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Research Article

An LMI-Based H_∞ Control Approach for Networked Control Systems with Deadband Scheduling Scheme

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Due to the bandwidth constraints in the networked control systems (NCSs), a deadband scheduling strategy is proposed to reduce the data transmission rate of network nodes. A discrete-time model of NCSs is established with both deadband scheduling and network-induced time-delay. By employing the Lyapunov functional and LMI approach, a state feedback H_∞ controller is designed to ensure the closed-loop system asymptotically to be stable with H_∞ performance index. Simulation results show that the introduced deadband scheduling strategy can ensure the control performance of the system and effectively reduce the node's data transmission rate.

1. Introduction

Networked control systems (NCSs) are control systems in which the control loop is closed over a wired or wireless communication network. They have received a great deal of attention in the recent years owing to their successful applications in a wide range of areas such as industrial automation, aerospace, and nuclear power station [1, 2]. Compared with the traditional point-to-point communication, NCSs have advantages of low cost, easy installation and maintenance, great reliability, and so forth. However, due to the introduction of the communication networks, they also incur some new issues such as network-induced delays, packet dropouts, and limited bandwidth resources, which make the analysis and design of NCSs become more complex [3, 4].

Many results for NCSs have been reported to handle network-induced delays, packet dropouts and communication constraints in the literature; see [5–12] and the references therein. It should be pointed out that most of the available results make use of time-driven sampling and communication scheme since it is relatively easy to implement, and there is a well-established system theory for periodic signals. However, time-driven communication scheme is not desirable in many control applications. For example, in the NCSs with limited bandwidth resources, frequent data transmission will increase the network collision probability when there are many nodes on the network, thereby increasing the communication delay and data dropouts and leading to the poor performance and instability of the systems [3]. On the other hand, as is well-known, in the wireless networked control systems (WNCSs), a main constraint of wireless devices is the limited battery life, and wireless transmission consumes significantly more energy than internal computation [13]; thus reducing the data transmission rate is particularly important in the WNCSs. For the above two cases, time-driven communication scheme is not suitable since its transmission rate is generally high which results in inefficient utilization of the limited resources, such as network bandwidth and energy. Therefore, how to design a reasonable scheduling strategy to reduce the use of constrained resources and ensure the performance of NCSs becomes one of the research hotspots.

Not only deadband scheduling techniques (i.e., by setting transmission deadband for the network node, the node will not transmit a new message if the node signal or signal change is within the deadband), which can effectively reduce the use of network bandwidth and energy consumption, but also the algorithms which are easy to realize have received an increasing attention in the recent years [14–18]. Besides, numerous other concepts have been proposed in the literature, such as send-on-delta sampling [19–21], event-based sampling [22–24], and event-triggered sampling [25, 26]. Despite the existence of many names, the basic principle is the same. In [14], a deadband method was introduced into the NCSs for the first time, in which the transmission deadbands were set in the sensor and controller to reduce the data transmission rate, and the deadband threshold optimization problem was also discussed. In [15], the relationship between the deadband threshold and the control performance was analyzed by simulation. The paper [23] used the deviation of two adjacent states beyond the deadband threshold to drive the nodes' data transmission and a dynamically selected deadband threshold value in accordance with the round-trip delay. In [16, 17], the stability of the system with deadband scheduling was investigated, but the network delay was not considered in [16] the nodes should be synchronized and the network delay should be measurable in [17]. The paper [18] proposed a signal difference-based transmission deadband scheduling strategy, a continuous-time model of WNCSs was established with both the probability distribution of delay and parametric uncertainties, and the H_∞ controller was designed. In [26], for a class of uncertain continuous-time NCSs with quantizations, the codesign for controller and event-triggering scheme was proposed by using a delay system approach.

Until now, although some important pieces of work have been reported on deadband scheduling schemes in NCSs, which have a great significance on both theoretical development and practical applications in NCSs, it is worth noting that the obtained results on deadband scheduling in NCSs mostly focus on the system simulation and performance analysis; few papers have solved the problems of controller design and synthesis, which are more useful and challenging than the issue of performance analysis. In addition, to the best of our knowledge, few related results have been established for discrete-time NCSs with deadband scheduling, which motivates the work of this paper.

In this paper, we propose a deadband scheduling scheme to save the limited bandwidth resources while guaranteeing the desired H_∞ control performance. Considering the influence of uncertain short time-delay, the NCSs with both deadband scheduling and time-delay is modeled as a discrete-time system with parameter uncertainties. By the Lyapunov functional and LMI approach, the H_∞ control problem is investigated. Finally, a numerical example is given to show the usefulness of the derived results.

The rest of this paper is organized as follows. Section 2 gives a discrete-time model of the closed-loop system. In Section 3, a state feedback H_∞ controller is designed to ensure the closed-loop system asymptotically to be stable with H_∞ performance index. Section 4 demonstrates the validness of the proposed method through a numerical example. Conclusions are given in Section 5.

Notation. The notations used throughout this paper are fairly standard. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space. $\|\cdot\|_2$ refers to the Euclidean vector norm. $l_2[0,\infty)$ is the space of square summable infinite sequence. I and 0 represent the identity matrix and zero matrix with appropriate dimensions, respectively. diag$\{\cdots\}$ stands for a diagonal matrix.

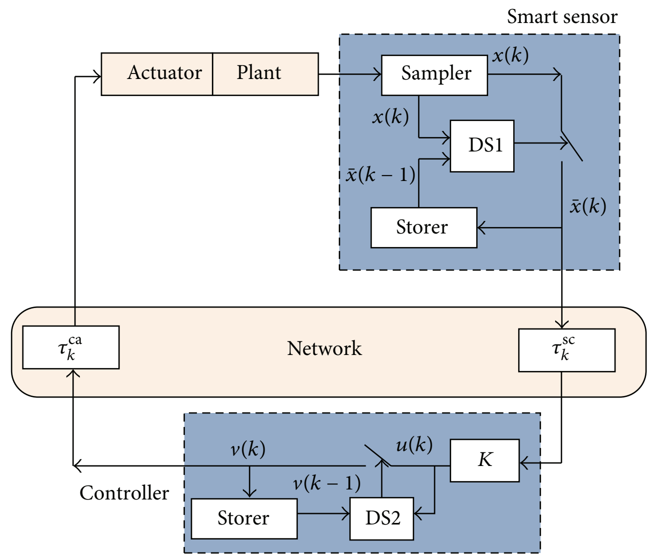

Figure 1: Model of networked control system with deadband scheduling.

The superscripts "T" and "-1" represent matrix transpose and inverse, and "$*$" denotes the term that is induced by symmetry.

2. Problem Description and Modeling

The networked control system with deadband scheduling in this paper is shown in Figure 1, where deadband schedulers (DS1, DS2) are set in the sensor and controller, respectively, τ_k^{sc} is sensor-to-controller delay, and τ_k^{ca} is controller-to-actuator delay.

Consider the following continuous plant model:

$$\dot{x}(t) = Ax(t) + B_1 v(t) + B_2 w(t),$$
$$y(t) = Cx(t), \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector of plant, $v(t) \in \mathbb{R}^p$ is the input vector, $y(t) \in \mathbb{R}^q$ is the output vector, and $w(t) \in \mathbb{R}^p$ is the disturbance input. A, B_1, B_2, and C are known real constant matrices with appropriate dimensions.

Make the following assumptions for the system.

(1) In the smart sensor, the sampler is time-driven, with a sampling period h; both the controller and actuator are event-driven.

(2) The total network-induced time-delay $\tau_k = \tau_k^{sc} + \tau_k^{ca}$ is time varying and nondeterministic, which satisfies $0 \le \tau_k \le h$.

Thus, the discretized equation of plant can be described as [27]

$$\begin{aligned} x(k+1) &= Gx(k) + H_0(\tau_k)v(k) \\ &\quad + H_1(\tau_k)v(k-1) + H_w w(k), \\ y(k) &= Cx(k), \end{aligned} \tag{2}$$

where $G = e^{Ah}$, $H_0(\tau_k) = \int_0^{h-\tau_k} e^{At}dtB_1$, $H_1(\tau_k) = \int_{h-\tau_k}^{h} e^{At}dt B_1$, and $H_w = \int_0^h e^{At}dtB_2$. By mathematical transformation, $H_0(\tau_k)$, $H_1(\tau_k)$ can be expressed as

$$\begin{aligned} H_0(\tau_k) &= H_0 + DF(\tau_k')E, \\ H_1(\tau_k) &= H_1 - DF(\tau_k')E, \end{aligned} \tag{3}$$

where

$$\begin{aligned} &H_0 = \int_0^{h/2} e^{As}ds \cdot B, \qquad H_1 = \int_{h/2}^{h} e^{As}ds \cdot B, \\ &\delta = \max_{\tau_k'} \left\| \int_0^{-\tau_k'} e^{As}ds \right\|_2 = \left\| \int_0^{h/2} e^{As}ds \right\|_2, \\ &D = \delta e^{A(h/2)}, \qquad E = B, \qquad \tau_k' \in \left[-\frac{h}{2}, \frac{h}{2}\right], \\ &F^T(\tau_k')F(\tau_k') \le I. \end{aligned} \tag{4}$$

Remark 1. Limited by space, the detailed discretization process for system (1) with uncertain short time-delay is omitted in this paper and can be found in [27]. From (2) and (3), we know that the continuous plant with uncertain short time-delay in NCSs can be modeled as a discrete linear system with parameter uncertainties.

2.1. Description of Deadband Schedulers. The signal will be transmitted only when the difference between the current signal and the previous transmission signal is greater than the error threshold. According to this, the deadband schedulers designed in this paper are the error threshold-based deterministic schedulers. The working mechanism of the deadband scheduler 1(DS1) in the sensor can be described as

$$\overline{x}_i(k) = \begin{cases} x_i(k), & |\overline{x}_i(k-1) - x_i(k)| > \delta_i |x_i(k)|, \\ \overline{x}_i(k-1), & |\overline{x}_i(k-1) - x_i(k)| \le \delta_i |x_i(k)|, \end{cases} \tag{5}$$

where $i = 1, 2, \ldots, n$; $\delta_i \in [0, 1]$, $\overline{x}_i(k)$, and $x_i(k)$ are given error threshold values, output signals, and input signals of DS1, respectively.

Set $\Lambda_1 = \operatorname{diag}\{\delta_1, \delta_2, \ldots, \delta_n\}$, $F_1(k) = \operatorname{diag}\{f_{11}(k), f_{12}(k), \ldots, f_{1n}(k)\}$, $f_{1i}(k) \in [-1, 1]$, $i = 1, 2, \ldots, n$; then from (5), the input-output relationship of DS1 can be converted to

$$\overline{x}(k) = x(k) + \Lambda_1 F_1(k)x(k), \tag{6}$$

where $F_1^T(k)F_1(k) \le I$.

Remark 2. In this paper, the effect of active packet dropouts under the deadband scheduling scheme is modeled as a bounded uncertain item of the transmission signal. The main advantages of this modeling method are as follows: (1) a non-linear relationship between the input and output of the deadband scheduler is converted to a linear relationship with uncertain parameters; (2) due to the bounded ranges of uncertain parameters related to the deadband threshold values, it is easier to merge scheduling policy parameters into the system model.

Similarly, the working mechanism of deadband scheduler 2(DS2) in the controller can be described as:

$$v_j(k) = \begin{cases} u_j(k), & |v_j(k-1) - u_j(k)| > \sigma_j |u_j(k)|, \\ v_j(k-1), & |v_j(k-1) - u_j(k)| \le \sigma_j |u_j(k)|, \end{cases} \tag{7}$$

where $j = 1, 2, \ldots, p$; $\sigma_j \in [0, 1]$, $v_j(k)$ and $u_j(k)$ are given error threshold values, output signals and input signals of DS2, respectively.

Set $\Lambda_2 = \operatorname{diag}\{\sigma_1, \sigma_2, \ldots \sigma_p\}$, $F_2(k) = \operatorname{diag}\{f_{21}(k), f_{22}(k), \ldots, f_{2p}(k)\}$, $f_{2j}(k) \in [-1, 1]$, $j = 1, 2, \ldots, p$, then from (7) the input-output relationship of DS2 can be converted to:

$$v(k) = u(k) + \Lambda_2 F_2(k)u(k), \tag{8}$$

where $F_2^T(k)F_2(k) \le I$.

From the above, we know that after the introduction of deadband schedulers into NCSs, the signals do not need to be transmitted at each sampling period so as to achieve the purpose of reducing data transmission rate and the effect of bandwidth constraints on the system. In addition, the principles of the considered schedulers are simple, which do not require a lot of computing and data storage.

2.2. The Model of Closed-Loop System. Employ a memoryless state feedback controller

$$u(k) = K\overline{x}(k), \tag{9}$$

where $\overline{x}(k) \in \mathbb{R}^n$, $u(k) \in \mathbb{R}^p$, and K is the state feedback gain with appropriate dimensions.

Selecting the augmented vector $z(k) = \left[x^T(k) \;\; v^T(k-1)\right]^T$ and synthesizing (2), (3), (6), (8), and (9), the closed-loop system can be described as

$$\begin{aligned} z(k+1) &= \Phi_k z(k) + \overline{H}_w w(k), \\ y(k) &= \overline{C}z(k), \end{aligned} \tag{10}$$

where

$$\Phi_k = \begin{bmatrix} G + \left(H_0 + DF\left(\tau_k'\right)E\right)\left(I + \Lambda_2 F_2(k)\right)K\left(I + \Lambda_1 F_1(k)\right) & H_1 - DF\left(\tau_k'\right)E \\ \left(I + \Lambda_2 F_2(k)\right)K\left(I + \Lambda_1 F_1(k)\right) & 0 \end{bmatrix}$$
$$= \overline{G} + \overline{H}K_1\overline{I} + \overline{D}F\left(\tau_k'\right)EK_1\overline{I} + \overline{D}F\left(\tau_k'\right)\overline{E}, \quad (11)$$

where

$$\overline{G} = \begin{bmatrix} G & H_1 \\ 0 & 0 \end{bmatrix}, \qquad \overline{C} = \begin{bmatrix} C & 0 \end{bmatrix},$$
$$\overline{H} = \begin{bmatrix} H_0 \\ I \end{bmatrix}, \qquad \overline{D} = \begin{bmatrix} D \\ 0 \end{bmatrix},$$
$$\overline{I} = \begin{bmatrix} I & 0 \end{bmatrix}, \qquad \overline{E} = \begin{bmatrix} 0 & -E \end{bmatrix},$$
$$\overline{H}_w = \begin{bmatrix} H_w \\ 0 \end{bmatrix}, \qquad K_1 = \left(I + \Lambda_2 F_2(k)\right)K\left(I + \Lambda_1 F_1(k)\right). \quad (12)$$

3. Design of H_∞ Controller

In this section, we will investigate H_∞ control problem for the closed-loop system (10). Throughout this paper, we will use the following lemmas.

3.1. Related Lemmas

Lemma 3 (see [28]). *For the given matrices A, $Q = Q^T$, and $P = P^T > 0$, $A^TPA + Q < 0$ hold if and only if*

$$\begin{bmatrix} Q & A^T \\ A & -P^{-1} \end{bmatrix} < 0, \qquad \begin{bmatrix} -P^{-1} & A \\ A^T & Q \end{bmatrix} < 0. \quad (13)$$

Lemma 4 (see [28]). *Let W, M, N, $F(k)$ be real matrices of appropriate dimensions with $F^T(k)F(k) \le I$ and $W = W^T$; then*

$$W + MF(k)N + N^TF^T(k)M^T < 0 \quad (14)$$

holds if and only if there exists a real scalar $\varepsilon > 0$, satisfying

$$W + \varepsilon MM^T + \varepsilon^{-1}N^TN < 0. \quad (15)$$

More especially, when $F(k)$ is a diagonal matrix, there also exists the following lemma.

Lemma 5 (see [29]). *Let W, M, N be real matrices of appropriate dimensions, let $F(k)$ be a diagonal matrix with $F^T(k)F(k) \le I$ and $W = W^T$; then the following two conditions are equivalent:*

(1) $W + MF(k)N + N^TF^T(k)M^T < 0$

(2) *there exists a matrix $S = S^T > 0$, satisfying $W + MSM^T + N^TS^{-1}N < 0$.*

Remark 6. Due to the introduction of a symmetric positive-definite matrix instead of a scalar in Lemma 5, problem solving is expected to have a less conservatism compared with Lemma 4.

We are now in a position to formulate H_∞ control problem for NCSs with both deadband scheduling and uncertain short time-delay. More specifically, given a disturbance attenuation level γ, we design a state feedback controller of the form (9) such that the closed-loop system (10) with $w(k) = 0$ is asymptotically stable and under zero initial condition; the output $y(k)$ satisfies $\|y\|_2 \le \gamma\|w\|_2$ for all nonzero $w(k) \in l_2[0,\infty)$.

3.2. Main Results. Based on Lyapunov functional method and H_∞ theory [28], the following conclusions can be obtained.

Theorem 7. *For a given scalar $\gamma > 0$, under the given deadband scheduling schemes (5) and (7), the closed-loop system (10) is asymptotically stable with H_∞ performance γ if there exist symmetric positive-definite matrices P, W_1, W_2, feedback gain matrix K, and scalar $\varepsilon_1 > 0$ such that*

$$\begin{bmatrix} \Pi_{11} & \overline{G} + \overline{H}K\overline{I} & \overline{H}_w & \Pi_{14} & \overline{H}K\Lambda_1 & 0 \\ * & \Pi_{22} & 0 & \left(EK\overline{I} + \overline{E}\right)^T & 0 & \left(K\overline{I}\right)^T \\ * & * & -\gamma^2 I & 0 & 0 & 0 \\ * & * & * & \Pi_{44} & EK\Lambda_1 & 0 \\ * & * & * & * & -W_1 & \left(K\Lambda_1\right)^T \\ * & * & * & * & * & -W_2 \end{bmatrix} < 0, \quad (16)$$

where

$$\Pi_{11} = -P^{-1} + \varepsilon_1\overline{D}\,\overline{D}^T + \left(\overline{H}\Lambda_2\right)W_2\left(\overline{H}\Lambda_2\right)^T,$$
$$\Pi_{14} = \left(\overline{H}\Lambda_2\right)W_2\left(E\Lambda_2\right)^T,$$
$$\Pi_{22} = -P + \overline{C}^T\overline{C} + \overline{I}^TW_1\overline{I},$$
$$\Pi_{44} = -\varepsilon_1 I + \left(E\Lambda_2\right)W_2\left(E\Lambda_2\right)^T. \quad (17)$$

Proof. (i) We first show that system (10) with $w(k) = 0$ is asymptotically stable. To the end, defining a Lyapunov functional as $V(k) = z^T(k)Pz(k)$, we have that

$$\begin{aligned} \Delta V(k) &= V(k+1) - V(k) \\ &= z^T(k+1)Pz(k+1) - z^T(k)Pz(k) \\ &= z^T(k)\Phi_k^TP\Phi_k z(k) - z^T(k)Pz(k) \\ &= z^T(k)\left(\Phi_k^TP\Phi_k - P\right)z(k). \end{aligned} \quad (18)$$

Obviously, if $\Phi_k^T P \Phi_k - P < 0$, then $\Delta V(k) < 0$, and the closed-loop system (10) is asymptotically stable.

(ii) Next, we prove that system (10) has H_∞ performance γ. Define

$$J = \sum_{k=0}^{\infty} \left[y^T(k) y(k) - \gamma^2 w^T(k) w(k) \right] = \sum_{k=0}^{\infty} \left[y^T(k) y(k) - \gamma^2 w^T(k) w(k) + \Delta V(k) \right] - V(\infty) + V(0). \quad (19)$$

Under zero initial conditions, we have that $V(0) = 0$, but $V(\infty) \geq 0$; therefore,

$$J \leq \sum_{k=0}^{\infty} \left[y^T(k) y(k) - \gamma^2 w^T(k) w(k) + \Delta V(k) \right] = \sum_{k=0}^{\infty} \left[z^T(k) \;\; w^T(k) \right] \Pi \begin{bmatrix} z(k) \\ w(k) \end{bmatrix}, \quad (20)$$

where

$$\Pi = \begin{bmatrix} \Phi_k^T P \Phi_k + \overline{C}^T \overline{C} - P & \Phi_k^T P \overline{H}_w \\ * & \overline{H}_w^T P \overline{H}_w - \gamma^2 I \end{bmatrix}. \quad (21)$$

If $\Pi < 0$, then $\Phi_k^T P \Phi_k + \overline{C}^T \overline{C} - P < 0$; therefore, $\Phi_k^T P \Phi_k - P < -\overline{C}^T \overline{C} < 0$; the condition in (i) holds, and system (10) with $w(k) = 0$ is asymptotically stable. In addition, If $\Pi < 0$, then $J < 0$; therefore, $\|y\|_2^2 = \sum_{k=0}^{\infty} y^T(k) y(k) < \gamma^2 \sum_{k=0}^{\infty} w^T(k) w(k) = \gamma^2 \|w\|_2^2$.

According to $\Pi < 0$, substituting $\Phi_k = \overline{G} + \overline{H} K_1 \overline{I} + \overline{D} F(\tau_k') E K_1 \overline{I} + \overline{D} F(\tau_k') \overline{E}$ into (21) and applying Lemma 3, we have that

$$\begin{bmatrix} -P^{-1} & \overline{G} + \overline{H} K_1 \overline{I} & \overline{H}_w \\ * & -P + \overline{C}^T \overline{C} & 0 \\ * & * & -\gamma^2 I \end{bmatrix} + \begin{bmatrix} \overline{D} \\ 0 \\ 0 \end{bmatrix} F(\tau_k') \left[0 \;\; E K_1 \overline{I} + \overline{E} \;\; 0 \right] + \left[0 \;\; E K_1 \overline{I} + \overline{E} \;\; 0 \right]^T F^T(\tau_k') \begin{bmatrix} \overline{D} \\ 0 \\ 0 \end{bmatrix}^T < 0. \quad (22)$$

Due to $F^T(\tau_k') F(\tau_k') \leq I$ and by the use of Lemma 4, we can get that

$$\begin{bmatrix} -P^{-1} & \overline{G} + \overline{H} K_1 \overline{I} & \overline{H}_w \\ * & -P + \overline{C}^T \overline{C} & 0 \\ * & * & -\gamma^2 I \end{bmatrix} + \varepsilon_1 \begin{bmatrix} \overline{D} \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} \overline{D} \\ 0 \\ 0 \end{bmatrix}^T + \varepsilon_1^{-1} \left[0 \;\; E K_1 \overline{I} + \overline{E} \;\; 0 \right]^T \left[0 \;\; E K_1 \overline{I} + \overline{E} \;\; 0 \right] < 0. \quad (23)$$

According to Lemma 3, (23) is equivalent to

$$\begin{bmatrix} -P^{-1} + \varepsilon_1 \overline{D}\,\overline{D}^T & \overline{G} + \overline{H} K_1 \overline{I} & \overline{H}_w & 0 \\ * & -P + \overline{C}^T \overline{C} & 0 & \left(E K_1 \overline{I} + \overline{E} \right)^T \\ * & * & -\gamma^2 I & 0 \\ * & * & * & -\varepsilon_1 I \end{bmatrix} < 0. \quad (24)$$

Substituting $K_1 = (I + \Lambda_2 F_2(k)) K (I + \Lambda_1 F_1(k))$ into (24), we have that

$$\begin{bmatrix} -P^{-1} + \varepsilon_1 \overline{D}\,\overline{D}^T & \overline{G} + \overline{H} (I + \Lambda_2 F_2(k)) K \overline{I} & \overline{H}_w & 0 \\ * & -P + \overline{C}^T \overline{C} & 0 & \left(E (I + \Lambda_2 F_2(k)) K \overline{I} + \overline{E} \right)^T \\ * & * & -\gamma^2 I & 0 \\ * & * & * & -\varepsilon_1 I \end{bmatrix}$$
$$+ \begin{bmatrix} \overline{H} (I + \Lambda_2 F_2(k)) K \Lambda_1 \\ 0 \\ 0 \\ E (I + \Lambda_2 F_2(k)) K \Lambda_1 \end{bmatrix} F_1(k) \left[0 \;\; \overline{I} \;\; 0 \;\; 0 \right] + \left[0 \;\; \overline{I} \;\; 0 \;\; 0 \right]^T F_1^T(k) \begin{bmatrix} \overline{H} (I + \Lambda_2 F_2(k)) K \Lambda_1 \\ 0 \\ 0 \\ E (I + \Lambda_2 F_2(k)) K \Lambda_1 \end{bmatrix}^T < 0. \quad (25)$$

Considering that $F_1(k)$ is a diagonal matrix with $F_1^T(k) F_1(k) \leq I$, and so employing Lemma 5, we can get that

$$\begin{bmatrix} -P^{-1}+\varepsilon_1\overline{D}\,\overline{D}^T & \overline{G}+\overline{H}\left(I+\Lambda_2F_2(k)\right)K\overline{I} & \overline{H}_w & 0 \\ * & -P+\overline{C}^T\overline{C} & 0 & \left(E\left(I+\Lambda_2F_2(k)\right)K\overline{I}+\overline{E}\right)^T \\ * & * & -\gamma^2 I & 0 \\ * & * & * & -\varepsilon_1 I \end{bmatrix} + \begin{bmatrix} \overline{H}\left(I+\Lambda_2F_2(k)\right)K\Lambda_1 \\ 0 \\ 0 \\ E\left(I+\Lambda_2F_2(k)\right)K\Lambda_1 \end{bmatrix} W_1^{-1} \begin{bmatrix} \overline{H}\left(I+\Lambda_2F_2(k)\right)K\Lambda_1 \\ 0 \\ 0 \\ E\left(I+\Lambda_2F_2(k)\right)K\Lambda_1 \end{bmatrix}^T + \begin{bmatrix} 0 & \overline{I} & 0 & 0 \end{bmatrix}^T W_1 \begin{bmatrix} 0 & \overline{I} & 0 & 0 \end{bmatrix} < 0. \tag{26}$$

According to Lemma 3, (26) is equivalent to

$$\begin{bmatrix} -P^{-1}+\varepsilon_1\overline{D}\,\overline{D}^T & \overline{G}+\overline{H}\left(I+\Lambda_2F_2(k)\right)K\overline{I} & \overline{H}_w & 0 & \overline{H}\left(I+\Lambda_2F_2(k)\right)K\Lambda_1 \\ * & -P+\overline{C}^T\overline{C}+\overline{I}^TW_1\overline{I}_1 & 0 & \left(E\left(I+\Lambda_2F_2(k)\right)K\overline{I}+\overline{E}\right)^T & 0 \\ * & * & -\gamma^2 I & 0 & 0 \\ * & * & * & -\varepsilon_1 I & E\left(I+\Lambda_2F_2(k)\right)K\Lambda_1 \\ * & * & * & * & -W_1 \end{bmatrix} < 0. \tag{27}$$

Similarly, we can eliminate $F_2(k)$. By use of Lemma 5, we have that

$$\begin{bmatrix} -P^{-1}+\varepsilon_1\overline{D}\,\overline{D}^T & \overline{G}+\overline{H}K\overline{I} & \overline{H}_w & 0 & \overline{H}K\Lambda_1 \\ * & -P+\overline{C}^T\overline{C}+\overline{I}^TW_1\overline{I}_1 & 0 & \left(EK\overline{I}+\overline{E}\right)^T & 0 \\ * & * & -\gamma^2 I & 0 & 0 \\ * & * & * & -\varepsilon_1 I & EK\Lambda_1 \\ * & * & * & * & -W_1 \end{bmatrix} + \begin{bmatrix} \overline{H}\Lambda_2 \\ 0 \\ 0 \\ E\Lambda_2 \\ 0 \end{bmatrix} W_2 \begin{bmatrix} \overline{H}\Lambda_2 \\ 0 \\ 0 \\ E\Lambda_2 \\ 0 \end{bmatrix}^T + \begin{bmatrix} 0 & K\overline{I} & 0 & 0 & K\Lambda_1 \end{bmatrix}^T W_2^{-1} \begin{bmatrix} 0 & K\overline{I} & 0 & 0 & K\Lambda_1 \end{bmatrix} < 0. \tag{28}$$

Furthermore, applying Lemma 3, (28) can be converted to (16).

The proof is completed. □

Remark 8. Notice that the matrix inequality (16) in Theorem 7 is a bilinear matrix inequality due to the existence of P^{-1}. Generally, it can be solved by the linear approach [30] or the cone complementarity linearization (CCL) method [31]. By contrast, the CCL result is less conservative [32] and so is employed in this paper.

Corollary 9. *The bilinear matrix inequality* (16) *can be transformed to the following objective function minimization problems by the CCL method.*

Find

$$\begin{aligned}
&P > 0, \quad X > 0, \quad W_1 > 0, \\
&W_2 > 0, \quad \varepsilon_1 > 0, K \\
\min \quad & \operatorname{Trace}(PX) \\
\text{s.t.} \quad & \begin{bmatrix} \widehat{\Pi}_{11} & \overline{G} + \overline{H}K\overline{I} & \overline{H}_w & \Pi_{14} & \overline{H}K\Lambda_1 & 0 \\ * & \Pi_{22} & 0 & \left(EK\overline{I} + \overline{E}\right)^T & 0 & \left(K\overline{I}\right)^T \\ * & * & -\gamma^2 I & 0 & 0 & 0 \\ * & * & * & \Pi_{44} & EK\Lambda_1 & 0 \\ * & * & * & * & -W_1 & \left(K\Lambda_1\right)^T \\ * & * & * & * & * & -W_2 \end{bmatrix} < 0, \\
& \begin{bmatrix} P & I \\ I & X \end{bmatrix} \geq 0,
\end{aligned} \tag{29}$$

where $\widehat{\Pi}_{11} = -X + \varepsilon_1 \overline{D}\,\overline{D}^T + (\overline{H}\Lambda_2)W_2(\overline{H}\Lambda_2)^T$.

Since Corollary 9 has turned the nonconvex feasibility problem of Theorem 7 into a minimization problem of nonlinear objective function with linear matrix inequalities constraints, it can be solved by the following iterative algorithm.

Algorithm 10.

Step 1. Find a set of feasible solutions $\Xi_0 = \{P_0, X_0, W_{10}, W_{20}, \varepsilon_{10}, K_0\}$, which satisfyies (29), and set the iterative number $l = 0$.

Step 2. Use LMI toolbox of mincx solver to solve the following linear objective function minimization problem:

$$\begin{aligned} \min \quad & \operatorname{Trace}\left(P_l X + P X_l\right) \\ \text{s.t.} \quad & (29). \end{aligned} \tag{30}$$

The solutions are set $\Xi^* = \{P^*, X^*, W_1^*, W_2^*, \varepsilon_1^*, K^*\}$.

Step 3. Substituting the solutions Ξ^* into the matrix inequality (16) in Theorem 7 to check if (16) is satisfied, then K^* becomes the state feedback gain matrix and the iteration terminates. Otherwise, enter into Step 4.

Step 4. If the iterative number satisfies $l \leq L$ (L is a predetermined iterative number upper bound), set $\Xi_{l+1} = \Xi^*$, $l = l + 1$, and return to Step 2 for the next iteration. Otherwise, enter into Step 1 and reselect a set of feasible solutions Ξ_0 to calculate.

Thus, a state feedback H_∞ controller can be obtained for NCSs with both deadband scheduling and uncertain short time-delay. More especially, if there are no deadband schedulers in the NCSs shown in Figure 1, the closed-loop system in (10) reads

$$\begin{aligned} z(k+1) &= \Phi_k z(k) + \overline{H}_w w(k), \\ y(k) &= \overline{C} z(k), \end{aligned} \tag{31}$$

where

$$\begin{aligned} \Phi_k &= \begin{bmatrix} G + \left(H_0 + DF\left(\tau_k'\right)E\right)K & H_1 - DF\left(\tau_k'\right)E \\ K & 0 \end{bmatrix} \\ &= \overline{G} + \overline{H}K\overline{I} + \overline{D}F\left(\tau_k'\right)EK\overline{I} + \overline{D}F\left(\tau_k'\right)\overline{E}. \end{aligned} \tag{32}$$

Then, we have the following corollary, which can be proved along similar lines as in the proof of Theorem 7.

Corollary 11. *Consider the NCSs in Figure 1, but without the deadband schedulers. For a given scalar $\gamma > 0$, the closed-loop system* (31) *is asymptotically stable with H_∞ performance γ if there exist symmetric positive-definite matrix P, feedback gain matrix K, and scalar $\varepsilon_1 > 0$ such that*

$$\begin{bmatrix} -P^{-1} + \varepsilon_1 \overline{D}\,\overline{D}^T & \overline{G} + \overline{H}K\overline{I} & \overline{H}_w & 0 \\ * & -P + \overline{C}^T\overline{C} & 0 & \left(EK\overline{I} + \overline{E}\right)^T \\ * & * & -\gamma^2 I & 0 \\ * & * & * & -\varepsilon_1 I \end{bmatrix} < 0. \tag{33}$$

Similarly, the bilinear matrix inequality (33) *can be solved by the above CCL method and the iterative algorithm in Corollary 9 and is thus omitted.*

4. Numerical Example

In this section, a numerical example is introduced to demonstrate the effectiveness of the proposed method. Consider a ball and beam system with [33]

$$\begin{aligned} \dot{x}(t) &= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} w(t), \\ y(t) &= \begin{bmatrix} 1 & 0 \end{bmatrix} x(t). \end{aligned} \tag{34}$$

Table 1: The feedback gain matrix K values for various Λ_1, Λ_2.

Case	Λ_1	Λ_2	K
1	Without scheduling	Without scheduling	$K = [-0.5462 \quad -1.0978]$
2	$[0.02 \quad 0; 0 \quad 0.02]$	0.02	$K = [-0.4942 \quad -1.0314]$
3	$[0.05 \quad 0; 0 \quad 0.05]$	0.05	$K = [-0.4393 \quad -0.9733]$
4	$[0.07 \quad 0; 0 \quad 0.07]$	0.05	$K = [-0.4185 \quad -0.9605]$
5	$[0.1 \quad 0; 0 \quad 0.1]$	0.07	No solution

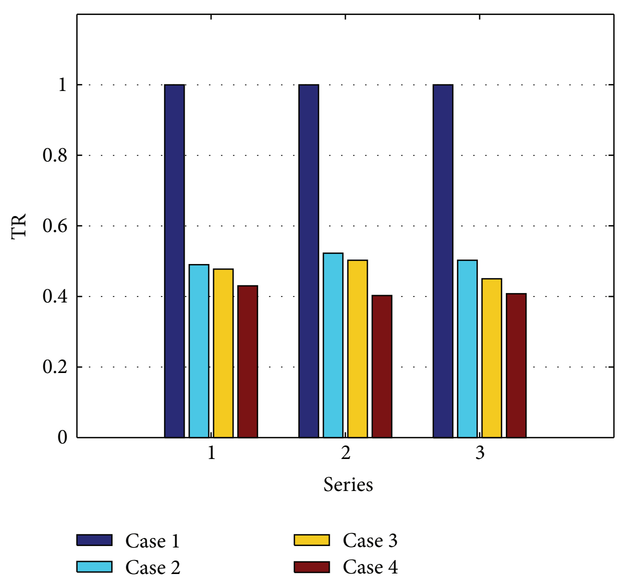

Figure 2: The performance of MTR.

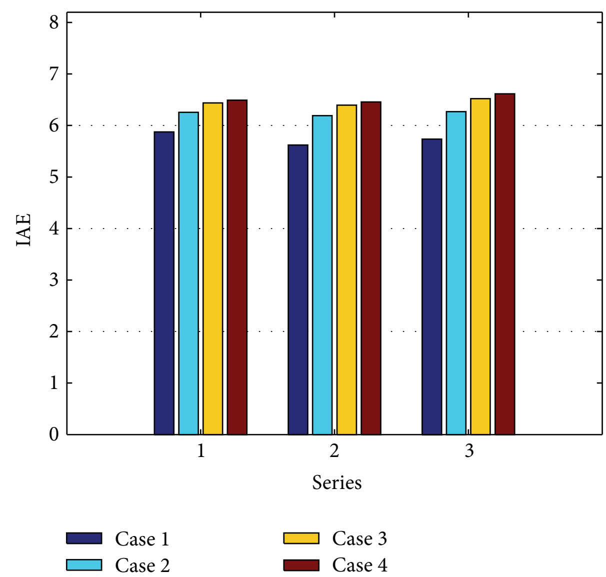

Figure 3: The performance of IAE.

In this example, we choose $h = 0.5s$, and $\tau_k \in [0, h]$ is time varying and nondeterministic. According to (2) and (3), we have that

$$G = \begin{bmatrix} 1 & 0.5 \\ 0 & 1 \end{bmatrix}, \qquad H_0 = \begin{bmatrix} 0.0313 \\ 0.25 \end{bmatrix},$$
$$H_1 = \begin{bmatrix} 0.0938 \\ 0.25 \end{bmatrix}, \qquad D = \begin{bmatrix} 0.2661 & 0.0665 \\ 0 & 0.2661 \end{bmatrix}, \tag{35}$$
$$E = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \qquad H_w = \begin{bmatrix} 0.125 \\ 0.5 \end{bmatrix}.$$

For a given disturbance attenuation level $\gamma = 5$, based on the LMI toolbox, and applying Corollaries 9 and 11, the feedback gain matrix K values are given with different error threshold values Λ_1, Λ_2 in Table 1. It is obvious from Table 1 that for a given level γ we can find the feasible feedback gain matrix K values when Λ_1, Λ_2 are within certain ranges.

In addition, we choose the initial value $x_0 = \begin{bmatrix} 2 & -0.5 \end{bmatrix}^T$, the disturbance

$$w(k) = \begin{cases} 0.1, & 50 \le k \le 60, \\ 0, & \text{other}, \end{cases} \tag{36}$$

and the total runtime 100 seconds. Define MTR $= n_{\text{sent}}/n_{\text{total}}$ and IAE $= \sum \|e(k)\|_2 \cdot h$, in which MTR denotes the mean data transmission rate (n_{sent} and n_{total} denote the number of data transmitted with and without deadband schedulers in the runtime, respectively.) and IAE denotes the control performance of the system. Under three different random time-delay sequences, the performance of MTR and IAE is shown for the system with the above four error threshold values in Figures 2 and 3, respectively. It can be easily seen that compared with the system without deadband schedulers (case 1), although the control performance of the system by using deadband scheduling scheme (case 2–case 4) is slightly worse (in Figure 3), the mean data transmission rate of the system is greatly reduced (in Figure 2).

Figures 4–6 show the simulation results for the system in which the error threshold values take $\Lambda_1 = \text{diag}\{0.07, 0.07\}$, $\Lambda_2 = 0.05$, the feedback gain matrix $K = [-0.4185 \quad -0.9605]$ according to Table 1, and the time-delay takes the first series. It can be seen that the closed-loop system is asymptotically stable (in Figure 4), and only part of the sampled data and control signal are transmitted with the proposed deadband scheduling scheme (in Figures 5 and 6, here the transmission interval of $\overline{x}_2(k)$ is similar to $\overline{x}_1(k)$ in DS1 and is thus omitted).

Furthermore, under zero initial conditions, we get that $\|y\|_2 = 0.7310$, $\|w\|_2 = 0.3317$, which yields $\gamma^* = 2.20$. It means that the practical disturbance attenuation level

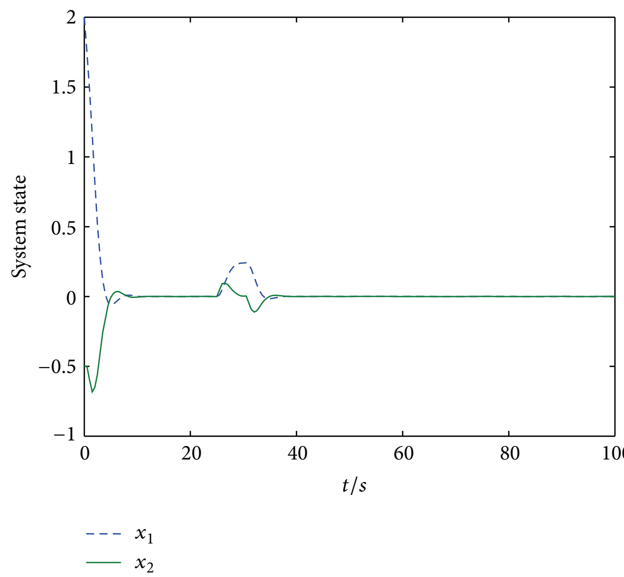

Figure 4: State response curves of the closed-loop system.

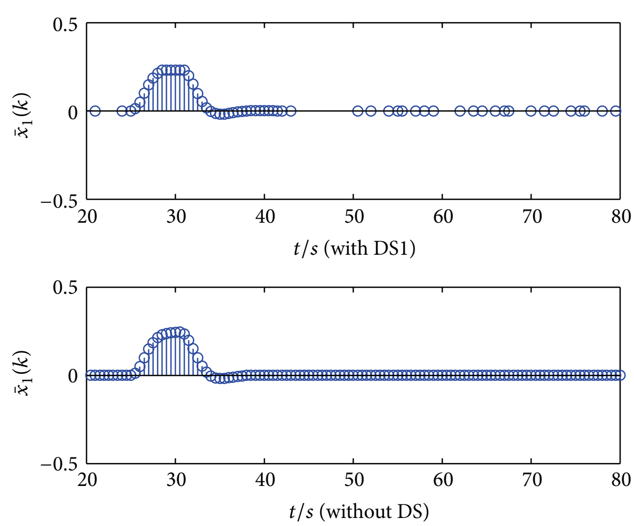

Figure 5: Comparisons of the transmission interval of $\bar{x}_1(k)$.

is smaller than the given level $\gamma = 5$, which shows the effectiveness of the proposed H_∞ controller design method.

5. Conclusions

In this paper, a discrete-time model for NCSs with both deadband scheduling and time-delay has been established and the H_∞ control problem has been investigated. Based on the LMI approach, a state feedback H_∞ controller has been designed to ensure the closed-loop system asymptotically to be stable with H_∞ performance index. A numerical example has been provided to show the validness of the derived results. Since the principles and algorithms of deadband schedulers in this paper are very simple, the smart sensor and controller are easy to implement. In addition, simulation results show

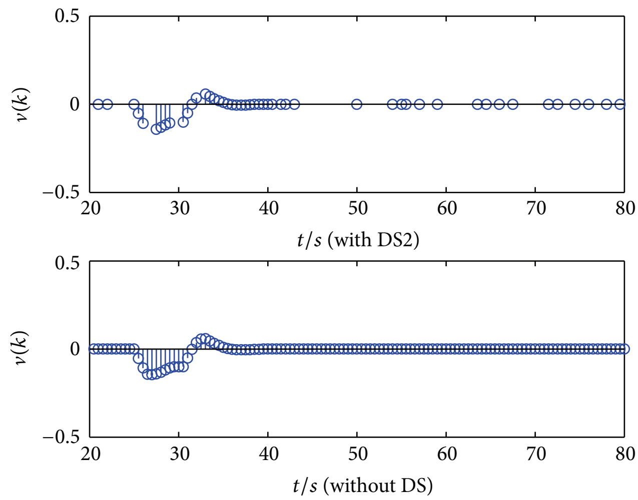

Figure 6: Comparisons of the transmission interval of $v(k)$.

that it can effectively reduce the node's data transmission rate, so it is very suitable for applying in the NCSs with limited bandwidth resources.

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