Fundamental Concepts in Modern Analysis

This page is intentionally left blank

Fundamental Concepts in Modern Analysis

Vagn Lundsgaard Hansen

Department of Mathematics
Technical University of Denmark

World Scientific
Singapore • New Jersey • London • Hong Kong

Published by

World Scientific Publishing Co. Pte. Ltd.

P O Box 128, Farrer Road, Singapore 912805

USA office: Suite 1B, 1060 Main Street, River Edge, NJ 07661

UK office: 57 Shelton Street, Covent Garden, London WC2H 9HE

British Library Cataloguing-in-Publication Data
A catalogue record for this book is available from the British Library.

FUNDAMENTAL CONCEPTS IN MODERN ANALYSIS

ISBN 981-02-3894-0

This book is printed on acid-free paper.

Printed in Singapore by Uto-Print

Preface

Many advanced mathematical disciplines, such as global differential geometry, the calculus of variations, dynamical systems and the theory of Lie groups, have a common foundation in general topology and analysis in normed vector spaces. The purpose of this book is to introduce students to basic parts of this foundation and to give them a firm basis for further studies of mathematics.

This book derives from a course at the advanced undergraduate or beginning graduate level offered to engineering students at the Technical University of Denmark. The intention of the course is to give the mathematically inclined and interested engineering students an opportunity to go into some depth with fundamental mathematical notions from analysis that are important not only from a mathematical point of view but also occur frequently in theoretical parts of the engineering sciences, and to introduce them to proofs in mathematics and to mathematical reasoning. It is my hope that the book will also appeal to university students in mathematics and in the physical sciences.

The book opens with a study of fundamental concepts from general topology: metric spaces, topological spaces, compactness, connectedness, function spaces. Then follows a study of fundamental concepts in analysis: normed vector spaces, differentiability in normed vector spaces, and the Inverse Function Theorem in Banach spaces. The theory developed is applied to lay the foundations of differentiable manifolds with a view towards global analysis and differential geometry. In the last two chapters we offer elementary introductions to singularity theory in finite dimensions, respectively Morse theory in infinite dimension.

Major parts of the book are a translation and revision of the lecture notes “Grundbegreber i den Moderne Analyse” for the above mentioned course, published in 1986 by the Department of Mathematics, Technical University of Denmark. The English translation of the first three chapters has been prepared with the very efficient help of Dan Erik Krarup Sørensen. The figures were drawn by Beth Beyerholm.

I am grateful to several people for valuable comments on the material in the book. In particular, I am indebted to the students who tested the material in practice. Among them, Jonas Bjerg, Peter Gross, Lars Gæde, Christian Henriksen, Jan Kristensen, Jens Christian Larsen, Anders Høst-Madsen, Thomas Randrup, Henrik Obbekær Rasmussen, Peter Røgen and Dan Erik Krarup Sørensen deserve particular mentioning for detailed comments. Jennifer Brockbank suggested many improvements in the translation of the first chapters.

My late colleague Niels Vigand Pedersen was a most valued discussion partner at the early stages of the Danish book.

It is a particular joy to thank my good colleague Poul Hjorth who has lectured on the material in the book and has contributed many valuable remarks. As a special favour, he has read most of the text and has suggested several improvements in the language. In this connection, I am also very grateful to Robert Sinclair.

Lyngby, February 1999 Vagn Lundsgaard Hansen

Contents