Hindawi Publishing Corporation
Mathematical Problems in Engineering
Volume 2007, Article ID 42072, 15 pages
doi:10.1155/2007/42072

Research Article

Analysis of Convective Straight and Radial Fins with Temperature-Dependent Thermal Conductivity Using Variational Iteration Method with Comparison with Respect to Finite Element Analysis

Safa Bozkurt Coşkun and Mehmet Tarik Atay

Received 13 December 2006; Revised 11 April 2007; Accepted 22 September 2007

Recommended by Josef Malek

In order to enhance heat transfer between primary surface and the environment, radiating extended surfaces are commonly utilized. Especially in the case of large temperature differences, variable thermal conductivity has a strong effect on performance of such a surface. In this paper, variational iteration method is used to analyze convective straight and radial fins with temperature-dependent thermal conductivity. In order to show the efficiency of variational iteration method (VIM), the results obtained from VIM analysis are compared with previously obtained results using Adomian decomposition method (ADM) and the results from finite element analysis. VIM produces analytical expressions for the solution of nonlinear differential equations. However, these expressions obtained from VIM must be tested with respect to the results obtained from a reliable numerical method or analytical solution. This work assures that VIM is a promising method for the analysis of convective straight and radial fin problems.

1. Introduction

Finned surfaces have enhanced heat transfer mechanism between the primary surface and its surrounding medium. Such heat transfer mechanisms are highly demanded with the developing technology. A fin array in conduction combined with radiation in a nonparticipating medium or basis of dynamics of heat transfer in a space radiator and basic one-dimensional radiating fins have been studied extensively [1–11]. A heat-rejecting system consisting of parallel tubes joined by web plates was studied by Bartas and Sellers [1]. Expression of the optimum proportion of triangular fins radiating to space at absolute zero was presented by Wilkins Jr [2]. As a structural element in the space craft applications, applications of the radiator were studied by Cockfield [4]. Optimization of mass and structure of a space radiator for a flight power system was considered by Keil [5]. Optimum shape and minimum mass of a thin film with diffuse reflecting surfaces were determined by Chung and Zhang [6]. Krishnaprakas and Narayana studied the optimum design of a longitudinal rectangular fin system with angle [7]. The topic of optimizing the design of heat tube/fin-type space radiators for the case of uniformly tapered fins for flat fins was considered by Naumann [8]. Since the temperature difference of the fin base and its tip is high in the actual situation, taking into consideration the variation of the conductivity is an important issue. For this purpose, Arslantürk [12] made an analysis including the effects of the variation of the thermal conductivity of the radial fin material by using Adomian decomposition method (ADM).

For the convective straight fins, a number of studies have also been conducted. Aziz and Hug [13] obtained a closed form solution for a straight convective fin with variable thermal conductivity using regular perturbation method. Yu and Chen [14] solved nonlinear conducting-convecting-radiating heat transfer equation assuming a linear variation for thermal conductivity. Razelos and Imre [15] took a variable convective heat transfer coefficient into account in fin problem. Laor and Kalman [16] analyzed different fins with temperature-dependent heat transfer coefficient. Arslantürk [17] used ADM to obtain analytical expressions for dimensionless temperature and fin efficiency with temperature-dependent thermal conductivity.

ADM has also been used for analyzing various nonlinear problems in heat transfer [18–22]. ADM is a technique for obtaining analytical expressions in the solution of nonlinear differential equations [23]. The method depends heavily on tedious work of finding of Adomian polynomials and then using them in the iteration of ADM approach to obtain an analytical expression as a solution. In brief, this is a cumbersome process especially in obtaining higher-order approximations.

In recent years, a solution technique called variational iteration method (VIM) [24] has been given great importance for solving nonlinear differential equations. VIM is a kind of variational-based analytical technique in efficient solution of nonlinear differential equations including boundary value and initial value problems, nonlinear system of differential equations, nonlinear partial differential equations such as linear fractional partial differential equations arising fluid mechanics, nonlinear thermoelasticity problems, nonlinear fluid flows in pipe-like domain problems, or solving integro-differential equations [25–30]. The reason for choosing VIM in present problem is that this solution technique can be used in many engineering problems effectively. VIM directly gives the solution of corresponding equation which is one of its advantages when compared to ADM solution for the second example. The formulation and solution processes of VIM are much easier when compared to decomposition methods, in this respect, VIM is an easy-to-apply method for the analysis of nonlinear problems in engineering.

In this study, analyses of convective straight and radial fins with temperature-dependent thermal conductivity are carried out by using VIM and FEM. The results obtained from both analyses are also compared with the available results in the literature obtained previously using ADM. Hence, the efficiency of the VIM solution technique can be illustrated with comparison with respect to ADM and FEM. Also by means of these comparisons, it can be shown that VIM is a better alternative in the solution of such problems.

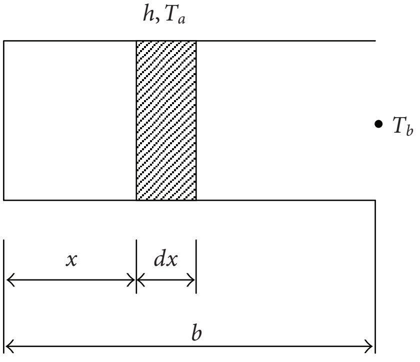

FIGURE 2.1. Geometry of a straight fin.

2. Convective straight and radial fins

2.1. Convective straight fins with temperature-dependent thermal conductivity. The straight fin in Figure 2.1 is considered by assuming a temperature-dependent thermal conductivity with an arbitrary cross-sectional area A_c, perimeter P and length b.

The temperature of the base surface where the fin is attached is T_b, surrounding fluid temperature is T_a. Fin's tip is insulated. One-dimensional energy-balance equation is

$$A_c \frac{d}{dx}\left[k(T)\frac{dT}{dx}\right] - Ph(T - T_a) = 0 \tag{2.1}$$

where $k(T)$ is temperature-dependent thermal conductivity. If thermal conductivity is assumed to be a linear function of temperature, it becomes as follows:

$$k(T) = k_a[1 + \lambda(T - T_a)], \tag{2.2}$$

where k_a is the thermal conductivity at the ambient fluid temperature of the fin and λ is a parameter defining the variation of thermal conductivity.

Introducing the following dimensionless parameters,

$$\theta = \frac{T - T_a}{T_b - T_a}, \qquad \xi = \frac{x}{b}, \qquad \beta = \lambda(T_b - T_a); \qquad \psi = \left(\frac{hPb^2}{k_a A_c}\right)^{1/2}, \tag{2.3}$$

(2.1) reduces to the following equation

$$\frac{d^2\theta}{d\xi^2} + \beta\theta\frac{d^2\theta}{d\xi^2} + \beta\left(\frac{d\theta}{d\xi}\right)^2 - \psi^2\theta = 0; \quad 0 \le \xi \le 1 \tag{2.4}$$

with the following boundary conditions:

$$\left.\frac{d\theta}{d\xi}\right|_{\xi=0} = 0, \quad \theta|_{\xi=1} = 1. \tag{2.5}$$

The computational domain $0 \le x \le b$ is transformed to $0 \le \xi \le 1$ by introducing the dimensionless parameters given in (2.3).

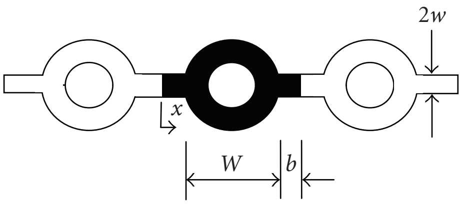

FIGURE 2.2. A heat pipe/fin radiating element.

2.2. Radial fins with temperature-dependent thermal conductivity. An example of heat pipe/fin space radiator is shown in Figure 2.2. Both surfaces of the fin are radiating to the outer space at a very low temperature, which is assumed equal to zero absolute. The fin has temperature-dependent thermal conductivity k, which depends on temperature linearly and fin is diffuse-grey with emissivity ε. The tube surfaces temperature and the base temperature T_b of the fin are constant, and the radiative exchange between the fin and the heat pipe is neglected. The temperature distribution within the fin is assumed to be one dimensional, because the fin is assumed to be thin. Hence, only fin tip length b is considered as the computational domain.

The energy balance equation for a differential element of the fin is given as

$$2w\frac{d}{dx}\left[k(T)\frac{dT}{dx}\right]-2\varepsilon\sigma T^4=0, \tag{2.6}$$

where $k(T)$ and σ are thermal conductivity and the Stefan-Boltzmann constant, respectively.

The thermal conductivity of the fin material is assumed to be a linear function of temperature according to

$$k(T)=k_b\left[1+\lambda(T-T_b)\right], \tag{2.7}$$

where k_b is the thermal conductivity at the base temperature of the fin and λ is the slope of the thermal conductivity-temperature curve.

Introducing the following dimensionless parameters

$$\theta=\frac{T}{T_b},\qquad \xi=\frac{x}{b},\qquad \beta=\lambda T_b,\qquad \psi=\frac{\varepsilon\sigma b^2T_b^3}{k_bw}, \tag{2.8}$$

the formulation of the fin problem reduces to the following equation:

$$\frac{d^2\theta}{d\xi^2}+\beta\theta\frac{d^2\theta}{d\xi^2}+\beta\left(\frac{d\theta}{d\xi}\right)^2-\psi\theta^4=0,\quad 0\le\xi\le 1, \tag{2.9}$$

with the following boundary conditions:

$$\left.\frac{d\theta}{d\xi}\right|_{\xi=0}=0,\quad \theta|_{\xi=1}=1. \tag{2.10}$$

As in straight fin, case computational domain is transformed to $0\le\xi\le 1$ by introducing the dimensionless parameters given in (2.8).

3. VIM formulation of the problem

According to VIM, the differential equation (2.9) may be considered

$$Lu + Nu = g(x), \tag{3.1}$$

where L is a linear operator, N is a nonlinear operator, and $g(x)$ is an inhomogeneous term.

Based on VIM, a correct functional can be constructed as follows:

$$u_{n+1}(x) = u_n(x) + \int_0^x \lambda\{Lu_n(\tau) + N\tilde{u}_n(\tau) - g(\tau)\}d\tau, \tag{3.2}$$

where λ is a general Lagrangian multiplier, which can be identified optimally via the variational theory, the subscript n denotes the nth-order approximation, $\tilde{u}$ is considered as a restricted variation, that is, $\delta\tilde{u} = 0$. Applying the formulation given above to differential equation (2.9), a new differential equation for λ can be obtained as follows:

$$\lambda''(\tau) = 0, \quad \text{when } \tau = \xi. \tag{3.3}$$

To solve (3.3), boundary conditions are obtained by integrating parts of (2.9) with respect to (3.2);

B.C.1: for $\delta\theta_n'(\xi)$,

$$\lambda(\tau) = 0, \quad \text{when } \tau = \xi; \tag{3.4}$$

B.C.2: for $\delta\theta_n(\xi)$,

$$(1 - \lambda'(\tau)) = 0, \quad \text{when } \tau = \xi. \tag{3.5}$$

Then, Lagrange Multiplier λ is obtained by assuming $L = d^2/d\xi^2$ with the restricted variation $\delta\tilde{u}_n = 0$.

If the above formulation is applied to (3.2), the following iteration formula can be obtained accordingly:

$$\theta_{n+1}(\xi) = \theta_n(\xi) + \int_0^\xi \lambda(\tau)\{L\theta_n(\tau) + N\tilde{\theta}_n(\tau)\}d\tau \tag{3.6}$$

with Lagrange multiplier as follows:

$$\lambda(\tau) = \tau - \xi. \tag{3.7}$$

The iteration formula given in (3.6) is a simple approximation. Further information about finding Lagrange multiplier λ and its related boundary conditions can be found in [24–30]. Especially, [24] is the pioneering work for the specific method VIM and other references include the applications of the method to different problems.

4. Solutions for fin temperature distribution

4.1. Straight fins. As a starting approximation for VIM solution, θ is assumed as constant, which was assumed as constant also in ADM solution [17]. First, three iterations of the VIM are

$$\theta_0 = A, \tag{4.1}$$

$$\theta_1 = A + \frac{1}{2}A\psi^2\xi^2, \tag{4.2}$$

$$\theta_2 = A + \frac{1}{2}A(1 - A\beta)\psi^2\xi^2 + \frac{1}{24}A(1 - 3A\beta)\psi^4\xi^4, \tag{4.3}$$

$$\begin{aligned}\theta_3 = A &+ \frac{1}{2}A\left(1 - A\beta + A^2\beta^2\right)\psi^2\xi^2 + \frac{1}{24}A\left(1 - 5A\beta + 9A^2\beta^2 - 3A^3\beta^3\right)\psi^4\xi^4 \\ &+ \frac{1}{720}A\left(1 - 18A\beta + 60A^2\beta^2 - 45A^3\beta^3\right)\psi^6\xi^6 - \frac{1}{1152}A^2\beta(-1 + 3A\beta)^2\psi^8\xi^8.\end{aligned} \tag{4.4}$$

An approximate expression for temperature distribution can be obtained by ignoring higher-order terms in the expression at the end of the seventh iteration. Hence, an approximate solution for θ becomes

$$\begin{aligned}\theta \cong A &+ \frac{A}{2}\left(1 - A\beta + A^2\beta^2 - A^3\beta^3 + A^4\beta^4 - A^5\beta^5 + A^6\beta^6\right)\psi^2\xi^2 \\ &+ \frac{A}{24}\left(1 - 5A\beta + 12A^2\beta^2 - 22A^3\beta^3 + 35A^4\beta^4 - 51A^5\beta^5\right. \\ &\qquad \left. + 63A^6\beta^6 - 45A^7\beta^7 + 30A^8\beta^8 - 18A^9\beta^9 + 9A^{10}\beta^{10}\right)\psi^4\xi^4 \\ &+ \frac{A}{720}\left(1 - 21A\beta + 123A^2\beta^2 - 415A^3\beta^3 + 1050A^4\beta^4 - 2205A^5\beta^5\right. \\ &\qquad \left. + 3735A^6\beta^6 - 4530A^7\beta^7 + 4617A^8\beta^8 - 4113A^9\beta^9 + 3180A^{10}\beta^{10}\right)\psi^6\xi^6 \\ &+ \frac{A}{40320}\left(1 - 85A\beta + 1174A^2\beta^2 - 7364A^3\beta^3 + 29799A^4\beta^4\right. \\ &\qquad - 90604A^5\beta^5 + 210484A^6\beta^6 - 364515A^7\beta^7 + 514868A^8\beta^8 \\ &\qquad \left. - 621369A^9\beta^9 + 649755A^{10}\beta^{10}\right)\psi^8\xi^8 \\ &+ \frac{A}{3628800}\left(1 - 341A\beta + 10845A^2\beta^2 - 125274A^3\beta^3 + 813763A^4\beta^4\right. \\ &\qquad - 3595442A^5\beta^5 + 11434800A^6\beta^6 - 26965949A^7\beta^7 + 50732463A^8\beta^8 \\ &\qquad \left. - 79893120A^9\beta^9 + 107847459A^{10}\beta^{10}\right)\psi^{10}\xi^{10}.\end{aligned} \tag{4.5}$$

In (4.1), coefficient A is the temperature at the fin tip, and must lie in the interval $[0,1]$. Value of A can be determined by applying the boundary conditions in (2.5)–(4.5). Since A is assumed as constant as an initial guess, it automatically satisfies the derivative boundary condition. As seen from the succeeding equations given in (4.1)–(4.5), additional terms include ξ with the power two or more. Hence, derivative boundary condition at $\xi = 0$ again automatically satisfies. Due to this fact, the only parameter A can be determined by means of the following boundary condition:

$$\theta|_{\xi=1} = 1. \tag{4.6}$$

4.2. Radial fins. As a starting approximation for VIM solution, θ is assumed as constant. The first two iterations are

$$\theta_0 = A, \tag{4.7}$$

$$\theta_1 = A + \frac{1}{2}A^4\psi\xi^2, \tag{4.8}$$

$$\begin{aligned}\theta_2 = A &+ \frac{1}{2}A^4\xi^2\psi - \frac{1}{2}A^5\beta\xi^2\psi - \frac{1}{24}A^7(-4+3A\beta)\psi^2\xi^4 \\ &+ \frac{1}{20}A^{10}\psi^3\xi^6 + \frac{1}{112}A^{13}\psi^4\xi^8 + \frac{1}{1440}A^{16}\psi^5\xi^{10}.\end{aligned} \tag{4.9}$$

VIM solution for radial fins is obtained at the end of the fourth iteration. As in straight fins, coefficient A is the temperature at the fin tip. The computational domain is defined by a nondimensional term ξ and the value of ξ is in the interval of $[0,1]$. Value of A can be determined again by applying the boundary conditions in (2.10) to (4.9).

5. Numerical results

5.1. Straight fins. The VIM results obtained from VIM analysis are compared with FEM results and also the results obtained from an approximate sixth iteration ADM expression given in [17]. FEM analyses of the problems are conducted using FlexPDE version 5. In the FEM analysis, quadratic basis functions and a modified Newton-Raphson algorithm are employed. A root-mean-square error criterion is used as a stopping criterion and the error values are changing between 10^{-7}–10^{-10}.

Between Figures 5.1–5.3, dimensionless temperature variations for straight fins for different ψ values are demonstrated. Between Figures 5.4–5.7, the behavior of fin tip temperature, A, is represented as with the increasing values of thermogeometric fin parameter. Results have shown that VIM expression still provides a good approximation for the temperature variation at the fin tip.

5.2. Radial fins. As in straight fins, the VIM results obtained from VIM analysis of radial fins are compared with FEM results and also the fifth iteration ADM results available in the literature [12].

Between Figures 5.8–5.11, dimensionless temperature variations for radial fins for different β values are shown. Between Figures 5.12–5.15, the behavior of fin tip temperature, A, is given with the increasing values of thermogeometric fin parameter. Results have shown that, VIM expression also provides a good approximation for the temperature variation at the fin tip.

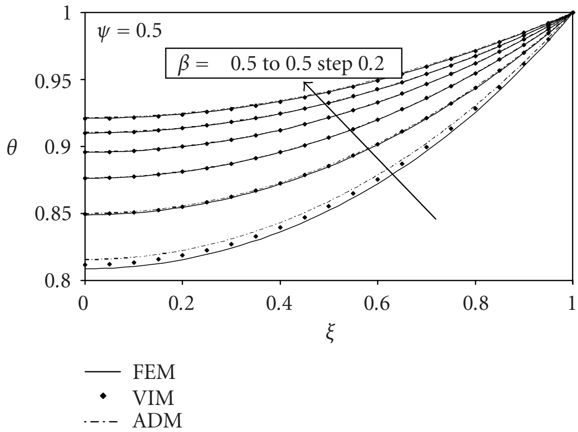

Figure 5.1. Comparison for dimensionless temperature variation for $\psi = 0.5$.

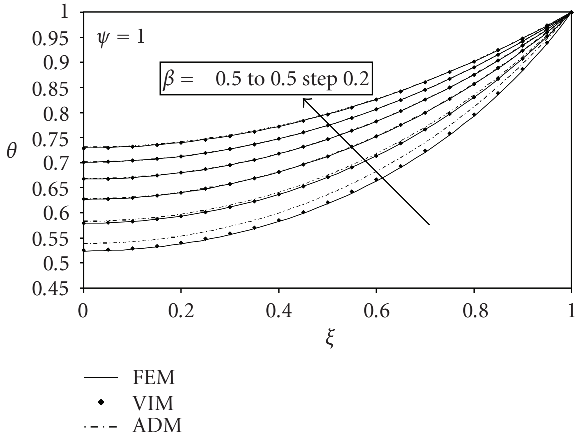

Figure 5.2. Comparison for dimensionless temperature variation for $\psi = 1.0$.

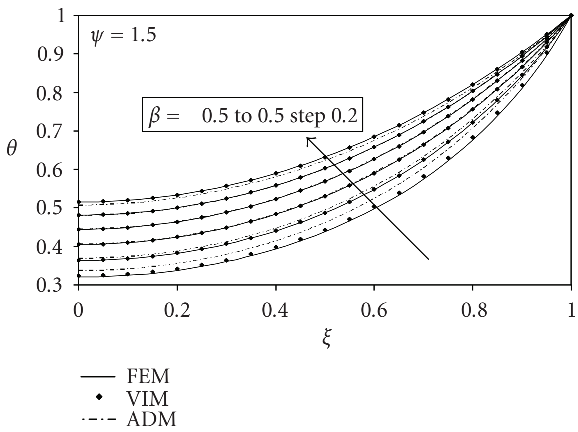

Figure 5.3. Comparison for dimensionless temperature variation for $\psi = 1.5$.

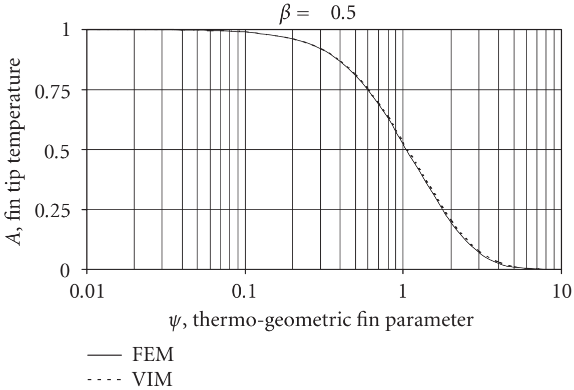

FIGURE 5.4. Variation of dimensionless fin tip temperature for $\beta = -0.5$.

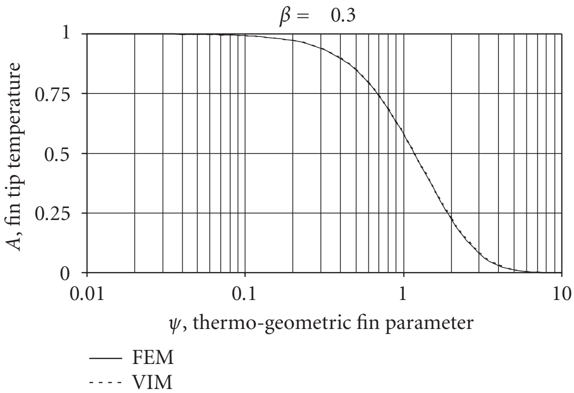

FIGURE 5.5. Variation of dimensionless fin tip temperature for $\beta = -0.3$.

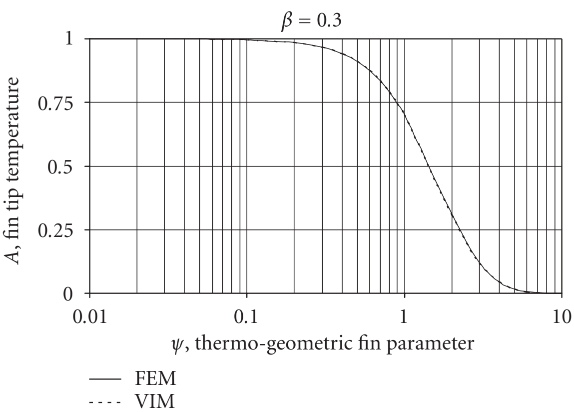

FIGURE 5.6. Variation of dimensionless fin tip temperature for $\beta = 0.3$.

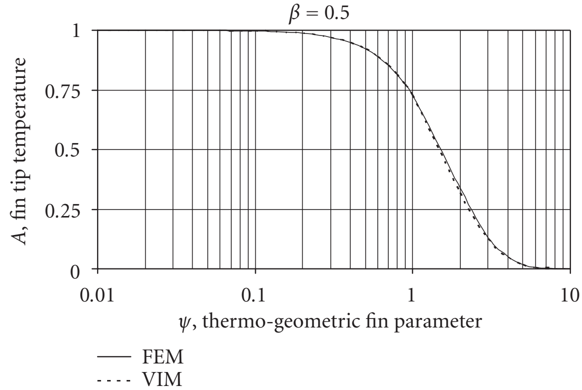

FIGURE 5.7. Variation of dimensionless fin tip temperature for $\beta = 0.5$.

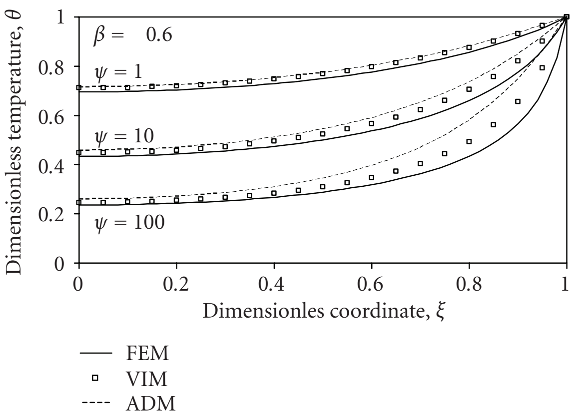

FIGURE 5.8. Comparison for dimensionless temperature variation for $\beta = -0.6$.

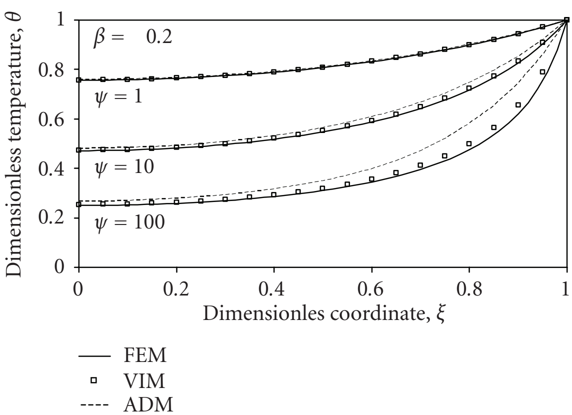

FIGURE 5.9. Comparison for dimensionless temperature variation for $\beta = -0.2$.

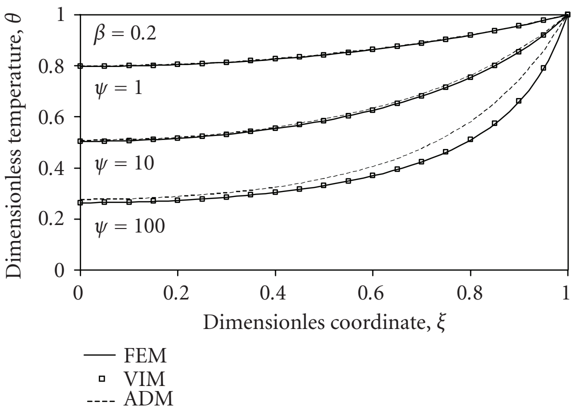

FIGURE 5.10. Comparison for dimensionless temperature variation for $\beta = 0.2$.

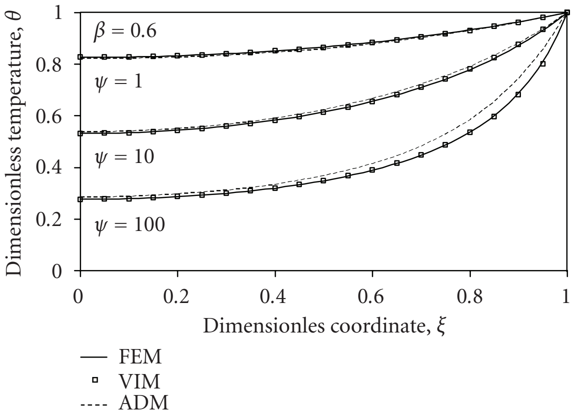

FIGURE 5.11. Comparison for dimensionless temperature variation for $\beta = 0.6$.

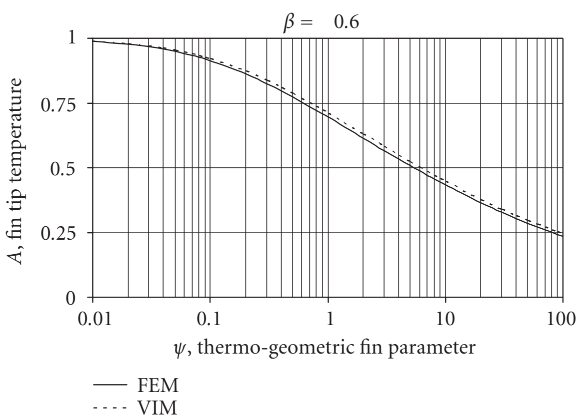

FIGURE 5.12. Variation of dimensionless fin tip temperature for $\beta = -0.6$.

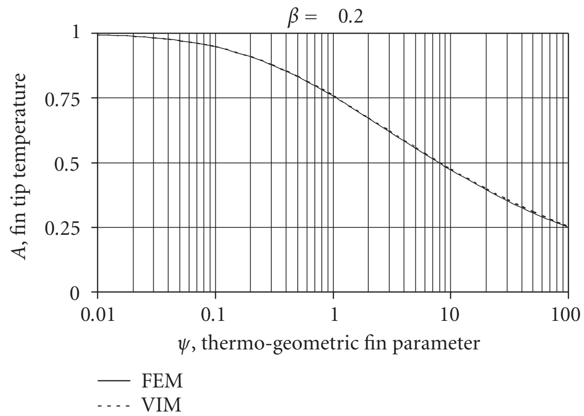

Figure 5.13. Variation of dimensionless fin tip temperature for $\beta = -0.2$.

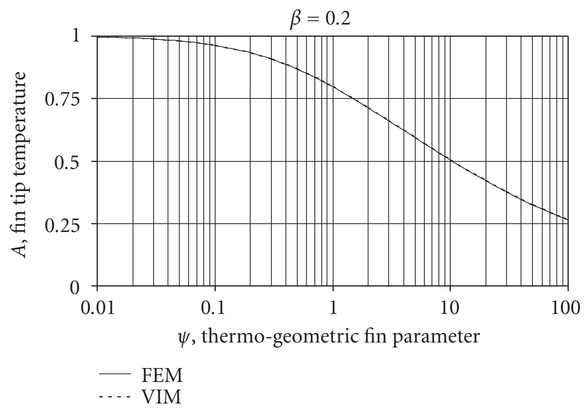

Figure 5.14. Variation of dimensionless fin tip temperature for $\beta = 0.2$.

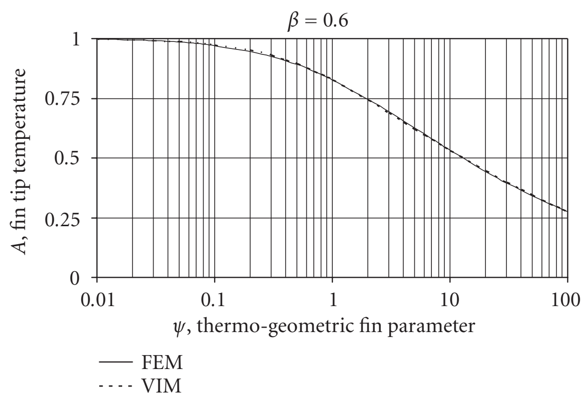

Figure 5.15. Variation of dimensionless fin tip temperature for $\beta = 0.6$.

6. Conclusion

VIM and FEM analyses of convective straight fins and radial fins with temperature-dependent thermal conductivity have been conducted in this study. VIM is a variational-based iterative technique and it is an effective method in the solution of nonlinear differential equations. In each iteration, the method gives directly the solution as a polynomial expression and this is the main advantage of the method when compared to ADM or FEM. For the problems considered in this study, the solution is obtained in the form of a higher-order polynomial ($n > 8$) in the space variable ξ. It can be clearly seen from the figures, VIM results seem much better than the results of ADM. With the increasing effect of variable thermal conductivity which leads to increasing nonlinearity in the equation, higher-order approximations may be required in VIM solution in order to reach an acceptable accuracy. However, VIM solutions for both problems give better results when compared to ADM solutions at the same order of approximation. If the nonlinearity in the equation to be solved increases significantly, more iteration can be required and this may be a time-consuming process. However, for the present study, obtained results are enough to come to a conclusion about the efficiency of the method. It is also observed that the value of thermogeometric fin parameter is another factor affecting the behavior of the solution. As a result, it can be concluded that VIM is an advantageous method when compared to ADM in view of formulation and solution processes.

Nomenclature

A: Integral constant
A_c: Cross-sectional area of the fin (m^2)
b: Fin base (m)
h: Heat transfer coefficient ($Wm^{-2}K^{-1}$)
k_a: Thermal conductivity at the ambient fluid temperature ($Wm^{-1}K^{-1}$)
k: Thermal conductivity of the fin material ($Wm^{-1}K^{-1}$)
k_b: Thermal conductivity at the base temperature ($Wm^{-1}K^{-1}$)
L: Linear differential operator
N: Nonlinear differential operator
T: Temperature (K)
T_b: Temperature at fin base (K)
x: Distance measured from the fin tip (m)
w: Semithickness of the fin (m)
P: Fin perimeter (m)
Q: Heat transfer rate (W)
W: Effective radiating width of the heat pipe (m)

References

[1] J. G. Bartas and W. H. Sellers, "Radiation fin effectiveness," *Journal of Heat Transfer, Series C*, vol. 82, pp. 73–75, 1960.

[2] J. E. Wilkins Jr., "Minimizing the mass of thin radiating fins," *Journal of Aerospace Science*, vol. 27, p. 145, 1960.

[3] B. V. Karlekar and B. T. Chao, “Mass minimization of radiating trapezoidal fins with negligible base cylinder interaction,” *International Journal of Heat and Mass Transfer*, vol. 6, no. 1, pp. 33–48, 1963.
[4] R. D. Cockfield, “Structural optimization of a space radiator,” *Journal of Spacecraft Rockets*, vol. 5, no. 10, p. 1240, 1968.
[5] H. Keil, “Optimization of a central-fin space radiator,” *Journal of Spacecraft Rockets*, vol. 5, no. 4, p. 463, 1968.
[6] B. T. F. Chung and B. X. Zhang, “Optimization of radiating fin array including mutual irradiations between radiator elements,” *ASME Journal of Heat Transfer*, vol. 113, p. 814, 1991.
[7] C. K. Krishnaprakas and K. B. Narayana, “Heat transfer analysis of mutually irradiating fins,” *International Journal of Heat and Mass Transfer*, vol. 46, p. 761, 2003.
[8] R. J. Naumann, “Optimizing the design of space radiators,” *International Journal of Thermophys*, vol. 25, no. 6, pp. 1929–1941, 2004.
[9] C. H. Chiu and C. K. Chen, “A decomposition method for solving the convective longitudinal fins with variable thermal conductivity,” *International Journal of Heat and Mass Transfer*, vol. 45, no. 10, pp. 2067–2075, 2002.
[10] C. H. Chiu and C. K. Chen, “Applications of adomian’s decomposition procedure to the analysis of convective-radiative fins,” *Journal of Heat Transfer*, vol. 125, no. 2, pp. 312–316, 2003.
[11] D. Lesnic and P. J. Heggs, “A decomposition method for power-law fin-type problems,” *International Communications in Heat and Mass Transfer*, vol. 31, no. 5, pp. 673–682, 2004.
[12] C. Arslantürk, “Optimum design of space radiators with temperature-dependent thermal conductivity,” *Applied Thermal Engineering*, vol. 26, no. 11-12, pp. 1149–1157, 2006.
[13] A. Aziz and S. M. E. Hug, “Perturbation solution for convecting fin with variable thermal conductivity,” *Journal of Heat Transfer*, vol. 97, p. 300, 1975.
[14] L. T. Yu and C. K. Chen, “Optimization of circular fins with variable thermal parameters,” *Journal of the Franklin Institute*, vol. 336, no. 1, pp. 77–95, 1999.
[15] P. Razelos and K. Imre, “The optimum dimensions of circular fins with variable thermal parameters,” *Journal of Heat Transfer*, vol. 102, p. 420, 1980.
[16] K. Laor and H. Kalman, “Performance and optimum dimensions of different cooling fins with a temperature-dependent heat transfer coefficient,” *International Journal of Heat and Mass Transfer*, vol. 39, no. 9, pp. 1993–2003, 1996.
[17] C. Arslantürk, “A decomposition method for fin efficiency of convective straight fins with temperature-dependent thermal conductivity,” *International Communications in Heat and Mass Transfer*, vol. 32, no. 6, pp. 831–841, 2005.
[18] D. Lesnic and L. Elliott, “The decomposition approach to inverse heat conduction,” *Journal of Mathematical Analysis and Applications*, vol. 232, no. 1, pp. 82–98, 1999.
[19] D. Lesnic, “Convergence of adomian’s decomposition method: periodic temperatures,” *Computers & Mathematics with Applications*, vol. 44, no. 1-2, pp. 13–24, 2002.
[20] D. Lesnic and P. J. Heggs, “A decomposition method for power-law fin-type problems,” *International Communications in Heat and Mass Transfer*, vol. 31, no. 5, pp. 673–682, 2004.
[21] C. H. Chiu and C. K. Chen, “A decomposition method for solving the convective longitudinal fins with variable thermal conductivity,” *International Journal of Heat and Mass Transfer*, vol. 45, no. 10, pp. 2067–2075, 2002.
[22] C. H. Chiu and C. K. Chen, “Application of the decomposition method to thermal stresses in isotropic circular fins with temperature-dependent thermal conductivity,” *Acta Mechanica*, vol. 157, no. 1–4, pp. 147–158, 2002.
[23] G. Adomian, *Solving Frontier Problems in Physics: The Decomposition Method*, Kluwer Academic, Dordrecht, The Netherlands, 1988.
[24] J. H. He, “Variational iteration method—a kind of non-linear analytical technique: some examples,” *International Journal of Nonlinear Mechanics*, vol. 34, no. 4, pp. 699–708, 1999.

[25] M. A. Abdou and A. A. Soliman, "New applications of variational iteration method," *Physica D*, vol. 211, no. 1-2, pp. 1–8, 2005.

[26] S. Momani and Z. Odibat, "Analytical approach to linear fractional partial differential equations arising in fluid mechanics," *Physics Letter A.*, vol. 355, no. 4-5, pp. 271–279, 2006.

[27] S. Momani, S. Abuasad, and Z. Odibat, "Variational iteration method for solving nonlinear boundary value problems," *Applied Mathematics and Computation*, vol. 183, no. 2, pp. 1351–1358, 2006.

[28] N. H. Sweilam and M. M. Khader, "Variational iteration method for one dimensional nonlinear thermoelasticity," *Chaos, Solitons and Fractals*, vol. 32, no. 1, pp. 145–149, 2007.

[29] E. M. Abulwafa, M. A. Abdou, and A. A. Mahmoud, "Nonlinear fluid flows in pipe-like domain problem using variational-iteration method," *Chaos, Solitons and Fractals*, vol. 32, no. 4, pp. 1384–1397, 2007.

[30] S. -Q. Wang and J.-H. He, "Variational iteration method for solving integrodifferential equations," *Physics Letter A*, 2007.

Safa Bozkurt Coşkun: Department of Civil Engineering, Nigde University, 51100 Nigde, Turkey
Email address: sbcoskun@nigde.edu.tr

Mehmet Tarik Atay: Department of Mathematics, Nigde University, 51100 Nigde, Turkey
Email address: ataymt@nigde.edu.tr


Advances in
Operations Research
Advances in
Decision Sciences
Journal of
Applied Mathematics
Algebra
Journal of
Probability and Statistics
The Scientific
World Journal
International Journal of
Differential Equations
Hindawi
Submit your manuscripts at
http://www.hindawi.com
International Journal of
Combinatorics
Advances in
Mathematical Physics
Journal of
Complex Analysis
Journal of
Mathematics
Mathematical Problems
in Engineering
Abstract and
Applied Analysis
Discrete Dynamics in
Nature and Society
International
Journal of
Mathematics and
Mathematical
Sciences
Journal of
Discrete Mathematics
Journal of
Function Spaces
International Journal of
Stochastic Analysis
Journal of
Optimization