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Research Article

A New Extended Soft Intersection Set to (M, N)-*SI* Implicative Fitters of *BL*-Algebras

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Molodtsov's soft set theory provides a general mathematical framework for dealing with uncertainty. The concepts of (M, N)-*SI* implicative (Boolean) filters of *BL*-algebras are introduced. Some good examples are explored. The relationships between (M, N)-*SI* filters and (M, N)-*SI* implicative filters are discussed. Some properties of (M, N)-*SI* implicative (Boolean) filters are investigated. In particular, we show that (M, N)-*SI* implicative filters and (M, N)-*SI* Boolean filters are equivalent.

1. Introduction

We know that dealing with uncertainties is a major problem in many areas such as economics, engineering, medical sciences, and information science. These kinds of problems cannot be dealt with by classical methods because some classical methods have inherent difficulties. To overcome them, Molodtsov [1] introduced the concept of a soft set as a new mathematical tool for dealing with uncertainties that is free from the difficulties that have troubled the usual theoretical approaches. Since then, especially soft set operations have undergone tremendous studies; for examples, see [2–5]. At the same time, soft set theory has been applied to algebraic structures, such as [6–8]. We also note that soft set theory emphasizes balanced coverage of both theory and practice. Nowadays, it has promoted a breath of the discipline of information sciences, decision support systems, knowledge systems, decision-making, and so on; see [9–13].

BL-algebras, which have been introduced by Hájek [14] as algebraic structures of basic logic, arise naturally in the analysis of the proof theory of propositional fuzzy logic. Turunen [15] proposed the concepts of implicative filters and Boolean filters in *BL*-algebras. Liu et al. [16, 17] applied fuzzy set theory to *BL*-algebras. After that, some researchers have further investigated some properties of *BL*-algebras. Further, Ma et al. investigated some kinds of generalized fuzzy filters *BL*-algebras and obtained some important results; see [18, 19]. Zhang et al. [20, 21] described the relations between pseudo-BL, pseudo-effect algebras, and BCC-algebras, respectively. The other related results can be found in [22, 23].

Recently, Çağman et al. put forward soft intersection theory; see [24, 25]. Jun and Lee [26] applied this theory to *BL*-algebras. Ma and Kim [27] introduced a new concept: (M, N)-soft intersection set. They introduced the concept of (M, N)-soft intersection filters of *BL*-algebras and investigated some related properties.

In this paper, we introduce the concept of (M, N)-soft intersection implicative filters of *BL*-algebras. Some related properties are investigated. In particular, we show that (M, N)-*SI* implicative filters and (M, N)-*SI* Boolean filters are equivalent.

2. Preliminaries

Recall that an algebra $L = (L, \leq, \wedge, \vee, \odot, \rightarrow, 0, 1)$ is a *BL-algebra* [14] if it is a bounded lattice such that the following conditions are satisfied:

(i) $(L, \odot, 1)$ is a commutative monoid,

(ii) $\odot$ and $\rightarrow$ form an adjoin pair, that is, $z \leq x \rightarrow y$ if and only if $x \odot z \leq y$ for all $x, y, z \in L$,

(iii) $x \wedge y = x \odot (x \rightarrow y)$,

(iv) $(x \rightarrow y) \vee (y \rightarrow x) = 1$.

In what follows, L is a BL-algebra unless otherwise is specified.

In any BL-algebra L, the following statements are true (see [14, 15]):

(a_1) $x \leq y \Leftrightarrow x \rightarrow y = 1$,

(a_2) $x \rightarrow (y \rightarrow z) = (x \odot y) \rightarrow z = y \rightarrow (x \rightarrow z)$,

(a_3) $x \odot y \leq x \wedge y$,

(a_4) $x \rightarrow y \leq (z \rightarrow x) \rightarrow (z \rightarrow y), x \rightarrow y \leq (y \rightarrow z) \rightarrow (x \rightarrow z)$,

(a_5) $x \rightarrow x' = x'' \rightarrow x$,

(a_6) $x \vee x' = 1 \Rightarrow x \wedge x' = 0$,

(a_7) $(x \rightarrow y) \odot (y \rightarrow z) \leq x \rightarrow z$,

(a_8) $x \leq y \Rightarrow x \rightarrow z \geq y \rightarrow z$,

(a_9) $x \leq y \Rightarrow z \rightarrow x \leq z \rightarrow y$,

(a_{10}) $x \vee y = ((x \rightarrow y) \rightarrow y) \wedge ((y \rightarrow x) \rightarrow x)$,

where $x' = x \rightarrow 0$.

A nonempty subset A of L is called a *filter* of L if it satisfies the following conditions: (I1) $1 \in A$, (I2) for all $x \in A$, for all $y \in L, x \rightarrow y \in A \Rightarrow y \in A$.

It is easy to check that a nonempty subset A of L is a filter of L if and only if it satisfies (I3) for all $x, y \in L$, $x \odot y \in A$, (I4) for all $x \in A$, for all $y \in L$, $x \leq y \Rightarrow y \in A$ (see [15]).

Now, we call a nonempty subset A of L an implicative filter if it satisfies (I1) and (I5) $x \rightarrow (z' \rightarrow y) \in A$, $y \rightarrow z \in A \Rightarrow x \rightarrow z \in A$.

A nonempty subset A of L is said to be a Boolean filter of L if it satisfies $x \vee x' \in A$, for all $x \in A$. (see [15–18]).

From now on, we let L be an BL-algebra, U an initial universe, E a set of parameters, $P(U)$ the power set of U, and $A, B, C \subseteq E$. We let $\emptyset \subseteq M \subset N \subseteq U$.

Definition 1 (see [1]). A soft set f_A over U is a set defined by $f_A : E \rightarrow P(U)$ such that $f_A(x) = \emptyset$ if $x \notin A$. Here f_A is also called an approximate function. A soft set over U can be represented by the set of ordered pairs $f_A = \{(x, f_A(x)) \mid x \in E, f_A(x) \in P(U)\}$. It is clear to see that a soft set is a parameterized family of subsets of U. Note that the set of all soft sets over U will be denoted by $S(U)$.

Definition 2 (see [9]). Let $f_A, f_B \in S(U)$.

(1) f_A is said to be a soft subset of f_B and denoted by $f_A \tilde{\subseteq} f_B$ if $f_A(x) \subseteq f_B(x)$, for all $x \in E$. f_A and f_B are said to be soft equally, denoted by $f_A = f_B$, if $f_A \tilde{\subseteq} f_B$ and $f_A \tilde{\supseteq} f_B$.

(2) The union of f_A and f_B, denoted by $f_A \tilde{\cup} f_B$, is defined as $f_A \tilde{\cup} f_B = f_{A \cup B}$, where $f_{A \cup B}(x) = f_A(x) \cup f_B(x)$, for all $x \in E$.

(3) The intersection of f_A and f_B, denoted by $f_A \tilde{\cap} f_B$, is defined as $f_A \tilde{\cap} f_B = f_{A \cap B}$, where $f_{A \cap B}(x) = f_A(x) \cap f_B(x)$, for all $x \in E$.

Definition 3 (see [26]). (1) A soft set f_L over U is called an SI-filter of L over U if it satisfies

(S_1) $f_L(x) \subseteq f_L(1)$ for any $x \in L$,

(S_2) $f_L(x \rightarrow y) \cap f_L(x) \subseteq f_L(y)$ for all $x, y \in L$.

(2) A soft set f_L over U is called an SI-*implicative filter* of L over U if it satisfies (S_1) and

(S_3) $f_L(x \rightarrow (z' \rightarrow y)) \cap f_l(y \rightarrow z) \subseteq f_L(x \rightarrow z)$, for all $x, y, z \in L$.

In [27], Ma and Kim introduced the concept of (M, N)-SI filters in BL-algebras.

Definition 4 (see [27]). A soft set f_S over U is called an (M, N)-soft intersection filter (briefly, (M, N)-SI filter) of L over U if it satisfies

(SI_1) $f_L(x) \cap N \subseteq f_L(1) \cup M$ for all $x \in L$,

(SI_2) $f_L(x \rightarrow y) \cap f_L(x) \cap N \subseteq f_L(y) \cup M$ for all $x, y \in L$.

Define an ordered relation "$\tilde{\subseteq}_{(M,N)}$" on $S(U)$ as follows. For any $f_L, g_L \in S(U)$, $\emptyset \subseteq M \subset N \subseteq U$, we define $f_L \tilde{\subseteq}_{(M,N)} g_L \Leftrightarrow f_L \cap N \tilde{\subseteq}_{(M,N)} g_L \cup M$.

And we define a relation "$=_{(M,N)}$" as follows: $f_L =_{(M,N)} g_L \Leftrightarrow f_L \tilde{\subseteq}_{(M,N)} g_L$ and $g_L \tilde{\subseteq}_{(M,N)} f_L$.

Definition 5 (see [27]). A soft set f_S over U is called an (M, N)-soft intersection filter (briefly, (M, N)-SI filter) of L over U if it satisfies

(SI_1') $f_L(x) \tilde{\subseteq}_{(M,N)} f_L(1)$ for all $x \in L$,

(SI_2') $f_L(x \rightarrow y) \cap f_L(x) \tilde{\subseteq}_{(M,N)} f_L(y)$ for all $x, y \in L$.

3. (M,N)-SI Implicative (Boolean) Filters

In this section, we investigate some characterizations of (M, N)-SI implicative filters of BL-algebras. Finally, we prove that a soft set in BL-algebras is an (M, N)-SI implicative filter if and only if it is an (M, N)-SI Boolean filter.

Definition 6. A soft set f_L over U is called an (M, N)-soft intersection implicative filter (briefly, (M, N)-SI implicative filter) of L over U if it satisfies (SI_1) and (SI_3) $f_L(x \rightarrow (z' \rightarrow y)) \cap f_L(y \rightarrow z) \cap N \tilde{\subseteq}_{(M,N)} f_L(y \rightarrow z) \cup M$ for all $x, y, z \in L$.

Remark 7. If f_L is an (M, N)-SI implicative filter of L over U, then f_L is an $(\emptyset, U)$-SI implicative filter of L. Hence every SI-implicative filter of L is an (M, N)-SI implicative filter of L, but the converse need not be true in general. See the following example.

Example 8. Assume that $U = D_2 = \{\langle x, y \rangle \mid x^2 = y^2 = e, xy = yx\} = \{e, x, y, yx\}$, dihedral group, is the universe set.

Let $L = \{0, a, b, 1\}$, where $0 < a < b < 1$. Then we define $x \wedge y = \min\{x, y\}, x \vee y = \max\{x, y\}$ and $\odot$ and $\rightarrow$ as follows:

$\odot$	0	a	b	1
0	0	0	0	0
a	0	a	a	a
b	0	a	a	b
1	0	a	b	1

$\longrightarrow$	0	a	b	1
0	1	1	1	1
a	0	1	1	1
b	0	b	1	1
1	0	a	b	1

(1)

Then $(L, \wedge, \vee, \odot, \rightarrow, 1)$ is a BL-algebra.

Let $M = \{e, y\}$ and $N = \{e, x, y\}$.

Define a soft set f_L over U by $f_L(1) = \{e, x\}$, $f_L(a) = f_L(b) = \{e, x, y\}$, and $f_L(0) = \{e, y\}$. Then one can easily check that f_L is an (M, N)-SI implicative filter of L over U, but it is not an SI implicative filter of L over U since $f_L(1) = \{e, x\} \not\supseteq f_L(a)$.

By means of "$\widetilde{\subseteq}_{(M,N)}$," we can obtain the following equivalent concept.

Definition 9. A soft set f_L over U is called an (M, N)-SI implicative filter of L over U if it satisfies (SI_1') and (SI_3') $f_L(x \rightarrow (z' \rightarrow y)) \cap f_L(y \rightarrow z) \widetilde{\subseteq}_{(M,N)} f_L(y \rightarrow z)$ for all $x, y, z \in L$.

From the above definitions, we have the following.

Proposition 10. *Every (M, N)-SI implicative filter of L over U is an (M, N)-SI filter, but the converse may not be true as shown in the following example.*

Example 11. Define $x \odot y = \min\{x, y\}$ and

$$x \longrightarrow y = \begin{cases} 1, & \text{if } x \leq y, \\ y, & \text{if } x > y. \end{cases} \quad (2)$$

Then $L = ([0, 1], \wedge, \vee, \odot, \rightarrow, 0, 1)$ is a BL-algebra.

Let $U = L$, $M = \{0.5, 0.75\}$, and $N = \{0.5, 0.75, 1\}$.

Define a soft set f_L over U by

$$f_L(x) = \begin{cases} \{0, 0.5\}, & \text{if } x \in \left[0, \frac{1}{2}\right], \\ \{0.5, 1\}, & \text{if } x \in \left[\frac{1}{2}, 1\right]. \end{cases} \quad (3)$$

Then one can easily check that f_L is an (M, N)-SI filter of L over U, but it is not an (M, N)-SI implicative filter of L over U. Since $f_L(2/3 \rightarrow ((1/3)' \rightarrow 1/4)) \cap f_L(1/4 \rightarrow 1/3) \cap N = f_L(1) \cap f_L(1) \cap N = \{0.5, 1\} \cap \{0.5, 0.75, 1\} = \{0.5, 1\}$ and $f_L(2/3 \rightarrow 1/4) \cup M = f_L(1/4) \cup M = \{0, 0.5\} \cup \{0.5, 0.75\} = \{0, 0.5, 0.75\}$, this implies that $f_L(2/3 \rightarrow ((1/3)' \rightarrow 1/4)) \cap f_L(1/4 \rightarrow 1/3) \cap N \not\subseteq f_L(x \rightarrow z) \cup M$.

Lemma 12 (see [27]). *If a soft set f_L over U is an (M, N)-SI filter of L, then for any $x, y, z \in L$ we have*

(1) $x \leq y \Rightarrow f_L(x) \widetilde{\subseteq}_{(M,N)} f_L(y)$,

(2) $f_L(x \rightarrow y) = f_L(1) \Rightarrow f_L(x) \widetilde{\not\subseteq}_{(M,N)} f_L(y)$,

(3) $f_L(x \odot y) =_{(M,N)} f_L(x) \cap f_L(y) =_{(M,N)} f_L(x \wedge y)$,

(4) $f_L(0) =_{(M,N)} f_L(x) \cap f_L(x')$,

(5) $f_L(x \rightarrow y) \cap f_L(y \rightarrow z) \widetilde{\subseteq}_{(M,N)} f_L(x \rightarrow z)$,

(6) $f_L(x) \cap f_L(y) \widetilde{\subseteq}_{(M,N)} f_L(y \odot z \rightarrow y \odot z)$,

(7) $f_L(x \rightarrow y) \widetilde{\subseteq}_{(M,N)} f_L((y \rightarrow z) \rightarrow (x \rightarrow z))$,

(8) $f_L(x \rightarrow y) \widetilde{\subseteq}_{(M,N)} f_L((z \rightarrow x) \rightarrow (z \rightarrow y))$.

Theorem 13. *Let f_L be an (M, N)-SI filter of L over U, then the following are equivalent:*

(1) *f_L is an (M, N)-SI implicative filter of L,*

(2) *$f_L(x \rightarrow z) \widetilde{\supseteq}_{(M,N)} f_L(x \rightarrow (z' \rightarrow z))$, for all $x, y, z \in L$,*

(3) *$f_L(x \rightarrow z) =_{(M,N)} f_L(x \rightarrow (z' \rightarrow z))$, for all $x, y, z \in L$,*

(4) *$f_L(x \rightarrow z) \widetilde{\supseteq}_{(M,N)} f_L(y \rightarrow (x \rightarrow (z' \rightarrow z))) \cap f_L(y)$, for all $x, y, z \in L$.*

Proof. (1) ⇒ (2) Assume that f_L is an (M, N)-SI filter of L over U. Putting $y = z$ in (SI_3), then

$$\begin{aligned} f_L(x \longrightarrow z) \cup M &= (f_L(x \longrightarrow z) \cup M) \cap M \\ &\supseteq \left(f_L\left(x \longrightarrow \left(z' \longrightarrow z\right)\right) \cap f_L(z \longrightarrow z) \cap N\right) \cup M \\ &= \left(f_L\left(x \longrightarrow \left(z' \longrightarrow z\right)\right) \cap f_L(1) \cap N\right) \cup M \\ &\supseteq f_L\left(x \longrightarrow \left(z' \longrightarrow z\right)\right) \cap (f_L(1) \cup M) \cap N \\ &\supseteq f_L\left(x \longrightarrow \left(z' \longrightarrow z\right)\right) \cap N; \end{aligned} \quad (4)$$

that is, $f_L(x \rightarrow z) \widetilde{\supseteq}_{(M,N)} f_L(x \rightarrow (z' \rightarrow z))$. Thus, (2) holds.

(2) ⇒ (3) By (a_1) and (a_2), $x \rightarrow z \leq z' \rightarrow (x \rightarrow z) = x \rightarrow (z' \rightarrow z)$; then it follows from Lemma 12 (1) that $f_L(x \rightarrow z) \widetilde{\subseteq}_{(M,N)} f_L(x \rightarrow (z' \rightarrow z))$. Thus, (3) holds.

(3) ⇒ (4) Assume that (4) holds. By Lemma 12 (5), we have $f_L(x \odot z' \rightarrow y) \cap f_L(y \rightarrow z) \widetilde{\subseteq}_{(M,N)} f_L(x \odot z' \rightarrow z)$. By (a_2), $f_L(x \rightarrow (z' \rightarrow y)) \cap f_L(y \rightarrow z) \widetilde{\subseteq}_{(M,N)} f_L(x \rightarrow (z' \rightarrow z))$.

(4) ⇒(1) Putting $y = 1$ in (4), we have

$$f_L(x \longrightarrow z) \widetilde{\supseteq}_{(M,N)} f_L\left(x \longrightarrow \left(z' \longrightarrow z\right)\right). \quad (5)$$

Hence

$$f_L(z \longrightarrow z) \widetilde{\supseteq}_{(M,N)} f_L\left(x \longrightarrow \left(z' \longrightarrow y\right)\right) \cap f_L(y \longrightarrow z). \quad (6)$$

Thus, (SI_3) holds. This shows that f_L is an (M, N)-SI implicative filter of L over U. □

Now, we introduce the concept of (M, N)-SI Boolean filters of BL-algebras.

Definition 14. Let f_L be an (M, N)-SI filter of L over U, then f_L is called an (M, N)-SI Boolean filter of L over U if it satisfies

(SI_4) $f_L(x \vee x') =_{(M,N)} f_L(1)$ for all $x \in L$.

Theorem 15. *A soft set f_L over U is an (M,N)-SI implicative filter of L if and only if it is an (M,N)-SI Boolean filter.*

Proof. Assume that f_L over U is an (M,N)-*SI* Boolean filter of L over U. Then

$$\begin{aligned} & f_L(x \longrightarrow z) \\ & \quad \tilde{\supseteq}_{(M,N)} f_L\left(\left(z \vee z'\right) \longrightarrow (x \longrightarrow z)\right) \cap f_L\left(z \vee z'\right) \\ & \quad =_{(M,N)} f_L\left(\left(z \vee z'\right) \longrightarrow (x \longrightarrow z)\right) \cap f_L(1) \\ & \quad \tilde{\supseteq}_{(M,N)} f_L\left(\left(z \vee z'\right) \longrightarrow (x \longrightarrow z)\right). \end{aligned} \tag{7}$$

By (a_{10}) and (a_1), we have

$$\begin{aligned} & \left(z \vee z'\right) \longrightarrow (x \rightarrow z) \\ & \quad = (z \longrightarrow (x \longrightarrow z)) \wedge \left(z' \longrightarrow (x \longrightarrow z)\right) \\ & \quad = z' \longrightarrow (x \longrightarrow z) = x \longrightarrow \left(z' \longrightarrow z\right). \end{aligned} \tag{8}$$

Hence $f_L(x \rightarrow z) \tilde{\supseteq}_{(M,N)} f_L(x \rightarrow (z' \rightarrow z))$. It follows from Theorem 13 that f_L is an (M,N)-*SI* implicative filter of L over U.

Conversely, assume that f_L is an (M,N)-*SI* implicative filter of L over U. By Theorem 13, we have

$$\begin{aligned} & f_L\left(\left(x' \longrightarrow x\right) \longrightarrow x\right) \\ & \quad =_{(M,N)} f_L\left(\left(x' \longrightarrow x\right) \longrightarrow \left(x' \rightarrow x\right)\right) = f_L(1), \\ & f_L\left(\left(x \longrightarrow x'\right) \longrightarrow x'\right) \\ & \quad =_{(M,N)} f_L\left(\left(x \longrightarrow x'\right) \longrightarrow \left(x'' \longrightarrow x'\right)\right) \\ & \quad = f_L\left(\left(x \longrightarrow x'\right) \longrightarrow (x \longrightarrow x)\right) = f_L(1). \end{aligned} \tag{9}$$

By Lemma 12, we have

$$\begin{aligned} & f_L\left(x \vee x'\right) \\ & \quad =_{(M,N)} f_L\left(\left(x \longrightarrow x'\right) \longrightarrow x'\right) \cap f_L\left(\left(x' \longrightarrow x\right) \longrightarrow x\right) \\ & \quad =_{(M,N)} f_L(1). \end{aligned} \tag{10}$$

Hence f_L is an (M,N)-*SI* Boolean filter of L over U. □

Remark 16. Every (M,N)-*SI* implicative filter and (M,N)-*SI* Boolean filter in *BL*-algebras are equivalent.

Next, we give some characterizations of (M,N)-*SI* implicative (Boolean) filters in *BL*-algebras.

Theorem 17. *Let f_L be an (M,N)-SI filter of L over U, then the following are equivalent:*

(1) *f_L is an (M,N)-SI implicative (Boolean) filter,*

(2) *$f_L(x) =_{(M,N)} f_L(x' \rightarrow x)$, for all $x \in L$,*

(3) *$f_L((x \rightarrow y) \rightarrow x) \tilde{\subseteq}_{(M,N)} f_L(x)$, for all $x, y \in L$,*

(4) *$f_L((x \rightarrow y) \rightarrow x) =_{(M,N)} f_L(x)$, for all $x, y \in L$,*

(5) *$f_L(x) \tilde{\supseteq}_{(M,N)} f_L(z \rightarrow ((x \rightarrow y) \rightarrow x)) \cap f_L(z)$, for all $x, y, z \in L$.*

Proof. (1) ⇒ (2). Assume that f_L is an (M,N)-*SI* implicative (Boolean) filter of L over U. By Theorem 13, we have

$$\begin{aligned} f_L(x) &= f_L(1 \longrightarrow x) =_{(M,N)} f_L\left(1 \rightarrow \left(x' \longrightarrow x\right)\right) \\ &= f_L\left(x' \longrightarrow x\right). \end{aligned} \tag{11}$$

Thus, (2) holds.

(2) ⇒ (3). By (a_1), (a_2), and (a_8), we have $x' \leq x \rightarrow y$ and so $(x \rightarrow y) \rightarrow x \leq x' \rightarrow x$. By Lemma 12, $f_L((x \rightarrow y) \rightarrow x) \tilde{\subseteq}_{(M,N)} f_L(x' \rightarrow x)$. Combining (2), $f_L(x) =_{(M,N)} f_L(x' \rightarrow x) \tilde{\supseteq}_{(M,N)} f_L((x \rightarrow y) \rightarrow x)$. Thus, (3) holds.

(3) ⇒ (4). Since $x \leq (x \rightarrow y) \rightarrow x$, then by Lemma 12 $f_L(x) \tilde{\subseteq}_{(M,N)} f_L((x \rightarrow y) \rightarrow x)$. Combining (3), $f_L(x) =_{(M,N)} f_L((x \rightarrow y) \rightarrow x)$.

(4) ⇒ (5). By (SI_2), $f_L((x \rightarrow y) \rightarrow x) \tilde{\supseteq}_{(M,N)} f_L(z \rightarrow ((x \rightarrow y) \rightarrow x)) \cap f_L(z)$. Combining (4), we have $f_L(x) \tilde{\supseteq}_{(M,N)} f_L(z \rightarrow ((x \rightarrow y) \rightarrow x)) \cap f_L(z)$. Thus, (5) holds.

(5) ⇒ (1). By (a_1), $z \leq x \rightarrow z$. By (a_8), $(x \rightarrow z)' \leq z'$ and so $z' \rightarrow (x \rightarrow z) \leq (x \rightarrow z)' \rightarrow (x \rightarrow z)$. Then by Lemma 12, $f_L(z' \rightarrow (x \rightarrow z)) \tilde{\subseteq}_{(M,N)} f_L((x \rightarrow z)' \rightarrow (x \rightarrow z)) =_{(M,N)} f_L(1 \rightarrow ((x \rightarrow z)' \rightarrow (x \rightarrow z))) \cap f_L(1)$. By (5), $f_L(x \rightarrow z) \tilde{\supseteq}_{(M,N)} f_L(z' \rightarrow (x \rightarrow z))$ and so $f_L(x \rightarrow z) \tilde{\supseteq}_{(M,N)} f_L(x \rightarrow (z' \rightarrow z))$. Therefore, it follows from Theorem 13 that f_L is an (M,N)-*SI* implicative filter of L. □

Finally, we investigate extension properties of (M,N)-*SI* implicative filters of *BL*-algebras.

Theorem 18 (extension property). *Let f_L and g_L be two (M,N)-SI filters of L over U such that $f_L(1) =_{(M,N)} g_L(1)$ and $f_L(x) \tilde{\subseteq}_{(M,N)} g_L(x)$ for all $x \in L$. If f_L is an (M,N)-SI implicative (Boolean) filter of L, then so is g_L.*

Proof. Assuming that f_L is an (M,N)-*SI* implicative (Boolean) filter of L over U, then $f_L(x \vee x') =_{(M,N)} f_L(1)$ for all $x \in L$. By hypothesis, $g_L(x \vee x') \tilde{\supseteq}_{(M,N)} f_L(x \vee x') =_{(M,N)} f_L(1) =_{(M,N)} g_L(1)$. By (SI_1'), we have $g_L(1) \tilde{\supseteq}_{(M,N)} g_L(x \vee x')$. Thus, $g_L(x \vee x') =_{(M,N)} g_L(1)$. Hence g_L is an (M,N)-*SI* implicative (Boolean) filter of L. □

4. Conclusions

In this paper, we introduce the concepts of (M,N)-*SI* implicative filters and (M,N)-*SI* Boolean filters of *BL*-algebras. Then we show that every (M,N)-*SI* Boolean filter is equivalent to (M,N)-*SI* implicative filters. In particular, some equivalent conditions for (M,N)-*SI* Boolean filters are obtained. We hope it can lay a foundation for providing a new soft algebraic tool in many uncertainties problems.

To extend this work, one can apply this theory to other fields, such as algebras, topology, and other mathematical branches. To promote this work, we can further investigate (M, N)-*SI* prime (semiprime) Boolean filters of *BL*-algebras. Maybe one can apply this idea to decision-making, data analysis, and knowledge based systems.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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