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Research Article

Parameters Identification and Synchronization of Chaotic Delayed Systems Containing Uncertainties and Time-Varying Delay

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Time delays are ubiquitous in real world and are often sources of complex behaviors of dynamical systems. This paper addresses the problem of parameters identification and synchronization of uncertain chaotic delayed systems subject to time-varying delay. Firstly, a novel and systematic adaptive scheme of synchronization is proposed for delayed dynamical systems containing uncertainties based on Razumikhin condition and extended invariance principle for functional differential equations. Then, the proposed adaptive scheme is used to estimate the unknown parameters of nonlinear delayed systems from time series, and a sufficient condition is given by virtue of this scheme. The delayed system under consideration is a very generic one that includes almost all well-known delayed systems (neural network, complex networks, etc.). Two classical examples are used to demonstrate the effectiveness of the proposed adaptive scheme.

1. Introduction

In recent years, delayed dynamical systems (so called, DDEs) have attracted lots of attention in the field of nonlinear dynamics, and the dynamical properties of DDEs have been extensively investigated. Strictly speaking, time delays are ubiquitous in real world due to the finite switching speed of amplifiers, finite signal propagation time in biological networks, finite chemical reaction times, memory effects, and so forth. Therefore, DDEs are used to model dynamical systems broadly in scientific and engineering areas, for instance, in population dynamics, biology, economy, neural networks, complex networks, and so on. It has been found that the presence of time delay(s) is often a source of complicated behaviors, for example, limit-cycle, loss of stability, bifurcation, and chaos [1–7]. Especially, one-dimensional delayed dynamical systems can generate high-dimensional chaos [8, 9]. Motivated by the study of chaotic phenomena, an increasing interest has been devoted to the study of chaos synchronization in delayed dynamical systems, for example, anticipating synchronization [10–15], generalized synchronization [16], phase synchronization [17], and complete synchronization [18–21]. Many applications of chaos synchronization in delayed dynamical systems have been found in many different areas including in secure communication, information science, optic systems, neural networks, and so forth. However, in most of the previous works, the considered systems are often specific, also the controllers are sometimes of limitation or too complicated to implement in nature; particularly, the controllers cannot be applied to delayed dynamical systems with time-varying time delay.

Very recently, many research of chaos synchronization has been devoted to neural networks or complex networks with time delay(s) and some theoretical and numerical results are obtained [22–27]. In [22], the authors considered the synchronization of two kinds of dynamical complex networks utilizing special matrix measure with constant linear coupling delays in the nodes of the whole networks. In [23], mathematical analysis is presented on the synchronization phenomena of linearly coupled systems described by ordinary differential equations with a single linear coupling delay. Reference [24] presented several delay-dependent conditions for continuous- and discrete-time complex dynamical network model with a single linear coupling delay using linear matrix inequalities. In [25], a strategy for synchronization of complex dynamical networks with a single linear coupling time-delay is proposed based on linear state feedback controllers. In [26], the authors studied the synchronization of neural networks subject to time-varying delays and sector nonlinearity based on the drive-response concept, where a complicated controller was designed to achieve synchronization. Cao et al. considered the synchronization of coupled identical neural networks with time-varying delay using a simple adaptive feedback scheme based on the invariant principle of functional differential equations, and a good result was obtained in [27]. But, to design the adaptive scheme, a strict constraint is added to the time-varying delay, and the state equation under consideration must be of specific form. However, there have been no previous reports of adaptive chaos synchronization of delayed dynamical systems with uncertain parameters and nonlinear time-varying delay.

An interesting application of chaos synchronization is to estimate parameters of a chaotic system from time series when partial information about the system is available [28–31]. In contrast to the large number of research papers on the parameter identification of chaotic systems without delay, only limited attention has been given to the parameter identification of delayed dynamical systems [32–38]. Among these works, most of them are limited to linear delayed system with constant delay. Actually, how to identify the unknown parameters from time series is still an open problem.

Motivated by the above discussion, in the present paper, we study parameters identification and synchronization of uncertain chaotic delayed systems with time-varying delay based on the famous Razumikhin condition and invariance principle of functional differential equations in the framework of Krasovskii-Lyapunov theory [39]. The system under consideration is a very general one that includes almost all well-known delayed systems, for example, Ikeda system [40], Mackey-Glass system [1], delayed Duffing system [7], Hopfield neural network [27], BAM neural network [41], cellular neural network [42], complex networks [22–24], and so on. The adaptive feedback controller utilized here is very simple, which is constructed by combination of adaptive scheme and linear feedback with the updated feedback strength.

This paper is organized as follows: problem statement and some preliminaries knowledge, including one lemma and two assumptions, are given in Section 2. Our main theoretical result is described in Section 3. In Section 4, two numerical examples, the delayed Rössler model [43] and delayed Hopfield neural network system [27], are employed to illustrate the effectiveness of the proposed adaptive scheme. Finally, conclusions and remarks are drawn in Section 5.

2. Problem Statement and Preliminaries

In this paper, we consider the general continuous-time delayed dynamical system with time-varying delay described by the following delayed differential equation:

$$\dot{x}(t) = Ax(t) + Bx(t-\tau) + Cf(x(t)) + Dg(x(t-\tau)), \tag{2.1}$$

where $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T \in R^n$ is the state vector, $A = (a_{ij})_{n\times n}$ and $B = (b_{ij})_{n\times n}$ are unknown constant matrices representing the linear parts of the system, $C = (c_{ij})_{n\times n}$ and $D = (d_{ij})_{n\times n}$ are unknown constant matrices representing the nonlinear parts of the system, $\tau = \tau(t) \geq 0$ is the time-varying delay, $f(x(t)) = [f_1(x(t)), f_2(x(t)), \ldots, f_n(x(t))]^T$ denotes the nonlinear function without delay, and $g(x(t)) = [g_1(x(t-\tau)), g_2(x(t-\tau)), \ldots, g_n(x(t-\tau))]^T$ denotes the nonlinear function with delay. Without loss of generality, we assume that the structure of the nonlinear dynamical system (2.1) is known and, furthermore, time series for all variables are available as the output of system (2.1). Let $\Omega \in R^n$ be a chaotic bounded set.

For the nonlinear vector functions $f(x)$ and $g(x)$ and time-varying delay $\tau(t)$, we have the following two assumptions.

Assumption 2.1. For any $x = (x_1, x_2, \ldots, x_n)^T$ and $y = (y_1, y_2, \ldots, y_n)^T \in \Omega$, there exist constants $l_i^1 > 0$ and $l_i^2 > 0$ $(n = 1, 2, \ldots, n)$ such that

$$\begin{aligned} |f_i(x) - f_i(y)| &\leq \sqrt{l_i^1} \max_{1\leq j\leq n}\{|x_j - y_j|\}, \quad i = 1, 2, \ldots, n, \\ |g_i(x) - g_i(y)| &\leq \sqrt{l_i^2} \max_{1\leq j\leq n}\{|x_j - y_j|\}, \quad i = 1, 2, \ldots, n. \end{aligned} \tag{2.2}$$

The above condition is the so-called uniform Lipschitz condition; $l_i^1 > 0$ and $l_i^2 > 0$ $(n = 1, 2, \ldots, n)$ refer to the uniform Lipschitz constants.

Assumption 2.2. $\tau = \tau(t) \geq 0$ is the smooth function of time t, and its derivative is bounded, that is, there exists some positive number $M > 0$ such that $|\dot{\tau}(t)| \leq M$.

Clearly, this assumption is certainly ensured if the delay $\tau(t)$ is constant.

Remark 2.3. Assumption 2.1 is a very loose constraint added to nonlinear vector functions $f(x)$ and $g(x)$. One can easily check that a wide variety of delayed dynamical systems satisfy the above condition; particularly, condition (2.2) will hold as long as the partial differential $\partial f_i/\partial x_j$ and $\partial g_i/\partial x_j$ $(i, j = 1, 2, \ldots, n)$ are bounded in $\Omega \in R^n$. Therefore, the class of systems in the form of (2.1) includes almost all well-known delayed systems, for example, Ikeda system [40], Mackey and Glass system [1], delayed Duffing system [7], Hopfield neural network [27], BAM neural network [41], cellular neural network [42], complex networks [22–24], and so forth.

Remark 2.4. Assumption 2.2 is a limitation of time-varying delay $\tau(t)$. Comparing to that of time-varying delay $\tau(t)$ in [27], this limitation is very loose.

We refer to system (2.1) as the drive system. An auxiliary system of variables $y(t) = (y_1(t), y_2(t), \ldots, y_n(t))^T \in R^n$ is introduced as the response system, whose evolution equations have identical form to system (2.1)

$$\dot{y}(t) = \overline{A}(t)y(t) + \overline{B}(t)y(t-\tau(t)) + \overline{C}(t)f\left(y(t)\right) + \overline{D}(t)g\left(y(t-\tau)\right) + u(t), \tag{2.3}$$

where $\overline{A} = (\overline{a}_{ij})_{n\times n}$, $\overline{B} = (\overline{b}_{ij})_{n\times n}$, $\overline{C} = (\overline{c}_{ij})_{n\times n}$, and $\overline{D} = (\overline{d}_{ij})_{n\times n}$ are the estimates of the unknown parameters matrices $A = (a_{ij})_{n\times n}$, $B = (b_{ij})_{n\times n}$, $C = (c_{ij})_{n\times n}$, and $D = (d_{ij})_{n\times n}$, respectively; $u(t)$ is a simple adaptive-feedback controller, $u(t) = K(y(t) - x(t))$ with K updated adaptively according to some updated law.

Defining the synchronization error as $e(t) = y(t) - x(t)$ and subtracting (2.1) from (2.3) yield the error system as follows:

$$\begin{aligned}\dot{e}(t) &= \overline{A}(t)y(t) + \overline{B}(t)y(t-\tau) + \overline{C}(t)f\left(y(t)\right) + \overline{D}(t)g\left(y(t-\tau)\right) + Ke(t)\\ &\quad - Ay(t) - By(t-\tau) - Cf\left(y(t)\right) - Dg\left(y(t-\tau)\right).\end{aligned} \tag{2.4}$$

Therefore, the task of this paper is to design a suitable adaptive scheme

$$\begin{gathered}\dot{\overline{A}} = \overline{A}\left(x, y, \overline{A}, \overline{B}, \overline{C}, \overline{D}, t\right), \qquad \dot{\overline{B}} = \overline{B}\left(x, y, \overline{A}, \overline{B}, \overline{C}, \overline{D}, t\right),\\ \dot{\overline{C}} = \overline{C}\left(x, y, \overline{A}, \overline{B}, \overline{C}, \overline{D}, t\right), \qquad \dot{\overline{D}} = \overline{D}\left(x, y, \overline{A}, \overline{B}, \overline{C}, \overline{D}, t\right),\\ \dot{K} = \overline{K}\left(x, y, \overline{A}, \overline{B}, \overline{C}, \overline{D}, t\right),\end{gathered} \tag{2.5}$$

such that $y(t)$ can track $x(t)$, that is, $\|e(t)\| \to 0$ as $t \to \infty$.

Furthermore, we introduce a lemma [44], which is needed in the proof of the main results.

Lemma 2.5. *For any vectors $x_1, x_2 \in R^n$ and any positive definite matrix $Q \in R_{n\times n}$, the following inequality holds:*

$$2x_1^T x_2 \leq x_1^T Q x_1 + x_2^T Q^{-1} x_2. \tag{2.6}$$

3. Adaptive Synchronization Scheme

In this section, we investigate the adaptive synchronization between the drive system (2.1) and the response system (2.3) in the framework of Krasovskii-Lyapunov theory [39]. The main result is described in the following theorem.

Theorem 3.1. *Under Assumption 2.1 and Assumption 2.2, system* (2.3) *can synchronize with system* (2.1) *if one designs* $\overline{A}$, $\overline{B}$, $\overline{C}$, *and* $\overline{D}$ *as*

$$\begin{aligned}
\dot{\overline{a}}_{ij}(t) &= -\alpha_{ij} e_i(t) y_j(t), \\
\dot{\overline{b}}_{ij}(t) &= -\beta_{ij} e_i(t) y_j(t-\tau(t)), \\
\dot{\overline{c}}_{ij}(t) &= -p_{ij} e_i(t) f_j(y(t)), \\
\dot{\overline{d}}_{ij}(t) &= -q_{ij} e_i(t) g_j(y(t-\tau(t))),
\end{aligned} \tag{3.1}$$

with the coupling strength $K = \operatorname{diag}(k_1, k_2, \ldots, k_n)^T$ *updated by*

$$\dot{k}_i = -\gamma_i e_i^2, \tag{3.2}$$

where $\alpha_{ij}, \beta_{ij}, p_{ij}, q_{ij}$, *and* γ_i $(i, j = 1, 2, \ldots, n)$ *are arbitrary positive constants.*

Proof. Construct a Lyapunov function of the form

$$\begin{aligned}
V(e(t)) = {} & \frac{1}{2} e^T(t) e(t) + \frac{1}{2} \int_{t-\tau(t)}^{t} e^T(s) e(s) ds + \frac{1}{2} \int_{t-\tau(t)}^{t} G^T(e(s)) G(e(s)) ds \\
& + \frac{1}{2} \sum_{i=1}^{n} \frac{1}{\gamma_i} (k_i + l)^2 + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{1}{\alpha_{ij}} (\overline{a}_{ij} - a_{ij})^2 + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{1}{\beta_{ij}} (\overline{b}_{ij} - b_{ij})^2 \\
& + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{1}{p_{ij}} (\overline{c}_{ij} - c_{ij})^2 + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{1}{q_{ij}} (\overline{d}_{ij} - d_{ij})^2,
\end{aligned} \tag{3.3}$$

where $G(e) = g(y) - g(x) = g(e+x) - g(x)$.

By differentiating the function V with respect to time along the trajectory of (2.4), one can obtain

$$
\begin{aligned}
\dot{V}(e(t)) &= e^T(t)\dot{e}(t) + \frac{1}{2}e^T(t)e(t) - \frac{1}{2}(1-\dot{\tau}(t))e^T(t-\tau(t))e(t-\tau(t)) \\
&\quad + \frac{1}{2}G^T(e(t))G(e(t)) - \frac{1}{2}(1-\dot{\tau}(t))G^T(e(t-\tau(t)))G(e(t-\tau(t))) \\
&\quad - \sum_{i=1}^{n}\sum_{j=1}^{n}(\overline{a}_{ij}-a_{ij})e_i(t)y_j(t) - \sum_{i=1}^{n}\sum_{j=1}^{n}\left(\overline{b}_{ij}-b_{ij}\right)e_i(t)y_j(t-\tau(t)) \\
&\quad - \sum_{i=1}^{n}\sum_{j=1}^{n}(\overline{c}_{ij}-c_{ij})e_i(t)f_j\left(y(t)\right) - \sum_{i=1}^{n}\sum_{j=1}^{n}\left(\overline{d}_{ij}-d_{ij}\right)e_i(t)g_j\left(y(t-\tau)\right) \\
&\quad - \sum_{i=1}^{n}(k_i+l)e_i^2 \\
&= e^T(t)\overline{A}(t)y(t) + e^T(t)\overline{B}(t)y(t-\tau(t)) + e^T(t)\overline{C}(t)f\left(y(t)\right) \\
&\quad + e^T(t)\overline{D}(t)g\left(y(t-\tau(t))\right) - e^T(t)Ax(t) + e^T(t)Bx(t-\tau(t)) \\
&\quad + e^T(t)Cf(x(t)) + e^T(t)Dg(x(t-\tau(t))) + e^T(t)u(t) - e^T(t)\overline{A}(t)y(t) \\
&\quad + e^T(t)Ay(t) - e^T(t)\overline{B}(t)y(t-\tau(t)) + e^T(t)By(t-\tau(t)) \\
&\quad - e^T(t)\overline{C}(t)f\left(y(t)\right) + e^T(t)Cf\left(y(t)\right) - e^T(t)\overline{D}(t)g\left(y(t-\tau(t))\right) \\
&\quad + e^T(t)Dg\left(y(t-\tau(t))\right) - e^T(t)Ke(t) - e^T(t)Le(t) + \frac{1}{2}e^T(t)e(t) \\
&\quad - \frac{1}{2}(1-\dot{\tau}(t))e^T(t-\tau(t))e(t-\tau(t)) + \frac{1}{2}G^T(e(t))G(e(t)) \\
&\quad - \frac{1}{2}(1-\dot{\tau}(t))G^T(e(t-\tau(t)))G(e(t-\tau(t))) \\
&= e^T(t)Ae(t) + e^T(t)Be(t-\tau(t)) + e^T(t)CF(e) + e(t)DG(e(t-\tau(t))) \\
&\quad - e^T(t)Le(t) + \frac{1}{2}e^T(t)e(t) - \frac{1}{2}(1-\dot{\tau}(t))e^T(t-\tau(t))e(t-\tau(t)) \\
&\quad + \frac{1}{2}G^T(e(t))G(e(t)) - \frac{1}{2}(1-\dot{\tau}(t))G^T(e(t-\tau(t)))G(e(t-\tau(t))),
\end{aligned}
\tag{3.4}
$$

where $F(e) = f(y) - f(x) = f(e+x) - f(x)$.

Recalling Assumption 2.1, we have the following two inequalities:

$$
\begin{aligned}
F^T(e(t))F(e(t)) &= \sum_{i=1}^{n}\left(f_i(y(t)) - f_i(y(t))\right)^2 \le nh_1 e^T(t)e(t), \\
G^T(e(t))G(e(t)) &= \sum_{i=1}^{n}\left(g_i(x(t)) - g_i(y(t))\right)^2 \le nh_2 e^T(t)e(t),
\end{aligned}
\tag{3.5}
$$

here $h_1 = \max\{l_1^1, l_2^1, \ldots, l_n^1\}$ and $h_2 = \max\{l_1^2, l_2^2, \ldots, l_n^2\}$. Substituting inequalities (3.5) into the right side of the above equality and applying Lemma 2.5, one can obtain

$$\begin{aligned}\dot{V}(e(t)) \le e^T(t)\left[A + \frac{1}{2}BB^T + \frac{1}{2}CC^T + \frac{1}{2}DD^T + \left(\frac{h_1 + h_2 + 1}{2} - L\right)I\right]e(t) \\ + \frac{M}{2}e^T(t-\tau(t))e(t-\tau(t)) + \frac{Mh_2}{2}e^T(t-\tau(t))e(t-\tau(t)).\end{aligned} \tag{3.6}$$

Applying the classical Razumikhin condition to inequality (3.6) and choosing

$$L = \lambda_{\max}\left[A + \frac{1}{2}BB^T + \frac{1}{2}CC^T + \frac{1}{2}DD^T\right] + \frac{h_1 + (M+1)h_2 + m + 1}{2} + 1, \tag{3.7}$$

one can obtain

$$\dot{V}(e(t)) \le -e^T(t)e(t). \tag{3.8}$$

Clearly, $\dot{V}(e(t)) = 0$ if and only if $e(t) = 0$. Therefore, $E = \{e = 0, \overline{A} = A_0, \overline{B} = B_0, \overline{C} = C_0, \overline{D} = D_0, K = K_0 \in R^n\}$ is the largest invariant set containing in $\{(e(t), \overline{A}, \overline{B}, \overline{C}, \overline{D}, K) \mid \dot{V} = 0\}$. According to the well-known invariant principle of functional differential equations [39], the trajectory of the argument system converges asymptotically to the largest invariant set E starting from any initial value as time tends to infinity, where the converged parameters A_0, B_0, C_0, D_0, and K_0 depends on initial values. This completes the proof. □

Remark 3.2. Note that this theorem only guarantees that $e(t) \to 0$, that is, $y(t) \to x(t)$, and $\dot{\overline{a}}_{ij} \to 0$, $\dot{\overline{b}}_{ij} \to 0$, $\dot{\overline{c}}_{ij} \to 0$, $\dot{\overline{d}}_{ij} \to 0$ as $t \to \infty$; therefore, it is not necessary that $A_0 = A$, $B_0 = B$, $C_0 = C$ and $D_0 = D$ since it is possible that $Ax(t) + Bx(t-\tau(t)) + Cf(x(t)) + Dg(x(t-\tau)) = A_0x(t) + B_0x(t-\tau(t)) + C_0f(x(t)) + D_0g(x(t-\tau))$ when $A_0 \ne A$, $B_0 \ne B$, $C_0 \ne C$, and $D_0 \ne D$. Another possible reason is that several sets of parameters A, B, C, D generate the same trajectory in response system (2.3), and $\overline{A}, \overline{B}, \overline{C}, \overline{D}$ converge to only one such set. Both of the above two reasons will make A_0, B_0, C_0, D_0 in the largest invariant set E not unique in general except for some special delayed systems.

Therefore, we have the following corollary.

Corollary 3.3. *The unknown parameters A, B, C, D of concerned delayed system are identifiable and $A_0 = A$, $B_0 = B$, $C_0 = C$, $D_0 = D$ if A_0, B_0, C_0, D_0 unique in the largest invariant set E.*

This corollary can be derived directly from Theorem 3.1.

4. Numerical Illustrations

In this section, two chaotic nonlinear systems with time-varying delay, that is, delayed Rössler system [43] and delayed Hopfield neural networks system [27], are employed to demonstrate the effectiveness of the proposed adaptive synchronization scheme.

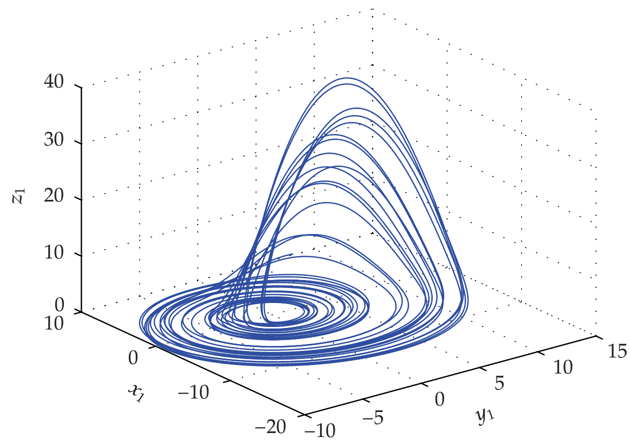

Figure 1: Chaotic attractor of delayed Rössler system for $A = 1$, $\Omega = 1$, $\tau_0 = 1$.

4.1. Delayed Rössler System

A delayed Rössler system is studied in [43] whose evolution equation is as follows (when $\tau_1 = \tau_2$ in [43]):

$$\begin{aligned} \dot{x}_1 &= -y_1 - z_1 + \alpha x_1(t-\tau), \\ \dot{y}_1 &= x_1 + a y_1, \\ \dot{z}_1 &= b + z_1(x_1 - c). \end{aligned} \tag{4.1}$$

When $\alpha = 0.5$, $a = b = 0.2$, $c = 5.7$, and $\tau = \tau_0 + A \sin \Omega t$, the delayed Rössler system is chaotic; see Figure 1.

Here we refer to system (4.1) with the parameters a, b, c being unknown as the drive system. The response system can be described as

$$\begin{aligned} \dot{x}_2 &= -y_2 - z_2 + \alpha x_2(t-\tau) + u_1(t), \\ \dot{y}_2 &= x_2 + \overline{a} y_2 + u_2(t), \\ \dot{z}_2 &= \overline{b} + z_2(x_2 - \overline{c}) + u_3(t). \end{aligned} \tag{4.2}$$

Clearly Assumption 2.1 and Assumption 2.2 are satisfied.

Defining the error state $e = [e_1, e_2, e_3]^T = [x_2 - x_1, y_2 - y_1, z_2 - z_1]^T$ and following the procedure proposed in the Section 2, we can design the adaptive scheme as follows:

$$\begin{aligned} u_1(t) &= k_1(x_2 - x_1), \\ u_2(t) &= k_2(y_2 - y_1), \\ u_3(t) &= k_3(z_2 - z_1), \end{aligned} \tag{4.3}$$

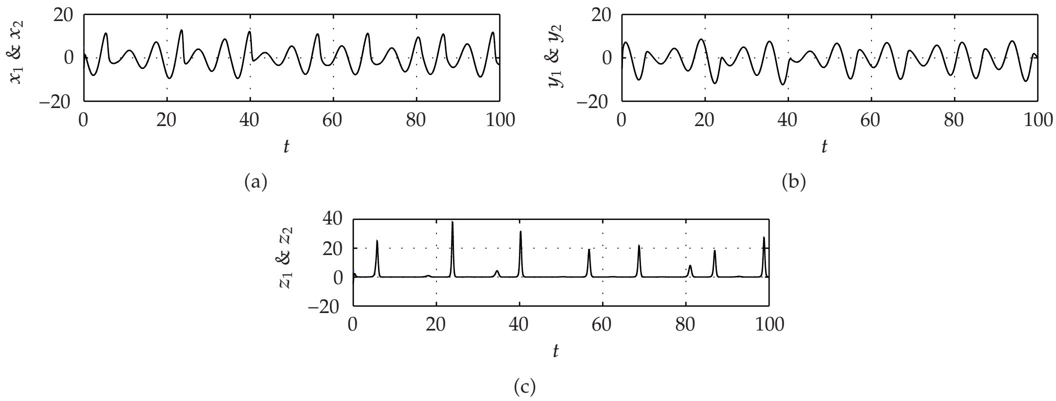

Figure 2: Numerically simulated time series x_1, y_1, z_1 (dotted line) and x_2, y_2, z_2 (black line).

with the feedback strength $K = \operatorname{diag}(k_1, k_2, k_3)$ updated by

$$\begin{aligned}
\dot{k}_1 &= -\gamma_1 e_1^2, \\
\dot{k}_2 &= -\gamma_2 e_2^2, \\
\dot{k}_3 &= -\gamma_3 e_3^2\,, \\
\dot{\bar{a}}(t) &= -\alpha_{22} e_2 y_2, \\
\dot{\bar{b}}(t) &= -p_{32} e_3, \\
\dot{\bar{c}}(t) &= -\alpha_{33} e_3 z_2,
\end{aligned} \tag{4.4}$$

where $\gamma_1, \gamma_2, \gamma_3, \alpha_{22}, \alpha_{33}$, and p_{32} are arbitrary positive constants.

In the following numerical simulations, we take $[x_1(t), y_1(t), z_1(t)]^T = [5, 5, 5]^T$, $[x_2(t), y_2(t), z_2(t)]^T = [-5, -5, -5]^T$, $[k_1(t), k_2(t), k_3(t)]^T = [0, 0, 0]^T$, and $[\bar{a}(t), \bar{b}(t), \bar{c}(t)]^T = [0, 0, 0]^T$ for $t \in [-2, 0]$. The results are shown in Figures 2, 3, 4, 5, and 6.

4.2. Hopfield Neural Network Model

This subsection considers a typical delayed Hopfield neural network system [27]

$$\dot{x} = Ax + Bx(t - \tau) + Cf(x) + Dg(x(t - \tau)), \tag{4.5}$$

where $x = [x_1, x_2]^T$ and $\tau = e^t/(1 + e^t)$. When $A = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$, $C = \begin{pmatrix} 2.0 & -0.1 \\ -5.0 & 3.2 \end{pmatrix}$, and $D = \begin{pmatrix} -1.6 & -0.1 \\ -0.18 & -2.4 \end{pmatrix}$, the delayed Hopfield neural network system (4.5) is chaotic.

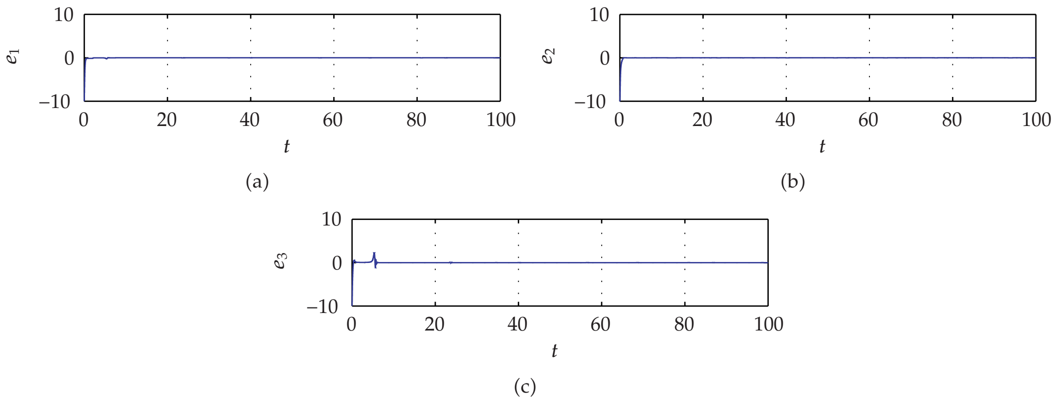

Figure 3: Numerically simulated error $e(t)$.

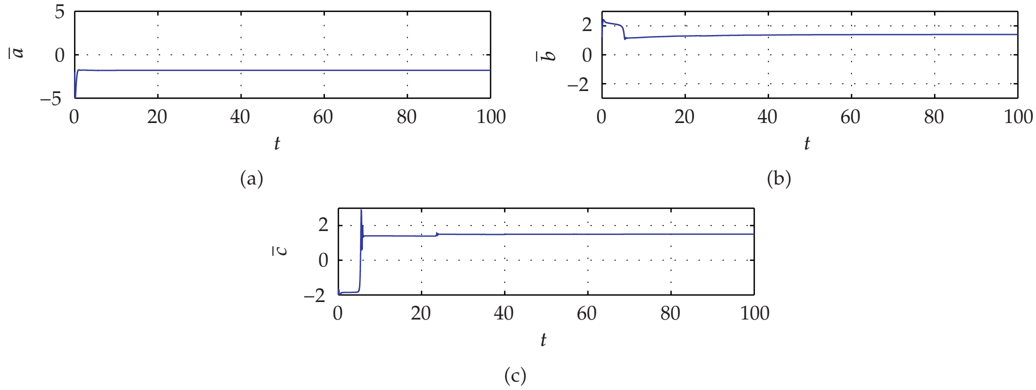

Figure 4: The temporal trajectory of $\overline{a}(t)$, $\overline{b}(t)$, and $\overline{c}(t)$.

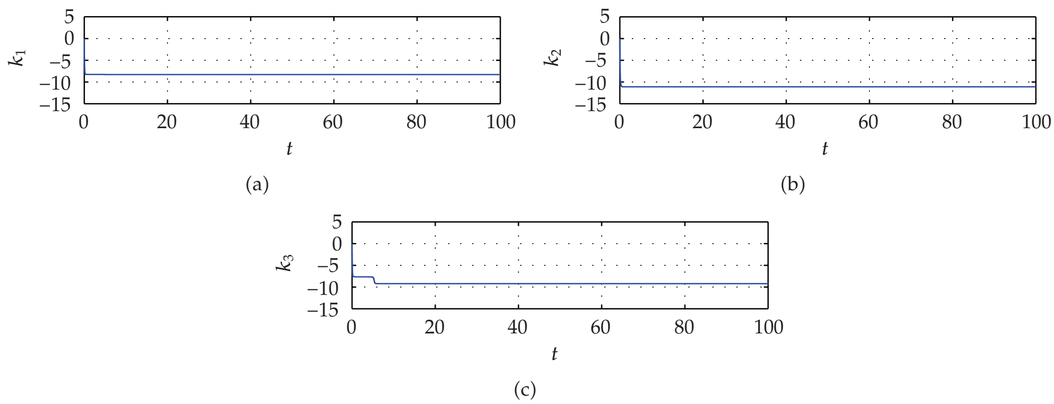

Figure 5: The evolution of feedback strength k.

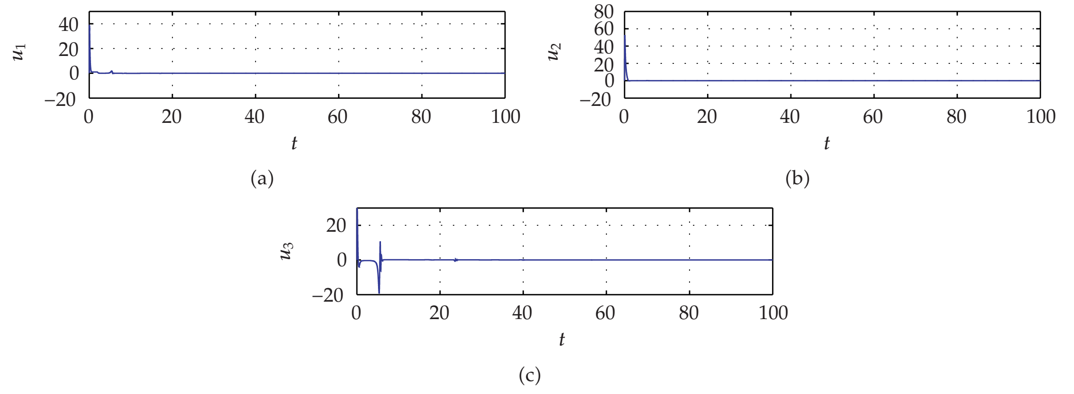

Figure 6: The evolution of adaptive controller $u(t)$.

We refer to system (4.5) with

$$A = \begin{pmatrix} a & 0 \\ 0 & -1 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \quad C = \begin{pmatrix} 2.0 & c \\ -5.0 & 3.2 \end{pmatrix}, \quad D = \begin{pmatrix} -1.6 & -0.1 \\ -0.18 & d \end{pmatrix}, \tag{4.6}$$

as the drive system, where parameters a, c, d are unknown. The response system has the following form:

$$\dot{y} = \overline{A}y + \overline{B}y(t-\tau) + \overline{C}f(y) + \overline{D}g(y(t-\tau)) + u(t), \tag{4.7}$$

where $y = [y_1, y_2]^T$, $u = [u_1, u_2]^T$, and $A = \begin{pmatrix} \bar{a} & 0 \\ 0 & -1 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$, $C = \begin{pmatrix} 2.0 & \bar{c} \\ -5.0 & 3.2 \end{pmatrix}$, and $D = \begin{pmatrix} -1.6 & -0.1 \\ -0.18 & \bar{d} \end{pmatrix}$.

It is easy to verify that Assumption 2.1 and Assumption 2.2 are satisfied. Defining $e = [e_1, e_2]^T = [y_1 - x_1, y_2 - x_1]^T$ and following the procedure proposed in the Section 2, the adaptive scheme can be designed as follows:

$$\begin{aligned} u_1(t) &= k_1(y_1 - x_1), \\ u_2(t) &= k_2(y_2 - x_2), \end{aligned} \tag{4.8}$$

with

$$\begin{aligned} \dot{k}_1 &= -\gamma_1 e_1^2, \\ \dot{k}_2 &= -\gamma_2 e_2^2, \\ \dot{\bar{a}} &= -\alpha_{11} e_1 y_1, \\ \dot{\bar{c}} &= -p_{12} e_1 \tanh y_2, \\ \dot{\bar{d}} &= -q_{22} e_2 \tanh(y_2(t-\tau)). \end{aligned} \tag{4.9}$$

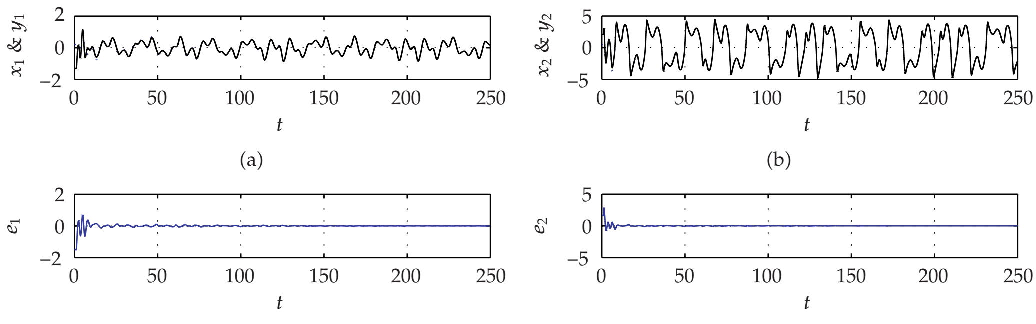

Figure 7: Numerically simulated time series x_1, x_2 (dotted line) and y_1, y_2 (black line) and error $e = [e_1, e_2]^T$.

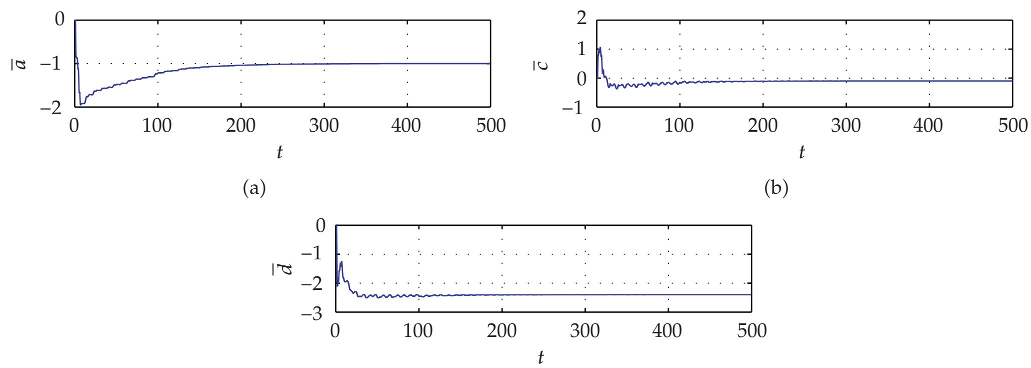

Figure 8: The temporal trajectory of $\overline{a}(t), \overline{c}(t)$, and $\overline{d}(t)$.

The initial values are chosen as $[x_1(t), x_2(t)]^T = [0.2, 0.5]^T$, $[y_1(t), y_2(t)]^T = [-1.3, 2.1]^T$, $[k_1(t), k_2(t)]^T = [0, 0]^T$ and $[\overline{a}(t), \overline{c}(t), \overline{d}(t)]^T = [0, 0, 0]^T$. The results are depicted in Figures 7–9.

By comparing Figures 4 and 8, one can see that in Figure 8 the estimates of the unknown parameters, $\overline{a}, \overline{c}, \overline{d}$, converge to the true value of $a = -1$, $c = -0.1$, and $d = -2.4$; however, for the delayed Rössler system, the estimates of unknown parameters, $\overline{a}, \overline{b}, \overline{c}$, converge to $a_0 = -1.85$, $b_0 = 1.4559$, and $c_0 = 1.3655$ which are not equal to the true value of a, b, c. It is reasonable because for Hopfield neural network system, $\overline{a}, \overline{c}, \overline{d}$ in the largest invariant set unique, which is equal to the true value of a, c, d; but for delayed Rössler system, $\overline{a}, \overline{b}, \overline{c}$ in the largest invariant set cannot be proven to be unique, which means what values $\overline{a}, \overline{b}, \overline{c}$ do converge to depend on the initial value of $\overline{a}, \overline{b}, \overline{c}$.

5. Concluding Remarks

The present paper dealt with the problem of parameters identification and synchronization of chaotic delayed systems containing uncertainties and time-varying delay. A simple but

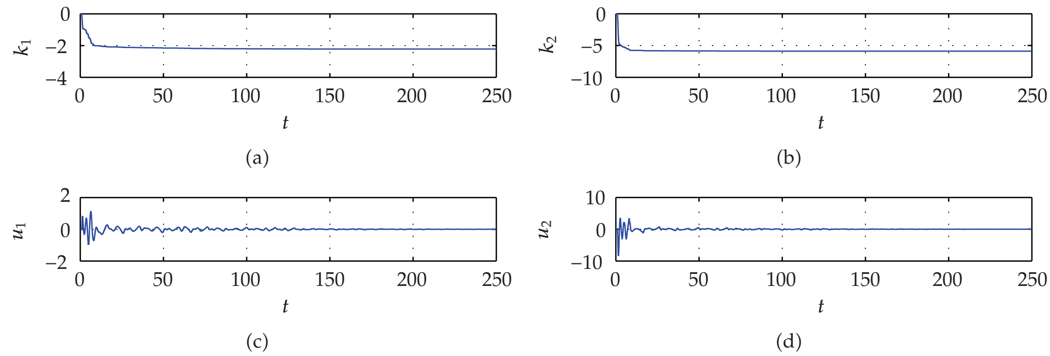

Figure 9: The evolution of feedback strength k and adaptive controller $u(t)$.

efficient adaptive regime was designed firstly to synchronize the chaotic dynamics between two coupled identical time-varying delayed chaotic systems, which is seriously proved in the framework of Krasovskii-Lyapunov theory [39] based on the famous Razumikhin condition and extended invariance principle for functional differential equations. Then, the proposed technique was utilized to estimate the unknown parameters containing in model. By virtue of the synchronization-based method, the unknown parameters of Rössler system were estimated exactly. But, because of the limitation of invariant principle, that is, it only guarantees the estimates of the unknown parameters to converge to the largest invariant set containing in $\{\dot{V} = 0\}$, hence, the synchronization-based adaptive schemes may fail if the largest invariant set containing in $\{(e(t), \overline{A}, \overline{B}, \overline{C}, \overline{D}, K) \mid \dot{V} = 0\}$ has more than one element. It is worth emphasizing that the largest invariant set depends tightly on the configuration of the system under consideration, which can be calculated by following some classical steps as given in [39] (or other literatures related to invariant principle theorem). The regime proposed here is rigorous and global. The effectiveness of the proposed scheme on synchronization and parameters identification is well demonstrated by the numerical examples.

Acknowledgments

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