B_1u_1(t - \tau_{i1})$$
  
+  $h'_{12}x_2(t - \tau_{c2} - \tau_{i1}),$  (1a)

$$y_1(t) = C_1 x_1 (t - \tau_{o1}),$$
 (1b)

S2: 
$$\dot{x}_{2}(t) = f_{2}(x_{2}(t - \tau_{s2})) + g_{2}(x_{2}(t - \tau_{s2}))$$
  
 $\cdot [u_{2}(t - \tau_{s2}) + h'_{2}x_{1}(t - \tau_{s2} - \tau_{s2})],$ 
(2a)

$$y_{2}(t) = C_{2}x_{2}(t - \tau_{o2}), \qquad (2b)$$

where 
$$x_1(t)$$
 and  $x_2(t)$  are the state vectors,  $u_1(t)$  and  $u_2(t)$  are  
the control input vectors, and  $y_1(t)$  and  $y_2(t)$  are the output  
vectors.  $f_2(\cdot)$  and  $g_2(\cdot)$  are nonlinear functions of the states  
 $x_2(t)$  of S2. E, A,  $A_1$ ,  $B_1$ ,  $C_1$ , and  $C_2$  are known as constant  
system matrices of appropriate dimensions and E is a singular  
matrix. State time delays  $\tau_{s1}$  and  $\tau_{s2}$ , interconnection time  
delays  $\tau_{c1}$  and  $\tau_{c2}$ , input time delays  $\tau_{i1}$  and  $\tau_{i2}$ , and output  
time delays  $\tau_{o1}$  and  $\tau_{o2}$  are assumed to be known. The time  
delays of interconnected state vectors  $h'_{12}x_2(t - \tau_{c2} - \tau_{i1})$   
and  $h'_{21}x_1(t - \tau_{c1} - \tau_{i2})$  are induced from multiple sensors

at different rates to accurately produce a reliable navigation solution.

The subsystem S1 is the time-delay singular system and subsystem S2 is the time-delay nonlinear subsystem. Before designing the controller, the decentralized modeling of the interconnected time-delay system is proposed in Figure 1. The notation  $L(\cdot)$  through this paper is a time lag operator; for example,  $L(\tau_{ii})u(t) = u(t - \tau_{ii})$ .

It is very difficult to directly design the tracker and observer for S1 and S2 because their system models are not nonsingular and linear models. To solve this problem, the previously proposed method in [21] and the OKID (observer/Kalman filter identification) method in [22] are appropriately utilized to make S1 and S2 become the equivalent linear time-delay nonsingular subsystems. As a result, the process becomes quite simple. Besides, as long as the designed tracker for each subsystem has the high-gain property, the designed global system will have the closed-loop decoupling property.

We will use the proposed schematic design in Figure 1 to construct the methodology of the decentralized control for the interconnected time-delay singular/nonlinear subsystems with the closed-loop decoupling property.

#### 3. Main Results

In this section, we construct the methodology of the decentralized control by using the design concept of the

observer-based suboptimal digital tracker to control timedelay singular subsystem and time-delay nonlinear subsystem, respectively. Before designing the controller, we need to obtain the equivalent time-delay linear nonsingular subsystem and the equivalent time-delay linear subsystem. The problem of decentralized stabilization is discussed in the appendix.

3.1. The Equivalent Time-Delay Linear Nonsingular Subsystems for the Time-Delay Singular/Nonlinear Subsystems. From the schematic design methodology of Figure 1, and by using the previous method in [20], we can make the time-delay singular subsystems (1a) and (1b) become the equivalent time-delay regular system as follows:

$$\dot{\tilde{x}}_{s}(t) = A_{s}\tilde{x}_{s}(t) + \widehat{A}_{d}\tilde{x}_{s}(t - \tau_{s1}) + B_{d}v_{c}(t - \tau_{i1}) + h'_{12}x_{2}(t - \tau_{c2} - \tau_{i1}),$$
(3a)

$$y_1(t) = C_{11}\tilde{x}_s(t - \tau_{o1}) - D_1v_c(t - \tau_{i1}),$$
 (3b)

where the parameters  $A_s$ ,  $\widehat{A}_d$ ,  $B_d$ ,  $C_{11}$ , and  $D_1$  and input  $v_c(t)$  can be referred to in [20].

*Remark A.0.* Notably, definitions of the regular pencil [23] and the standard pencil [24] are satisfied on no state delay term in systems (1a) and (1b). If  $A_1$  exists, then definitions of the regular pencil and the standard pencil do not guarantee that systems (1a) and (1b) can be decomposed into the equivalent time-delay regular system.

Similarly, the time-delay nonlinear subsystems (2a) and (2b) can transform the equivalent time-delay linear subsystem by OKID method [21, 22] as follows:

$$\begin{aligned} x_{d2} \left( k_2 T_2 + T_2 \right) &= G_{d2} x_{d2} \left( k_2 T_2 \right) + H_{d2} u_{d2} \left( k_2 T_2 - \tau_{i2} \right), \end{aligned} \tag{4a} \\ y_{d2} \left( k_2 T_2 \right) &= C_{d2} x_{d2} \left( k_2 T_2 - \tau_{o2} \right), \end{aligned} \tag{4b}$$

where  $G_{d2}$ ,  $H_{d2}$ , and  $C_{d2}$  are the identified parameters by OKID method. The corresponding continuous-time system of (4a) and (4b) is described by

$$x_{c2}(t) = A_{c2}x_{c2}(t) + B_{c2}u_2(t - \tau_{i2}), \qquad (5a)$$

$$y_{c2}(t) = C_{c2} x_{c2} (t - \tau_{o2}).$$
 (5b)

Notably,  $A_{c2}$ ,  $B_{c2}$ , and  $C_{c2}$  are known as constant system matrices of appropriate dimensions.

The equivalent subsystems (3a), (3b), and (5a) and (5b) will be applied to the observer-based suboptimal digital tracker [21] for the singular/nonlinear subsystem in the next subsection and finally we proposed the schematic design methodology of decentralized control for the interconnected time-delay singular/nonlinear subsystems with closed-loop decoupling property.

3.2. The Observer-Based Suboptimal Digital Tracker Design [21]. Consider the continuous time-delay singular subsystems (3a) and (3b) or the time-delay subsystems (5a) and

(5b). Here, we take the time-delay singular subsystems (3a) and (3b) to design the observer-based suboptimal digital tracker and the design results are similar to the time-delay subsystems (5a) and (5b).

Consider the continuous time-delay singular subsystems (3a) and (3b) without the time delay of interconnected state vector  $h'_{12}x_2(t - \tau_{c2} - \tau_{i1})$ . By [21],  $T_1$  is the sampling period. Let the state delay time be given by  $\tau_{s1} = \rho_1 T_1 + \Gamma_1$ , where  $0 \le \Gamma_1 < T_1$  and  $\rho_1 \ge 0$  is an integer, and let the input delay time be given by  $\tau_{i1} = \eta_1 T_1 + \sigma_1$ , where  $0 \le \sigma_1 < T_1$  and  $\eta_1 \ge 0$  is an integer. The time-delay singular subsystems (3a) and (3b), by both the Newton extrapolation method and the Chebyshev quadrature method [25, 26], become

$$\begin{split} \widetilde{x}_{ds} \left( (k+1) T_{1} \right) \\ &= G \widetilde{x}_{ds} \left( kT_{1} \right) + \widehat{G}_{1}^{(1)} \widetilde{x}_{ds} \left( kT_{1} - \rho_{1}T_{1} + T_{1} \right) \\ &+ \widehat{G}_{1}^{(2)} \widetilde{x}_{ds} \left( kT_{1} - \rho_{1}T_{1} \right) + \widehat{G}_{1}^{(3)} \widetilde{x}_{ds} \left( kT_{1} - \rho_{1}T_{1} - T_{1} \right) \\ &+ H_{1}^{(0)} v_{d} \left( kT_{1} - \eta_{1}T_{1} \right) + H_{1}^{(1)} v_{d} \left( kT_{1} - \eta_{1}T_{1} - T_{1} \right), \end{split}$$

$$(6)$$

where

$$G = e^{A_s T}, \qquad \widehat{G}_1^{(1)} = \frac{T_1}{2} \left[ Q_1^{(2)} + Q_1^{(3)} \right] \widehat{A}_d,$$
$$\widehat{G}_1^{(2)} = T_1 \left[ Q_1^{(1)} - Q_1^{(3)} \right] \widehat{A}_d, \qquad \widehat{G}_1^{(3)} = \frac{T_1}{2} \left[ Q_1^{(3)} - Q_1^{(2)} \right] \widehat{A}_d,$$
$$H_1^{(0)} = \left[ G^{1-\gamma_1} - I_n \right] A_s^{-1} B_d, \qquad H_1^{(1)} = \left[ G - G^{1-\gamma_1} \right] A_s^{-1} B_d,$$
(7)

in which

$$\gamma_{1} = \frac{\sigma_{1}}{T_{1}}, \qquad \beta_{1} = \frac{\Gamma_{1}}{T_{1}}, \qquad Q_{1}^{(1)} = [G - I_{n}] (A_{s}T_{1})^{-1},$$

$$Q_{1}^{(2)} = [Q_{1}^{(1)} - (1 - \beta_{1}) I_{n} - \beta_{1}G] (A_{s}T_{1})^{-1}, \qquad (8)$$

$$Q_{1}^{(3)} = [2Q_{1}^{(2)} - (1 - \beta_{1})^{2} I_{n} - \beta_{1}^{2}G] (A_{s}T_{1})^{-1}.$$

Some terms in (6) may be combined because of the same delay, so (6) can be reduced to

$$\begin{split} \widetilde{x}_{ds}\left(\left(k+1\right)T_{1}\right) \\ &= \overline{G}\widetilde{x}_{ds}\left(kT_{1}\right) + \sum_{i=1}^{M_{1}}\overline{G}_{i}\widetilde{x}_{ds}\left(kT_{1}-iT_{1}\right) \\ &+ \overline{H}v_{d}\left(kT_{1}\right) + \sum_{j=1}^{M_{2}}\overline{H}_{j}v_{d}\left(kT_{1}-jT_{1}\right). \end{split}$$

(9)

The time-delay state  $x_1(t - \tau_{o1})$  for  $kT_1 \le t - \tau_{o1} < (k + 1)T_1$ must be evaluated as follows:

$$\begin{split} \widetilde{x}_{s}\left(t-\tau_{o1}\right) \\ &= e^{A_{s}\left(t-\tau_{o1}-kT_{1}\right)}\widetilde{x}_{ds}\left(kT_{1}\right) \\ &+ \int_{kT_{1}}^{t-\tau_{o1}} e^{A_{s}\left(t-\tau_{o1}-\lambda\right)}\widehat{A}_{d}\widetilde{x}_{ds}\left(\lambda-\tau_{s1}\right)d\lambda \\ &+ \int_{kT_{1}}^{t-\tau_{o1}} e^{A_{s}\left(t-\tau_{o1}-\lambda\right)}B_{d}\nu_{d}\left(\lambda-\tau_{i1}\right)d\lambda \\ &= \delta_{1}\left(t-\tau_{o1}-kT_{1}\right)\widetilde{x}_{ds}\left(kT_{1}\right) \\ &+ \left[\widehat{\delta}_{1}^{(1)}\left(t-\tau_{o1}-kT_{1}\right)\widetilde{x}_{ds}\left(kT_{1}-\rho_{1}T_{1}+T_{1}\right)\right. \\ &+ \left.\widehat{\delta}_{1}^{(2)}\left(t-\tau_{o1}-kT_{1}\right)\widetilde{x}_{ds}\left(kT_{1}-\rho_{1}T_{1}\right) \\ &+ \left.\widehat{\delta}_{1}^{(3)}\left(t-\tau_{o1}-kT_{1}\right)\widetilde{x}_{ds}\left(kT_{1}-\rho_{1}T_{1}-T_{1}\right)\right] \\ &+ \left[\varphi_{1}^{(0)}\left(t-\tau_{o1}-kT_{1}\right)\nu_{d}\left(kT_{1}-\eta_{1}T_{1}\right) \\ &+ \varphi_{1}^{(1)}\left(t-\tau_{o1}-kT_{1}\right)\nu_{d}\left(kT_{1}-\eta_{1}T_{1}-T_{1}\right)\right], \end{split}$$

$$(10)$$

where

$$\begin{split} &\delta_1 \left( t - \tau_{o1} - kT_1 \right) = e^{A_s (t - \tau_{o1} - kT_1)}, \\ &\widehat{\delta}_1^{(1)} \left( t - \tau_{o1} - kT_1 \right) = \frac{T_1}{2} \left[ q_1^{(2)} + q_1^{(3)} \right] \widehat{A}_d, \\ &\widehat{\delta}_1^{(2)} \left( t - \tau_{o1} - kT_1 \right) = T_1 \left[ q_1^{(1)} - q_1^{(3)} \right] \widehat{A}_d, \\ &\widehat{\delta}_1^{(3)} \left( t - \tau_{o1} - kT_1 \right) = \frac{T_1}{2} \left[ q_1^{(3)} - q_1^{(2)} \right] \widehat{A}_d, \end{split}$$

 $\varphi_1^{(0)}(t-\tau_{o1}-kT_1)$  $= \begin{cases} O_{n \times m}, & t - \tau_{o1} < \sigma_{1} \\ \left[ e^{-A_{s}\sigma_{1}} \delta_{1} \left( t - \tau_{o1} - kT_{1} \right) - I_{n} \right] A_{s}^{-1} B_{d}, & t - \tau_{o1} \ge \sigma_{1}, \end{cases}$  $\varphi_1^{(1)}(t-\tau_{o1}-kT_1)$  $= \begin{cases} \left[ \delta_1 \left( t - \tau_{o1} - kT_1 \right) - I_n \right] A_s^{-1} B_d, & t - \tau_{o1} < \sigma_1 \\ \delta_1 \left( t - \tau_{o1} - kT_1 \right) \left[ I_n - e^{-A_s \sigma_1} \right] A_s^{-1} B_d, & t - \tau_{o1} \ge \sigma_1, \end{cases}$ 

in which

$$q_{1}^{(1)} = \left[\delta_{1}\left(t - \tau_{o1} - kT_{1}\right) - I_{n}\right]\left(A_{s}T_{1}\right)^{-1},$$

$$q_{1}^{(2)} = \left[q_{1}^{(1)} - \left(\frac{t - \tau_{o1} - kT_{1}}{T_{1}} - \beta_{1}\right)I_{n} - \beta_{1}\delta_{1}\left(t - \tau_{o1} - kT_{1}\right)\right]\left(A_{s}T_{1}\right)^{-1},$$

$$q_{1}^{(3)} = \left[2q_{1}^{(2)} - \left(\frac{t - \tau_{o1} - kT_{1}}{T_{1}} - \beta_{1}\right)^{2}I_{n} + \beta_{1}^{2}\delta_{1}\left(t - \tau_{o1} - kT_{1}\right)\right]\left(A_{s}T_{1}\right)^{-1}.$$
(12)

Some terms in (10) may be combined as in (9), and (10) can be rewritten as

$$\begin{split} \tilde{x}_{s}\left(t-\tau_{o1}\right) &= \overline{\delta}_{0}\left(t-\tau_{o1}-kT_{1}\right)\tilde{x}_{ds}\left(kT_{1}\right) \\ &+ \sum_{i=1}^{M_{1}} \overline{\delta}_{i}\left(t-\tau_{o1}-kT_{1}\right)\tilde{x}_{ds}\left(kT_{1}-iT_{1}\right) \\ &+ \overline{\varphi}_{0}\left(t-\tau_{o1}-kT_{1}\right)v_{d}\left(kT_{1}\right) \\ &+ \sum_{j=1}^{M_{2}} \overline{\varphi}_{j}\left(t-\tau_{o1}-kT_{1}\right)v_{d}\left(kT_{1}-jT_{1}\right). \end{split}$$
(13)

Then, the output (3b) can be rewritten as

$$y_{1}(t) = C_{11}\tilde{x}_{s}(t - \tau_{o1}) - D_{1}v_{c}(t - \tau_{i1})$$

$$= C_{11}\overline{\delta}_{0}(t - \tau_{o1} - kT_{1})\tilde{x}_{ds}(kT_{1})$$

$$+ \sum_{i=1}^{M_{1}}C_{11}\overline{\delta}_{1}(t - \tau_{o1} - kT_{1})\tilde{x}_{ds}(kT_{1} - iT_{1})$$

$$+ C_{11}\overline{\varphi}_{0}(t - \tau_{o1} - kT_{1})v_{d}(kT_{1})$$

$$+ \sum_{j=1}^{M_{2}}C_{11}\overline{\varphi}_{j}(t - \tau_{o1} - kT_{1})v_{d}(kT_{1} - jT_{1})$$

$$- \left[D_{1}^{(0)}v_{d}(kT_{1} - \eta_{1}T_{1}) + D_{1}^{(1)}v_{d}(kT_{1} - \eta_{1}T_{1} - T_{1})\right],$$
(14)

where  $D_1^{(0)} = D_1^* (B_d^T B_d)^{-1} H_1^{(0)}, D_1^{(1)} = D_1^* (B_d^T B_d)^{-1} H_1^{(1)}$ , and  $D_1^* = [D_1 \ O]^T$ . Similarly, some terms in (14) can be combined so (14) can

be rewritten as

$$y_{1}(t) = C_{11}\overline{\delta}_{0} \left(t - \tau_{o1} - kT_{1}\right) \widetilde{x}_{ds} \left(kT_{1}\right) + \sum_{i=1}^{M_{1}} C_{11}\overline{\delta}_{1} \left(t - \tau_{o1} - kT_{1}\right) \widetilde{x}_{ds} \left(kT_{1} - iT_{1}\right) + C_{11}\overline{\varphi}_{0}^{*} \left(t - \tau_{o1} - kT_{1}\right) v_{d} \left(kT_{1}\right) + \sum_{j=1}^{M_{2}} C_{11}\overline{\varphi}_{j}^{*} \left(t - \tau_{o1} - kT_{1}\right) v_{d} \left(kT_{1} - jT_{1}\right),$$
(15)

where  $\overline{\varphi}_0^*, \overline{\varphi}_j^*, \overline{\varphi}_0$ , and  $\overline{\varphi}_j$  are the summation of multiple inputdelay terms.

In the following work, we use (13) and (15) to derive the equivalent extended delay-free system as follows:

$$X_d\left(\left(k+1\right)T_1\right) = \widehat{G}_e X_d\left(kT_1\right) + \widehat{H}_e v_d\left(kT_1\right), \qquad (16a)$$

$$y_d(kT_1) = \widehat{C}_e X_d(kT_1), \qquad (16b)$$

where

$$X_{d}(kT_{1}) = \left[\tilde{x}_{ds}(kT_{1}) \ \tilde{x}_{ds}(kT_{1} - T_{1}) \cdots \tilde{x}_{ds}(kT_{1} - M_{1}T_{1}) \ v_{d}(kT_{1} - T_{1}) \cdots v_{d}(kT_{1} - M_{2}T_{1}) \ r^{*}(kT_{1})\right]^{T}$$
(17)

means the extended virtual state vector.

By the previous method [21], we derive the observerbased suboptimal tracker for the time-delay singular system with unavailable states using the equivalent extended delayfree system. The extended observer-based suboptimal digital tracker can be represented as

$$\begin{aligned} \widehat{X}_{d}\left(\left(k+1\right)T_{1}\right) \\ &= \widehat{G}_{o}\widehat{X}_{d}\left(kT_{1}\right) + \widehat{H}_{o}\nu_{d}\left(kT_{1}\right) + L_{d}\left[y_{d}\left(kT_{1}\right) - \widehat{C}_{e}\widehat{X}_{d}\left(kT_{1}\right)\right], \end{aligned}$$
(18a)

$$v_d(kT_1) = -\widehat{K}(kT_1)\widehat{X}_d(kT_1), \qquad (18b)$$

where  $\widehat{X}_d(kT_1)$  is the estimate of the extended state  $X_d(kT_1)$  in (17) and

$$\widehat{G}_{o} = \widehat{G}_{e} - L_{d}\widehat{C}_{e}\widehat{G}_{e},$$

$$\widehat{H}_{o} = \widehat{H}_{e} - L_{d}\widehat{C}_{e}\widehat{H}_{e},$$

$$\widehat{K}(kT_{1}) = [K_{d}(kT_{1}) \quad F_{d}(kT_{1}) \quad E_{d}(kT_{1})],$$
(19)

in which

$$K_{d}(kT_{1}) = \begin{bmatrix} K_{d}^{(0)}(kT_{1}) & K_{d}^{(1)}(kT_{1}) & \cdots & K_{d}^{(M_{1})}(kT_{1}) \end{bmatrix},$$
  

$$F_{d}(kT_{1}) = \begin{bmatrix} F_{d}^{(1)}(kT_{1}) & \cdots & F_{d}^{(M_{2})}(kT_{1}) \end{bmatrix}.$$
(20)

The details of the parameters can be referred to in [21]. Here, the observer-based suboptimal tracker has been completely obtained. Figure 2 presents the realization of decentralized control for the interconnected time-delay singular/nonlinear subsystems.

From Figures 1 and 2, the design procedure can be summarized as the following steps.

*Step 1.* Perform the previously proposed method [21] and the OKID method [22] to determine the equivalent time-delay linear subsystems in Figure 1.

*Step 2.* Design the observer-based suboptimal digital trackers from the equivalent time-delay linear subsystems obtained in Step 1.

*Step 3.* Perform the observer-based suboptimal digital trackers obtained in Step 2. The decentralized control for the interconnected time-delay singular/nonlinear subsystems is shown in Figure 2.

#### 4. An Illustrative Example

Consider the time-delay system consisting of two interconnected MIMO subsystems shown as

S1: 
$$E\dot{x}(t) = Ax_1(t) + A_1x_1(t - \tau_{s1}) + B_1u_1(t - \tau_{i1})$$
  
+  $h'_{12}x_2(t - \tau_{c2} - \tau_{i1}),$ 

$$y_1(t) = C_1 x_1 (t - \tau_{o1}),$$
 (21b)

S2: 
$$\dot{x}_{2}(t) = f_{2}(x_{2}(t - \tau_{s2})) + g_{2}(x_{2}(t - \tau_{s2}))$$
  
 $\cdot [u_{2}(t - \tau_{i2}) + h'_{21}x_{1}(t - \tau_{c1} - \tau_{i2})],$ 

$$y_{2}(t) = C_{2}x_{2}(t - \tau_{o2}),$$
(22b)

where

$$u_{1}(t) = \begin{bmatrix} u_{1,1}(t) \\ u_{1,2}(t) \end{bmatrix}, \qquad u_{2}(t) = \begin{bmatrix} u_{2,1}(t) \\ u_{2,2}(t) \end{bmatrix},$$

$$x_{1}(t) = \begin{bmatrix} x_{1,1}(t) \\ x_{1,2}(t) \\ x_{1,3}(t) \\ x_{1,4}(t) \\ x_{1,5}(t) \\ x_{1,6}(t) \end{bmatrix}, \qquad x_{2}(t) = \begin{bmatrix} x_{2,1}(t) \\ x_{2,2}(t) \\ x_{2,3}(t) \\ x_{2,4}(t) \end{bmatrix}.$$
(23)

The first subsystem S1 of the large-scale system is given as follows:

$$E = \begin{bmatrix} 1 & 2 & 1 & 1 & -3 & -2 \\ 0 & 2 & 2 & 1 & -3 & -3 \\ 1 & 2 & 1 & 1 & -3 & -2 \\ 1 & 2 & 1 & 3 & -5 & -4 \\ 0 & 2 & 1 & 1 & -2 & -2 \\ 1 & 0 & 0 & 0 & -1 & 0 \end{bmatrix}, \qquad A = I_6$$



FIGURE 2: The decentralized control for the interconnected time-delay singular/nonlinear subsystems.



FIGURE 3: Two-link robot.

 $A_1$ 

$$= \begin{bmatrix} 1.6207 & 0 & 0 & 0 & -1.6207 & 0 \\ 0 & 0.8103 & -0.4052 & 0 & -1.6207 & 0 \\ 0 & 0 & 0.8103 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.6207 & -3.2413 & 0 \\ 0 & 0 & 0 & 0 & 1.6207 & 0 \\ 0 & 0 & 0 & 0 & -3.2413 & 1.6207 \end{bmatrix}$$
$$B_{1} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 1 & 0 & 0 \end{bmatrix}^{T},$$
$$C_{1} = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \end{bmatrix},$$
$$\tau_{s1} = 0, \qquad \tau_{i1} = \tau_{o1} = 0.5 \times T_{1}.$$

Let the sampling period  $T_1 = 0.01$  sec and the initial condition is  $x_c(0) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$ .

The second subsystem S2 of the large-scale system is given by two-link robot [27, 28], which is described as shown in Figure 3.

The dynamic equation of the two-link robot system can be expressed as follows:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \Gamma, \qquad (25)$$

(24)

where

$$M(q) = \begin{bmatrix} (m_1 + m_2) l_1^2 & m_2 l_1 l_2 (s_1 s_2 + c_1 c_2) \\ m_2 l_1 l_2 (s_1 s_2 + c_1 c_2) & m_2 l_2^2 \end{bmatrix},$$

$$C(q, \dot{q}) = m_2 l_1 l_2 (c_1 s_2 - s_1 c_2) \begin{bmatrix} 0 & -\dot{q}_2 \\ -\dot{q}_1 & 0 \end{bmatrix},$$

$$G(q) = \begin{bmatrix} -(m_1 + m_2) l_1 g_r s_1 \\ -m_2 l_2 g_r s_2 \end{bmatrix},$$
(26)

and  $q = [q_1 \ q_2]^T$ ,  $q_1, q_2$  are the angular positions, M(q) is the moment of inertia,  $C(q, \dot{q})$  includes Ceoriolis and centripetal forces, G(q) is the gravitational force, and  $\Gamma$  is the applied torque vector. Here, we use the short hand notations  $s_i = \sin(q_i)$  and  $c_i = \cos(q_i)$ . The nominal parameters of the system are given as follows: the link masses  $m_1 = 5 \text{ kg}, m_2 = 2.5 \text{ kg}$ , the length  $l_1 = l_2 = 0.5 \text{ m}$ , and the gravitational acceleration  $g_r = 9.81 \text{ ms}^{-2}$ . Rewrite (25) in the following form:

$$\ddot{q} = M^{-1}\left(q\right)\left(\Gamma - C\left(q, \dot{q}\right)\dot{q} - G\left(q\right)\right).$$
<sup>(27)</sup>

Let  $x_1$  and  $f_1(x_1)$  represent the state of the system and the nonlinear function of the state  $x_1$ , respectively. And the notation is shown as follows:

$$x_{2}(t) \equiv \begin{bmatrix} x_{2,1} & x_{2,2} & x_{2,3} & x_{2,4} \end{bmatrix}^{T} = \begin{bmatrix} q_{1} & \dot{q}_{1} & q_{2} & \dot{q}_{2} \end{bmatrix}^{T},$$
  
$$f_{2}(x_{2}(t)) \equiv \begin{bmatrix} f_{2,1} & f_{2,2} & f_{2,3} & f_{2,4} \end{bmatrix}^{T},$$
 (28)

where  $f_{2,1} = x_{2,2}, f_{2,3} = x_{2,4}$ , and  $[f_{2,2}, f_{2,4}]^T = M^{-1}(-C[x_{2,2}, x_{2,4}]^T - G)$ . Also, let  $u_2 \equiv \Gamma$ , in which  $\Gamma = [\Gamma_1, \Gamma_2]^T$ .

Calculate the inverse of the matrix M, and then we can have  $M^{-1} = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}$  such that

$$g_{2}(x_{2}(t)) = \begin{bmatrix} 0 & 0 \\ p_{11} & p_{12} \\ 0 & 0 \\ p_{21} & p_{22} \end{bmatrix}.$$
 (29)

Therefore, the dynamic equation of the two-link robot system can be reformulated as follows:

$$\dot{x}_{2}(t) = f_{2}(x_{2}(t - \tau_{s2})) + g_{2}(x_{2}(t - \tau_{s2}))u_{2}(t - \tau_{i2}),$$
(30a)
$$y_{2}(t) = C_{2}x_{2}(t - \tau_{o2}),$$
(30b)

where  $C_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ , the sampling period  $T_2 = 0.02$  sec, and the initial condition  $x_2(0) = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}^T$ .

Combining the above systems with the nonlinear interconnected terms, the large-scale system can then be shown in Figures 1 and 2, where the nonlinear interconnected terms  $h'_{12}(x_{d2}(t))$  and  $h'_{21}(x_{d1}(t))$  are given as  $\begin{bmatrix} x^2_{d2,3}\cos(x_{d2,1})\\ \sin^2(x_{d2,2}) \end{bmatrix}$  and  $\begin{bmatrix} x^2_{d1,1}\\ x_{d1,3}\sin(x_{d1,2}) \end{bmatrix}$ , respectively. The time delays of the nonlinear interconnected terms are  $\tau_{c1} = 3 \times T_2$  and  $\tau_{c2} = 1 \times T_1$ , where  $T_1 = 0.01$  sec and  $T_2 = 0.02$  sec.

Based on Section 3.1 [20], the time-delay singular subsystem S1 can be transformed into the equivalent time-delay regular system as follows:

S1: 
$$\tilde{x}_{s}(t) = A_{s}\tilde{x}_{s}(t) + \widehat{A}_{d}\tilde{x}_{s}(t - \tau_{s1}) + B_{d}v_{c}(t - \tau_{i1}) + h'_{12}x_{2}(t - \tau_{c2} - \tau_{i1}),$$
  
(31a)

$$y_1(t) = C_{11} \tilde{x}_s (t - \tau_{o1}) - D_1 v_c (t - \tau_{i1}),$$
 (31b)

where

$$A_{s} = \begin{bmatrix} 1 & 0 & 0 & -0.5001 \\ 0 & 0.4999 & -0.25 & -0.4999 \\ 0 & 0 & 0.4999 & 0 \\ 0 & 0 & 0 & 0.4999 \end{bmatrix},$$
  
$$\widehat{A}_{d} = \begin{bmatrix} 0.4472 & 0 & 0 & -0.2236 \\ 0 & 0.2236 & -0.1118 & -0.2236 \\ 0 & 0 & 0.2236 & 0 \\ 0 & 0 & 0 & 0.2236 \end{bmatrix},$$
  
$$B_{d} = \begin{bmatrix} 0.4999 & 0.5001 \\ -0.25 & -0.25 \\ 0.4999 & 0.4999 \\ 0.4999 & -0.4999 \end{bmatrix},$$

$$C_{11} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}, \quad D_1 = \begin{bmatrix} 0 & 2 \\ 0.5 & 1.5 \end{bmatrix}.$$
(32)

By OKID [21, 22] in Figure 1, the identified subsystem S2 is given as

S2: 
$$x_{d2} (k_2 T_2 + T_2) = G_{d2} x_{d2} (k_2 T_2)$$
  
+  $H_{d2} u_{d2} (k_2 T_2 - \tau_{i2}),$   
 $y_{d2} (k_2 T_2) = C_{d2} x_{d2} (k_2 T_2 - \tau_{o2}),$  (33b)

where

$$G_{d2} = \begin{bmatrix} 1.16 \times 10^{0} & -5.39 \times 10^{-2} & -8.39 \times 10^{-2} & 1.15 \times 10^{-2} & 1.48 \times 10^{-8} \\ 5.81 \times 10^{-2} & 1.07 \times 10^{0} & -9.69 \times 10^{-3} & -8.64 \times 10^{-2} & -3.85 \times 10^{-8} \\ 1.83 \times 10^{-1} & 2.44 \times 10^{-3} & 9.23 \times 10^{-1} & -5.00 \times 10^{-2} & -9.23 \times 10^{-9} \\ 6.99 \times 10^{-3} & 1.90 \times 10^{-1} & 5.30 \times 10^{-2} & 7.99 \times 10^{-1} & -5.21 \times 10^{-8} \\ -8.04 \times 10^{-9} & 3.30 \times 10^{-9} & 1.38 \times 10^{-8} & -1.29 \times 10^{-8} & -7.21 \times 10^{-3} \end{bmatrix},$$

$$H_{d2} = \begin{bmatrix} -4.08 \times 10^{-6} & 2.11 \times 10^{-4} \\ -1.46 \times 10^{-4} & 2.92 \times 10^{-4} \\ 2.32 \times 10^{-5} & -2.69 \times 10^{-4} \\ 1.98 \times 10^{-4} & -3.77 \times 10^{-4} \\ -6.01 \times 10^{-12} & 1.85 \times 10^{-8} \end{bmatrix},$$

$$C_{d2} = \begin{bmatrix} 6.43 \times 10^{-1} & -8.50 \times 10^{-1} & 5.29 \times 10^{-1} & -6.39 \times 10^{-1} & -1.52 \times 10^{-8} \\ 8.72 \times 10^{-1} & 7.84 \times 10^{-1} & 7.35 \times 10^{-1} & 4.80 \times 10^{-1} & -2.02 \times 10^{-8} \end{bmatrix},$$

in which the input time-delay  $\tau_{i2} = 0.5 \times T_2$  and output timedelay  $\tau_{o2} = 0.5 \times T_2$ . Following the proposed method in this paper, let the

Following the proposed method in this paper, let the reference inputs  $r(t) = [0.5 \sin(t) \ 0.5 \cos(t)]^T$  and apply them to subsystem S1 and subsystem S2, respectively. We obtain the observer gain matrix  $L_d$  for S1 and S2 as follows:

S1: 
$$L_{d1} = \begin{bmatrix} -7.19 \times 10^{-3} & -1.26 \times 10^2 & 1.00 \times 10^0 \\ 1.02 \times 10^0 & 2.60 \times 10^2 & -1.00 \times 10^0 \\ 1.98 \times 10^0 & -1.57 \times 10^{-2} & -1.26 \times 10^2 \\ -3.97 \times 10^0 & 2.83 \times 10^{-1} & 2.58 \times 10^2 \\ 2.49 \times 10^{-1} & 1.97 \times 10^0 & -1.23 \times 10^0 \\ -2.49 \times 10^{-1} & -3.94 \times 10^0 & 1.18 \times 10^0 \end{bmatrix}$$

$$\begin{array}{cccc} -1.77 \times 10^{2} & -8.82 \times 10^{-2} & 1.96 \times 10^{0} \\ 3.62 \times 10^{2} & 8.81 \times 10^{-2} & -3.93 \times 10^{0} \\ -1.94 \times 10^{-3} & -1.94 \times 10^{-3} & 0 & 0 \\ 1.09 \times 10^{-4} & 1.15 \times 10^{-4} & 0 & 0 \end{array} \right]^{T},$$

$$(35)$$

S2: 
$$L_{d2} = \begin{bmatrix} -3.90 \times 10^{-1} & -3.39 \times 10^{-1} & -4.23 \times 10^{-1} \\ -3.28 \times 10^{-1} & 3.97 \times 10^{-1} & -3.61 \times 10^{-1} \\ 3.25 \times 10^{-1} & 4.90 \times 10^{-8} & -1.01 \times 10^{-1} \\ -3.67 \times 10^{-1} & 7.07 \times 10^{-10} & -8.27 \times 10^{-2} \end{bmatrix}$$



FIGURE 4: (a) Output responses of the subsystem S1: output  $y_{d11}(t)$  and reference  $r_{11}(t)$ . (b) Output responses of the subsystem S1: output  $y_{d12}(t)$  and reference  $r_{12}(t)$ .



FIGURE 5: (a) Output responses of the subsystem S2: output  $y_{d21}(t)$  and reference  $r_{21}(t)$ . (b) Output responses of the subsystem S2: output  $y_{d22}(t)$  and reference  $r_{22}(t)$ .

$$7.80 \times 10^{-6} \quad -9.87 \times 10^{-6} \quad 0 \quad 0 \\ -7.04 \times 10^{-6} \quad 2.65 \times 10^{-5} \quad 0 \quad 0 \end{bmatrix}^{T}.$$
(36)

Finally, the scheme of Figure 2 is implemented. For simplification, the numerical analysis is not presented and Figures 4 and 5 show the results of the simulation.

In order to confirm the independence of the control for the two subsystems, the time-varying optimal digital



FIGURE 6: The unanticipated failure occurs without fault-tolerant control during  $t = 4 \sim 6$  sec. (a) Output responses of the subsystem S1: output  $y_{d11}(t)$  and reference  $r_{11}(t)$ . (b) Output responses of the subsystem S1: output  $y_{d12}(t)$  and reference  $r_{12}(t)$ .



FIGURE 7: The unanticipated failure occurs without fault-tolerant control during  $t = 4 \sim 6$  sec. (a) Output responses of the subsystem S2: output  $y_{d21}(t)$  and reference  $r_{21}(t)$ . (b) Output responses of the subsystem S2: output  $y_{d22}(t)$  and reference  $r_{22}(t)$ .

controller of the subsystem S2 is reduced by multiplying a scalar 0.97 during 4 sec to 6 sec in this simulation. Although the time-varying optimal digital controller of the subsystem S2 is reduced, the tracking performance of the subsystem S1 will not be affected by this condition and the results are shown in Figures 6 and 7.

To show the effectiveness of the proposed method, we compare it with the observer/Kalman filter identification (OKID) method in the simulation for the subsystem S2.

Following [20, 21], let the subsystem S2 be excited by the control force u(t) with white noise  $u(t) = [u_1(t) \ u_2(t)]^T$  having zero mean and covariance diag  $[cov(u_1(t)) \ cov(u_2(t))] =$  diag [0.2 0.2]. Then, the comparisons between the actual outputs and the OKID method for subsystem S2 are shown in Figure 8, and the comparisons between the actual outputs and the proposed method for subsystem S2 are shown in Figure 9.



FIGURE 8: (a) The comparison between the system output  $y_{d21}(k_2T_2)$  and its observer-based output  $y_{okid21}(k_2T_2)$  by OKID for the subsystem S2. (b) The comparison between the system output  $y_{d22}(k_2T_2)$  and its observer-based output  $y_{okid22}(k_2T_2)$  by OKID for the subsystem S2.



FIGURE 9: (a) The comparison between the system output  $y_{d21}(k_2T_2)$  and its observer-based output  $y_{id21}(k_2T_2)$  by the proposed method for the subsystem S2. (b) The comparison between the system output  $y_{d22}(k_2T_2)$  and its observer-based output  $y_{id22}(k_2T_2)$  by the proposed method for the subsystem S2.

From the comparison between Figures 8 and 9, the effectiveness of the proposed method is better than OKID method in the tracking error performance.

#### 5. Conclusion

This paper presents a systematical methodology of the decentralized control for the interconnected time-delay singular/nonlinear subsystems with closed-loop decoupling property. We use the observer-based suboptimal digital tracker with high gain property to keep the good tracking performance. Moreover, the decoupling property performs very well such that even if some unanticipated fault occurs in some of subsystems, it still will not affect the tracking performance of each subsystem. The proposed methods depend on the decentralized modeling of the interconnected sampled-data time-delay subsystems in Section 2 and the controller design is suitable to time-delay singular/nonlinear subsystems in Section 3. Thus, the proposed method can deal with the signal quantization and sensor delay but cannot deal with intermittent measurements and missing/fading measurements. In future works, we will pay more attention to fault-tolerant control, intermittent measurements, and missing/fading measurements by using the proposed methods.

#### Appendix

#### The Decentralized Control Stabilization

The necessary and sufficient conditions for the decentralized stabilization are presented in [29]. Here, we provide the proof for the decentralized stabilization and more details can be seen [29]. The following proofs are cited from [29].

Consider the given system  $\overline{\Sigma}$ :

$$\overline{\Sigma}: \quad x (kT + T) = Ax (kT) + \sum_{i=1}^{r} B_i u_i (kT),$$

$$y_i (kT) = C_i x (kT), \quad i = 1, ..., v.$$
(A.1)

The decentralized stabilization problem for  $\overline{\Sigma}$  is to find controllers  $\Sigma_i$ , i = 1, ..., v, such that the poles of the closed loop system are in the desired locations in the open unit disc. In order to provide an easier bookkeeping, we define the following matrices:

$$B = \begin{bmatrix} B_1 \cdots B_\nu \end{bmatrix}, \qquad C = \begin{bmatrix} C_1^T \cdots C_\nu^T \end{bmatrix}^T,$$
  

$$K = \operatorname{diag} \begin{bmatrix} K_1 \cdots K_\nu \end{bmatrix}, \qquad L = \operatorname{diag} \begin{bmatrix} L_1 \cdots L_\nu \end{bmatrix}, \qquad (A.2)$$
  

$$M = \operatorname{diag} \begin{bmatrix} M_1 \cdots M_\nu \end{bmatrix}, \qquad N = \operatorname{diag} \begin{bmatrix} N_1 \cdots N_\nu \end{bmatrix}.$$

Definition A.1. Consider system  $\overline{\Sigma}$ ;  $\lambda \in C$  is called a decentralized fixed mode if for all block diagonal matrices H one has det $(\lambda I - A - BHC) = 0$ .

**Lemma A.2.** Necessary and sufficient condition for the existence of a decentralized feedback control law for the system  $\overline{\Sigma}$  such that the closed loop system is asymptotically stable is that

*all the fixed modes of the system are asymptotically stable (in the unit disc).* 

*Proof.* We first establish necessity. Assume local controllers  $\Sigma_i$  together stabilize  $\overline{\Sigma}$  then for any  $|\lambda| \ge 1$  there exists a  $\delta$  such that  $(\lambda + \delta)I - K$  is invertible and the closed loop system replacing K with  $K - \delta I$  is still asymptotically stable. This choice is possible because if  $\lambda I - K$  is invertible obviously we can choose  $\delta = 0$ . If  $\lambda I - K$  is not invertible, by small enough choice of  $\delta$  we can make sure that  $(\lambda + \delta)I - K$  is invertible and the closed loop system replacing K with  $K - \delta I$  is still asymptotically stable. But the closed loop system when  $K - \delta I$  is in the loop is asymptotically stable. In particular, it cannot have a pole in  $\lambda$ . So

$$\det \left(\lambda I - A - B\left[M\left(\lambda I - (K - \delta I)\right)^{-1}L + N\right]C\right) \neq 0.$$
(A.3)

Hence the block diagonal matrix  $S = M(\lambda I - (K - \delta I))^{-1}L + N$  has the property that

$$\det \left(\lambda I - A - BSC\right) \neq 0. \tag{A.4}$$

Thus  $\lambda$  is not a fixed mode. Since this argument is true for any  $\lambda$  on or outside the unit disc, this implies that all the fixed modes must be inside the unit disc. This proves the necessity of the Lemma A.2.

Next, we establish sufficiency. To prove that we can actually stabilize the system, we use a recursive argument. Assume the system has an unstable eigenvalue in  $\mu$ . Since  $\mu$  is not a fixed mode there exists  $N_i$  such that

$$A + \sum_{i=1}^{\nu} B_i N_i C_i \tag{A.5}$$

no longer has an eigenvalue in  $\mu$ . Let k be the smallest integer such that an unstable eigenvalue of A is no longer an eigenvalue of

$$A + \sum_{i=1}^{k} B_i N_i C_i, \tag{A.6}$$

while  $N_i$  can be chosen small enough not to introduce additional unstable eigenvalues. Then for the system

$$\left(A + \sum_{i=1}^{k-1} B_i N_i C_i, B_k, C_k\right),$$
(A.7)

an unstable eigenvalue is both observable and controllable. But this implies that there exists a dynamic controller which moves this eigenvalue in the open unit disc without introducing new unstable eigenvalues. Through a recursion, we can move all eigenvalues one-by-one in the open unit disc and in this way find a decentralized controller which stabilizes the system. This proves the sufficiency of Lemma A.2.

#### **Conflict of Interests**

The authors declare that there is no conflict of interests regarding the publication of this paper.

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