

## Research Article

# An Optimized Grey Dynamic Model for Forecasting the Output of High-Tech Industry in China

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The grey dynamic model by convolution integral with the first-order derivative of the 1-AGO data and  $n$  series related, abbreviated as GDMC(1,  $n$ ), performs well in modelling and forecasting of a grey system. To improve the modelling accuracy of GDMC(1,  $n$ ),  $n$  interpolation coefficients (taken as unknown parameters) are introduced into the background values of the  $n$  variables. The parameters optimization is formulated as a combinatorial optimization problem and is solved collectively using the particle swarm optimization algorithm. The optimized result has been verified by a case study of the economic output of high-tech industry in China. Comparisons of the obtained modelling results from the optimized GDMC(1,  $n$ ) model with the traditional one demonstrate that the optimal algorithm is a good alternative for parameters optimization of the GDMC(1,  $n$ ) model. The modelling results can assist the government in developing future policies regarding high-tech industry management.

## 1. Introduction

Grey system theory [1, 2], a rising interdisciplinary in uncertainty, takes the uncertain systems of “small samples and incomplete information” with “partial known information and partial unknown information” as the research object. This theory extracts valuable information mainly by generating and developing the “partial” known information. Then, the accurate demonstration of the system running behavior and the evolution rule is done and effective monitoring is achieved. Grey system theory can be used like this to deal with the practical problems in a situation where the sample data size is not large enough to demonstrate the statistical law or the deterministic law of the system behavior.

At present, the most widely used grey model GM(1, 1) [1, 2]—a first-order one-variable grey differential equation—is proposed based on the aforementioned principle. Its modelling principle does not depend on distribution information from the raw data but on the application of a first-order accumulative generation operation (1-AGO), to make the sequence display grey exponent law behavior as a whole. Based on this, a first-order grey differential equation is constructed and solved. The forecast values are then derived from

the first-order inverse accumulative generating operation (1-AGO). Due to the fact that the construction of GM(1, 1) does not require a large sample and it is easy to build and calculate, GM(1, 1) and its improved variant models have been widely used [3–6].

The GM(1,  $n$ ) model [1], with  $n-1$  relative variables acting as an associated series besides the predicted series, is a basic grey multivariable model. The forecasts of a time series  $X_1^{(0)}$  may be considerably improved by using information coming from some associated series  $X_i^{(0)}$ ,  $i = 2, 3, \dots, n$ . This is particularly true if changes in  $X_1^{(0)}$  tend to be anticipated by changes in  $X_i^{(0)}$ ,  $i = 2, 3, \dots, n$ . Nevertheless, the solution of the whitening differential equation for GM(1,  $n$ ) is rough [7]—it can easily result in large errors in actual forecasting applications. Thus, for a long time, this model has been little used. In fact, the only existing research applications have proceeded on the basis of improved models [8–10]. The grey model with convolution integral, GMC(1,  $n$ ), has been proposed by Tien [11]. This is a new model which seeks to improve the traditional GM(1,  $n$ ) model. Theoretically, the modelling values produced by GMC(1,  $n$ ) are the exact solution of the traditional GM(1,  $n$ ) model and the grey control parameter

(similar to that of the GM(1, 1) model) introduced into the model. When the number of variables in the model  $n = 1$ , the GMC(1,  $n$ ) model reduces to GM(1, 1). Using GMC(1,  $n$ ) greatly improves the forecasting accuracy of the multivariable grey model and has been successfully applied to various different fields [11–13]. The multivariable grey model GDMC(1,  $n$ ) [14] is based on the model GMC(1,  $n$ ). The higher frequency components of the 1-AGO data of the associated series were significant for the grey prediction system and the first-order derivatives of 1-AGO data of the associated series were added into the GDMC(1,  $n$ ) model.

In this study,  $n$  interpolation coefficients are introduced into the background values of the variables in GDMC(1,  $n$ ) model. Then, by aiming to minimize the modelling error, the optimal parameters are solved using the particle swarm optimization algorithm. Thereby, the adaptability and forecasting accuracy of GDMC(1,  $n$ ) to real data are enhanced. The remainder of this paper is organized as follows. Section 2 explains the modelling methods, including the traditional GDMC(1,  $n$ ) and the optimization method. In Section 3, the traditional GDMC(1,  $n$ ) and the optimized GDMC(1,  $n$ ) are applied to forecast the output of high-tech industry in China. Finally, conclusions are presented in Section 4.

## 2. Methodology

**2.1. The Representation of the Grey Dynamic Model with Convolution Integral.** Suppose that pairs of observations  $X^{(0)} = (X_1^{(0)}, X_2^{(0)}, \dots, X_n^{(0)})$  are available at equispaced time intervals consisting of  $n - 1$  inputs  $X_2^{(0)}, X_3^{(0)}, \dots, X_n^{(0)}$  and an output  $X_1^{(0)}$  from some dynamic system. The existing GDMC(1,  $n$ ) modelling process [14] is carried out as follows. Consider the original predicted series

$$X_1^{(0)} = \{X_1^{(0)}(rp + 1), X_1^{(0)}(rp + 2), \dots, X_1^{(0)}(rp + r)\} \quad (1)$$

and the original associated series

$$X_i^{(0)} = \{X_i^{(0)}(1), X_i^{(0)}(2), \dots, X_i^{(0)}(r)\}, \quad i = 2, 3, \dots, n. \quad (2)$$

Then the 1-AGO data for  $X_1^{(0)}, X_2^{(0)}, \dots, X_n^{(0)}$  are given by the following equations, respectively:

$$X_1^{(1)}(rp + t) = \sum_{j=1}^t X_1^{(0)}(rp + j), \quad t = 1, 2, \dots, r, \quad (3)$$

$$X_i^{(1)}(t) = \sum_{j=1}^t X_i^{(0)}(j), \quad t = 1, 2, \dots, r, \quad j = 2, 3, \dots, n. \quad (4)$$

The grey forecasting model based on the predicted 1-AGO series

$$X_1^{(1)} = \{X_1^{(1)}(rp + 1), X_1^{(1)}(rp + 2), \dots, X_1^{(1)}(rp + r)\} \quad (5)$$

and the associated 1-AGO series

$$X_i^{(1)} = \{X_i^{(1)}(1), X_i^{(1)}(2), \dots, X_i^{(1)}(r)\}, \quad i = 2, 3, \dots, n, \quad (6)$$

is given by the following differential equation:

$$\begin{aligned} & \frac{dX_1^{(1)}(rp + t)}{dt} + b_1 X_1^{(1)}(rp + t) \\ &= b_2 + \sum_{i=2}^n \left[ b_{2i-1} \frac{dX_i^{(1)}(t)}{dt} + b_{2i} X_i^{(1)}(t) \right], \end{aligned} \quad (7)$$

where  $b_1$  and  $b_2$  are the developmental coefficient and the grey control parameter, respectively, and  $(b_{2i-1}, b_{2i})$  are the associated coefficients corresponding to the associated series  $X_i^{(0)}$ ,  $i = 2, 3, \dots, n$ , respectively,  $r$  is the number of entries for model building,  $rp$  is a period of delay, and  $b_1, b_2, \dots, b_{2n}$  are model parameters to be estimated.

Equation (7) is called the grey dynamic model by convolution integral with the first-order derivative of the 1-AGO data and  $n$  series related, abbreviated as GDMC(1,  $n$ )—the “1” represents the first-order derivative of the 1-AGO series of  $X_1^{(1)}$  and the “ $n$ ” represents the total of  $n$  relative series introduced into the grey differential equation.

**2.2. The Evaluation of Parameters  $b_1, b_2, \dots$ , and  $b_{2n}$ .** The grey derivative for the first-order grey differential equation with 1-AGO is conventionally represented as

$$\frac{dX_1^{(1)}(rp + t)}{dt} = \lim_{\Delta t \rightarrow 0} \frac{X_1^{(1)}(rp + t) - X_1^{(1)}(rp + t - \Delta t)}{\Delta t}. \quad (8)$$

When  $\Delta t \rightarrow 1$ ,

$$\begin{aligned} \frac{dX_1^{(1)}(rp + t)}{dt} &= \frac{\Delta X_1^{(1)}(rp + t)}{\Delta t} \\ &= X_1^{(1)}(rp + t) - X_1^{(1)}(rp + t - 1) \\ &= X_1^{(0)}(rp + t). \end{aligned} \quad (9)$$

The background value of the grey derivative  $dX_1^{(1)}(rp + t)/dt$  is taken as the mean of  $X_1^{(1)}(rp + t)$  and  $X_1^{(1)}(rp + t - 1)$ . Those of the associated series  $X_i^{(1)}(t)$  are also taken as the mean of  $X_i^{(1)}(t)$  and  $X_i^{(1)}(t - 1)$  for  $i = 2, 3, \dots, n$ , respectively, for the determination of the model parameters in GDMC(1,  $n$ ).

The least-squares solution for the model parameters in the GMC(1,  $n$ ) in (7) for  $t$  from 1 to  $r$  is given by

$$(b_1, b_2, \dots, b_{2n})^T = (B^T B)^{-1} B^T Y_N, \quad (10)$$

where

$$B = \begin{bmatrix} -W_1(rp+1) & 1 & X_2^{(0)}(1) & W_2(1) & X_3^{(0)}(1) & W_2(1) & \cdots & X_n^{(0)}(1) & W_n(1) \\ -W_1(rp+2) & 1 & X_2^{(0)}(2) & W_2(2) & X_3^{(0)}(2) & W_3(2) & \cdots & X_n^{(0)}(2) & W_n(2) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ -W_1(rp+r) & 1 & X_2^{(0)}(r) & W_2(r) & X_3^{(0)}(r) & W_3(r) & \cdots & X_n^{(0)}(r) & W_n(r) \end{bmatrix}, \quad (11)$$

$$W_i(t) = \frac{1}{2} (X_i^{(1)}(t) + X_i^{(1)}(t-1)) \quad \text{for } i = 1, 2, \dots, n,$$

$$Y_N = (X_1^{(0)}(rp+2), X_1^{(0)}(rp+3), \dots, X_1^{(0)}(rp+r))^T.$$

2.3. *The Evaluation of  $\widehat{X}_1^{(0)}$ .* In summary, in the right-hand side of (7), the discrete function  $f(t)$  can be obtained as

$$f(t) = b_2 + \sum_{i=2}^n \left[ b_{2i-1} \frac{dX_i^{(1)}(t)}{dt} + b_{2i} X_i^{(1)}(t) \right], \quad (12)$$

$$t = 0, 1, \dots, r + rf.$$

The 1-AGO modelling values of the predicted series can be derived using the initial condition  $\widehat{X}_1^{(1)}(rp+1) = X_1^{(1)}(rp+1)$  as

$$\begin{aligned} \widehat{X}_1^{(1)}(rp+t) \\ = X_1^{(1)}(rp+1) e^{-b_1(t-1)} + \int_1^t e^{-b_1(t-\tau)} f(\tau) d\tau. \end{aligned} \quad (13)$$

The second term of the right-hand side in (13) can be evaluated approximately by the two-point Gauss numerical integration [14] with the linear assumption on  $f(t)$  between any two neighboring times; we have

$$\widehat{X}_1^{(1)}(rp+1) = X_1^{(1)}(rp+1) = X_1^{(0)}(rp+1), \quad (14)$$

$$\begin{aligned} \widehat{X}_1^{(1)}(rp+t) \\ = X_1^{(0)}(rp+1) e^{-b_1(t-1)} + u(t-2) \\ \times \sum_{\tau=2}^t \left\{ \frac{1}{2} \omega_1 e^{-b_1(t+0.5-\tau+0.5\lambda_1)} \right. \\ \times [0.5(f(\tau) + f(\tau-1)) \\ \left. + 0.5\lambda_1(f(\tau) - f(\tau-1))] \right\}, \quad (15) \\ + \frac{1}{2} \omega_2 e^{-b_1(t+0.5-\tau+0.5\lambda_2)} \\ \times [0.5(f(\tau) + f(\tau-1)) \\ \left. + 0.5\lambda_2(f(\tau) - f(\tau-1))] \right\}, \\ t = 2, 3, \dots, r + rf, \end{aligned}$$

where the coefficients  $\omega_1$  and  $\omega_2$  are both equal to 1, the nodes  $\lambda_1$  and  $\lambda_2$  are equal to  $-1/\sqrt{3}$  and  $1/\sqrt{3}$ , respectively,

and  $u(t-2)$  is the unit step function. Applying 1-IAGO to (15) yields the following modelled values together with the forecasts:

$$\begin{aligned} \widehat{X}_1^{(1)}(rp+1) &= X_1^{(1)}(rp+1) = X_1^{(0)}(rp+1), \\ \widehat{X}_1^{(0)}(rp+t) &= \widehat{X}_1^{(1)}(rp+t) - \widehat{X}_1^{(1)}(rp+t-1), \quad (16) \\ t &= 2, 3, \dots, r + rf, \end{aligned}$$

where  $rf$  is the number of entries to be forecasted or indirectly measured.

Assuming the system parameters in (7) to be constants in the postsampling period, then, by using the postsampling data combined with the data given for the corresponding associated series as a new input series, the corresponding forecasts or values of indirect measurement for the predicted series can be derived.

It is obvious that, when the number of associated series is zero, that is, when  $n = 1$ , (7) reduces to the grey single variable forecasting model GM(1, 1).

#### 2.4. Evaluation of the Modelling and Forecasting Accuracy.

To evaluate the accuracy of the grey models Tien [14], who proposed the GDMC(1,  $n$ ) model, applied the root mean squared percentage error (RMSPE) to the priori sample period (RMSPEPR) and postsample period (RMSPEPO), respectively. Generally, the RMSPEPR and RMSPEPO are defined, respectively, as

$$\text{RMSPEPR} = \sqrt{\frac{1}{r} \sum_{t=rp+1}^{rp+r} \frac{[\widehat{X}_1^{(0)}(t) - X_1^{(0)}(t)]^2}{[X_1^{(0)}(t)]^2}} \times 100\%, \quad (17)$$

$$\text{RMSPEPO} = \sqrt{\frac{1}{rf} \sum_{t=rp+rf+1}^{rp+rf+rf} \frac{[\widehat{X}_1^{(0)}(t) - X_1^{(0)}(t)]^2}{[X_1^{(0)}(t)]^2}} \times 100\%. \quad (18)$$

#### 2.5. Optimization of the GDMC(1, $n$ ) Model Based on PSO Algorithm.

In this study, to enhance the modelling and forecasting accuracy of the GDMC(1,  $n$ ) model,  $n$  interpolation coefficients  $\rho_i \in [0, 1]$  ( $i = 1, 2, \dots, n$ ) are introduced into the background values of each of the variables

in GDMC(1,  $n$ ). Then the optimal  $\rho_i$ ,  $i = 1, 2, \dots, n$ , are calculated with the objective of minimizing RMSPEPR.

Based on the above method, the background value of the grey derivative  $dX_1^{(1)}(rp+t)/dt$  is taken as the weighted mean of  $X_1^{(1)}(rp+t)$  and  $X_1^{(1)}(rp+t-1)$ ; namely,  $W_1(t) = \rho_1 X_1^{(1)}(rp+t) + (1-\rho_1)X_1^{(1)}(rp+t-1)$ . Those of the associated series  $X_i^{(1)}(t)$  are also taken as the weighted means of  $X_i^{(1)}(t)$  and  $X_i^{(1)}(t-1)$ ; that is,  $W_i(t) = \rho_i X_i^{(1)}(t) + (1-\rho_i)X_i^{(1)}(t-1)$  for  $i = 2, 3, \dots, n$  in the determination of the model parameters in GDMC(1,  $n$ ). When  $\rho_1 = \rho_2 = \dots = \rho_n = 1/2$ , the optimized GDMC(1,  $n$ ) model is reduced to a traditional GDMC(1,  $n$ ) model.

The particle swarm optimization (PSO) algorithm [15] is a population-based heuristic algorithm that simulates the social behavior as birds flocking to a promising position to achieve precise objectives in a multidimensional space. Each particle is a potential solution to the optimization problem. A particle represents a point in an  $n$ -dimensional space, and its status is characterized through its position and velocity. The position for the particle  $i$  can be represented as  $X_i = (X_{i1}, X_{i2}, \dots, X_{in})$ . The velocity of this particle can be represented by another  $n$ -dimensional vector  $v_i = (v_{i1}, v_{i2}, \dots, v_{in})$ . The best previous position of the  $i$ th particle can be represented as  $P_i = (P_{i1}, P_{i2}, \dots, P_{in})$ . The best position in the entire swarm is denoted as  $P_g = (P_{g1}, P_{g2}, \dots, P_{gn})$ . To search for the optimal solution, each particle changes its velocity and position according to the following two formulas:

$$\begin{aligned} v_{ij}(t+1) &= \omega v_{ij}(t) + c_1 \text{rand}(\cdot) [P_{ij}(t) - X_{ij}(t)] \\ &\quad + c_2 \text{rand}(\cdot) [P_{gj}(t) - X_{ij}(t)], \\ X_{ij}(t+1) &= X_{ij}(t) + v_{ij}(t+1), \end{aligned} \quad (19)$$

where  $t$  means the number of iterations.  $\omega$  is called the inertia weight that controls the impact of previous velocity of particle on its current one. In a general way, we let  $\omega_{\min} = 0.4$ ,  $\omega_{\max} = 0.9$ .  $c_1$  and  $c_2$  are positive constant parameters called acceleration factors which control the maximum step size. Typical values for  $c_1$  and  $c_2$  are  $c_1 = c_2 = 2$ .  $\text{rand}(\cdot)$  is a random number between zero and one.

To avoid the phenomenon of ‘‘shock’’ when particles near global optimal solution, we can employ a linear gradient strategy for the inertia weight:

$$\omega = \omega_{\max} - \frac{\omega_{\max} - \omega_{\min}}{t_{\max}} \cdot t, \quad (20)$$

where  $t_{\max}$  is the total number of iterations.

This study uses the PSO algorithm to solve the optimal background value coefficients for GDMC(1,  $n$ ) model. The specific steps are as follows.

*Step 1.* Generate  $m$  particles randomly in the  $n$ -dimensional space. The position and velocity for the particle  $i$  can be represented as

$$\begin{aligned} \rho_i &= (\rho_{i1}, \rho_{i2}, \dots, \rho_{in}), & v_i &= (v_{i1}, v_{i2}, \dots, v_{in}), \\ & & i &= 1, 2, \dots, m. \end{aligned} \quad (21)$$

*Step 2.* Set the initial position of the particles as  $\rho_1 = (0, 0, \dots, 0)$  and generate an initial velocity vector  $v_1$  randomly in the interval  $(0, 1.1]$ .

*Step 3.* According to (10), substitute  $\rho_i$  into the matrix  $B$  and establish the fitness function

$$F(\rho_i) = \sqrt{\frac{1}{r} \sum_{t=rp+1}^{rp+r} \frac{[\widehat{X}_1^{(0)}(t) - X_1^{(0)}(t)]^2}{[X_1^{(0)}(t)]^2}}. \quad (22)$$

*Step 4.* Calculate the fitness value for each particle.

*Step 5.* Compare each particle's fitness to the global best position  $p_{\text{best}}$  to adapt to the value; if it is the optimum, it can be taken as the best position; if not, turn to Step 6.

*Step 6.* Update particles velocity and position according to the evolution equation according to (19).

*Step 7.* If stopping criteria are met, show the output and its fitness, which corresponds to the optimal parameters and the RMSPEPR, or else go back to Step 3.

### 3. Forecasting the Output of High-Tech Industry in China

Forecasting the output of high-tech industry is essential for the development of projects and policy making. As Chinese high-tech industry is neonatal and so the data relating to existing economic indices are limited, it is difficult to apply existing statistical methods of analysis and forecasting to them. As a result, little research has been conducted on quantitative forecasting of Chinese high-tech industry. In this section, the advantage of the optimized GDMC(1,  $n$ ) model (abbreviated as OGDMC(1,  $n$ )) over the traditional one is demonstrated by a real case study of high-tech industry in China.

*3.1. Variables and Data.* In economics theory, human resources and capital investment are the crucial factors of the economic system output [16]. The economic output of high-tech industry is taken as the predicted series and is denoted as  $X_1$ . And the annual average employment and investment are adopted as the relative variables of the output of high-tech industry and are denoted as  $X_2$  and  $X_3$ , respectively. The original data relating to the output, average employment, and investment of the high-tech industry in China for 2003–2010 are shown in Table 1. These data are all collected from the China Statistics Yearbook on High Technology Industry (2004–2011).

*3.2. Modelling GDMC(1, 3) and OGDMC(1, 3).* In the following, the industrial output value  $X_1$  is used as the forecasting variable. At the same time, with the average employment  $X_2$  and the investment  $X_3$  as relative variables, multivariable forecasting models GDMC(1, 3) and OGDMC(1, 3) models are established. The data of  $X_1$ ,  $X_2$ , and  $X_3$  from 2003 to 2010

TABLE 1: Industrial output value, average employment, and investment of high-tech industry in China.

Time (Year)	Industrial output value (RMB 100 million yuan)	Average employment (10000 persons)	Investment (RMB 100 million yuan)
3 (2003)	20556	477	1423
4 (2004)	27769	587	1790
5 (2005)	34367	663	2144
6 (2006)	41996	744	2761
7 (2007)	50461	843	3388
8 (2008)	57087	945	4169
9 (2009)	60430	958	4882
10 (2010)	74709	1092	6945

TABLE 2: The values of parameters  $n, r,$  and  $rp$  in (7), the estimates of model parameters  $b_1, b_2,$  and  $(b_{2i-1}, b_{2i})$  in (10), and the value of RMSPEPR in (17).

Parameter	Value	Parameter	Value
$n$	3	$b_3$	23.28
$r$	8	$b_4$	17.78
$rp$	0	$b_5$	9.30
$b_1$	0.10822	$b_6$	-3.20
$b_2$	-5083.77	RMSPEPR (%)	7.07

TABLE 3: The discrete function  $f(t)$  for GDMC(1, 3) in (12).

$t$	$f(t)$	$t$	$f(t)$
1	23173.06	5	68109.22
2	33851.44	6	81192.52
3	43829.28	7	89520.70
4	55835.86	8	108987.60

is employed as the original modelling series to construct GDMC(1, 3) and OGDMC(1, 3) models.

3.2.1. *GDMC(1, 3)*. Applying the GDMC(1, 3) model given by (7)–(16), the values of the parameters  $n, r,$  and  $rp$  in (7) and the estimates of the model parameters  $b_1, b_2,$  and  $(b_{2i-1}, b_{2i})$  in (10) can be obtained (Table 2). The resulting GDMC(1, 2) model from (7) has the form

$$\begin{aligned} & \frac{dX_1^{(1)}(t)}{dt} + 0.10822X_1^{(1)}(t) \\ &= -5083.77 + 23.28 \frac{dX_2^{(1)}(t)}{dt} + 17.78X_2^{(1)}(t) \\ &+ 9.30 \frac{dX_3^{(1)}(t)}{dt} - 3.20X_3^{(1)}(t), \\ & t = 1, 2, \dots, 8. \end{aligned} \tag{23}$$

To sum up the right-hand side of (7), the discrete function  $f(t)$  in (12) for the GMC(1, 2) model is obtained and tabulated (Table 3). The values of RMSPEPR in (17), respectively, are also listed in Table 2.

TABLE 4: The values of parameters  $n, r, rf,$  and  $rp$  in (7), the estimates of model parameters  $b_1, b_2,$  and  $(b_{2i-1}, b_{2i})$  in (10), the optimized parameters  $\rho_1, \rho_2,$  and  $\rho_3,$  and the values of RMSPEPR in (17).

Parameter	Value	Parameter	Value
$n$	3	$b_5$	8.23
$r$	8	$b_6$	-1.68
$rp$	0	$\rho_1$	0.17624
$b_1$	-0.34721	$\rho_2$	0.14669
$b_2$	-0.45756	$\rho_3$	1
$b_3$	34.90	RMSPEPR (%)	1.86
$b_4$	18.76		

TABLE 5: The discrete function  $f(t)$  for OGDMC(1, 3) in (12).

$t$	$f(t)$	$t$	$f(t)$
1	27896.61	5	93126.25
2	42762.85	6	113838.20
3	57161.09	7	129932.20
4	74385.06	8	160408.60

TABLE 6: Levels of accuracy and their critical values.

Level	MRE	ADGI	RSD	PSE
1	0.01	0.90	0.35	0.95
2	0.05	0.80	0.50	0.80
3	0.10	0.70	0.65	0.70
4	0.20	0.60	0.80	0.60

3.2.2. *OGDMC(1, 3)*. Applying the OGDMC(1, 3) model given by (7)–(16) and the optimized method, the values of the parameters  $n, r,$  and  $rp$  in (7), the estimates of the model parameters  $b_1, b_2,$  and  $(b_{2i-1}, b_{2i})$  in (10), and the optimized parameters  $\rho_1, \rho_2,$  and  $\rho_3$  can be obtained (listed in Table 4). The resulting OGDMC(1, 3) model from (7) has the form

$$\begin{aligned} & \frac{dX_1^{(1)}(t)}{dt} + 0.22310X_1^{(1)}(t) \\ &= -7021.86 + 34.90 \frac{dX_2^{(1)}(t)}{dt} + 18.76X_2^{(1)}(t) \\ &+ 8.23 \frac{dX_3^{(1)}(t)}{dt} - 1.68X_3^{(1)}(t), \\ & t = 1, 2, \dots, 8. \end{aligned} \tag{24}$$

To sum up the right-hand side of (7), the discrete function  $f(t)$  in (12) for the OGDMC(1, 2) model is obtained and tabulated (Table 5). The values of RMSPEPR in (17) are also listed in Table 4.

3.3. *Evaluation of the Grey Forecasting Models*. In this study, the most commonly used four measurements in grey theory, including mean relative error (MRE), absolute degree of grey incidence (ADGI), ratio of standard deviation (RSD), and probability of small error (PSE) [2, 17], are used to evaluate the accuracy of the models involved. The levels of accuracy and their critical values are given in Table 6 [17]. When all



TABLE 7: The modelling results obtained using the GDMC(1, 3) and OGDMC(1, 3) models.

Time (Year)	Actual value	GDMC(1, 3)		OGDMC(1, 3)	
		Modelling value	Relative error	Modelling value	Relative error
1 (2003)	20556	20556	0	20556	0
2 (2004)	27769	24823.94	0.0609	27308.16	0.0095
3 (2005)	34367	32072.61	0.0277	34970.11	0.0073
4 (2006)	41996	39183.89	0.0226	42101.73	0.0008
5 (2007)	50461	46668.65	0.0217	49775.32	0.0039
6 (2008)	57087	53891.31	0.0138	57470.03	0.0016
7 (2009)	60430	58551.09	0.0064	62549.36	0.0072
8 (2010)	74709	67230.30	0.0204	72731.92	0.0054
MRE		0.0217		0.0045	
ADGI		0.94		1.00	
RSD		0.12		0.06	
PSE		1.00		1.00	

the measured values in a forecasting model meet the requirements of the critical values listed in the table, the model is applicable to the prediction. If they fall within the ranges of levels 1 and 2, this suggests that the model has high forecasting accuracy.

3.4. Empirical Results and Discussion. The modelling results for China's high-tech industry using the two grey models described above are shown in Table 7 together with the four accuracy measurements. The results show that the GDMC(1, 3) model and the OGDMC(1, 3) model fall within levels 2 and 1, respectively. The OGDMC(1, 3) model reduces the MRE of the GDMC(1, 3) model from 0.0217 to 0.0045, increases the ADGI of the GDMC(1, 3) model from 0.94 to 1.00, and also reduces the RSD of the GDMC(1, 3) model from 0.12 to 0.06. This indicates that the OGDMC(1,  $n$ ) model can improve the modelling accuracy of the traditional model GDMC(1,  $n$ ) significantly.

Figure 1 demonstrates the degree of closeness between the modelling results of the models and the real values of the output of high-tech industry in China. Clearly, the model OGDMC(1, 3) is closer to the actual data than the traditional one. Moreover, as can be seen from a histogram of the modelling residual errors (Figure 2), the residual error from the OGDMC(1, 3) model is notably smaller than that of the GDMC(1, 3). The reason for this lies in the fact that, through optimization of the background value, the modelling ability of the GDMC(1, 3) model can be further improved in the OGDMC(1, 3) model. The model coefficients  $b_3$ ,  $b_4$ ,  $b_5$ , and  $b_6$  can reveal the role and importance of employment and investment and their impact on the output of high-tech industry.

### 4. Conclusions

This study presents a PSO-algorithm-based optimization method for the grey dynamic model by convolution integral with the first-order derivative of the 1-AGO data and  $n$  series related. According to empirical modelling results of the high-tech industry in China, the modelling accuracy of the

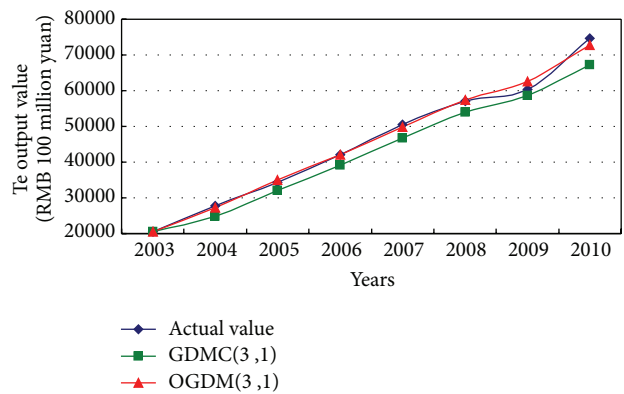


FIGURE 1: The output value and the modelling curves obtained using the GDMC(1, 3) and OGDMC(1, 3) models.

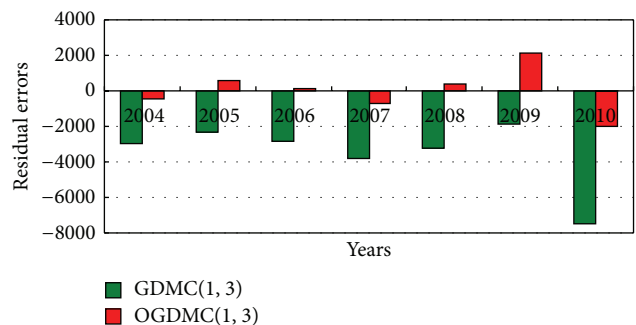


FIGURE 2: The residual errors occurring in the GDMC(1, 3) and OGDMC(1, 3) models.

traditional GDMC(1,  $n$ ) model can be effectively increased using a background value optimization method as proposed in this study. This is important in practice as the government of high-tech industry needs to make decisions based on traditional parameter solutions which may be constrained by an incorrect local optimal solution.

Due to the lack of more additional data published by China's Statistic Department at present, the out-of-sample

and the time-delayed forecasting results of this study have not fully been validated. Therefore, in future work, the out-of-sample and the time-delayed forecasting using both the GDMC(1,  $n$ ) and the OGDMC(1,  $n$ ) needs to be taken into account after more data is released by China's Statistic Department. In addition, the grey forecasting model established in this study merely considers two basic input factors, that is, capital and labor, while ignoring the technical level. The main reason lies in the fact that sample data are rather limited in the short development of China's high-tech industries. Moreover, it is challengeable to estimate the technology level of high-tech industries using small sample data. To address this problem, further researches are also needed.

### Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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