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Research Article

Integer-Valued Moving Average Models with Structural Changes

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It is frequent to encounter integer-valued time series which are small in value and show a trend having relatively large fluctuation. To handle such a matter, we present a new first order integer-valued moving average model process with structural changes. The models provide a flexible framework for modelling a wide range of dependence structures. Some statistical properties of the process are discussed and moment estimation is also given. Simulations are provided to give additional insight into the finite sample behaviour of the estimators.

1. Introduction

Integer-valued time series occur in many situations, often as counts of events in consecutive points of time, for example, the number of births at a hospital in successive months, the number of road accidents in a city in successive months, and big numbers even for frequently traded stocks. Integer-valued time series represent an important class of discrete-valued time series models. Because of the broad field of potential applications, a number of time series models for counts have been proposed in literature. McKenzie [1] introduced the first order integer-valued autoregressive, INAR(1), model. The statistical properties of the INAR(1) are discussed in McKenzie [2], Al-Osh and Alzaid [3]. The model is further generalized to a pth-order autoregression, INAR(p), by Alzaid and Al-Osh [4] and Du and Li [5]. The qth-order integer-valued moving average model, INMA(q), was introduced by Al-Osh and Alzaid [6] and in a slightly different form by McKenzie [7]. Ferland et al. [8] proposed an integer-valued GARCH model to study overdispersed counts, and Fokianos and Fried [9], Weiß [10], and Zhu and Wang [11–13] made further studies. Györfi et al. [14] proposed a nonstationary inhomogeneous INAR(1) process, where the autoregressive type coefficient slowly converges to one. Bakouch and Ristić [15] introduced a new stationary integer-valued autoregressive process of the first order with zero truncated Poisson marginal distribution. Kachour and Yao [16] introduced a class of autoregressive models for integer-valued time series using the rounding operator. Kim and Park [17] proposed an extension of integer-valued autoregressive INAR models by using a signed version of the thinning operator. Zheng et al. [18] proposed a first order random coefficient integer-valued autoregressive model and got its ergodicity, moments, and autocovariance functions of the process. Gomes and Canto e Castro [19] presented a random coefficient autoregressive process for count data based on a generalized thinning operator. Existence and weak stationarity conditions for these models were established. A simple bivariate integer-valued time series model with positively correlated geometric marginals based on the negative binomial thinning mechanism was presented by Ristić et al. [20], and some properties of the model are also considered. Pedeli and Karlis [21] considered a bivariate INAR(1) (BINAR(1)) process where cross correlation is introduced through the use of copulas for the specification of the joint distribution of the innovations.

Structural changes in economic data frequently correspond to instabilities in the real world. However, most work in this area has been concentrated on models without structural changes. It seems that the integer-valued autoregressive moving average (INARMA) model with break point has not attracted too much attention. For instance, a new method for modelling the dynamics of rain sampled by a tipping bucket rain gauge was proposed by Thyregod et al. [22].

The models take the autocorrelation and discrete nature of the data into account. First order, second order, and threshold models are presented together with methods to estimate the parameters of each model. Monteiro et al. [23] introduced a class of self-exciting threshold integer-valued autoregressive models driven by independent Poisson-distributed random variables. Basic probabilistic and statistical properties of this class of models were discussed. Moreover, parameter estimation was also addressed. Hudecová [24] suggested a procedure for testing a change in the autoregressive models for binary time series. The test statistic is a maximum of normalized sums of estimated residuals from the model, and thus it is sensitive to any change which leads to a change in the unconditional success probability. Structural change is a statement about parameters, which only have meaning in the context of a model. In our discussion, we will focus on structural change in the simple count data model, the first order integer-valued moving average model, whose coefficient varies with the value of innovation. One of the leading reasons is that piecewise linear functions can offer a relatively simple approximation to the complex nonlinear dynamics.

The rest of this paper is divided into four sections. In Section 2, we give the definition and basic properties of the new INMA(1) model with structural changes. Section 3 discusses the estimation of the unknown parameters. We test the accuracy of the estimation via simulations in Section 4. Section 5 includes some concluding remarks.

2. Definition and Basic Properties

Definition 1. Let $\{X_t\}$ be a process with state space $\mathbb{N}_0$; let $0 < \alpha_i < 1$, $i = 1, \ldots, m$, and τ_i, $i = 1, \ldots, m-1$, be positive integers. The process $\{X_t\}$ is said to be first order integer-valued moving average model with structural change (INMASC(1)) if X_t satisfies the following equation:

$$X_t = \begin{cases} \alpha_1 \circ \varepsilon_{t-1} + \varepsilon_t, & \text{for } \tau_0 \le \varepsilon_{t-1} \le \tau_1 \\ \alpha_2 \circ \varepsilon_{t-1} + \varepsilon_t, & \text{for } \tau_1 < \varepsilon_{t-1} \le \tau_2 \\ \vdots & \\ \alpha_{m-1} \circ \varepsilon_{t-1} + \varepsilon_t, & \text{for } \tau_{m-2} < \varepsilon_{t-1} \le \tau_{m-1} \\ \alpha_m \circ \varepsilon_{t-1} + \varepsilon_t, & \text{for } \tau_{m-1} < \varepsilon_{t-1} < \tau_m, \end{cases} \tag{1}$$

where $\{\varepsilon_t\}$ is a sequence of independent and identically distributed Poisson random variables with mean λ and $\tau_0 := 0$, $\tau_m := \infty$.

The aim of this section is to provide expressions for the moments and stationary of INMASC(1) model. For this purpose, we introduce the following notations:

$$\begin{gathered} p_i := P(\tau_{i-1} < \varepsilon_t \le \tau_i), \qquad u_i := E(\tau_{i-1} < \varepsilon_{t-1} \le \tau_i), \\ \sigma_i^2 := \operatorname{Var}(\tau_{i-1} < \varepsilon_{t-1} \le \tau_i), \qquad q_i := 1 - p_i, \\ I_{t-1,i} := \begin{cases} 1, & \text{if } \tau_{i-1} < \varepsilon_{t-1} \le \tau_i \\ 0, & \text{otherwise}, \end{cases} \qquad i = 1, \ldots, m. \end{gathered} \tag{2}$$

Theorem 2. *The numerical characteristics of $\{X_t\}$ are as follows:*

$$\begin{aligned} &(i)\ \mu_X := E(X_t) = \sum_{i=1}^{m} p_i \alpha_i u_i + \lambda, \\ &(ii)\ \sigma_X^2 := \operatorname{Var}(X_t) \\ &\quad = \sum_{i=1}^{m} p_i \alpha_i \left[\alpha_i \left(u_i^2 + \sigma_i^2\right) + (1-\alpha_i) u_i\right] \\ &\quad - \left(\sum_{i=1}^{m} p_i \alpha_i u_i\right)^2 + \lambda, \\ &(iii)\ \gamma_X(k) := \operatorname{cov}(X_t, X_{t-k}) \\ &\quad = \begin{cases} \sum_{i=1}^{m} p_i \alpha_i \left(u_i^2 + \sigma_i^2 - \lambda u_i\right), & k = 1 \\ 0, & k \ge 2. \end{cases} \end{aligned} \tag{3}$$

Proof. (i) It is easy to get the mean and variance of X_t by using the law of iterated expectations:

$$\begin{aligned} E(X_t) &= E\left[I_{t-1,1}(\alpha_1 \circ \varepsilon_{t-1}) + \cdots + I_{t-1,m}(\alpha_m \circ \varepsilon_{t-1}) + \varepsilon_t\right] \\ &= E\{E[I_{t-1,1}(\alpha_1 \circ \varepsilon_{t-1}) \\ &\quad + \cdots + I_{t-1,m}(\alpha_m \circ \varepsilon_{t-1}) \mid \varepsilon_{t-1}]\} + E(\varepsilon_t) \\ &= \alpha_1 E(I_{t-1,1}\varepsilon_{t-1}) + \cdots + \alpha_m E(I_{t-1,m}\varepsilon_{t-1}) + E(\varepsilon_t) \\ &= \sum_{i=1}^{m} p_i \alpha_i u_i + \lambda. \end{aligned} \tag{4}$$

(ii) Moreover,

$$\begin{aligned} &\operatorname{Var}(X_t) \\ &= \operatorname{Var}\left(I_{t-1,1}(\alpha_1 \circ \varepsilon_{t-1}) + \cdots + I_{t-1,m}(\alpha_m \circ \varepsilon_{t-1}) + \varepsilon_t\right) \\ &= \sum_{i=1}^{m} \operatorname{Var}\left(I_{t-1,i}(\alpha_i \circ \varepsilon_{t-1})\right) + \operatorname{Var}(\varepsilon_t) \\ &\quad + 2\sum_{i<j} \operatorname{cov}\left(I_{t-1,i}(\alpha_i \circ \varepsilon_{t-1}), I_{t-1,j}(\alpha_j \circ \varepsilon_{t-1})\right) \\ &= \sum_{i=1}^{m} \{\operatorname{Var}(E[I_{t-1,i}(\alpha_i \circ \varepsilon_{t-1}) \mid \varepsilon_{t-1}]) \\ &\quad + E(\operatorname{Var}[I_{t-1,i}(\alpha_i \circ \varepsilon_{t-1}) \mid \varepsilon_{t-1}])\} + \lambda \\ &\quad + 2\sum_{i<j} \{E[I_{t-1,i}(\alpha_i \circ \varepsilon_{t-1}) I_{t-1,j}(\alpha_j \circ \varepsilon_{t-1})] \\ &\quad - 2E[I_{t-1,i}(\alpha_i \circ \varepsilon_{t-1})] E[I_{t-1,j}(\alpha_j \circ \varepsilon_{t-1})]\} \\ &= \sum_{i=1}^{m} \left[\alpha_i^2 \operatorname{Var}(I_{t-1,i}\varepsilon_{t-1}) + \alpha_i (1-\alpha_i) E(I_{t-1,i}\varepsilon_{t-1})\right] + \lambda \\ &\quad - 2\sum_{i<j} E\{E[I_{t-1,i}(\alpha_i \circ \varepsilon_{t-1}) \mid \varepsilon_{t-1}]\} \\ &\quad \times E\{E[I_{t-1,j}(\alpha_j \circ \varepsilon_{t-1}) \mid \varepsilon_{t-1}]\} \end{aligned}$$

$$= \sum_{i=1}^{m} \{ [\alpha_i^2 E(I_{t-1}^2 \varepsilon_{t-1}^2) - E^2(I_{t-1,i}\varepsilon_{t-1})] + p_i\alpha_i(1-\alpha_i)u_i \} + \lambda - 2\sum_{i<j}\alpha_i\alpha_j E(I_{t-1,i}\varepsilon_{t-1}) E(I_{t-1,j}\varepsilon_{t-1})$$
$$= \sum_{i=1}^{m} \{ [p_i\alpha_i^2(u_i^2+\sigma_i^2) - p_i^2\alpha_i^2u_i^2] + p_i\alpha_i(1-\alpha_i)u_i \} + \lambda - 2\sum_{i<j} p_i\alpha_i u_i p_j\alpha_j u_j$$
$$= \sum_{i=1}^{m} p_i\alpha_i [\alpha_i(u_i^2+\sigma_i^2) + (1-\alpha_i)u_i] - \left(\sum_{i=1}^{m} p_i\alpha_i u_i\right)^2 + \lambda. \quad (5)$$

(iii) Note the correlation between $\alpha_i \circ \varepsilon_{t-1}$ and ε_{t-1}; we have

$$\operatorname{cov}(X_t, X_{t-1}) = \operatorname{cov}\left(\sum_{i=1}^{m} I_{t-1,i}(\alpha_i \circ \varepsilon_{t-1}) + \varepsilon_t, \sum_{j=1}^{m} I_{t-2,j}(\alpha_j \circ \varepsilon_{t-2}) + \varepsilon_{t-1}\right)$$
$$= \sum_{i=1}^{m} \operatorname{cov}(I_{t-1,i}(\alpha_i \circ \varepsilon_{t-1}) + \varepsilon_t, \varepsilon_{t-1})$$
$$= \sum_{i=1}^{m} \{ E[I_{t-1,i}(\alpha_i \circ \varepsilon_{t-1})\varepsilon_{t-1}] - E[I_{t-1,i}(\alpha_i \circ \varepsilon_{t-1})] E(\varepsilon_{t-1}) \}$$
$$= \sum_{i=1}^{m} \{ E[E(I_{t-1,i}(\alpha_i \circ \varepsilon_{t-1})\varepsilon_{t-1} \mid \varepsilon_{t-1})] - E[E(I_{t-1,i}(\alpha_i \circ \varepsilon_{t-1}) \mid \varepsilon_{t-1})] E(\varepsilon_{t-1}) \}$$
$$= \sum_{i=1}^{m} \alpha_i [E(I_{t-1,i}\varepsilon_{t-1}^2) - \lambda\alpha_i E(I_{t-1,i}\varepsilon_{t-1})]$$
$$= \sum_{i=1}^{m} p_i\alpha_i (u_i^2 + \sigma_i^2 - \lambda u_i). \quad (6)$$

□

Theorem 3. *Let X_t be the process defined by the equation in* (1)*; then the $\{X_t\}$ is a covariance stationary process.*

Proof. Both the unconditional mean and the unconditional variance of the $\{X_t\}$ are finite constant. And the autocovariance function does not change with time. Thus $\{X_t\}$ is a stationary process. □

Theorem 4. *Suppose $\{X_t\}$ is INMASC*(1) *process. Then*

(i) $\sqrt{T}(\overline{X} - \mu_X) \xrightarrow{L} N(0, \sigma_X^2 + 2\gamma_X(1))$;

(ii) $E(X_t^k \mid I_{t-1,i} = 1) < \infty, k = 1, 2, 3, i = 1, \ldots, m$.

Proof. (i) From definition and Theorem 2, we have that $(X_1, \ldots, X_i)$ and $(X_j, X_{j+1}, \ldots)$ are independent whenever $j - i > 1$. According to Theorem 9.1 of DasGupta [25], the process $\{X_t\}$ is a stationary 1-dependent sequence. Therefore we can complete the proof.

(ii) For $k = 1$, it follows that

$$E(X_t) \le \max\{E[I_{t-1,i}(\alpha_i \circ \varepsilon_{t-1}) + \varepsilon_t], \ i = 1, \ldots, m\} \le \max\{E(\alpha_i \circ \varepsilon_{t-1}) + E(\varepsilon_t), \ i = 1, \ldots, m\} \le \lambda(\alpha_{\max} + 1) < \infty, \alpha_{\max} = \max(\alpha_1, \ldots, \alpha_m). \quad (7)$$

For $k = 2$,

$$E(X_t^2) \le \max\{E[I_{t-1,i}(\alpha_i \circ \varepsilon_{t-1}) + \varepsilon_t]^2, \ i = 1, \ldots, m\}$$
$$= \max\{E[I_{t-i,1}(\alpha_i \circ \varepsilon_{t-1})]^2 + E(\varepsilon_t^2) + 2E[I_{t-i,1}(\alpha_i \circ \varepsilon_{t-1})\varepsilon_t], \ i = 1, \ldots, m\}$$
$$\le \max\{E[(\alpha_i \circ \varepsilon_{t-1})]^2 + E(\varepsilon_t^2) + 2E[(\alpha_i \circ \varepsilon_{t-1})\varepsilon_t], \ i = 1, \ldots, m\}$$
$$= \max\{[(\lambda + \lambda^2)\alpha_i^2 + \lambda\alpha_i(1-\alpha_i)] + (\lambda + \lambda^2) + \lambda^2\alpha_i, \ i = 1, \ldots, m\}$$
$$\le 2(\lambda + \lambda^2)\alpha_{\max} + 0.25\lambda + \lambda^2\alpha_{\max} < \infty. \quad (8)$$

For $k = 3$,

$$E(X_t^3) \le \max\{E[I_{t-1,i}(\alpha_i \circ \varepsilon_{t-1}) + \varepsilon_t]^3, \ i = 1, \ldots, m\}$$
$$= \max\{E[I_{t-1,i}(\alpha_i \circ \varepsilon_{t-1})]^3 + E(\varepsilon_t^3) + 3E[I_{t-1,i}^2(\alpha_i \circ \varepsilon_{t-1})^2\varepsilon_t] + 3E[I_{t-1,i}(\alpha_i \circ \varepsilon_{t-1})\varepsilon_t^2], \ i = 1, \ldots, m\}$$
$$\le \max\{E(\alpha_i \circ \varepsilon_{t-1})^3 + E(\varepsilon_t^3) + 3E[(\alpha_i \circ \varepsilon_{t-1})^2\varepsilon_t] + 3E[(\alpha_i \circ \varepsilon_{t-1})\varepsilon_t^2], \ i = 1, \ldots, m\}$$
$$\le \max\{[\alpha_i^3\tau_1 + 3\alpha_i^2(1-\alpha_i)\tau_2 + (\alpha_i - 3\alpha_i^2(1-\alpha_i) - \alpha_i^3)\lambda] + \tau_1 + 3\{[\alpha_i^2\tau_2 + \alpha_i(1-\alpha_i)\lambda]\lambda\} + 3\lambda\alpha_i\tau_2, \ i = 1, \ldots, m\}$$
$$\le \lambda\alpha_{\max}[\alpha_{\max}^2(\tau_1 - 1 - 3\lambda) + 3\tau_2(\alpha_{\max} + 1) + 3\lambda + 1] + \tau_1 < \infty, \quad (9)$$

where $\tau_1 := \lambda^3 + 3\lambda^2 + \lambda$, $\tau_2 := \lambda^2 + \lambda$, and $\alpha_{\max} = \max(\alpha_1, \ldots, \alpha_m)$. Then note that $E(X_t^k) < \infty$ implies $E(X_t^k \mid I_{t-1,i} = 1) < \infty$ for $k = 1, 2, 3, i = 1, 2, \ldots, m$. □

Theorem 5. *Let $\{X_t\}$ be a INMASC(1) process according to Definition 1. Let $\overline{X}$ be the sample mean of $\{X_t\}$; then the stochastic process $\{X_t\}$ is ergodic in the mean.*

Proof. Since $\gamma_X(k) \to 0, k \to \infty$.

From Theorem 7.1.1 in Brockwell and Davis [26], we get

$$\operatorname{Var}\left(\overline{X}_T\right) = E\left(\overline{X}_T - \mu_X\right)^2 \longrightarrow 0. \quad (10)$$

Then $\overline{X}_T$ converges in probability to μ_X. Therefore, the process $\{X_t\}$ is ergodic in the mean. □

Theorem 6. *Suppose $\{X_t\}$ is a INMASC(1) process; then*

$$P\left(\left|\widehat{\gamma}_X(k) - \gamma_X(k)\right| \geq \varepsilon\right) \xrightarrow{P} 0, \quad (11)$$

where $\widehat{\gamma}_X(k) := (1/T)\sum_{t=1}^{T-k}(X_{t+k} - \overline{X}_T)(X_t - \overline{X}_T)$.

The proof of Theorem 6 is similar to Theorem 4 given in Yu et al. [27]. It is easy to verify; we skip the details.

3. Estimation of Parameters

In this paper, we consider one method, namely, moment estimation. An advantage of the method is that it is simple and often produces good results. The estimation problem of INMASC(1) parameters is complex. In fact, for the INMASC(1) processes, the conditional distribution of the X_t given ε_{t-1} is the convolution of the distribution of the arrival process ε_t and one thinning operation $\alpha_i \circ \varepsilon_{t-1}$. On the other hand, there are too many unknown parameters of the model, such as λ, α_i, p_i, u_i, and σ_i^2, $i = 1, \ldots, m$, whereas the number of moment conditions is small.

Therefore we cannot estimate all the parameters unless additional assumptions are made. Then, we assume that the number of break point m is two and assume that the value of break point τ_i, $i = 1, \ldots, m$, and the mean of innovation λ are also known. Thus, here we estimate INMASC(1) model with two break points. Under these assumptions, all the parameters λ, p_i, u_i, and σ_i^2, $i = 1, 2, 3$, are known. We only need to estimate the autoregressive coefficients α_1, α_2, and α_3. Using the sample mean and sample covariance function, we can get the moment estimators via solving the following equations:

$$\begin{aligned}\widehat{\gamma}(0) &= \sum_{i=1}^{3} p_i\alpha_i\left[\alpha_i\left(u_i^2 + \sigma_i^2\right) + \left(1 - \alpha_i\right)u_i\right] \\ &\quad - \left(\sum_{i=1}^{3} p_i\alpha_i u_i\right)^2 + \lambda \\ \widehat{\gamma}(1) &= \sum_{i=1}^{3} p_i\alpha_i\left(u_i^2 + \sigma_i^2 - \lambda u_i\right) \\ \overline{X} &= \sum_{i=1}^{3} p_i\alpha_i u_i + \lambda.\end{aligned} \quad (12)$$

Table 1: Bias and mean square error for models A, B, and C.

Model	Parameter	Sample size 50	200	500	
A	α_1	0.0267	0.0097	0.0034	Bias
		(0.3948)	(0.0753)	(0.0453)	MSE
	α_2	0.0645	0.0115	0.0025	Bias
		(0.4731)	(0.1314)	(0.0376)	MSE
	α_3	0.0417	0.0083	0.0046	Bias
		(0.2908)	(0.0811)	(0.0342)	MSE
B	α_1	0.0335	0.0127	0.0036	Bias
		(0.4623)	(0.1803)	(0.0745)	MSE
	α_2	0.0297	0.0103	0.0054	Bias
		(0.2806)	(0.3449)	(0.0847)	MSE
	α_3	0.0251	0.0081	0.0024	Bias
		(0.3408)	(0.0372)	(0.0165)	MSE
C	α_1	0.0736	0.0178	0.0068	Bias
		(1.0435)	(0.3562)	(0.0357)	MSE
	α_2	0.0582	0.0215	0.0049	Bias
		(0.4127)	(0.0433)	(0.0212)	MSE
	α_2	0.0237	0.0081	0.0031	Bias
		(0.3205)	(0.0547)	(0.0274)	MSE

If you want to estimate all parameters, you can use GMM method based on probability generating functions introduced by BräKnnäK and Hall [28]. But they found covariance matrix of estimators depends on z and the orders besides the model parameters in a highly complex way. Thus we do not use this method here. In next section, simulations are provided to give insight into the finite sample behaviour of these estimators.

4. Simulation Study

Consider the following INMASC(1) model:

$$X_t = \begin{cases}\alpha_1 \circ \varepsilon_{t-1} + \varepsilon_t, & \text{for } \varepsilon_{t-1} \leq \tau_1 \\ \alpha_2 \circ \varepsilon_{t-1} + \varepsilon_t, & \text{for } \tau_1 < \varepsilon_{t-1} \leq \tau_2 \\ \alpha_3 \circ \varepsilon_{t-1} + \varepsilon_t, & \text{for } \tau_2 < \varepsilon_{t-1},\end{cases} \quad (13)$$

where $\{\varepsilon_t\}$ is a sequence of i.i.d. For fixed t, ε_t follows a Poisson distribution with mean λ.

The parameters values considered in this model are listed as follows:

(model A) $(\alpha_1, \alpha_2, \alpha_3) = (0.1, 0.1, 0.1)$, with $\tau_1 = 3$, $\tau_2 = 10$, $\lambda = 1$;

(model B) $(\alpha_1, \alpha_2, \alpha_3) = (0.2, 0.3, 0.1)$, with $\tau_1 = 8$, $\tau_2 = 17$, $\lambda = 10$;

(model C) $(\alpha_1, \alpha_2, \alpha_3) = (0.4, 0.3, 0.1)$, with $\tau_1 = 21$, $\tau_2 = 43$, $\lambda = 50$.

We use the above models to generate data and then use moment methods to estimate the parameters. We computed the empirical bias and the mean square error (MSE) based on 300 replications for each parameter combination. These values are reported within parenthesis in Table 1.

From the results in Table 1, we can see moment estimation is good estimation methods producing estimators whose bias and MSEs are small when the sample sizes are larger. In addition, this method is fast and easy to implement. It is perhaps not surprising that the MSEs are larger when these sample sizes are smaller. As to be expected, both the bias and the MSEs converge to zero with increasing sample size T.

5. Conclusion

Based on some limitations of the present count data models, a new INMA model is introduced to model structural changes. Expressions for mean, variance, and autocorrelation functions are given. Stationary and other basic statistical properties are also obtained. We derived moment estimators of the unknown parameters. Furthermore, we constructed several simulations to evaluate the performance of the estimators of model parameters.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper

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References

[1] E. McKenzie, "Some simple models for discrete variate time series," *Water Resources Bulletin*, vol. 21, no. 4, pp. 635–644, 1985.

[2] E. McKenzie, "Autoregressive moving-average processes with negative-binomial and geometric marginal distributions," *Advances in Applied Probability*, vol. 18, no. 3, pp. 679–705, 1986.

[3] M. A. Al-Osh and A. A. Alzaid, "First order integer-valued autoregressive INAR(1) process," *Journal of Time Series Analysis*, vol. 8, no. 3, pp. 261–275, 1987.

[4] A. A. Alzaid and M. Al-Osh, "An integer-valued pth-order autoregressive structure INAR(p) process," *Journal of Applied Probability*, vol. 27, no. 2, pp. 314–324, 1990.

[5] J. G. Du and Y. Li, "The integer-valued autoregressive (INAR(p)) model," *Journal of Time Series Analysis*, vol. 12, no. 2, pp. 129–142, 1991.

[6] M. Al-Osh and A. A. Alzaid, "Integer-valued moving average (INMA) process," *Statistical Papers*, vol. 29, no. 4, pp. 281–300, 1988.

[7] E. McKenzie, "Some ARMA models for dependent sequences of Poisson counts," *Advances in Applied Probability*, vol. 20, no. 4, pp. 822–835, 1988.

[8] R. Ferland, A. Latour, and D. Oraichi, "Integer-valued GARCH process," *Journal of Time Series Analysis*, vol. 27, no. 6, pp. 923–942, 2006.

[9] K. Fokianos and R. Fried, "Interventions in INGARCH processes," *Journal of Time Series Analysis*, vol. 31, no. 3, pp. 210–225, 2010.

[10] C. H. Weiß, "The INARCH(1) model for overdispersed time series of counts," *Communications in Statistics: Simulation and Computation*, vol. 39, no. 6, pp. 1269–1291, 2010.

[11] F. Zhu, "A negative binomial integer-valued GARCH model," *Journal of Time Series Analysis*, vol. 32, no. 1, pp. 54–67, 2011.

[12] F. Zhu, "Modeling overdispersed or underdispersed count data with generalized Poisson integer-valued GARCH models," *Journal of Mathematical Analysis and Applications*, vol. 389, no. 1, pp. 58–71, 2012.

[13] F. Zhu and D. Wang, "Diagnostic checking integer-valued ARCH(p) models using conditional residual autocorrelations," *Computational Statistics and Data Analysis*, vol. 54, no. 2, pp. 496–508, 2010.

[14] L. Györfi, M. Ispány, G. Pap, and K. Varga, "Poisson limit of an inhomogeneous nearly critical INAR(1) model," *Acta Universitatis Szegediensis*, vol. 73, no. 3-4, pp. 789–815, 2007.

[15] H. S. Bakouch and M. M. Ristić, "Zero truncated Poisson integer-valued AR(1) model," *Metrika*, vol. 72, no. 2, pp. 265–280, 2010.

[16] M. Kachour and J. F. Yao, "First-order rounded integer-valued autoregressive ((RINAR(1)) process," *Journal of Time Series Analysis*, vol. 30, no. 4, pp. 417–448, 2009.

[17] H.-Y. Kim and Y. Park, "A non-stationary integer-valued autoregressive model," *Statistical Papers*, vol. 49, no. 3, pp. 485–502, 2008.

[18] H. Zheng, I. V. Basawa, and S. Datta, "First-order random coefficient integer-valued autoregressive processes," *Journal of Statistical Planning and Inference*, vol. 137, no. 1, pp. 212–229, 2007.

[19] D. Gomes and L. Canto e Castro, "Generalized integer-valued random coefficient for a first order structure autoregressive (RCINAR) process," *Journal of Statistical Planning and Inference*, vol. 139, no. 12, pp. 4088–4097, 2009.

[20] M. M. Ristić, A. S. Nastić, K. Jayakumar, and H. S. Bakouch, "A bivariate INAR(1) time series model with geometric marginals," *Applied Mathematics Letters*, vol. 25, no. 3, pp. 481–485, 2012.

[21] X. Pedeli and D. Karlis, "A bivariate INAR(1) process with application," *Statistical Modelling*, vol. 11, no. 4, pp. 325–349, 2011.

[22] P. Thyregod, J. Carstensen, H. Madsen, and K. Arnbjerg-Nielsen, "Integer valued autoregressive models for tipping bucket rainfall measurements," *Environmetrics*, vol. 10, no. 4, pp. 395–411, 1999.

[23] M. Monteiro, M. G. Scotto, and I. Pereira, "Integer-valued self-exciting threshold autoregressive processes," *Communications in Statistics: Theory and Methods*, vol. 41, no. 15, pp. 2717–2737, 2012.

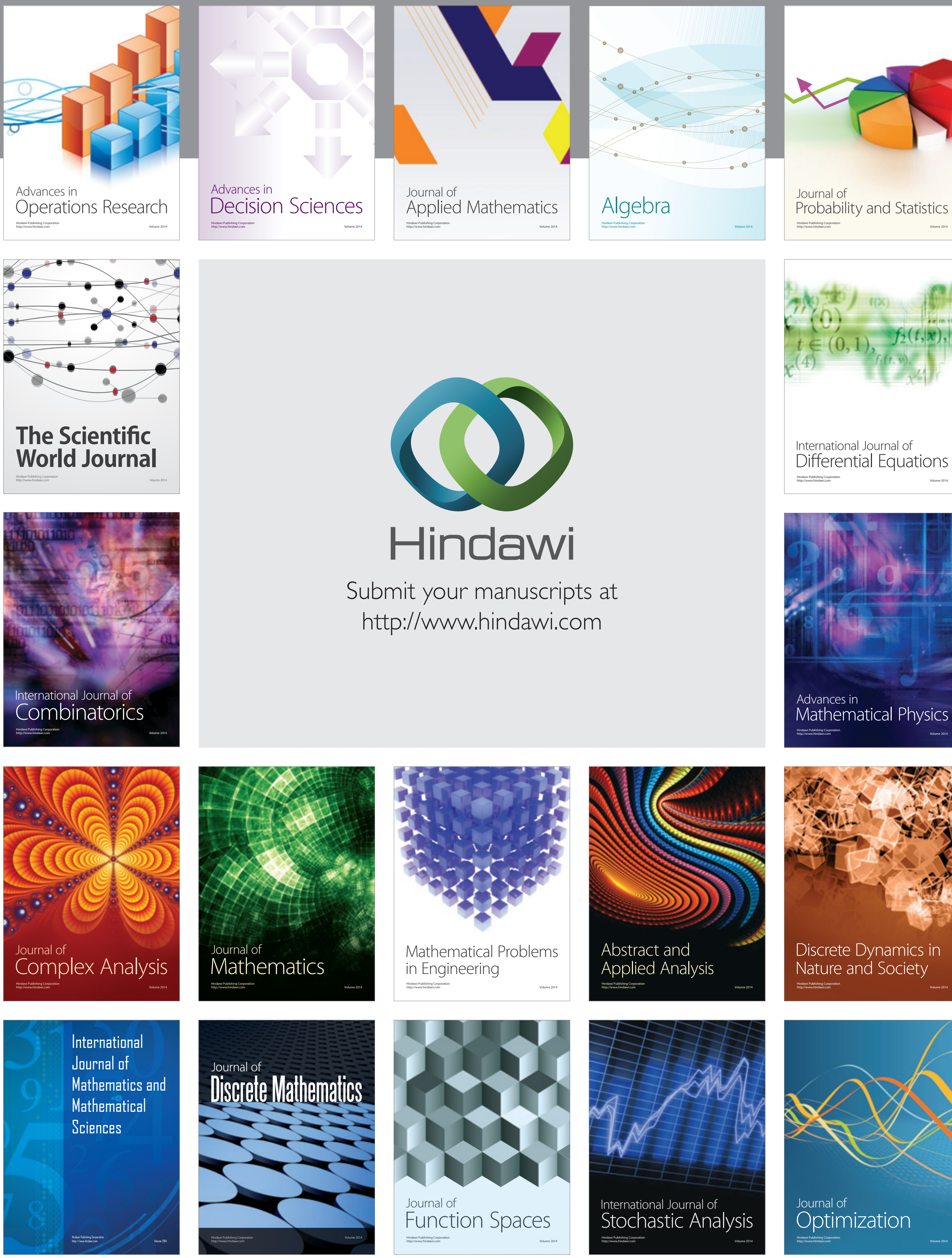
[24] Š. Hudecová, "Structural changes in autoregressive models for binary time series," *Journal of Statistical Planning and Inference*, vol. 143, no. 10, pp. 1744–1752, 2013.

[25] A. DasGupta, *Asymptotic Theory of Statistics and Probability*, Springer, New York, NY, USA, 2008.

[26] P. J. Brockwell and R. A. Davis, *Time Series: Theory and Methods*, Springer, New York, NY, USA, 1991.

[27] K. Yu, D. Shi, and H. Zou, "The random coefficient discretevalued time series model," *Statistical Research*, vol. 28, no. 4, pp. 106–112, 2011.

[28] K. BräKnnäK and A. Hall, "Estimation in integer-valued moving average models," *Applied Stochastic Models in Business and Industry*, vol. 17, no. 3, pp. 277–291, 2001.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