Hindawi Publishing Corporation
International Journal of Optics
Volume 2011, Article ID 629605, 9 pages
doi:10.1155/2011/629605

Research Article

Higher-Order Amplitude Squeezing in Six-Wave Mixing Process

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Received 28 April 2010; Accepted 11 April 2011

Academic Editor: A. Cartaxo

We investigate theoretically the generation of squeezed states in spontaneous and stimulated six-wave mixing process quantum mechanically. It has been found that squeezing occurs in field amplitude, amplitude-squared, amplitude-cubed, and fourth power of field amplitude of fundamental mode in the process. It is found to be dependent on coupling parameter "g" (characteristics of higher-order susceptibility tensor) and phase values of the field amplitude under short-time approximation. Six-wave mixing is a process which involves absorption of three pump photons and emission of two probe photons of the same frequency and a signal photon of different frequency. It is shown that squeezing is greater in a stimulated interaction than the corresponding squeezing in spontaneous process. The degree of squeezing depends upon the photon number in first and higher orders of field amplitude. We study the statistical behaviour of quantum field in the fundamental mode and found it to be sub-Poissonian in nature. The signal-to-noise ratio has been studied in different orders. It is found that signal-to-noise ratio is higher in lower orders. This study when supplemented with experimental observations offers possibility of improving performance of many optical devices and optical communication networks.

1. Introduction

Over the past three decades, particular attention has been focused on theoretical investigations and experimental observations in generation of squeezed light, for improving the performance of many optical devices and optical communication networks. The concept of squeezed light is concerned with reduction of quantum fluctuations in one of the quadrature, at the expense of increased fluctuations in the other quadrature. In general, the two important nonclassical effects, squeezing and antibunching (or Sub-Poissonian photon statistics), are not interrelated; that is, some states exist that exhibit the first but not the second and vice versa. However, squeezing can be detected using simple photon counting in higher-order sub-Poissonian statistics.

A lot of work has appeared in the literature on the theoretical and experimental investigations on generation of squeezed states of electromagnetic field. Mandel [1] found squeezed state of the second harmonic when a beam of light propagates through a nonlinear crystal. Later, Hillery [2] defined amplitude-squared squeezing and showed that amplitude-squared squeezed states can be of use in reducing noise in the output of certain nonlinear optical devices. Hong and Mandel [3, 4] introduced the notion of Nth-order squeezing as a generalization of the second-order squeezing. Zhan [5] proposed the generation of amplitude-cubed squeezing in the fundamental mode in second and third harmonic generation. Jawahar and Jaiswal [6] extended the results obtained by Zhan for amplitude-cubed squeezing in the fundamental mode during second and third harmonic generations to kth order. The significant experimental observations include gravity wave detection [7–10], in optical communication [11], in nanodisplacement measurement [12], and in optical storage [13], and interferometer enhancement [14, 15]. The experimental detections and applications confirm the importance of the theoretical investigations into various optical processes such as four- and six-wave mixing [16–20], eight-wave mixing [21], higher-order harmonic generation [22–25], parametric amplification [26], Raman [27] and hyper-Raman processes [28], and so forth. Higher-order sub-Poissonian statistics have been studied by a number of authors such as those in

[29–31]. The conversion of higher-order squeezed light into nonclassical light with high sub-Poissonian statistics and its experimental detection has been discussed in [31–33].

Recently, Giri and Gupta [19] have investigated the squeezing effects in six-wave mixing process. In this paper, we propose a different model for the same interaction process. Also, this paper shows one of the distinguished examples of nonlinear processes when light exhibits both squeezing and sub-Poissonian photon statistics at the same time. Squeezing in field amplitude, amplitude-squared, amplitude-cubed, and in fourth-order amplitude has been studied in fundamental mode for the proposed model. The photon statistics and dependence of squeezing on photon number have also been investigated.

2. Definition of Squeezing and Higher-Order Squeezing

Squeezing is a purely quantum mechanical phenomenon which cannot be explained on the basis of classical physics. The coherent states do not exhibit nonclassical effects, but a superposition of coherent states can exhibit normal squeezing, higher-order squeezing, and sub-Poissonian photon statistics. A coherent state changes to a superposition of coherent states when it interacts with a non linear medium. Squeezed states of an electromagnetic field are the states with reduced noise below the vacuum limit in one of the canonical conjugate quadratures. Normal squeezing is defined in terms of the operators

$$X_1 = \frac{1}{2}\left(A + A^{\dagger}\right), \qquad X_2 = \frac{1}{2i}\left(A - A^{\dagger}\right), \tag{1}$$

where X_1 and X_2 are the real and imaginary parts of the field amplitude, respectively. A and $A^{\dagger}$ are slowly varying operators defined by

$$A = ae^{i\omega t}, \qquad A^{\dagger} = a^{\dagger}e^{-i\omega t}. \tag{2}$$

The operators X_1 and X_2 obey the commutation relation

$$[X_1, X_2] = \frac{i}{2} \tag{3}$$

which leads to the uncertainty relation ($\hbar = 1$)

$$\Delta X_1 \Delta X_2 \geq \frac{1}{4}. \tag{4}$$

A quantum state is squeezed in X_i variable if

$$\Delta X_i < \frac{1}{2} \quad \text{for } i = 1 \text{ or } 2. \tag{5}$$

Amplitude-squared squeezing is defined in terms of operators Y_1 and Y_2 as

$$Y_1 = \frac{1}{2}\left[A^2 + A^{\dagger 2}\right], \qquad Y_2 = \frac{1}{2i}\left[A^2 - A^{\dagger 2}\right]. \tag{6}$$

The operators Y_1 and Y_2 obey the commutation relation $[Y_1, Y_2] = i(2N + 1)$, where N is the usual number operator which leads to the uncertainty relation

$$\Delta Y_1 \Delta Y_2 \geq \left\langle \left(N + \frac{1}{2}\right) \right\rangle. \tag{7}$$

Amplitude-squared squeezing is said to exist in Y_i variable if

$$(\Delta Y_i)^2 < \left\langle \left(N + \frac{1}{2}\right) \right\rangle \quad \text{for } i = 1 \text{ or } 2. \tag{8}$$

Amplitude-cubed squeezing is defined in terms of the operators

$$Z_1 = \frac{1}{2}\left(A^3 + A^{\dagger^3}\right), \qquad Z_2 = \frac{1}{2i}\left(A^3 - A^{\dagger^3}\right). \tag{9}$$

The operators Z_1 and Z_2 obey the commutation relation

$$[Z_1, Z_2] = \frac{i}{2}\left(9N^2 + 9N + 6\right). \tag{10}$$

Relation (10) leads to the uncertainty relation

$$\Delta Z_1 \Delta Z_2 \geq \frac{1}{4}\left(9N^2 + 9N + 6\right). \tag{11}$$

Amplitude-cubed squeezing exists when

$$(\Delta Z_i)^2 < \frac{1}{4}\left\langle \left(9N^2 + 9N + 6\right) \right\rangle \quad \text{for } i = 1 \text{ or } 2. \tag{12}$$

Real and imaginary parts of fourth-order amplitude are given as

$$F_1 = \frac{1}{2}\left(A^4 + A^{\dagger 4}\right), \qquad F_2 = \frac{1}{2i}\left(A^4 - A^{\dagger 4}\right). \tag{13}$$

The operators F_1 and F_2 obey the commutation relation

$$[F_1, F_2] = \frac{i}{2}\left(16N^3 + 24N^2 + 56N + 24\right) \tag{14}$$

and satisfy the uncertainty relation ($\hbar = 1$)

$$\Delta F_1 \Delta F_2 \geq \frac{1}{4}\left\langle \left(16N^3 + 24N^2 + 56N + 24\right) \right\rangle. \tag{15}$$

Fourth-order squeezing exists when

$$(\Delta F_i)^2 < \frac{1}{4}\left\langle \left(16N^3 + 24N^2 + 56N + 24\right) \right\rangle \quad \text{for } i = 1 \text{ or } 2. \tag{16}$$

3. Squeezing in Fundamental Mode in Six-Wave Mixing Process

The model considers the process involving absorption of three pump photons of frequency ω_1 each, going from state $|1\rangle$ to state $|2\rangle$ and emission of two probe photons from state $|2\rangle$ to state $|3\rangle$ with frequency ω_2 each. The atomic system returns to its original state by emitting one signal photon of frequency ω_3 from state $|3\rangle$ to $|1\rangle$. The process is shown in Figure 1.

The Hamiltonian for this process is as follows ($\hbar = 1$)

$$H = \omega_1 a^{\dagger} a + \omega_2 b^{\dagger} b + \omega_3 c^{\dagger} c + g\left(a^3 b^{\dagger 2} c^{\dagger} + a^{\dagger 3} b^2 c\right), \tag{17}$$

in which g is a coupling constant. $A = ae^{i\omega_1 t}$, $B = be^{i\omega_2 t}$, and $C = ce^{i\omega_3 t}$, respectively, are the slowly varying operators for the three modes at ω_1, ω_2, and ω_3. $a(a^{\dagger}), b(b^{\dagger}), c(c^{\dagger})$ are the

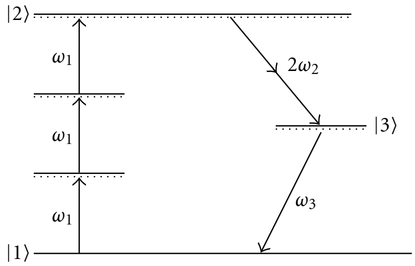


Figure 1: Six-wave interaction model.

usual annihilation (creation) operators associated with the relation $3\omega_1 = 2\omega_2 + \omega_3$.

The Heisenberg equation of motion for mode A is

$$\dot{A} = \frac{\partial A}{\partial t} + i[H, A]. \tag{18}$$

Using (17) in (18), we obtain

$$\dot{A} = -3igA^{\dagger 2}B^2C. \tag{19}$$

Similarly, we obtained the relations for $\dot{B}$ and $\dot{C}$ as

$$\begin{aligned} \dot{B} &= -2igA^3B^{\dagger}C^{\dagger}, \\ \dot{C} &= -igA^3B^{\dagger 2}. \end{aligned} \tag{20}$$

Expanding $A(t)$ using Taylor's series expansion by assuming the short-time interaction of waves with the medium and retaining the terms up to $|gt|^2$, we obtain

$$\begin{aligned} A(t) = A &- 3igtA^{\dagger 2}B^2C + \frac{3}{2}g^2t^2 \\ &\times \Big(6A^{\dagger}A^2B^{\dagger 2}B^2C^{\dagger}C + 6AB^{\dagger 2}B^2C^{\dagger}C \\ &\quad - 4A^{\dagger 2}A^3B^{\dagger}BC^{\dagger}C \\ &\quad - 2A^{\dagger 2}A^3C^{\dagger}C - A^{\dagger 2}A^3B^{\dagger 2}B^2C^{\dagger}C \\ &\quad - 4A^{\dagger 2}A^3B^{\dagger}BC^{\dagger}C - 2A^{\dagger 2}A^3\Big). \end{aligned} \tag{21}$$

The real quadrature component for squeezing of field amplitude in fundamental mode A is given as

$$X_{1A}(t) = \frac{1}{2}\Big[A(t) + A^{\dagger}(t)\Big]. \tag{22}$$

For spontaneous interaction, we consider the quantum state as a product of coherent state for the fundamental mode A and the vacuum state for the modes B and C, that is,

$$|\psi\rangle = |\alpha\rangle_A|0\rangle_B|0\rangle_C, \tag{23}$$

where α is the complex field amplitude of the fundamental mode. Using (21)–(23), we obtain the expectation values as

$$\begin{aligned} &\langle\psi|X_{1A}^2(t)|\psi\rangle \\ &= \frac{1}{4}\Big[\alpha^2 + \alpha^{*2} + 2|\alpha|^2 + 1 \\ &\qquad -6g^2t^2\Big(\alpha^2|\alpha|^4 + \alpha^2|\alpha|^2 + \alpha^{*2}|\alpha|^4 + \alpha^{*2}|\alpha|^2 + 2|\alpha|^6\Big)\Big], \end{aligned} \tag{24}$$

$$\begin{aligned} &\langle\psi|X_{1A}(t)|\psi\rangle^2 \\ &= \frac{1}{4}\Big[\alpha^2 + \alpha^{*2} + 2|\alpha|^2 - 6g^2t^2\Big(\alpha^2|\alpha|^4 + \alpha^{*2}|\alpha|^4 + 2|\alpha|^6\Big)\Big]. \end{aligned} \tag{25}$$

Therefore,

$$\begin{aligned} [\Delta X_{1A}(t)]^2 &= \langle X_{1A}^2(t)\rangle - \langle X_{1A}(t)\rangle^2 \\ &= \frac{1}{4}\Big[1 - 6g^2t^2\Big(\alpha^2|\alpha|^2 + \alpha^{*2}|\alpha|^2\Big)\Big], \end{aligned} \tag{26}$$

$$[\Delta X_{1A}(t)]^2 - \frac{1}{4} = -3g^2t^2|\alpha|^4\cos 2\theta, \tag{27}$$

where θ is the phase angle, with $\alpha = |\alpha|e^{i\theta}$ and $\alpha^* = |\alpha|e^{-i\theta}$.

The right-hand side of the expression (27) is negative, indicating that squeezing will occur in the first-order amplitude in the fundamental mode in six-wave mixing process for which $\cos 2\theta > 0$ for spontaneous interaction.

In parallel to the spontaneous interaction, the stimulated emission is caused due to the coupling of the atom to the other states of the field. Therefore, the study of squeezing in stimulated interaction in six-wave mixing process requires initial quantum state as a product of coherent states for modes 1, 2 and vacuum state for 3, that is,

$$|\psi\rangle = |\alpha\rangle_A|\beta\rangle_B|0\rangle_C. \tag{28}$$

Retaining the terms up to g^2t^2, we obtain

$$\begin{aligned} &\langle\psi|X_{1A}(t)|\psi\rangle^2 \\ &= \frac{1}{4}\Big[\alpha^2 + \alpha^{*2} + 2|\alpha|^2 - 3g^2t^2\Big(\alpha^2|\alpha|^4 + \alpha^{*2}|\alpha|^4 + 2|\alpha|^6\Big) \\ &\qquad \times\Big(|\beta|^4 + 4|\beta|^2 + 2\Big)\Big], \\ &\langle\psi|X_{1A}^2(t)|\psi\rangle \\ &= \frac{1}{4}\Big[\alpha^2 + \alpha^{*2} + 2|\alpha|^2 + 1 - 3g^2t^2 \\ &\qquad \times\Big(\alpha^2|\alpha|^4 + \alpha^2|\alpha|^2 + \alpha^{*2}|\alpha|^4 + \alpha^{*2}|\alpha|^2 + 2|\alpha|^6\Big) \\ &\qquad \times\Big(|\beta|^4 + 4|\beta|^2 + 2\Big)\Big]. \end{aligned} \tag{29}$$

Therefore,

$$[\Delta X_{1A}(t)]^2 - \frac{1}{4} = -\frac{3}{2}g^2t^2|\alpha|^4\Big(|\beta|^4 + 4|\beta|^2 + 2\Big)\cos 2\theta, \tag{30}$$

which is negative, indicating that squeezing will occur for those values of θ for which $\cos 2\theta > 0$, in the fundamental mode in stimulated interaction under short-time approximation. The effect of the stimulated interaction is represented by the factor $(|\beta|^4 + 4|\beta|^2 + 2)$.

Using (21) and (23), the second-order amplitude is expressed as

$$A^2(t) = A^2 - 6igt\left(A^{\dagger 2}A + A^{\dagger}\right)B^2C - 3g^2t^2 \times \left(A^{\dagger 2}A^4 + A^{\dagger}A^3\right)\left(B^{\dagger 2}B^2 + 4B^{\dagger}B + 2\right). \quad (31)$$

For second-order squeezing, the real quadrature component for the fundamental mode is expressed as

$$Y_{1A}(t) = \frac{1}{2}\left[A^2(t) + A^{\dagger 2}(t)\right]. \quad (32)$$

Using (23) and (31) in (32), we get the expectation values in spontaneous six-wave mixing process as

$$\begin{aligned}\langle \psi | Y_{1A}(t) | \psi \rangle^2 &= \frac{1}{4}\Big[\alpha^4 + \alpha^{*4} + 2|\alpha|^4 - 12g^2t^2 \\ &\quad \times \left(\alpha^4|\alpha|^4 + \alpha^4|\alpha|^2 + \alpha^{*4}|\alpha|^4 + \alpha^{*4}|\alpha|^2 + 2|\alpha|^8 + 2|\alpha|^6\right)\Big], \\ \langle \psi | Y_{1A}^2(t) | \psi \rangle &= \frac{1}{4}\Big[\alpha^4 + \alpha^{*4} + 2|\alpha|^4 + 4|\alpha|^2 + 2 - 12g^2t^2 \\ &\quad \times \left(\alpha^4|\alpha|^4 + 3\alpha^4|\alpha|^2 + 2\alpha^4 + \alpha^{*4}|\alpha|^4 + 3\alpha^{*4}|\alpha|^2 + 2\alpha^{*4} + 2|\alpha|^8 + 4|\alpha|^6\right)\Big].\end{aligned} \quad (33), (34)$$

Therefore,

$$\begin{aligned}[\Delta Y_{1A}(t)]^2 &= \langle Y_{1A}^2(t)\rangle - \langle Y_{1A}(t)\rangle^2 \\ &= \frac{1}{4}\Big[4|\alpha|^2 + 2 - 24g^2t^2\left(\alpha^4|\alpha|^2 + \alpha^4 + \alpha^{*4}|\alpha|^2 + \alpha^{*4} + |\alpha|^6\right)\Big].\end{aligned} \quad (35)$$

The number of photons in mode A may be expressed as

$$\begin{aligned}N_{1A}(t) &= A^{\dagger}(t)A(t) \\ &= A^{\dagger}A + 3igt\left(A^3B^{\dagger 2}C^{\dagger} - A^{\dagger 3}B^2C\right) \\ &\quad - 3g^2t^2A^{\dagger 3}A^3\left(B^{\dagger 2}B^2 + 4B^{\dagger}B + 2\right) \\ &\quad + 9g^2t^2\left(A^{\dagger 2}A^2 + 4A^{\dagger}A + 2\right)B^{\dagger 4}B^4C^{\dagger}C.\end{aligned} \quad (36)$$

Thus, using condition (23), we get

$$\left\langle N_{1A}(t) + \frac{1}{2}\right\rangle = \left[|\alpha|^2 + \frac{1}{2} - 6g^2t^2|\alpha|^6\right]. \quad (37)$$

Subtracting (37) from (35), we get

$$[\Delta Y_{1A}(t)]^2 - \left\langle N_{1A}(t) + \frac{1}{2}\right\rangle = -12g^2t^2\left(|\alpha|^6 + |\alpha|^4\right)\cos 4\theta. \quad (38)$$

Using initial condition (28), we obtain squeezing for the stimulated process as

$$[\Delta Y_{1A}(t)]^2 - \left\langle N_{1A}(t) + \frac{1}{2}\right\rangle = -6g^2t^2\left(|\alpha|^6 + |\alpha|^4\right) \times \left(|\beta|^4 + 4|\beta|^2 + 2\right)\cos 4\theta. \quad (39)$$

The right-hand sides of (38) and (39) are negative for all values of θ for which $\cos 4\theta > 0$ and thus shows the existence of squeezing in the second order of the field amplitude in spontaneous and stimulated interaction under short-time approximation.

Using (21), cubed-amplitude is expressed as

$$\begin{aligned}A^3(t) &= A^3 - 3igt\left(3A^{\dagger 2}A^2 + 6A^{\dagger}A + 2\right)B^2C \\ &\quad - 18g^2t^2\left(A^{\dagger 4}A + 3A^{\dagger 3}\right)B^4C^2 - \frac{3}{2}g^2t^2 \\ &\quad \times \left(3A^{\dagger 2}A^5 + 6A^{\dagger}A^4 + 2A^3\right)\left(B^{\dagger 2}B^2 + 4B^{\dagger}B + 2\right),\end{aligned} \quad (40)$$

and the real quadrature component for third-order squeezing in the fundamental mode is expressed as

$$\begin{aligned}Z_{1A}(t) &= \frac{1}{2}\left[A^3(t) + A^{\dagger 3}(t)\right] \\ &= \frac{1}{2}\Big[A^3 + A^{\dagger 3} - 3igt\left(A^{\dagger 2}A^2 + 6A^{\dagger}A + 2\right)B^2C \\ &\quad + 3igt\left(A^{\dagger 2}A^2 + 6A^{\dagger}A + 2\right)B^{\dagger 2}C^{\dagger} \\ &\quad - 18g^2t^2\left(A^{\dagger 4}A + 3A^{\dagger 3}\right)B^4C^2 \\ &\quad - 18g^2t^2\left(A^{\dagger}A^4 + 3A^3\right)B^{\dagger 4}C^{\dagger 2} - \frac{3}{2}g^2t^2 \\ &\quad \times \left(3A^{\dagger 5}A^2 + 6A^{\dagger 4}A + 2A^{\dagger 3} + 3A^{\dagger 2}A^5 + 6A^{\dagger}A^4 + 2A^3\right)\Big].\end{aligned} \quad (41)$$

Using (23) and (41), we get the expectation values for spontaneous interaction as

$$\begin{aligned}\langle \psi | Z_{1A}(t) | \psi \rangle^2 &= \frac{1}{4}\Big[\alpha^6 + \alpha^{*6} + 2|\alpha|^6 - 6g^2t^2 \\ &\quad \times \left(3|\alpha|^4 + 6|\alpha|^2 + 2\right)\left(\alpha^6 + \alpha^{*6} + 2|\alpha|^6\right)\Big],\end{aligned} \quad (42)$$

$$\langle\psi\,|\,Z_{1A}^2(t)\,|\,\psi\rangle = \frac{1}{4}\Big[\alpha^6+\alpha^{*6}+2|\alpha|^6+9|\alpha|^4+18|\alpha|^2+6 - 6g^2t^2\Big(3|\alpha|^4+15|\alpha|^2+20\Big)\left(\alpha^6+\alpha^{*6}\right) + 6|\alpha|^{10}+30|\alpha|^8+40|\alpha|^6\Big]. \tag{43}$$

Subtracting (42) from (43), we get

$$[\Delta Z_{1A}(t)]^2 = \frac{1}{4}\Big[9|\alpha|^4+18|\alpha|^2+6-54g^2t^2 \times\Big(|\alpha|^2+2\Big)\left(\alpha^6+\alpha^{*6}\right)+2|\alpha|^8+4|\alpha|^6\Big]. \tag{44}$$

Using (23) and (36), we have

$$\frac{1}{4}\langle 9N_{1A}^2(t)+9N_{1A}(t)+6\rangle = \frac{1}{4}\Big[9|\alpha|^4+18|\alpha|^2+6-108g^2t^2\Big(|\alpha|^8+2|\alpha|^6\Big)\Big]. \tag{45}$$

Subtracting (45) from (44), we get

$$[\Delta Z_{1A}(t)]^2 - \frac{1}{4}\langle 9N_{1A}^2(t)+9N_{1A}(t)+6\rangle = -27g^2t^2\Big(|\alpha|^8+2|\alpha|^6\Big)\cos 6\theta. \tag{46}$$

Using (28), we obtain the stimulated process as

$$[\Delta Z_{1A}(t)]^2 - \frac{1}{4}\langle 9N_{1A}^2(t)+9N_{1A}(t)+6\rangle = -\frac{27}{2}g^2t^2\Big(|\alpha|^8+2|\alpha|^6\Big)\Big(|\beta|^4+4|\beta|^2+2\Big)\cos 6\theta. \tag{47}$$

The right-hand sides of (46) and (47) are negative, for all values of θ for which $\cos 6\theta > 0$, indicating the existence of squeezing in cubed amplitude in the fundamental mode in the spontaneous and stimulated processes.

For fourth-order squeezing, amplitude is expressed as

$$A^4(t) = A^4 - 12igt\Big(A^{\dagger 2}A^3+3A^{\dagger}A^2+2A\Big)B^2C - 6g^2t^2 \times\Big(A^{\dagger 2}A^6+3A^{\dagger}A^5+2A^4\Big)\Big(B^{\dagger 2}B^2+4B^{\dagger}B+2\Big). \tag{48}$$

The real quadrature component for fourth-order squeezing in fundamental mode is given as

$$F_{1A}(t) = \frac{1}{2}\Big[A^4(t)+A^{\dagger 4}(t)\Big]. \tag{49}$$

Using (23) and (48) in (49), we get the expectation values as

$$\langle\psi|F_{1A}(t)|\psi\rangle^2 = \frac{1}{4}\Big[\alpha^8+\alpha^{*8}+2|\alpha|^8-24g^2t^2 \times\Big\{\Big(|\alpha|^4+3|\alpha|^2+2\Big)\left(\alpha^8+\alpha^{*8}\right) + 2|\alpha|^{12}+6|\alpha|^{10}+4|\alpha|^8\Big\}\Big], \tag{50}$$

$$\langle\psi\,|\,F_{1A}^2(t)\,|\,\psi\rangle = \frac{1}{4}\Big[\alpha^8+\alpha^{*8}+2|\alpha|^8+16|\alpha|^6+72|\alpha|^4 + 96|\alpha|^2+24-24g^2t^2 \times\Big\{\Big(|\alpha|^4+7|\alpha|^2+14\Big)\left(\alpha^8+\alpha^{*8}\right) + 2|\alpha|^{12}+18|\alpha|^{10}+64|\alpha|^8+68|\alpha|^6\Big\}\Big]. \tag{51}$$

Therefore, subtracting (50) from (51), we obtain

$$[\Delta F_{1A}(t)]^2 = \frac{1}{4}\Big[16|\alpha|^6+72|\alpha|^4+96|\alpha|^2+24-24g^2t^2 \times\Big\{\Big(4|\alpha|^2+12\Big)\left(\alpha^8+\alpha^{*8}\right)+12|\alpha|^{10} + 60|\alpha|^8+68|\alpha|^6\Big\}\Big]. \tag{52}$$

Using (23) and (36), we have

$$\frac{1}{4}\Big\langle 16N_{1A}^3(t)+24N_{1A}^2(t)+56N_A+24\Big\rangle = \frac{1}{4}\Big[16|\alpha|^6+72|\alpha|^4+96|\alpha|^2+24 - 24g^2t^2\Big(12|\alpha|^{10}+60|\alpha|^8+68|\alpha|^6\Big)\Big]. \tag{53}$$

Subtracting (53) from (52), we get

$$[\Delta F_{1A}(t)]^2 - \frac{1}{4}\Big\langle 16N_{1A}^3(t)+24N_{1A}^2(t)+56N_A+24\Big\rangle = -48g^2t^2\Big(|\alpha|^{10}+3|\alpha|^8\Big)\cos 8\theta. \tag{54}$$

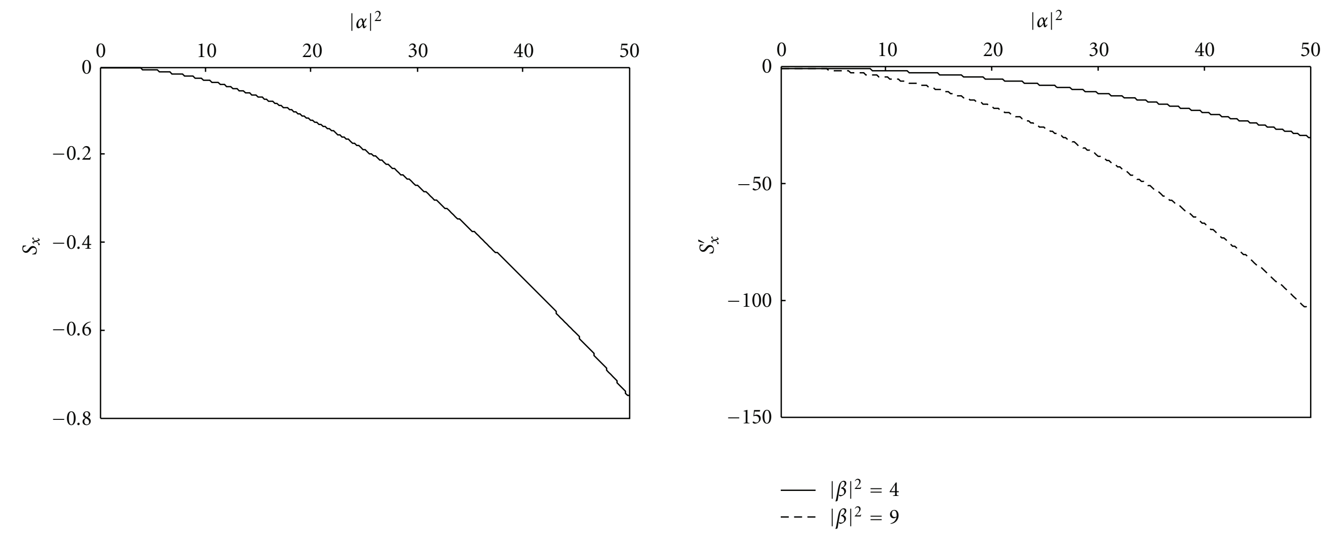
Using (28), we obtain the stimulated process as

$$[\Delta F_{1A}(t)]^2 - \frac{1}{4}\Big\langle 16N_{1A}^3+24N_{1A}^2(t)+56N_{1A}(t)+24\Big\rangle = -24g^2t^2\Big(|\alpha|^{10}+3|\alpha|^8\Big)\Big(|\beta|^4+4|\beta|^2+2\Big)\cos 8\theta. \tag{55}$$

The right-hand sides of (54) and (55) are negative, for all values of θ for which $\cos 8\theta > 0$, indicating the existence of squeezing in fourth-order field amplitude in the fundamental mode in the spontaneous and stimulated processes, respectively.

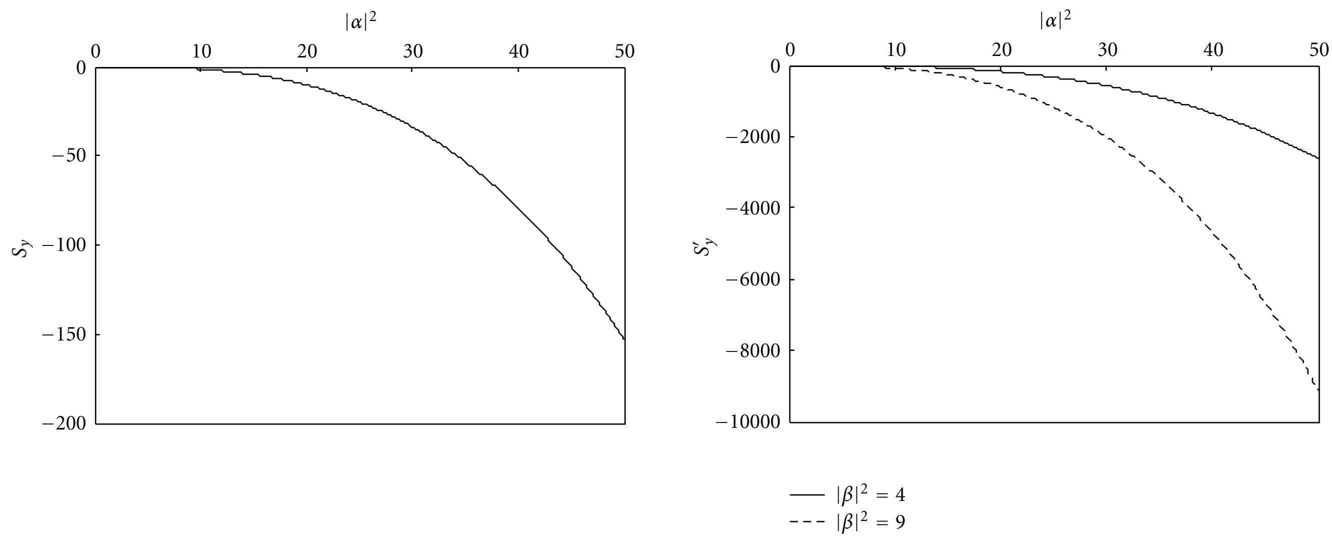
Using (23) and (36), the statistics of fundamental mode in six-wave mixing is found to be sub-Poissonian, given as

$$\langle\Delta N_{1A}\rangle^2 - \langle N_{1A}\rangle = -12g^2t^2|\alpha|^6. \tag{56}$$



(a) (b)

Figure 2: Dependence of first-order squeezing (a) s_X with $|\alpha|^2$ in spontaneous and (b) s'_X with $|\alpha|^2$ and $|\beta|^2$ in stimulated six-wave mixing process.



(a) (b)

Figure 3: Dependence of amplitude-squared squeezing (a) s_y with $|\alpha|^2$ in spontaneous and (b) s'_y with $|\alpha|^2$ and $|\beta|^2$ in stimulated six-wave mixing process.

4. Signal-to-Noise Ratio

Signal-to-noise ratio is defined as ratio of the magnitude of the signal to the magnitude of the noise. With the approximations $\theta = 0$ and $|gt|^2 \ll 1$, the maximum signal-to-noise ratio (in decibels) in field amplitude and higher orders is given in the following.

Using (25) and (26), signal-to-noise ratio in field amplitude is defined as

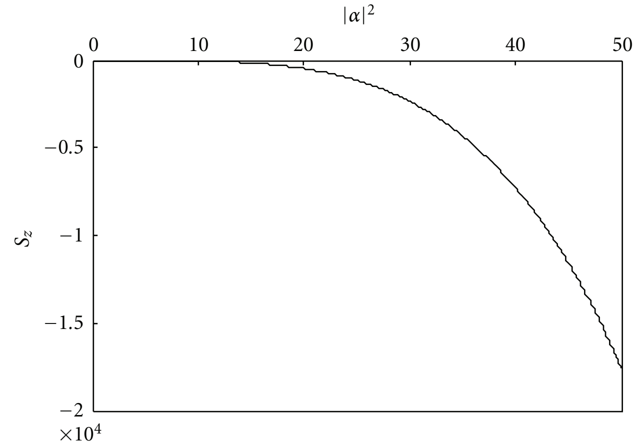
$$\mathrm{SNR}_1 = 20 * \log_{10}\frac{\langle X_{1A}(t)\rangle^2}{[\Delta X_{1A}(t)]^2} = 20 * \log_{10}\left(2|\alpha|^2\right). \quad (57)$$

Using (33) and (35), SNR in amplitude-squared squeezing is given as

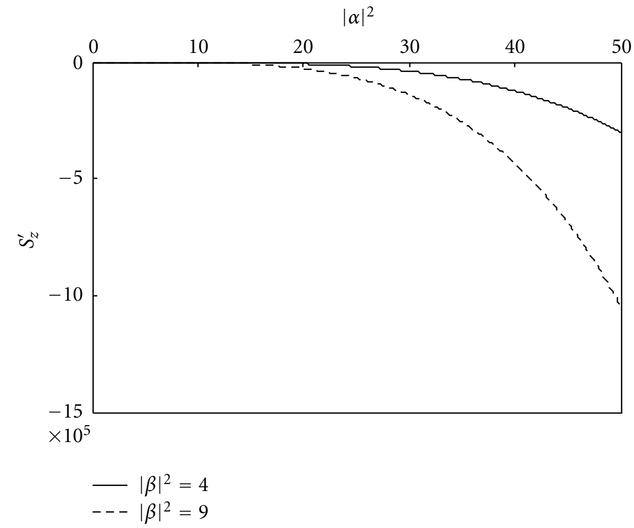
$$\mathrm{SNR}_2 = 20 * \log_{10}\frac{\left(2|\alpha|^4 + |\alpha|^2\right)}{\left(3|\alpha|^2 + 2\right)}. \quad (58)$$

Using (42) and (44), SNR in amplitude-cubed squeezing is expressed as

$$\mathrm{SNR}_3 = 20 * \log_{10}\frac{\left(3|\alpha|^4 + 6|\alpha|^2 + 2\right)}{\left(9|\alpha|^2 + 18\right)}. \quad (59)$$



(a)



(b)

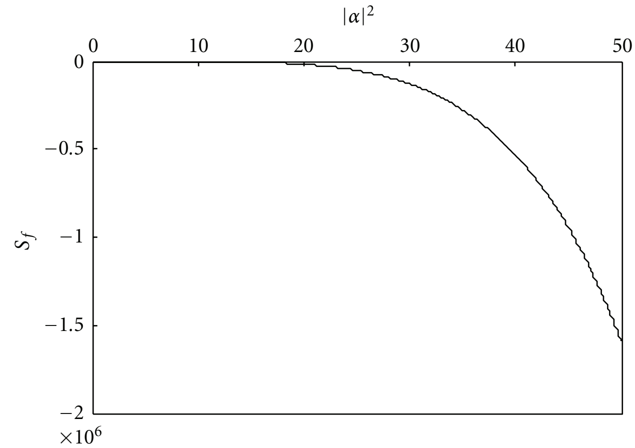
Figure 4: Dependence of amplitude-cubed squeezing (a) S_z with $|\alpha|^2$ in spontaneous and (b) S'_z with $|\alpha|^2$ and $|\beta|^2$ in stimulated six-wave mixing process.

Using (50) and (52), SNR in fourth-order squeezing is expressed as

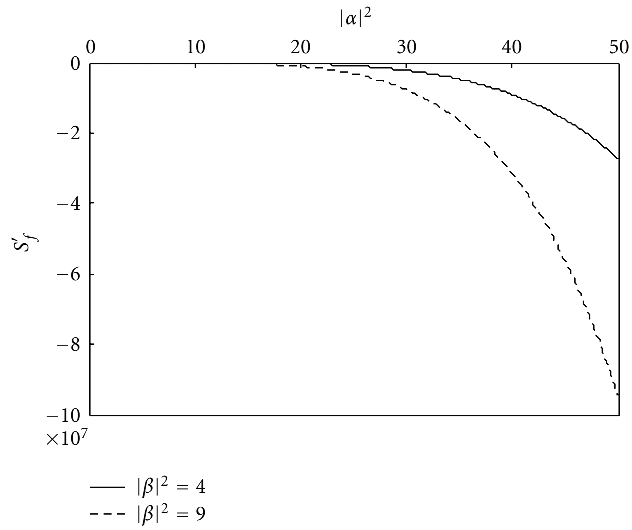
$$\mathrm{SNR}_4 = 20 * \log_{10} \frac{\left(|\alpha|^6 + 3|\alpha|^4 + 2|\alpha|^2\right)}{\left(5|\alpha|^4 + 21|\alpha|^2 + 17\right)}. \qquad (60)$$

5. Results

The results show the presence of squeezing in field amplitude, amplitude-squared, amplitude-cubed, and fourth-order field amplitude of fundamental mode in six-wave mixing process. To study squeezing, we denote the right-hand sides of relations (27), (38),(46), and (54) by S_x, S_y, S_z, and S_f for spontaneous and right-hand sides of relations (30), (39), (47), and (55) by S'_x, S'_y, S'_z, and S'_f for stimulated interaction for field amplitude, amplitude-squared, amplitude-cubed,



(a)



(b)

Figure 5: Dependence of fourth-order field amplitude squeezing (a) S_f with $|\alpha|^2$ in spontaneous and (b) S'_f with $|\alpha|^2$ and $|\beta|^2$ in stimulated six-wave mixing process.

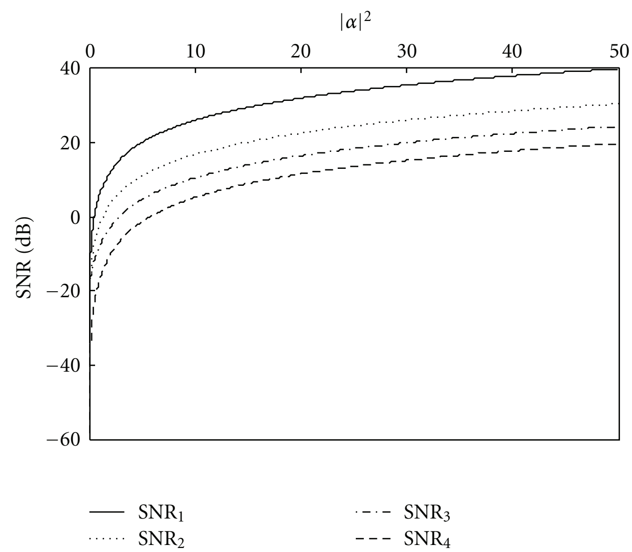


Figure 6: Signal-to-noise ratio for different order squeezing.

and for fourth-order field amplitude, respectively. Taking $|gt|^2 = 10^{-4}$ and $\theta = 0$, the variations of S_x, S_y, S_z, and S_f with photon number $|\alpha|^2$ for spontaneous interaction and of S'_x, S'_y, S'_z, and S'_f with $|\alpha|^2$and $|\beta|^2$for stimulated interaction are shown from Figures 2, 3, 4, and 5.

A comparison between results of spontaneous and stimulated processes shows the occurrence of multiplication factor $(|\beta|^4 + 4|\beta|^2 + 2)$. It implies that squeezing in the fundamental mode in stimulated interaction is greater than corresponding squeezing in spontaneous interaction. It is also seen that maximum squeezing occurs when $\theta = 0$. The signal-to-noise ratio is found to be higher in lower orders as shown in Figure 6.

6. Conclusion

Figures 2, 3, 4, and 5 show that squeezing increases nonlinearly with $|\alpha|^2$, which is directly dependent upon the number of photons. The squeezing in any order during stimulated interaction (Figures 5(b), 4(b), 3(b), and 2(b)) is higher than the squeezing in corresponding order in spontaneous (Figures 5, 4, 3, and 2) interaction by a factor $(|\beta|^4+4|\beta|^2+2)$. The squeezing is higher in higher orders in both processes. Thus, the higher-order squeezing associated with higher order nonlinear optical processes makes it possible to achieve significant noise reduction.

It has also been found that the fundamental mode of field amplitude shows sub-Poissonian behavior as shown in relation (56). The signal-to-noise ratio is higher in lower orders squeezed states as reported earlier for Raman process [34].

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