# The de-noising of sonic echo test data through wavelet transform reconstruction

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This paper presents the results of feasibility study into the application of the wavelet transform signal processing method to sonic based non-destructive testing techniques. Finite element generated data from cast in situ foundation piles were collated and processed using both continuous and discrete wavelet transform techniques. Results were compared with conventional Fourier based methods. The discrete Daubechies wavelets and the continuous Mexican hat wavelet were used and their relative merits investigated. It was found that both the continuous Mexican hat and discrete Daubechies D8 wavelets were significantly better at locating the pile toe compared than the Fourier filtered case. The wavelet transform method was then applied to field test data and found to be successful in facilitating the detection of the pile toe.

Keywords: Non-destructive testing, wavelet transform, signal analysis, low strain testing

## 1. Introduction

An investigation was conducted into the application of wavelet transform techniques to sonic based non-destructive testing. Specifically, the test methods investigated were of the impact integrity type. Here, an instrumented hammer excites the structure under test and the recorded response is interpreted as an indirect measurement of its integrity. Conventionally the resultant traces from these tests are interpreted either in the Fourier domain (transient dynamic response, modal analysis, etc.) or time domain (sonic pulse, sonic echo, etc.), or both. Finite element generated data from cast in situ foundation piles were collated and processed using both continuous and discrete wavelet transform techniques. Field data has been obtained from an industrial collaborator. Results were compared with conventional Fourier based methods and examples of each are given below.

The wavelet transform (WT) was introduced in the early 1980's [13,15] as a new signal processing technique that overcomes one of the inherent problems of conventional Fourier techniques, namely the retention of temporal information. It provides a method for the decomposition of non-stationary signals such that scale characteristics and feature location can be highlighted simultaneously through the unfolding of a one-dimensional signal into two dimensions. Since its introduction the wavelet transform has become a popular tool in the fields of science and engineering. Especially medical sciences [27,32], geophysics [10,17], automated speech analysis [16], fluid turbulence [11, 33] and image compression/processing [18]. No literature has been found concerning the WT's use in 'real' impact testing. However, researchers in Taiwan have published results suggesting that when laboratory testing concrete cylinders of up to 160 cm length using a dropped mass the signal-to-noise ratio can be reduced by filtering in the wavelet domain rather than the Fourier one [31].

The short term Fourier transform (STFT) also provides some temporal resolution along with frequency data, but here the frequency resolution is compromised by the temporal resolution (i.e., the finer the temporal resolution the coarser the frequency resolution must be – and *vice versa*). Daubechies [9] has made a comparison between the Gabor transform (a special case STFT) and the WT and points out that, by its nature, the WT is more appropriate for applications where more precise time resolution is required for higher frequencies than for lower ones. This has been supported by empirical evidence where it has been shown that the

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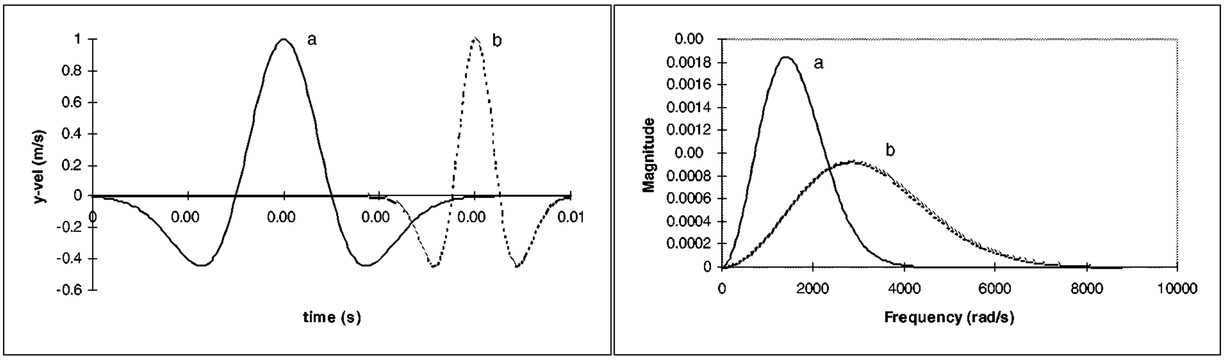


Fig. 1. The Mexican hat wavelet for two dilations and locations and, right, their spectra.

WT is better for the detection of discontinuities or singularities in time as here high frequency terms dominate [5,7].

A function is described as a wavelet if it adheres to three primary conditions:

1. It must have finite energy.
2. If $g(\omega)$ is the Fourier transform of the wavelet $g(t)$, then the *admissibility condition* must hold, i.e., the functions mean should be zero. Hence,

$$C_g = \int_0^\infty \frac{|\widehat{g}(\omega)|^2}{\omega}\,\mathrm{d}\omega < \infty \quad \text{when } a \in +R. \tag{1}$$

3. For complex wavelet functions the Fourier transform should be real and vanish for $\omega \leqslant 0$.

The wavelet is chosen to be compact in both time and Fourier domains, e.g., the Mexican hat wavelet of Fig. 1. This figure also illustrates how the mother wavelet $g(t)$ is shifted and its width altered in order to analyse a trace in both time and scale. That is, it has a dilation parameter $a$ and a translational parameter $b$ which are altered to unfold the signal from a 1-D trace to a 2-D map. The wavelets' dilation or width is altered through the general equation

$$g_a(t) = \frac{1}{a^n} g\left(\frac{t}{a}\right), \tag{2}$$

where $n$ is commonly set to 0, $1/2$ or 1 [21]. We will set $n$ to one half which will ensure the $L^2$ norm of the wavelet is independent of $a$, i.e., the wavelet energy remains constant for all $a$ scales. The wavelet transform is then described as the convolution product of the wavelet with the trace to be analysed for the various values of $a$ and $b$ thus:

$$W(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} s(t) g\left(\frac{t-b}{a}\right) \mathrm{d}t. \tag{3}$$

A more in depth description of the wavelet transform has been published by the authors elsewhere where some of its more useful properties are described [1,3].

## 2. Wavelet selection

Mallat [19], Grossmann and Morlet [15], three of the pioneers of modern wavelet analysis, have co-authored papers reporting the WT's usefulness in detecting singularities and discontinuities in traces when compared to Fourier based methods. Related research in vibration analysis has recently been published by Staszewski [28–30] and Newland [25].

A multitude of wavelets, which satisfy the above constraints, are available to the researcher. Each will have different mathematical properties which render it more suitable for given applications. Staszewski employs the Morlet wavelet for detection of system non-linearities through the identification of damping and stiffness parameters for multi-degree-of-freedom systems [28,30] during transient testing. The Morlet wavelet has good support in both frequency and time domains which allows the decoupling of the system's various modes of vibration with respect to time. This makes it very effective for this application.

Newland applies his own wavelet, the harmonic wavelet, to the analysis of bending wave propagation within a struck mild steel beam [25]. This wavelet is similar in shape, and therefore properties, as the Morlet wavelet except that, being defined in the Fourier domain, it can be forced to be orthogonal for each 'a' scale. Again this demonstrates the wavelets ability to separate the spectral components of the propagating pulse. This is important in an application such as this as the group velocity of bending waves within a beam is frequency dependent and so, with multiple reflections also occurring, the time domain response record becomes too complex for direct interpretation.

For a problems involving large scale heavily damped systems such as sonic pile testing, in practice, there are rarely multiple longitudinal reflections and the frequency dependence of group velocities is negligible. Thus, as long as feature types can be differentiated through scale, the temporal isolation of these trace features are of a greater importance than the decoupling of their frequency components. For the investigation detailed herein, therefore, the Mexican hat wavelet, shown in Fig. 1, was used. This is the second derivative of a Gaussian function and has been shown to be an effective wavelet for feature location in the fields of medicine [6], geophysics [12] and turbulence analysis [22,33]. This wavelet has a number of advantages over those mentioned above for this application. Specifically, although compactness in the frequency domain is compromised, the wavelet has far better temporal support. This leads to improved signal reconstruction properties post filtering. The wavelet is also real and so the polarity of the coefficients in the transform space represents the polarity of the features detected and no complex phase investigation is necessary.

The Mexican hat function is given as:

$$g(t) = \left(1 - t^2\right) \exp\left(\frac{-t^2}{2}\right). \tag{4}$$

## 3. Feature detection

Figure 2 shows the application of the continuous wavelet transform to low strain integrity test data for an eleven metre, cast in situ, pile in a stiff/very stiff clay environment. The data has been generated through the finite element method and validated through comparison to data from the EPSRC Blythe test site supplied by Testconsult Ltd. This data was Fourier filtered at source and so the finite element data was necessary to compare like with like.

The wavelet domain data is presented in the form of a *scaleogram* (the lower plot of Fig. 2). This filled contour plot has the translation, $b$, value along the $x$-axis and the dilation, $a$, value along the $y$-axis. By convention the $y$-axis is a minus log scale so that smaller $a$ values, i.e., smaller wavelet widths, are at the top of the graph. This leads to discontinuities in the one-dimensional trace effectively being pointed to by the

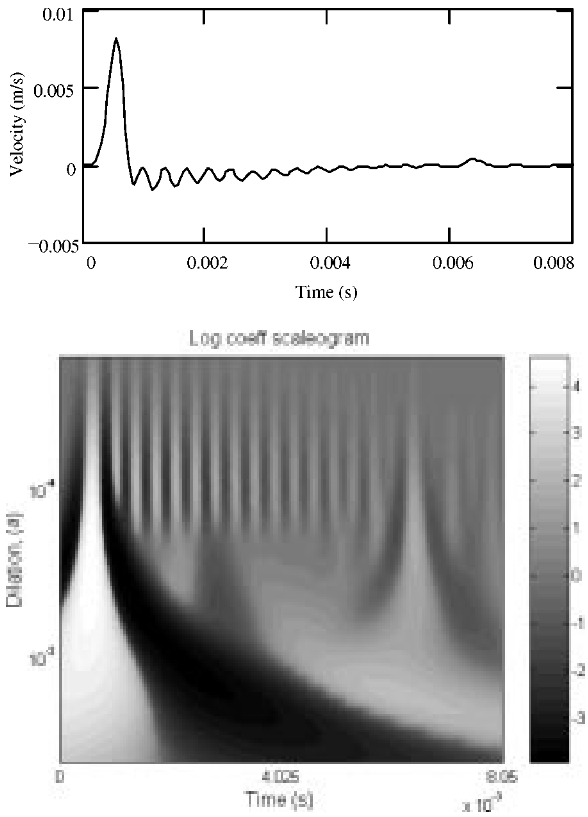


Fig. 2. The Fourier filtered velocity trace (top) and wavelet transform scaleogram (bottom) of an eleven metre pile in stiff/very stiff clay.

two-dimensional scaleogram. These and other properties of the continuous wavelet transform have been presented by the authors elsewhere along with other examples of its use in low-strain integrity testing [3,34].

The scaleogram correctly shows a feature at around 11 m ($3800\,\mathrm{ms}^{-1} \times 0.0058/2$). This feature has $a$ scales of around 0.5 ms. For the Mexican hat function the $a$ value represents the width of the wavelet from its centre to the first point at which it crosses the $x$-axis. As 0.5 ms is also, approximately, the rise time of the input pulse we can say with some confidence that the wavelet transform is indicating the position of the pile toe. We can also see that the detection of the pile toe has been facilitated by the scaleogram when compared to the Fourier filtered trace.

## 4. Noise reduction

We can see from the scaleogram in Fig. 2 that, for the feature in question, $a$ scales of less than 0.0001 s can be ignored. Given that the original signal can be

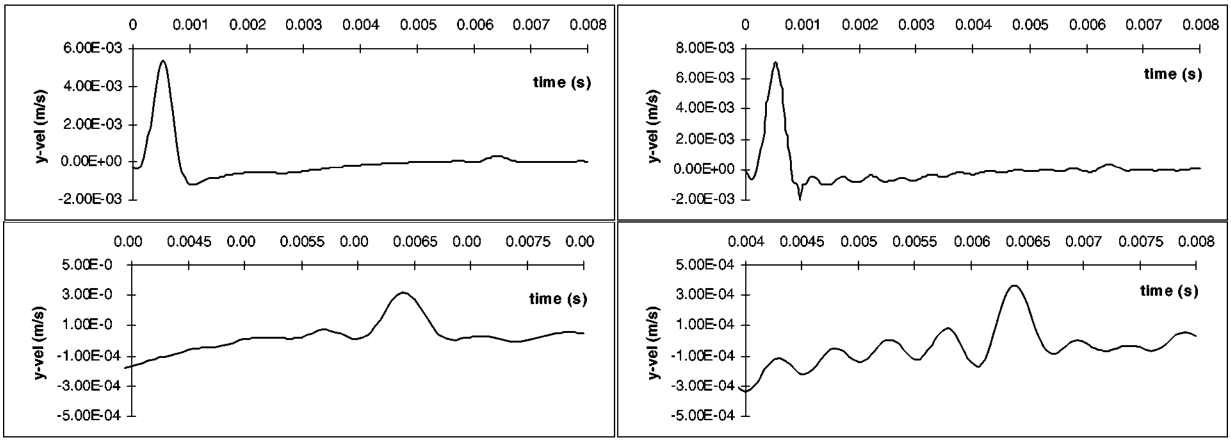


Fig. 3. Wavelet (left) and Fourier (right) filtered traces for finite element generated pile test data.

reconstructed using Eq. (5), below, it is clear that by picking our limits for $a$ judiciously we can effectively filter in the wavelet domain rather than the Fourier one

$$s(t) = \frac{1}{C_g} \int_{+\infty}^{+\infty} \int_0^{+\infty} \frac{1}{\sqrt{a}} \overline{g}\left(\frac{t-b}{a}\right) W(a,b) \frac{\mathrm{d}a\,\mathrm{d}b}{a^2} \quad \text{when } a \in +R. \tag{5}$$

Figure 3 contains the reconstructed traces of both wavelet and Fourier filtered traces. For the Fourier low pass filter the cut-off frequency is set to 2.25 kHz as is the case in the field. As can be seen in the figure, the time averaged Fourier filtered trace contains spurious frequency components towards the end of the trace whereas in the wavelet filtered trace, because it is time dependent and therefore not global, these frequencies are not present. This leads to the reflection from the toe being easier to identify and locate.

## 5. The discrete wavelet transform

The implementation of the wavelet transform (WT) for discrete signals can be performed by the discrete wavelet transform (DWT). The DWT allows for the decomposition of the signal into its wavelet components such that the first wavelet dilation has the wavelet centred and covering the entire signal. At each subsequent *level* the dilation is halved so smaller features within the signal are resolved. As the dilation is halved so the translation step is halved so avoiding overlap in the locations analysed thus avoiding redundancy in the transform set – illustrated in Fig. 4. The number of required

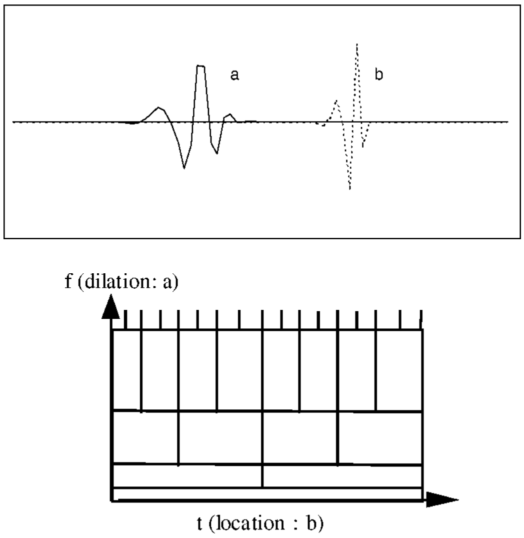


Fig. 4. A Daubechies, D(8), wavelet (b) having undergone translation and dilation from wavelet (a), top. Also a diagrammatic representation of the frequency (dilation) and time (location) resolutions for an orthonormal, complete wavelet with location and dilation halving at each consecutive level, bottom. Each enclosed area contains one wavelet.

operations for the DWT are, therefore, less those that for the continuous case so reducing computation time and the number of resultant coefficients.

In this investigation the Daubechies wavelets of orders 4, 6, 8, 12, 16 and 20 were considered. This wavelet system has been chosen because it is orthonormal and complete [24]. This allows for the regeneration of the original signal from the decomposed data and, due to its orthonormality, it lends itself to the discrete wavelet transform. Initial results from the D(4) analysing wavelet and a further discussion of its merits have been published at conference [3]. However,

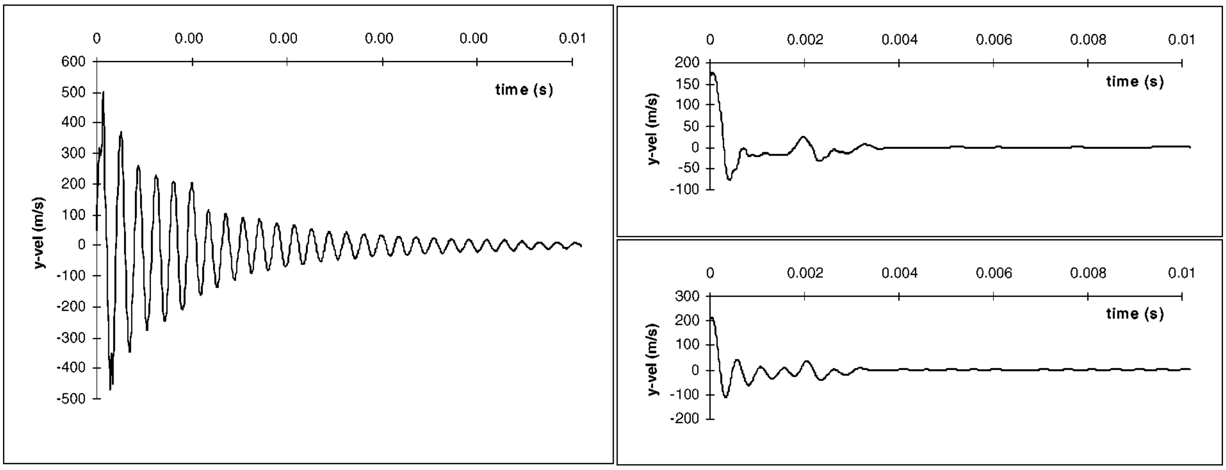


Fig. 5. Field test result (left) and when filtered: wavelet (top, right) and Fourier (bottom, right).

following a parametric study involving the wavelets mentioned above the D(8) wavelet was chosen as the most effective. Figure 4 illustrates two Daubechies D(8) wavelets with the second being a translated and dilated version of the first. Not surprisingly the D(8) wavelet has a shape nearest to the Gaussian-like input pulse obtained in low strain impact testing.

Figure 5 shows results from the favoured D(8) wavelet where a signal has been filtered on the first six levels of eleven. This can be compared directly with a Fourier filtered version of the data (2.25 kHz low pass). The signal is a 'real' trace for a 4 m reinforced concrete, cast in situ, pile installed in soft clay (supplied by Technotrade Ltd). The reflection from the pile toe can be seen at 0.002 seconds. As can be seen, as with the continuous case (Fig. 3), the time localised nature of the wavelet filtered response has eliminated the additional sinusoids found in the Fourier filtered trace.

## 6. Concluding remarks

The work detailed above has shown the wavelet transform to be of potential use in facilitating the interpretation of non-destructive testing signals. The main conclusions that can be drawn from the author's study are:

1. 2D unfolding of a 1D time signal improves feature detection within low strain NDT signals.
2. Wavelet filtering can eliminate spurious signal components found in Fourier filtered traces.
3. Discrete wavelet transforms successfully improved field test data resolution when compared to the Fourier filtered case.

In conclusion, this study has shown the wavelet transform to be a powerful tool for the analysis of transient, low-strain NDT signals. Present research focuses on the following areas:

1. The determination of location dependency on the discrete wavelet transform. Specifically, a study of the effect of low temporal resolution at large $a$ scales.
2. The investigation of the use of less implemented transform methods, e.g., the matching pursuit transform [20] and wavelet packet transform [8].
3. The in situ testing of piled foundations in differing soil types to measure the system's dependency and effectiveness.

Although the authors believe that the wavelet transform will provide a useful additional tool for NDT signal analysis, many issues on the practicalities of the technique [4] must be addressed before it can become industrially useful.

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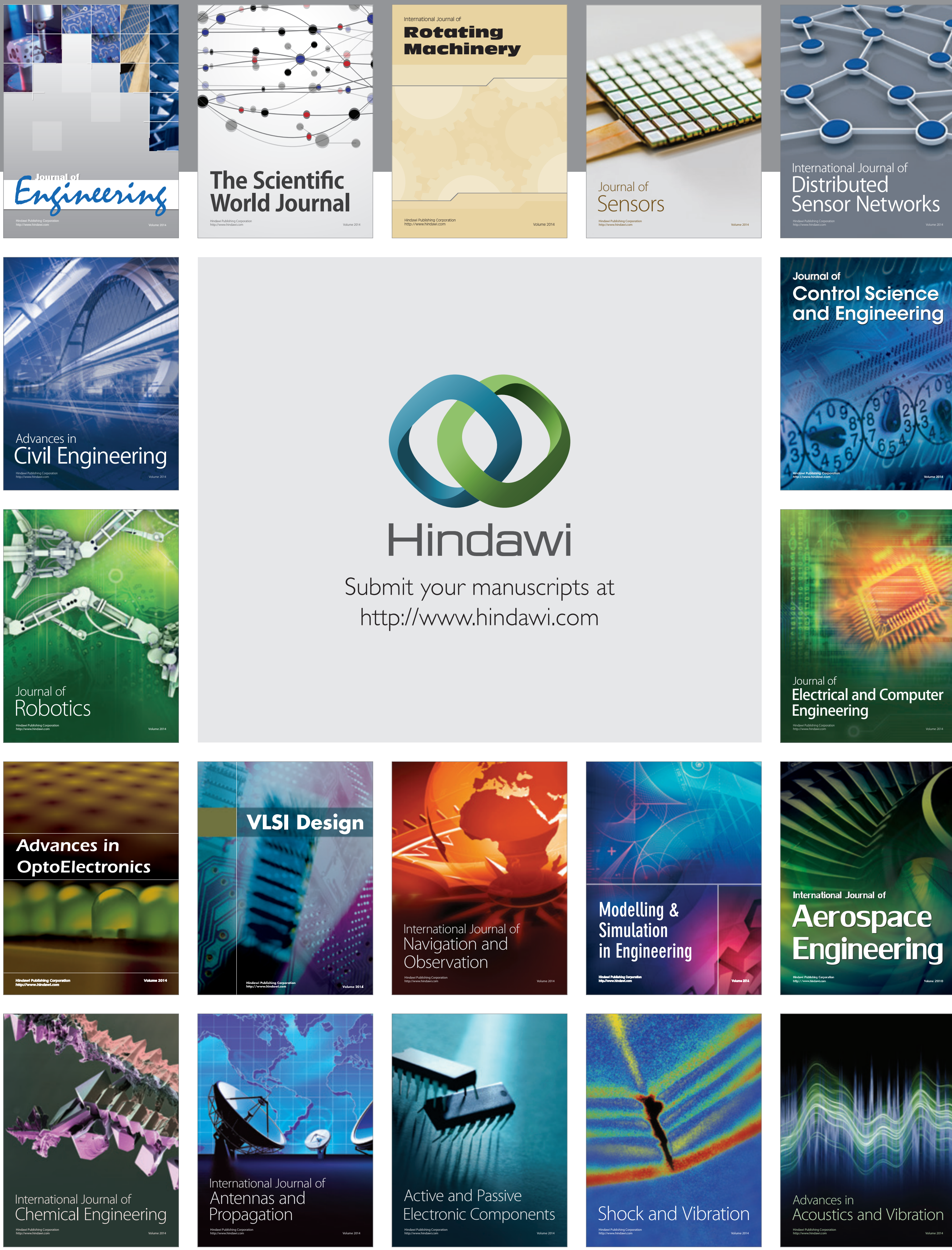
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