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*Research Article*

# Coefficient Inequalities for a Subclass of $p$-Valent Analytic Functions

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The aim of this paper is to study the problem of coefficient bounds for a newly defined subclass of $p$-valent analytic functions. Many known results appear as special consequences of our work.

## 1. Introduction

Let $\mathcal{A}(p)$ denote the class of functions $f(z)$ of the form

$$f(z)=z^p+\sum_{n=p+1}^{\infty}a_nz^n,\quad (p\in\mathbb{N}), \tag{1}$$

which are analytic and multivalent in the open unit disk $\mathbb{U}=\{z : |z|<1\}$. Also let $\mathcal{S}_p^*$ and $\mathcal{K}_p$ denote the well-known classes of $p$-valent starlike functions and $p$-valent convex functions, respectively.

For $f(z)\in\mathcal{A}(p)$ given by (1) and $g(z)\in\mathcal{A}(p)$ given by

$$g(z)=z^p+\sum_{n=p+1}^{\infty}b_nz^n,\quad (p\in\mathbb{N}), \tag{2}$$

the Hadamard product (or convolution) of $f(z)$ and $g(z)$ is given by

$$(f*g)(z)=z^p+\sum_{n=p+1}^{\infty}a_nb_nz^n,\quad (z\in\mathbb{U}). \tag{3}$$

Motivated by Ruscheweyh operator [1], Goel and Sohi [2] introduced a differential operator $D^{\delta+p-1}$ for $p$-valent analytic functions given by

$$D^{\delta+p-1}f(z)=\frac{z}{(1-z)^{\delta+p}}*f(z)=z^p+\sum_{n=p+1}^{\infty}\varphi_n(\delta)a_nz^n, \tag{4}$$

with $\delta>-p$,

$$\varphi_n(\delta)=\frac{(\delta+p)_{n-p}}{(n-p)!}, \tag{5}$$

and $(x)_n$ is a Pochhammer symbol given by

$$(x)_n=\begin{cases}1, & n=0,\\ x(x+1)(x+2)\cdots(x+n-1), & n\in\mathbb{N}.\end{cases} \tag{6}$$

It is obvious that when $\delta$ is any integer greater than $-p$,

$$D^{\delta+p-1}f(z)=\frac{z^p\left(z^{\delta-1}f(z)\right)^{(\delta+p-1)}}{(\delta+p-1)!}. \tag{7}$$

The following identity can be easily established:

$$(\delta+p)D^{\delta+p}f(z)=\delta D^{\delta+p-1}f(z)+z\left(D^{\delta+p-1}f(z)\right)'. \tag{8}$$

Using the generalized Ruscheweyh operator, we define a subclass of $\mathcal{VD}_p^{\lambda}(\delta, b, \alpha, \beta)$ of $p$-valent analytic functions as follows.

*Definition 1.* An analytic $p$-valent function $f(z)$ of the form (1) belongs to the class $\mathcal{VD}_p^{\lambda}(\delta, b, \alpha, \beta)$, if and only if

$$\operatorname{Re}\left\{e^{i\lambda}\left(1-\frac{2}{b}+\frac{2}{b}\frac{D^{\delta+p}f(z)}{D^{\delta+p-1}f(z)}\right)\right\} > \alpha\left|\frac{2}{b}\left(\frac{D^{\delta+p}f(z)}{D^{\delta+p-1}f(z)}-1\right)\right|+\beta\cos\lambda, \tag{9}$$

where $\alpha \geq 0$, $b \in \mathbb{C}\setminus\{0\}$, $\delta > -p$, $\lambda$ is real with $|\lambda| < (\pi/2)$, and $0 \leq \beta < 1$.

By giving specific values to $\alpha$, $\beta$, $\lambda$, $p$, $b$, and $\delta$ in $\mathcal{VD}_p^{\lambda}(\delta, b, \alpha, \beta)$, we obtain many important subclasses studied by various authors in earlier papers; see for details [3–6]; we list some of them as follows:

(i) $\mathcal{VD}_1^{\lambda}(0,2,0,0) \equiv \mathcal{S}_{\lambda}^{*}$ and $\mathcal{VD}_1^{\lambda}(1,1,0,0) \equiv \mathcal{K}_{\lambda}$, studied by Spacek [7] and Robertson [8], respectively; for the advancement work see [9–11];

(ii) $\mathcal{VD}_1^{0}(0,2,\alpha,\beta) \equiv \mathcal{SD}(\alpha,\beta)$ and $\mathcal{VD}_1^{0}(1,1,\alpha,\beta) \equiv \mathcal{KD}(\alpha,\beta)$, studied by both Owa et al. and Shams et al. [12, 13];

(iii) $\mathcal{VD}_1^{\lambda}(0,2,1,0) \equiv \mathcal{USP}(\lambda)$ and $\mathcal{VD}_1^{\lambda}(1,1,1,0) \equiv \mathcal{UCSP}(\lambda)$, introduced by Ravichandran et al. [14];

(iv) $\mathcal{VD}_1^{0}(\delta,b,\alpha,\beta) \equiv \mathcal{VD}(\delta,b,\alpha,\beta)$, considered by Latha [15];

(v) $\mathcal{VD}_1^{0}(0,2,0,\beta) \equiv \mathcal{S}^{*}(\beta)$ and $\mathcal{VD}_1^{0}(1,1,0,\beta) \equiv \mathcal{K}(\beta)$, the well-known classes of starlike and convex functions of order $\beta$.

From the above special cases we note that this class provides a continuous passage from the class of starlike functions to the class of convex functions.

We will assume throughout our discussion, unless otherwise stated, that $\alpha \geq 0$, $0 \leq \beta < 1$, $\delta > -1$, $\lambda$ is real with $|\lambda| < (\pi/2)$, and $b \in \mathbb{C}\setminus\{0\}$.

## 2. Main Results

**Theorem 2.** *Let $f(z) \in \mathcal{VD}_p^{\lambda}(\delta, b, \alpha, \beta)$ with $0 \leq \alpha \leq \beta$. Then $f(z) \in \mathcal{VD}_p^{\lambda}(\delta, b, 0, \zeta)$, where $\zeta = (\beta-\alpha)/(1-\alpha)$.*

*Proof.* Let $f(z) \in \mathcal{VD}_p^{\lambda}(\delta, b, \alpha, \beta)$. Then we obtain

$$\begin{aligned}\operatorname{Re} e^{i\lambda}\left\{1-\frac{2}{b}+\frac{2}{b}\frac{D^{\delta+p}f(z)}{D^{\delta+p-1}f(z)}\right\} &> \alpha\left|\frac{2}{b}\left(\frac{D^{\delta+p}f(z)}{D^{\delta+p-1}f(z)}-1\right)\right|+\beta\cos\lambda \\ &> \alpha\operatorname{Re} e^{i\lambda}\left(1-\frac{2}{b}+\frac{2}{b}\frac{D^{\delta+p}f(z)}{D^{\delta+p-1}f(z)}\right) \\ &\quad -\alpha\operatorname{Re} e^{i\lambda}+\beta\cos\lambda,\end{aligned} \tag{10}$$

and this implies

$$\operatorname{Re}\left[e^{i\lambda}\left(1-\frac{2}{b}+\frac{2}{b}\frac{D^{\delta+p}f(z)}{D^{\delta+p-1}f(z)}\right)\right] > \zeta\cos\lambda. \tag{11}$$

Also if $0 \leq \alpha \leq \beta$, then we can easily obtain

$$0 \leq \zeta < 1, \tag{12}$$

and this completes the proof. □

**Theorem 3.** *If $f(z) \in \mathcal{VD}_p^{\lambda}(\delta, b, \alpha, \beta)$, then*

$$|a_2| \leq \frac{|b|\,|\eta|}{|1-\alpha|}, \tag{13}$$

*and*

$$\left|a_{n+p-1}\right| \leq \frac{(\delta+p)\,|b|\,|\eta|}{(n-1)\,|1-\alpha|\,\varphi_{n+p-1}(\delta)} \times\prod_{j=1}^{n-2}\left(1+\frac{(\delta+p)\,|b|\,|\eta|}{j\,|1-\alpha|}\right), \quad n \geq 3, \tag{14}$$

*where $\varphi_{n+p-1}(\delta)$ is given by* (5) *and*

$$\eta = (1-\beta)\cos\lambda + i(1-\alpha)\sin\lambda. \tag{15}$$

*Proof.* Let $f(z) \in \mathcal{VD}_p^{\lambda}(\delta, b, \alpha, \beta)$. Then by Theorem 2, we have

$$\operatorname{Re}\left[e^{i\lambda}\left(1-\frac{2}{b}+\frac{2}{b}\frac{D^{\delta+p}f(z)}{D^{\delta+p-1}f(z)}\right)\right] > \left(\frac{\beta-\alpha}{1-\alpha}\right)\cos\lambda, \quad (z \in E). \tag{16}$$

Let us define $p(z)$ by

$$e^{i\lambda}\left(1-\frac{2}{b}+\frac{2}{b}\frac{D^{\delta+p}f(z)}{D^{\delta+p-1}f(z)}\right) = \left[\left(\frac{1-\beta}{1-\alpha}\right)p(z)+\left(\frac{\beta-\alpha}{1-\alpha}\right)\right]\cos\lambda + i\sin\lambda. \tag{17}$$

Then $p(z)$ is analytic in $E$ with $p(0) = 1$ and $\operatorname{Re} p(z) > 0$, $z \in E$. Let

$$p(z) = 1 + \sum_{n=1}^{\infty} p_n z^n, \quad z \in E. \tag{18}$$

Then (17) becomes

$$1 - \frac{2}{b} + \frac{2}{b}\frac{D^{\delta+p} f(z)}{D^{\delta+p-1} f(z)} = 1 + \frac{(1-\beta)\cos\lambda + i(1-\alpha)\sin\lambda}{e^{i\lambda}(1-\alpha)} \sum_{n=1}^{\infty} p_n z^n. \tag{19}$$

That is,

$$2e^{i\lambda}(1-\alpha)\left[D^{\delta+p} f(z) - D^{\delta+p-1} f(z)\right] = b\eta D^{\delta+p-1} f(z) \sum_{n=1}^{\infty} p_n z^n, \tag{20}$$

where $\eta$ is given by (15). Using (8) in (20), we obtain

$$2e^{i\lambda}(1-\alpha)\left[z\left(D^{\delta+p-1} f(z)\right)' - pD^{\delta+p-1} f(z)\right] = b(\delta+p)\eta D^{\delta+p-1} f(z) \sum_{n=1}^{\infty} p_n z^n, \tag{21}$$

or, equivalently,

$$2e^{i\lambda}(1-\alpha)\left[\sum_{k=p+1}^{\infty}(k-p)\varphi_k(\delta) a_k z^k\right] = b(\delta+p)\eta\left[z^p + \sum_{k=p+1}^{\infty}\varphi_k(\delta) a_k z^k\right]\left(\sum_{n=1}^{\infty} p_n z^n\right). \tag{22}$$

Comparing the coefficients of $z^{n+p-1}$ on both sides,

$$2e^{i\lambda}(1-\alpha)(n-1)\varphi_{n+p-1}(\delta) a_{n+p-1} = b(\delta+p)\eta\left\{p_1 a_{n+p-2}\varphi_{n+p-2}(\delta) + \cdots + p_{n-1}\right\}. \tag{23}$$

Taking absolute on both sides and then applying the coefficient estimates $|p_n| \le 2$ for Caratheodory functions [3], we have

$$\left|a_{n+p-1}\right| \le \frac{|b|(\delta+p)|\eta|}{(n-1)|1-\alpha|\varphi_{n+p-1}(\delta)} \times \left\{1 + \varphi_{p+1}(\delta)\left|a_{p+1}\right| + \cdots + \varphi_{n+p-2}(\delta)\left|a_{n+p-2}\right|\right\}. \tag{24}$$

We apply mathematical induction on (24). So for $n = 2$,

$$\left|a_{p+1}\right| \le \frac{|b||\eta|}{|1-\alpha|}, \tag{25}$$

which shows that (13) is true. For $n = 3$,

$$\left|a_{p+2}\right| \le \frac{|b||\eta|(\delta+p)}{|1-\alpha|(2)\varphi_{p+2}(\delta)}\left\{1 + \varphi_{p+1}(\delta)\left|a_{p+1}\right|\right\}, \tag{26}$$

and using (13), we have

$$\left|a_{p+2}\right| \le \frac{|b||\eta|(\delta+p)}{(1-\alpha)(2)\varphi_{p+2}(\delta)}\left\{1 + \frac{|b||\eta|(\delta+p)}{(1-\alpha)}\right\}. \tag{27}$$

Therefore, (14) holds for $n = 3$.

Assume that (14) is true for $n = k$; that is,

$$\left|a_{k+p-1}\right| \le \frac{|b||\eta|(\delta+p)}{|1-\alpha|(k-1)\varphi_{k+p-1}(\delta)}\prod_{j=1}^{k-2}\left(1 + \frac{|b||\eta|(\delta+p)}{|1-\alpha| j}\right). \tag{28}$$

Consider

$$\begin{aligned}\left|a_{k+p}\right| &\le \frac{|b||\eta|(\delta+p)}{|1-\alpha|(k)\varphi_{k+p}(\delta)} \times \left\{\left(1 + \frac{|b||\eta|(\delta+p)}{|1-\alpha|}\right) + \frac{|b||\eta|(\delta+p)}{|1-\alpha|(2)} \times \left(1 + \frac{|b||\eta|(\delta+p)}{|1-\alpha|}\right) + \cdots + \frac{|b||\eta|(\delta+p)}{|1-\alpha|(k-1)} \times \prod_{j=1}^{k-2}\left(1 + \frac{|b||\eta|(\delta+p)}{|1-\alpha| j}\right)\right\} \\ &= \frac{|b||\eta|(\delta+p)}{|1-\alpha|(k)\varphi_{k+p}(\delta)}\prod_{j=1}^{k-1}\left(1 + \frac{|b||\eta|(\delta+p)}{|1-\alpha| j}\right).\end{aligned} \tag{29}$$

Therefore, the result is true for $n = k + 1$, and hence by using mathematical induction, (14) holds true for all $n \ge 3$.

If we put $\lambda = 0$, $p = 1$, $b = 2$, and $\delta = 0$ in Theorem 3, we obtain the result proved in [12]. □

**Corollary 4.** *If $f(z) \in \mathcal{SD}(\alpha, \beta)$, then*

$$\left|a_2\right| \le \frac{2(1-\beta)}{|1-\alpha|}, \qquad \left|a_n\right| \le \frac{2(1-\beta)}{(n-1)|1-\alpha|}\prod_{j=1}^{n-2}\left(1 + \frac{2(1-\beta)}{j|1-\alpha|}\right), \quad (n \ge 3). \tag{30}$$

*If one takes $\alpha = 0$ in Corollary 4, one obtains the following inequality:*

$$\left|a_n\right| \le \frac{1}{(n-1)!}\prod_{j=2}^{n}(j - 2\beta), \quad (n \ge 2), \tag{31}$$

*which was proved by Robertson [16].*

*By setting $\lambda = 0$, $p = 1$, $b = 1$, and $\delta = 1$ in Theorem 3, one obtains the result proved in [12].*

**Corollary 5.** *If $f(z) \in \mathscr{KD}(\alpha, \beta)$, then*

$$|a_2| \le \frac{(1-\beta)}{|1-\alpha|},$$
$$|a_n| \le \frac{2(1-\beta)}{n(n-1)|1-\alpha|}\prod_{j=1}^{n-2}\left(1+\frac{2(1-\beta)}{j|1-\alpha|}\right), \quad (n \ge 3). \tag{32}$$

*Letting $\alpha = 0$ in Corollary 5, one gets the following inequality proved by Robertson [16]:*

$$|a_n| \le \frac{1}{n!}\prod_{j=2}^{n}(j-2\beta), \quad (n \ge 2). \tag{33}$$

**Theorem 6.** *If $f(z) \in \mathscr{A}(p)$ and satisfies*

$$\begin{aligned}
&2(\alpha+1)(\delta+p)|b|\sin\frac{\lambda}{2} + \beta(\delta+p)|b|\cos\lambda \\
&\quad + \sum_{k=p+1}^{\infty}\varphi_k(\delta)|a_k| \\
&\quad \times\left[(\alpha+1)\left\{2(\delta+p)|b|\sin\frac{\lambda}{2} + 2(k-p)\right\}\right. \\
&\qquad \left. -\beta(\delta+p)|b|\cos\lambda + (\delta+p)|b|\right] \\
&< (\delta+p)|b|,
\end{aligned} \tag{34}$$

*where $\varphi_k(\delta)$ is given by (5), then $f(z) \in \mathscr{VD}_p^{\lambda}(\delta, b, \alpha, \beta)$.*

*Proof.* Suppose (34) holds. Also let us suppose

$$Q(z) = e^{i\lambda}\left(1 - \frac{2}{b} + \frac{2}{b}\frac{D^{\delta+p}f(z)}{D^{\delta+p-1}f(z)}\right). \tag{35}$$

Then

$$\begin{aligned}
&|Q(z)-1| \\
&= \left|\frac{\left(be^{i\lambda} - 2e^{i\lambda} - b\right)D^{\delta+p-1}f(z) + 2e^{i\lambda}D^{\delta+p}f(z)}{bD^{\delta+p-1}f(z)}\right|.
\end{aligned} \tag{36}$$

Using (8) and then simplifications gives

$$\begin{aligned}
&|Q(z)-1| \\
&= \left|\left(\left\{b\delta\left(e^{i\lambda}-1\right) + bp\left(e^{i\lambda}-1\right) - 2pe^{i\lambda}\right\}\right.\right. \\
&\qquad \left.\times D^{\delta+p-1}f(z) + 2e^{i\lambda}z\left(D^{\delta+p-1}f(z)\right)'\right) \\
&\qquad \left.\times\left(b(\delta+p)D^{\delta+p-1}f(z)\right)^{-1}\right| \\
&\le \left(2(\delta+p)|b|\sin\frac{\lambda}{2} + \left|\sum_{k=p+1}^{\infty}\varphi_k(\delta)a_kz^k\right|\right. \\
&\qquad \left.\times\left[2(\delta+p)|b|\sin\frac{\lambda}{2} + (k-p)\right]\right) \\
&\qquad \times\left((\delta+p)|b|\left(1 - \sum_{k=p+1}^{\infty}\varphi_k(\delta)|a_k|\right)\right)^{-1}.
\end{aligned} \tag{37}$$

Now consider

$$\begin{aligned}
&\alpha|Q(z)-1| - \operatorname{Re}|Q(z)-1| + \beta\cos\lambda \\
&\quad \le (\alpha+1)|Q(z)-1| + \beta\cos\lambda \\
&\quad \le \left(\left((\alpha+1)\left\{2(\delta+p)|b|\sin\frac{\lambda}{2} + \left|\sum_{k=p+1}^{\infty}\varphi_k(\delta)a_kz^k\right|\right.\right.\right. \\
&\qquad \left.\left.\times\left[2(\delta+p)|b|\sin\frac{\lambda}{2} + (k-p)\right]\right\}\right) \\
&\qquad \left.\times\left((\delta+p)|b|\left(1 - \sum_{k=p+1}^{\infty}\varphi_k(\delta)|a_k|\right)\right)^{-1}\right) \\
&\qquad + \beta\cos\lambda.
\end{aligned} \tag{38}$$

The last expression is bounded by 1 if

$$\begin{aligned}
&2(\alpha+1)(\delta+p)|b|\sin\frac{\lambda}{2} + \beta(\delta+p)|b|\cos\lambda \\
&\quad + \sum_{k=p+1}^{\infty}\varphi_k(\delta)|a_k| \\
&\quad \times\left[(\alpha+1)\left\{2(\delta+p)|b|\sin\frac{\lambda}{2} + 2(k-p)\right\}\right. \\
&\qquad \left. -\beta(\delta+p)|b|\cos\lambda + (\delta+p)|b|\right] < (\delta+p)|b|,
\end{aligned} \tag{39}$$

and this completes the proof. □

For $\lambda = 0$, $\delta = -p + 1$, $b = 2$, and $\alpha = 0$ in Theorem 6, we obtain the following.

**Corollary 7.** *If $f(z) \in \mathscr{A}(p)$ and satisfies*

$$\sum_{k=p+1}^{\infty} (k+1-p-\beta)\left|a_k\right| < (1-\beta), \tag{40}$$

*then $f(z) \in \mathscr{S}_p^*(\beta)$, the class of p-valent starlike functions of order $\beta$.*

*For $\lambda = 0$, $\delta = -p+2$, $b = 1$, and $\alpha = 0$ in Theorem 6, one has the following.*

**Corollary 8.** *If $f(z) \in \mathscr{A}(p)$ and satisfies*

$$\sum_{k=p+1}^{\infty} (k-p+1)(k+1-p-\beta)\left|a_k\right| < (1-\beta), \tag{41}$$

*then $f(z) \in \mathscr{K}_p(\beta)$, the class of p-valent convex functions of order $\beta$.*

*Further for $p = 1$ in both the last two corollaries, one obtains the results for the classes $\mathscr{S}^*(\beta)$ and $\mathscr{K}(\beta)$ which was proved by Merkes et al. [17] and Silverman [18], respectively.*

## Conflict of Interests

The authors declare that they have no conflict of interests. Please consider this paper for further process.

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