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*Research Article*

# Geometrically Tunable Transverse Electric Field in Multilayered Structures

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Appearance of a transverse component in dc electric field with respect to the applied current is investigated in periodic multilayer composite structures made of nanometer-to-micrometer scale alternating layers of two different homogeneous and isotropic conducting materials. Dependence of the transverse electric field on geometrical orientation of the layers is examined using the coordinate transformation approach. Electric field bending angle as a function of the layers' resistivity ratio is studied in detail. It is shown that both the direction and the magnitude of the field can be changed using orientation angle of the layers as a tuning parameter.

## 1. Introduction

Recent realization of metamaterial devices based on cloaking and concentration effects for a static magnetic field [1–5] became one of the most up-to-date applications of transformation electrodynamics [6–9]. Analogous methods have been widely used to design devices of various type controlling manipulation of heat [10–13], acoustic [14], and material [15] waves. In [16], an analytical design of a multilayered composite structure allowing redirection (bending) of dc magnetic induction vector with respect to the applied external magnetic field has been presented. The aim of this paper is an investigation of dc electric field redirection relative to the direction of the applied electric current in stratified structures. Using coordinate transformation approach analogous to that in [16], we will show the possibility for the appearance of transverse (with respect to the current) component of the field in such a periodic multilayer structure. The ratio of the transverse and longitudinal components is very sensitive to geometrical orientation of the alternating isotropic layers. It means that both the direction and magnitude of the field can be changed using orientation angle of the layers as a tuning parameter. We hope that geometrically tunable anisotropic resistivity in multilayered structures predicted in this paper can find use in various devices based on the control and manipulations of dc electric field.

## 2. Model and Theory

Consider a periodic multilayer structure consisting of alternating layers of two different materials with isotropic resistivity $\rho_1$ and $\rho_2$, static dielectric permittivity $\varepsilon_1$, $\varepsilon_2$, and thickness $l_1$, $l_2$ in micrometer-to-nanometer scale, as both $l_1$ and $l_2$ are much larger than the lattice constant of the constituent components of the medium. The structure in the presence of dc electric current can be considered as an effective anisotropic homogeneous medium (a metamaterial) with electric field vector

$$\mathbf{E} = \widehat{\rho}_{\text{ef}}\mathbf{J}, \tag{1}$$

where $\mathbf{J}$ is the applied electric current density, $\widehat{\rho}_{\text{ef}} = \widehat{\sigma}_{\text{ef}}^{-1}$ is the effective resistivity tensor, and $\widehat{\sigma}_{\text{ef}}$ is the effective conductivity tensor of the composite structure. In the coordinate system

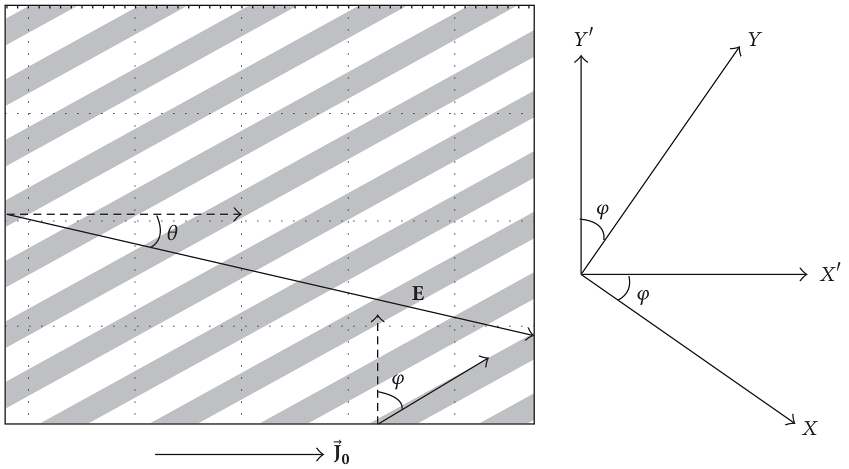

Figure 1: Cross section of the multilayer composite in the $x'y'$-plane with layers rotated on angle $\varphi$ around the $z$-axis. $\theta$ is the bending angle of dc electric field. The $x$-axis is perpendicular to the plane of the layers.

$xyz$ with the $x$-axis perpendicular to the plane of the layers, tensor $\widehat{\rho}_{\text{ef}}$ and effective permittivity tensor $\widehat{\varepsilon}_{\text{ef}}$ have the form

$$\widehat{\rho}_{\text{ef}} = \begin{pmatrix} \rho_{xx} & 0 & 0 \\ 0 & \rho_{yy} & 0 \\ 0 & 0 & \rho_{zz} \end{pmatrix}, \qquad \widehat{\varepsilon}_{\text{ef}} = \begin{pmatrix} \varepsilon_{xx} & 0 & 0 \\ 0 & \varepsilon_{yy} & 0 \\ 0 & 0 & \varepsilon_{zz} \end{pmatrix}, \tag{2}$$

where

$$\rho_{xx} = \frac{(\rho_1 l_1 + \rho_2 l_2)}{d}, \qquad \rho_{yy} = \rho_{zz} = \frac{\rho_1 \rho_2 d}{\rho_1 l_2 + \rho_2 l_1}, \tag{3a}$$

$$\varepsilon_{xx} = \frac{\varepsilon_1 \varepsilon_2 d}{\varepsilon_1 l_2 + \varepsilon_2 l_1}, \qquad \varepsilon_{yy} = \varepsilon_{zz} = \frac{(\varepsilon_1 l_1 + \varepsilon_2 l_2)}{d}. \tag{3b}$$

and $d = l_1 + l_2$ is the period of the structure. Note that parallel to the plane of the layers components $\rho_{yy}$ and $\rho_{zz}$ are always less than the transverse component $\rho_{xx}$.

An additional anisotropy in the resistivity and permittivity can be introduced rotating the layers in the structure around the $z$-axis on an angle $\varphi$ (see Figure 1). Such a rotation can be realized using coordinate transformations [16]

$$x' = x\cos\varphi + y\sin\varphi, \qquad y' = -x\sin\varphi + y\cos\varphi, \qquad z' = z, \tag{4}$$

where $x'y'z'$ is the reference coordinate system fixed to the sample of rectangular parallelepiped form while $xyz$ system of the coordinate axes is connected with the layers which are arranged in the direction perpendicular to the $x$-axis. The angle $\varphi$ is assumed to be positive if the rotation is in counterclockwise direction.

A static electric field in a source free space is described by Maxwell equations [17]

$$\operatorname{curl}\mathbf{E} = 0, \qquad \operatorname{div}\mathbf{J} = 0. \tag{5}$$

To keep (5) form-invariant under transformations (4), the effective conductivity and resistivity tensors should be transformed as

$$\widehat{\sigma'} = \frac{\widehat{A}\widehat{\sigma}_{\text{ef}}\widehat{A^T}}{\det\left|\widehat{A}\right|}, \qquad \widehat{\rho'} = \widehat{A}\widehat{\rho}_{\text{ef}}\widehat{A^T}\det\left|\widehat{A}\right|, \tag{6}$$

respectively, where $\widehat{A}$ is the Jacobian transformation matrix

$$\widehat{A} = \begin{pmatrix} \cos\varphi & \sin\varphi & 0 \\ -\sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{7}$$

and $\widehat{A^T} = \widehat{A^{-1}}$ is the transpose of $\widehat{A}$. Then for the effective resistivity tensor in the coordinate system $x'y'z'$ we obtain

$$\widehat{\rho'} = \begin{pmatrix} \rho_{xx}\cos^2\varphi + \rho_{yy}\sin^2\varphi & \left(\rho_{yy} - \rho_{xx}\right)\sin\varphi\cos\varphi & 0 \\ \left(\rho_{yy} - \rho_{xx}\right)\sin\varphi\cos\varphi & \rho_{xx}\sin^2\varphi + \rho_{yy}\cos^2\varphi & 0 \\ 0 & 0 & \rho_{zz} \end{pmatrix}. \tag{8}$$

Let the static current of density $\mathbf{J_0}$ be applied along the $x'$ axis. Then from (3a) and (8) one can conclude that the electric field vector $\mathbf{E}^{/} = \widehat{\rho}'\mathbf{J}_0$ inside the composite is bent with respect to $\mathbf{J}_0$ on the angle $\theta$, where

$$\tan\theta = \frac{E_{y'}}{E_{x'}} = -\frac{\beta\tan\varphi}{1+\beta+\tan^2\varphi}, \tag{9}$$

$$\beta \equiv \frac{l_1 l_2 (\rho_1 - \rho_2)^2}{\rho_1\rho_2 d^2}. \tag{10}$$

The negative sign in (9) indicates that, for a positive angle $\varphi$, the vector $\mathbf{E}^{/}$ is bending in clockwise direction (see Figure 1). Transversal component of the field is described by

$$E_{y'} = \rho'_{yx} J_0 = -\frac{1}{2}\beta\rho_{yy} J_0 \sin 2\varphi, \tag{11}$$

and for a given value of the current density it can easily be tuned by changing the layers' orientation angle $\varphi$. It is important to note that not only the direction, but also the magnitude of the field is a function of the angle $\varphi$:

$$E' = J_0\left(\rho_{xx}^2\cos^2\varphi + \rho_{yy}^2\sin^2\varphi\right)^{1/2}. \tag{12}$$

Consequently, $E'$ decreases monotonically with increasing $\varphi$ from $E' = \rho_{xx}J_0$ at $\varphi = 0$ to the value $E' = \rho_{yy}J_0$ at $\varphi = \pi/2$. It is easy to see that for any value of $\varphi$, in the range $0 < \varphi < \pi/2$, the absolute value of the bending angle is less than $\varphi$, and that with increasing $\varphi$ $|\theta|$ initially increases from zero and runs up to the maximum value at

$$\varphi_{\max} = \tan^{-1}\left[(1+\beta)^{1/2}\right], \tag{13}$$

given by

$$|\theta|_{\max} = \tan^{-1}\left[\frac{(1+\beta)^{-1/2}\beta}{2}\right], \tag{14}$$

and then it decreases monotonically and vanishes at $\varphi = \pi/2$. For all possible values of parameter $\beta$, $\varphi_{\max}$ is always larger than $\pi/4$.

Note also that the bending angle $\theta$ is an odd function of $\varphi$: $\theta(-\varphi) = -\theta(\varphi)$. It means that using samples with opposite rotated configuration of the layers, we can redirect the electric field vector upwards or downwards with respect to the applied current direction.

The dependence of $|\theta|$ on the thickness ratio $l_1/l_2$ at given values of $\varphi$, $\rho_1$, and $\rho_2$ is analogous to the behavior of the function $|\theta(\varphi)|$: $|\theta|$ increases with increasing $l_1/l_2$, has a maximum at $l_1/l_2 = 1$, and then decreases monotonically.

## 3. Electric Field Vector Redirection Details

For simplicity, we will further restrict ourselves to the consideration of the case when different layers of the structure have the same thickness, that is, when the function $|\theta(l_1/l_2)|$ has a maximum, and assume that $\rho_1 > \rho_2$. Then (9) can be rewritten in the form

$$\tan\theta = -\frac{\tan\varphi}{1+4\gamma(1-\gamma)^{-2}/\cos^2\varphi}, \tag{15}$$

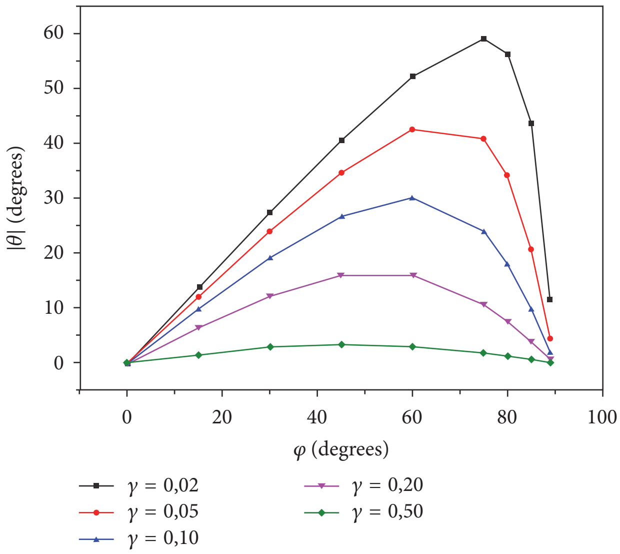

Figure 2: Dependence of the electric field bending angle $|\theta|$ on the layers' orientation angle $\varphi$ at five different values of the resistivity ratio $\gamma = \rho_2/\rho_1$.

where $\gamma = \rho_2/\rho_1$. One can see that at a given value of the angle $\varphi$, the bigger $|\theta|$ is, the less $\gamma$ is (i.e., the larger the contrast between resistivity values of individual layers is). We have to note that this conclusion is true not only at $l_1 = l_2$ but also for any value of the ratio $l_1/l_2$. Note also that, with decreasing $\gamma$, $\varphi_{\max}$ [see (13)] is shifted monotonically to the higher values while as a function of $l_1/l_2$ it has a maximum given by $\tan\varphi_{\max} = (\rho_1+\rho_2)/2(\rho_1\rho_2)^{1/2}$ at $l_1 = l_2$.

In Figure 2, the dependence of the bending angle $|\theta|$ on the angle $\varphi$ (in degrees) is demonstrated for five different values of the ratio $\gamma$, in the region $\varphi \le \pi/2$. Note that in the case when the second layer in a periodic unit of the multilayer structure is made from a perfect electrical conductor ($\rho_2 \to 0$), the absolute value of the bending angle coincides with the layers' orientation angle $\varphi$. It may be interesting to note also that in the opposite case when one of the layers in the periodic unit is an ideal isolating material ($\rho_1 \to \infty$), the bending angle has the same absolute value: $\theta = -\varphi$. In both these limiting cases, the parameter $\gamma \to 0$.

## 4. Conclusions

The possibility of redirection of dc electric field with respect to the applied current is studied in periodic multilayer structures consisting of alternating layers of two isotropic materials with different resistivity values. It is shown that not only the direction but also the magnitude of the field can be changed using the layers' orientation angle $\varphi$ as a tuning parameter.

In this paper, we have found effective resistivity tensor of the composite structure using method of effective anisotropic homogeneous medium. Further, using coordinate transformation approach analogous to that in transformation optics, an analytical expression for the electric field bending angle $\theta$ has been found for arbitrary orientation of the layers with respect to the applied static current. It is shown that both the magnitude and sign of $\theta$ are very sensitive to the orientation of the layers. Dependence of $\theta$ on the layers' orientation angle $\varphi$ is examined in detail for different values of the resistivity ratio. It is shown that with increasing $\varphi$, the absolute value of the bending angle increases from zero and reaches a maximum at $\varphi = \varphi_{\max}$ (which is always larger than $\pi/4$) and then decreases monotonically and vanishes at $\varphi = \pi/2$. Unlike that, with increasing $\varphi$ the magnitude of the field decreases monotonically. In two opposite particular cases at which the resistivity ratio $\rho_2/\rho_1 \ll 1$, the bending angle $\theta = -\varphi$. For a given value of the thickness ratio $l_1/l_2$ and a given orientation of the layers relative to the applied current, the larger the bending angle is the bigger the contrast between resistivity values of the alternating layers is.

The possibility of the tunable field bending in multilayered structures by changing $\varphi$ can find widespread applications in various devices based on the control and manipulation of dc electric field such as solar cells, batteries, multilayered thin-film sensors, and flat-panel displays and in computer chips.

## Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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