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Book Review

Idempotency, edited by J. Gunawardena, 1998, Cambridge University Press, 443 pp., ISBN 0-521-55344-X, $80.

This book arose from the workshop on Idempotency held at Hewlett-Packard's Basic Research Institute in the Mathematical Sciences (BRIMS) in October 1994.

J. Gunawardena (who contributed an introductory survey paper) edited this collection of 24 papers on different aspects of idempotency. The authors have written compelling and authoritative contributions to the subject that has grown rapidly over the past 10 years. Although the idempotent analysis is a relatively new field of research (the concept was introduced by V. Kolokoltsov and V. Maslov in the late 80s [6]), it has deep roots in mathematical literature [2,3] and extensive areas of applications in optimisation theory, computer science, discrete event systems (DES), nonlinear partial differential equations and many other fields [1,5–12]. As a young discipline, the idempotent analysis still suffers from the lack of unity between different authors even in the key notations in this field. Although the book under review is not free from this deficiency, it benefits greatly from the fact that most of the papers contain brief introduction and comprehensive surveys of literature. Starting from the basic notion of dioids as idempotent semirings (a semiring is idempotent if $a \oplus a = a$ is valid for all elements a of this set), the reader is exposed gradually to a number of important subclasses of these algebraic structures such as quantales, tropical dioids and max-plus dioids that are used throughout the book. Deep algebraic roots of idempotency in the theory of lattice ordered groups are well referenced and the basic tools of studying topological spaces from algebraic point of view are explained. Due to the multidisciplinary nature of the book, these features of the book are important indeed. They help to keep the contents of the book pretty much self-contained and accessible to a wider community of readers.

Idempotency theory has an intrinsic connection with computer science. Not surprisingly, a large amount of contributions to the volume are devoted to theoretical computer science. One such paper is the paper by J.-E. Pin on tropical semirings, a convenient algebraic tool to decide whether a collection of objects is finite or infinite. Properties of pseudorings and pseudomodules as idempotent structures are studied in the paper by E. Wagneur. Idempotent algebraic structures such as tropical dioids $\mathbb{N}_{\min}$ play a critical role in answering the question of whether a given rational set has the finite power property. An important new result on this property for a class of monoids is announced in the paper by F. d'Alessandro and J. Sakarovitch.

It would be fair to say that automata theory is one of the most popular applications of idempotency in computer science. Exploiting an association of finite automata with mathematical models of computing devices, the paper by H. Leung deals with nondeterministic behaviour of finite automata in connection with their inner structure. The author of this paper puts forward the idea of considering the finite power property as a special case of the limitedness problem and proposes a novel topological approach to the solution of this problem. An important generalisation of finite automata is given via distance automata which not only can decide whether inputs are accepted, but can also assign a measure of cost for the effort to accept the input. The paper by D. Krob gives a survey on automata-theoretic aspects of a special type of semirings, called min–max-plus.

In addition to their contributions to theoretical computer science, the results obtained in the above papers have a number of applications. For example, techniques based on min–max-plus computations have proved to be useful in the analysis of DES, nonlinear partial differential equations such as the Hamilton–Jacobi–Bellman (HJB) equation, in the formal language theory and other areas.

Some papers in the volume give excellent examples of an intrinsic blend between computer science, systems theory, and other fields of engineering and mathematics. Indeed, using semirings G. Cohen, S. Gaubert and J.-P. Quadrat develop a new algebraic approach to modelling of dynamic behaviour of Continuous Timed Petri Nets (CTPN) and successfully apply their technique to the solution of resource optimisation problems.

Semirings used in the study of dynamical systems, automata theory, and formal languages have well-known limitations. A "many point" object generalisation of semirings has been proposed in the paper by G. Mascari and M. Pedicini. The authors of this paper attack a challenging problem in studying complex systems by modelling the internal structure of the system and its dynamical evolution in an integrated view. Their paper is a nice example of bridging a gap between mathematics and computer science by exploring an intrinsic connection between the mathematical structures (often encounted in the idempotent analysis and category theory) used in computer science to model the dynamical behaviour of algorithms and those considered in mathematics within which specific algorithms are designed. The authors deal with structural aspects of systems by means of a logical approach based on partially additive categories and developed within C^*-algebras. In another paper of the volume S. Gaubert and J. Mairesse use the logical modelling (in terms of trace monoids) to explore how to model a specific class of dynamical systems. They concentrate on task resource models and provide an algebraic framework for dealing with a number of scheduling problems.

Many systems appearing in manufacturing and business (e.g. automated manufacturing systems, office information systems), communication (e.g. air traffic control systems), computer science (e.g. computer networks) accept a description in terms of DES. These systems (that are often formed by interconnecting several subsystems with well-understood dynamics) are typically driven by asynchronous occurences of discrete events resulting from the interactions between the subsystems. Idempotency theory has substantially contributed to the study of such systems and a number of papers in the volume under review are devoted to this topic. Examples from studying manufacturing systems and systems with supervisory control are given in the introductory paper by J. Gunawardena who explains how such systems can be effectively modelled by max-plus matrices. D. Cofer and V. Garg study the supervisory control of logical DES and timed DES via automata known as finite state machines and timed event graphs (the latter is a special class of Petri nets), using the fact that both types of automata can be described effectively in terms of dioids. In the paper by R. Cuninghame-Green the author investigates the connection between maxipolynomials and DES in the case when the output of a DES has properties expressed in terms of maxpolynomials.

F. Baccelli and J. Mairesse argue in their paper that a usual characteristic of DES is the existence of some sources of randomness affecting their behaviour. They conclude that a natural framework to study them is the one of stochastic (often open) DES. The authors show that ergodic theorems on discrete event networks, such as queueing networks and/or Petri nets, are a generalisation of some stochastic operators. They also provide a good survey of the main ergodic theory techniques which are used in the study of iterates of monotone and homogeneous stochastic operators.

If M is a topological space and $f: M \to M$ is a continuous map, one of the fundamental questions to ask is the question about the behaviour of iterates $f^k(x)$ as $k \to \infty$. If f is nonexpansive with respect to a metric ρ on M (i.e. if $\rho(f(x), f(y)) \leq \rho(x, y) \forall x, y \in M$), then some information on the behaviour of these iterates can be provided. R. Nussbaum in his paper states interesting results on nonexpansive maps and indicates some specific classes of maps for which these results can be effectively applied. Such properties as nonexpansiveness and periodicity play an important role in the context of the theory of topical functions (a generalisation of Perron–Frobenius theory). In fact, topical functions can provide fundamental blocks for DES and a number of optimisation problems from deterministic optimal control and Markov Decision Processes can be treated efficiently within this theory.

Many DES, such as stochastic event graphs, timed automata, and mini–max systems can be modelled via stochastic recursive systems. Although qualitative theorems characterising the asymptotic behaviour of such systems are known, efficient quantitative methods are still lacking in the literature. By investigating the effectiveness of two approaches to estimate the behaviour of recursive systems (parallel simulation and exact Markovian analysis), B. Gaujal and A. Jean-Marie show that the analysis by means of Markov chains (based on the evolution equations) provides an alternative way to analyse systems where the "standard" approaches can fail. They also show how to represent a system in order to make computation procedures more efficient.

There are various approaches to analyse DES such as those based on Markov chains, queueing networks, Petri nets, formal languages theory, perturbation analysis, object-oriented technique, dioid algebra. In the paper by A. Gürel, O. Pastravanu and F. Lewis the authors focus on a specific sub-class of DES known as discrete event manufacturing systems (DEMS). By applying a system-theoretic approach, they present a DEMS control strategy which incorporates the classic principles of control.

In the late 80s V. Maslov observed that the solving operator of the Bellman's evolution equation is linear in the space of functions taking values in a certain idempotent semiring. Since that time the idempotency analysis has become an important tool in nonlinear differential equations and the close connection between functional analytical technique over dioids and viscocity solution theory has been recently revealed [4,9]. A number of papers in the volume are devoted to the development of this tool. In particular, a stochastic version of the Cauchy problem for the HJB equation is thoroughly analysed in the paper by V. Kolokoltsov and the theory of generalised solutions for this problem is developed on the basis of the methods of idempotent analysis. The problem is set in the context of the theory of stochastic optimisation.

Applications of idempotent analysis to optimisation problems go back to the 50s when it was noticed that some discrete optimisation problems can be linearised over a suitable dioid (see [2] and references therein). Since that time tools and methods of idempotent analysis in optimisation problems have been extensively developed. A number of interesting results in this direction are presented in the book under review. In particular, E. Walkup and G. Borriello propose a general method for optimising linear max-plus systems. V. Kolokoltsov and V. Maslov derive the differential equation describing the continuous dynamics of Pareto sets in multicriteria optimisation problems and use idempotent structures for the solution of this equation. In the paper by S. Samborskii the technique of the HJB equation is applied to the analysis of the Lagrange problem of variational calculus from the point of view of idempotent analysis.

The idea of considering the idempotent optimisation theory as the large deviation limit of classical probability theory goes back to Maslov's constructions of an idempotent measure theory with measures taking values in $\mathbb{R}_{\min}$ and an observation of the intrinsic connection between idempotent optimisation theory and classical probability theory [8]. A number of papers in the volume develop further this idea. In particular, following the theory of idempotent Maslov measures, a formalism analogous to probability calculus is obtained in the paper by M. Akian, J.-P. Quadrat and M. Viot for optimisation by replacing the classical structure of real numbers $(\mathbb{R}, +, \times)$ by the idempotent semifield obtained by endowing the set $\mathbb{R} \cup \{+\infty\}$ with the "min" and "+" operations. Using various transformations, P. del Moral in his paper shows that the Maslov optimisation processes and Markov stochastic processes can be mapped into each other. He clarifies the relationship between optimisation and estimation problems by introducing some transformations between performance and probability measures. One of these transformations, namely the Log–Exp transform, provides a bijective correspondence between Maslov performances and deterministic optimisation problems on the one hand, and Markov probabilities and filtering problems on the other hand. As it is explained in the paper by P. del Moral and G. Salut, this idea leads to a very powerful approach in the numerical solution of many infinite-dimensional problems.

A quite general view on the subject of idempotency is provided in the two concluding papers of the volume. In his paper V. Maslov gives a new interpretation of Gibbs' and Mott-Smith's theories and generalises his result in a nontrivial way to describe second-kind phase transitions. Finally, inspired by the correspondence principle of quantum mechanics G. Litvinov and V. Maslov argue that the successive application of the idempotent correspondence principle leads to a diversity of results, among them a methodology for constructing unifying algorithms for scientific and technical computation as well as a hardware realisation of these algorithms (see also [7]). These ideas put new horizons for applications of idempotent analysis.

The book could not cover all aspects of idempotency. It does, however, leave the reader aware of main tools of idempotent analysis and its intrinsic connection with other branches of applied and pure mathematics, as well as computer science, engineering and theoretical physics.

A work of this size can rarely reach the press without a few misprints and inaccuracies. For example, a number of references on pp. 41–49 (such as AQV, Kolb, Kro, Morb, MP, MSc) are given incorrectly, while others (such as Rom67, Vor67) are incomplete. The nonstationary problem (5.2) appears unexpectedly on p. 29, since the reader is promised an example of nondifferentiable solutions to stationary Hamilton–Jacobi–Bellman equation (5.1). Different references to the same paper are given on p. 47 (Ref. MSb) and p. 380 (Ref. [14]).

Despite these glitches, I found the book interesting. I am confident that the book under review would be a very valuable addition to the bookshelf of anybody who wishes to work in this field or wants to know more about the subject. Those researchers who are familiar with the subject will find a variety of new challenging problems (e.g., pp. 25–27, 110, 237). Inspiring collection of applications should secure the book a good place in the existing literature on the topic (in particular, [5–10]) and the variety of multidisciplinary approaches within the common topic of idempotency should be of interest to applied and pure mathematicians, theoretical physicists, engineers and computer scientists.

References

[1] C.G. Cassandras, *Discrete Event Systems: Modeling and Performance Analysis*, Aksen-Irwin, Homewood, IL, 1993.

[2] R.A. Cuninghame-Green, *Minimax Algebra*, Springer-Verlag, Berlin, 1979.

[3] L. Fuchs, *Partially Ordered Algebraic Systems*, Pergamon Press, Oxford, 1963.

[4] M. Gondran, Minplus analysis, *Comptes Rendus, De L'Academie Des Sciences. Serie I, Mathematique*, **323**, 371, 1996.

[5] V.N. Kolokoltsov and V.P. Maslov, *Idempotent Analysis and its Applications*, Kluwer Academic Publisher, Boston, 1997.

[6] V.N. Kolokoltsov and V.P. Maslov, Idempotent analysis as a tool of control theory and optimal synthesis I and II, *Functional Analysis and Its Applications*, **23**, 1–14; 53–62, 1989.

[7] G.L. Litvinov and V.P. Maslov, Idempotent mathematics: a correspondence principle and its applications to computing, *Russian Mathematical Survey*, **51**, 1210–1211, 1996.
[8] V.P. Maslov, *Operational Methods*, Mir, Moscow, 1976.
[9] V. Maslov and S. Samborskii (Eds.), Idempotent analysis, *Advances in Soviet Mathematics*, Vol. 13, American Mathematical Society, Providence, R.I., 1992.
[10] V. Maslov and V. Kolokoltsov, *Idempotent Analyisis and its application to optimal control theory*, Nauka, Moscow, 1994.
[11] R.V.N. Melnik, Dynamic system evolution and Markov chain approximation, *Discrete Dynamics in Nature and Society*, **2**, 7–39, 1998.
[12] J. Rhodes (Ed.), *Monoids and Semigroups with Applications*, World Scientific, Singapore, New Jersey, 1991.

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