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Decision-making in incomplete markets with ambiguity—a case study of a gas field acquisition

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We apply utility indifference pricing to solve a contingent claim problem, valuing a connected pair of gas fields where the underlying process is not standard Geometric Brownian Motion and the assumption of complete markets is not fulfilled. First, empirical data are often characterized by time-varying volatility and fat tails; therefore, we use Gaussian generalized autoregressive score (GAS) and GARCH models, extending them to Student's t -GARCH and t -GAS. Second, an important risk (reservoir size) is not hedgeable. As a result, markets are incomplete which makes preference free pricing impossible and thus standard option pricing methodology inapplicable. Therefore, we parametrize the investor's risk preference and use utility indifference pricing techniques. We use Least Squares Monte Carlo simulations as a dimension reduction technique in solving the resulting stochastic dynamic programming problems. Moreover, an investor often only has an approximate idea of the true probabilistic model underlying variables, making model ambiguity a relevant problem. We show empirically how model ambiguity affects project values, and importantly, how option values change as model ambiguity gets resolved in later phases of the projects. We show that traditional valuation approaches will consistently underestimate the value of project flexibility and in general lead to overly conservative investment decisions in the presence of time-dependent stochastic structures.

Keywords: Real option valuation; Option pricing; Least Squares Monte Carlo; Incomplete market; Model ambiguity; Strategic decision-making; Energy market

JEL Classification: C61, D81, G1, G31, G34, Q40

1. Introduction

Firms need project evaluation techniques for many purposes: capital budgeting assessment, risk management, mergers and acquisitions (M&A) activities and so forth. The most popular and well-adopted evaluation method over the past decades is the net present value (NPV) approach, for whose calculations only one time discount rate and a series of future cash flows are required. The NPV approach is simple and straightforward, but to achieve that needs strong assumptions and suffers from rigidity and inflexibility. Problems arise when investors believe that they may benefit from the flexibilities embedded in the projects: within a NPV framework, there is no way of quantifying the benefits of such flexibilities. As a consequence, NPV structurally underestimates the value of projects with flexible investment opportunities. Real option valuation (ROV), which quantifies the value of embedded flexibilities through option pricing techniques, is a more appropriate tool for projects with flexibilities, for instance, a not-to-exceed value for M&A activities. Before applying any option evaluation methods, additional analytical procedures need to be carefully executed.

The investor first has to reformulate the development plan into a strategic one, which highlights all the inherent managerial flexibilities embedded in the investment project. Next, in order to determine an optimal investment strategy, the investor has to consider mainly three aspects: the dynamics of the underlying asset returns, the constraints on the investment strategy, and the value of the investor's strategy. Each aspect affects the final decisions significantly and has to be carefully taken into consideration.

The so-called real option problem embedded in investment projects is of particular concern for the energy industry. The energy industry is highly capital intensive, but subject to numerous uncertainties. Some of them are market related, such as gas or oil price uncertainty, but some have no correlation with any tradable asset, for example uncertainty about reservoir size facing companies having to decide on exploration or bringing discovered fields in production. Yet investment decisions are made on daily basis based on valuation results where projects are evaluated by simply discounting their future cash flows under various mutually exclusive scenarios. But this approach ignores the value of flexibility that the ability to respond to new information coming on stream during the project life span

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gives rise to. If there is any value to this flexibility, ignoring the value of flexibility then structurally undervalues projects that offer flexibility when compared with projects that involve more rigid, irreversible choices. Further complications arise when there is ambiguity on the precise probabilities to be attached to various possible outcomes of random variables. In this paper, we show how ROV offers a way out of those problems. We demonstrate this approach in valuing a set of connected gas fields based on real North Sea data.

Real option approaches have been known for a long time, but have by and large been dismissed in practice; there exists a highly sophisticated literature, but one focusing on obtaining analytical results which therefore has to make very restrictive assumptions, such as risk neutral preferences and constant variance Geometric Brownian Motion (GBM) processes. We allow for much less restrictive assumptions on both scores: we introduce different attitudes towards risk, and various models of stochastic volatility. The resulting problems require solving quintessentially non-linear stochastic dynamic optimization problems (SDP), and the numerical problems solving them become rapidly insurmountable as the problem's dimensionality increases: SDP is plagued by the curse of dimensionality. We demonstrate, however, that a dimension reduction approach long used in the solution of problems posed by the valuation of American options (i.e. options with endogenous exercise timing) can also be applied successfully to the stochastic dynamic programming problems arising in complex high dimensionality real option problems. We analyse a real-world case study, the valuation of two connected offshore gas fields in the presence of price uncertainty with variable volatility (using t -GAS models, on which more below), inter-temporally correlated uncertainty and even model ambiguity concerning the reservoir size of the two connected gas fields. Our analysis shows substantial payoffs to explicitly introducing asymmetric stochastic variance modelling, substantial option values in the presence of unhedgeable risks (although in that case we show them to depend on investor's risk preferences) and the importance for decision-making of taking into account (declining) model ambiguity. Before going into the existing literature on real options, we elaborate on the major differences of our analysis with standard option pricing models, focusing on market incompleteness, more realistic models of volatility than the constant variance GBM model used routinely, and finally on the important topic of model ambiguity.

1.1. Incomplete markets

One difficulty of applying standard option techniques to real-life problems is that the decision-maker often is facing an asset pricing problem in an incomplete market setting, where not all underlying risks are hedgeable through the market. For instance, in the gas field valuation problem considered in this paper, we have two major uncertainties: the reservoir size[†] and the gas price. Reservoir uncertainty cannot be hedged away in any existing market. In fact, even price risk cannot be fully

[†]Geophysicists are not able to measure the exact reservoir size of one gas field before any development starts. The best approximation of reservoir size is a truncated lognormal distribution. For more research on reservoir distribution, see [Hanna et al. \(2011\)](#).

hedged since the derivative market built on the Dutch gas contracts is still young and immature.[‡] Another cause of market incompleteness is the stochastic volatility characterizing the underlying process driving asset returns (gas prices), because the dynamics of the second moment of the process cannot be hedged through the market either. Therefore, classical option pricing models such as the Black–Scholes formula ([Black and Scholes 1973](#)), are not applicable, since they are based on the assumption of complete markets. The traditional option pricing methods are based on ruling out arbitrage between the option(s) considered and an equivalent replicating portfolio consisting of traded financial instruments, but such a replicating portfolio does not exist in an incomplete markets environment. One consequence of the absence of replicating portfolios is that preference free (risk neutral) pricing becomes impossible: an individual's risk preference has to be parametrized and taken into account. This is exactly what we do in applying what is called utility indifference pricing (UIP) to the real options problem considered.

1.2. Heteroskedasticity

The volatility of underlying processes obviously matters for option pricing problems. [Van Wijnbergen and Zhao \(2016\)](#) show in an application of ROV to an energy-related project similar to the one analysed in this paper that a Gaussian GARCH specification outperforms one in which one assumes constant variance in modelling the dynamics of the underlying asset returns, in this case gas prices. [Van Wijnbergen and Zhao \(2016\)](#) have also shown that switching from constant variance to Gaussian GARCH has a dramatic impact on option values, so modelling the structure of volatility clearly matters. In this paper, we again consider GARCH models as well as a more general volatility modelling approach, generalized autoregressive score (GAS) models, which are capable of capturing some unique characteristics of the latent volatilities of the time series. This GAS model family, first proposed and developed by [Harvey \(2013\)](#) and [Creal et al. \(2013\)](#), is a more general set-up compared to GARCH models. By adding the first derivatives of its likelihood function as the latent factor of dynamics, this model specification takes full advantage of the changing directions of likelihood. [Creal et al. \(2013\)](#) claims that the GAS framework demonstrates superior features and better empirical fit over Gaussian GARCH models, which is consistent with our findings in this paper for Dutch gas prices. Note that GARCH(1,1) processes can be seen as a special case of a more general GAS(1,1) structure.

Furthermore, the diagnostic test on residuals from both Gaussian GARCH and Gaussian GAS models rejects the normality assumption. This is commonly found in financial data, where one often encounters what is referred to as 'black swans': extreme outcomes happen more often than implied by normal distributions, with consequent failure of normality tests. Therefore, we switch to Student's t -GARCH and t -GAS variants to capture the fat-tail characteristics of the data. The estimated degrees of freedom for both models are smaller than 4, which

[‡]The Dutch gas spot market is called title transfer facility (TTF). The first TTF Natural Gas Options were launched by ICE in December 2011.

strongly confirms the fat-tail hypothesis. Hence a simple Gaussian assumption would unduly underestimate the high occurrence of extreme events and lead to incorrect characterization of the stochastic structure generating the data.

To our knowledge, this is the first study to derive option pricing results under a stochastic process of t -GAS structure; we compare the t -GAS based results with option valuations based on constant variance and results based on Gaussian GARCH assumptions. Even though we solve the problem within a real-option set-up, the results are doubtlessly relevant to the valuation of finance options traded in the market.

1.3. Model ambiguity

Another important issue in decision-making problem involves model ambiguity. This occurs when the decision-maker is uncertain about the true probabilistic model generating the data, which is often referred to as Knightian uncertainty (Knight 1921) or model ambiguity. Note that a decision problem with ambiguity is different from one under risk: the latter refers to a decision problem with the true probability distribution known and the former is one without the true probability known. Model ambiguity is a realistic and robust assumption because an investor often does not have access to the true probabilistic model underlying relevant variables and may only have an approximation for the true one at the best.

In applications like the one in this paper concerning gas field evaluation, model ambiguity occurs often due to the relatively unsophisticated existing technology for reservoir size estimation. Geophysicists estimate parameters for reservoir distribution based on imperfect exploration data, often supplemented by insights derived from their own past experiences, which makes model ambiguity a particularly important issue for valuation problems. Mathematically, model ambiguity possibly presents new complications, such as time inconsistency; we discuss this issue in detail in the numerical methods and the results sections below (Sections 4.4 and 5.3).

1.4. Guide to the reader

This paper is arranged as follows. Section 2 reviews related literature on real options, model ambiguity, GARCH and GAS models. Our representative gas field case study problem is described in section 3. The econometric models and option pricing models are explained in the following section 4. Section 5 demonstrates the results and section 6 concludes.

2. Literature review

2.1. Real option valuation

McDonald and Siegel (1986) initiated the application of option pricing technology to decisions involving irreversible 'real' projects. They solve for optimal investment rules using a contingent claim setting and find significantly positive value of waiting. Following their work, Pindyck (1991) provides

methodology for practitioners. He emphasizes two major characteristics of investment opportunities: irreversible expenditure and postponement of execution. These features both have a profound effect on investment decision, and share similarities with financial options.

Borison (2005) criticizes existing applications of real option theory for requiring assumptions that are not realistic in practice, thereby invalidating the pricing methodologies chosen. He surveys the applicability and assumptions of all existing approaches, including the classic approach (Brennan and Schwartz 1985, Amram and Kulatilaka 1999), the subjective approach (Howell 2001), the MAD approach (Copeland *et al.* 1994), the revised classic approach (Dixit and Pindyck 1994), and the integrated approach (Smith and Nau 1995, Smith and McCardle 1998). All of them except the last one assume market completeness (hedgeable risks), which is however rarely the case in real-life problems. Furthermore, the first two approaches explicitly assume the underlying asset follows a constant variance GBM process, which is also not always a good approximation of real life situations as we demonstrate in our analysis of natural gas prices.

2.2. Discount rate

In an incomplete market setting, a proper discount rate should not only reflect the decision-maker's risk aversion and her time discounted value, but also the structure of the uncertainty embedded in the project itself. In practice, the appropriate choice of discount rates is not always clear to the decision-maker, managers are often struggling in determining the time discount factor, especially for individual projects. According to a survey conducted by Mukerji and Tallon (2001), the most popular valuation method chosen by CFOs is discounted cash flow, i.e. the NPV method, but when applying this approach, the CFOs are not clear on the choice of discount rate. They typically use one of the following four discount rates: the acquiring firm's weighted average cost of capital, the acquiring firm's cost of equity, the target's weighted average cost of capital, or other rates such as the target's cost of equity. Each discount rate has its pros and cons, and the choice may also depend on the (size of the) M&A project itself. This apparently confuses CFOs and may lead to biased (too optimistic or pessimistic) results. And anyhow project structures may be such that the use of any constant discount rate is inappropriate because the risk structure changes over time.

Borison (2005) points out that the weighted average cost-of-capital (WACC) is often used without clearly identifying its risk coverage, i.e. whether it reflects private risk[†] only or the overall risk of the investment. Therefore, we decompose the discount rate and discuss how each aspect inherent in discount rate determination affects the decision-making process.

2.3. Dynamic processes of underlying assets

As is explained in Van Wijnbergen and Zhao (2016), the dynamic of gas prices follows a complicated structure with

[†]As defined in Amram and Kulatilaka (2000), 'Risks not captured in the price fluctuations of traded securities are known as private risks'.

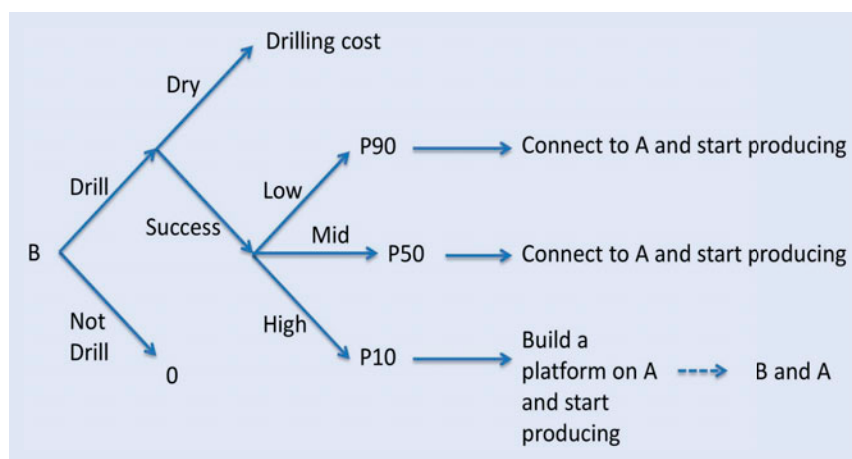


Figure 1. Development plan.

time-varying volatilities, which cannot be captured by a GBM process. Therefore, the classic approach, the subjective approach and the revised classic approaches mentioned before become inapplicable, since they rely on the GBM assumption for their pricing formulas.

In this paper, we consider GAS/GARCH models accounting for volatility, where both are able to reproduce the volatile volatility. Creal *et al.* (2013) explain that GAS models can be specialized into GARCH models by selecting appropriate factors. They also compare different dynamic copula models and conclude that the likelihood information is extensively exploited under a GAS framework. As shown in Andres (2014), the model with dynamic scores outperforms autoregressive conditional duration (ACD) models in terms of the rate of convergence and reliability. Note that an ACD model, as proposed in Engle and Russell (1998), is analogous to a Gaussian GARCH model.

Furthermore, financial data often contain fat-tails: extreme outcomes happen too often for a normal distribution to be capable of accounting for the outliers. By applying GARCH and GAS models to global equity returns, Creal *et al.* (2011) find that t -GAS produces highly persistent estimated factors and improves the log-likelihood substantially.

2.4. Real options and incomplete markets

As mentioned above and in Borison (2005), most real option approaches assume market completeness, which results in problematic applications when there are unhedgeable risks. For example, the subjective approach (Howell 2001) uses subjective probability; therefore, it is incapable of shedding light on the market trading price. The MAD approach (Copeland and Antikarov 2001) argues that traditional NPV serves as an unbiased replicating portfolio; however, the no-arbitrage assumption cannot be satisfied with this argument only, arbitrage opportunities may persist due to the use of subjective data. Several attempts have been made to resolve the incomplete market problem. For example, Smith and Nau (1995) and Smith and McCardle (1998) attempt to remedy the issue by assuming some of the risks are hedgeable and some are not and

apply a mixed method, combining a zero arbitrage approach for the hedgeable risks and UIP for the unhedgeable risks, an approach also used in Van Wijnbergen and Zhao (2016). Carmona (2009) states the effectiveness of UIP mechanisms for option pricing problems in an incomplete market, where risk preferences are built into the model to acknowledge attitudes to risks. UIP comes down to determining a price at which an investor becomes indifferent between on the one hand paying that price and receiving an uncertain claim, vs. not paying that price but also not receiving that claim, all the time maintaining an optimal trading strategy. See in particular Carmona (2009) for an extensive introduction to and coverage of the approach. In this paper, we opt for UIP for all risk dimensions since even the gas prices are characterized by unhedgeable volatility risk factors.

2.5. Model ambiguity

The concern for modelling ambiguity can be traced back to Knight (1921), who describes ambiguity as ‘uncertainty’ (hence the oft used term ‘Knightian uncertainty’). The essential difference between risk and Knightian uncertainty (or ambiguity, as we refer to it here) is whether the true probability is known or unknown. The breakthrough made by Gilboa and Schmeidler (1989) solves the ambiguity problem numerically through a maxmin utility with multiple priors, by showing that under certain conditions an ambiguity averse agent will focus on the worst case scenario, again assuming an optimal trading strategy throughout.

Camerer and Weber (1992) give an extensive survey on ambiguity aversion, including both theoretical and empirical papers. In earlier experimental studies, e.g. Heath and Tversky (1991), subjects were shown to exhibit strong ambiguity aversion in many circumstances. However, the results for the effect of ambiguity on asset prices are not always coherent. For example, Camerer and Kunreuther (1989) show that even though ambiguity has changed the market structure, it did not affect asset prices systematically. But Sarin and Weber (1993) claim that ambiguity drives prices down, slightly but significantly. Chen and Epstein (2002) investigate the effect

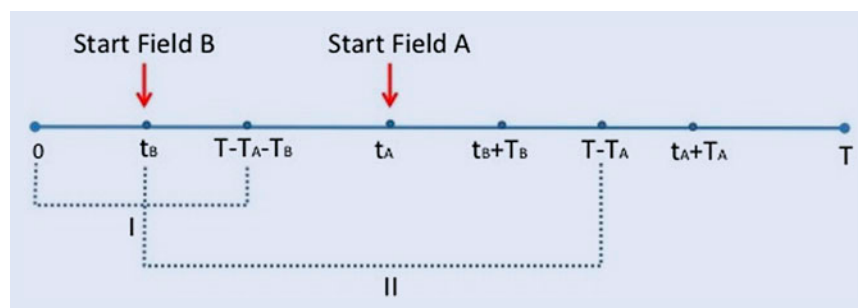


Figure 2. Development timeline.

of ambiguity also by considering multiple-priors utility. Their continuous time based approach allows them to decompose excess returns into a risk premium and an ambiguity premium. Maccheroni *et al.* (2006) consider models of decision-making under ambiguity that take the form of so-called variational preferences, which include both multiple preference parameters and multiple priors as special cases. Furthermore, market incompleteness and model ambiguity may mutually reinforce each other: Mukerji and Tallon (2001) argue that the markets are less complete due to the effect of ambiguity aversion.

Several authors have looked at ambiguity in a real options context. Assuming a standard GBM process, Nishimura and Ozaki (2007) find that higher risk leads to higher valuation of the investment opportunity; however, higher Knightian uncertainty results in a lower project value. We find similar results in this paper. However, their results are based on the simplifying but severely restrictive assumption that the decision-maker is risk neutral. This paper relaxes this assumption and allows the risk preference to play a role in valuation, a key issue in an incomplete markets setting. Trojanowska and Kort (2010) consider a finite investment problem, as opposed to the perpetual one studied by McDonald and Siegel (1986), Dixit and Pindyck (1994) and Nishimura and Ozaki (2007). Again, given an ambiguity averse investor and a maxmin strategy, a higher degree of ambiguity yields a lower risk- and ambiguity aversion adjusted NPV. Different from Nishimura and Ozaki (2007) and Trojanowska and Kort (2010), Thijssen (2011) studies an irreversible investment problem under ambiguity by assuming that ‘ambiguity is solely about the appropriate rate at which the cash-flows should be discounted’. Despite the different model setting, Thijssen (2011) reaches a similar conclusion, that ambiguity delays the investments.

Hellmann and Thijssen (2015) investigate a two-player investment game under ambiguity. A sequential game with stochastic payoffs and a with a first mover advantage is carefully studied. Under such a setting, they conclude that the worst-case prior is equivalent to the ‘lowest trend’ of the diffusion process (they assume a GBM process) if the leader knows she will be the leader. However, there is an opposing force under which the other player might also invest early in which case it is not necessarily true anymore that the lowest trend is the worst-case prior. We analyse a single player valuation problem, but similar time consistency problems can arise in a single player model too.† In our case, such time consistency

problems cannot arise because of the strong monotonicity in our different priors: the same prior from a given set of priors (or any linear combination thereof) is always inferior under optimizing behaviour conditional on a given prior because of their symmetry combined with positive risk aversion.

Cheng and Riedel (2013) focus on the time-varying feature of ambiguity like we do also in the second half of section 5.3 below, because ‘it is rational to change one’s belief about the worst drift’. We also investigate time-varying ambiguity and in particular demonstrate the effect of resolving ambiguity over time on project valuations if it is known in advance that resolution will take place as the project unfolds. For more recent references on indifference valuation, incomplete markets and ambiguity, see Laeven and Stadje (2014).

3. Problem description

The investment problem we analyse in this study corresponds to a real-world valuation problem of a set of two undeveloped gas fields located (geographically) close to each other in the North Sea. The investor needs precise valuations for the project for acquisition reasons. The recoverable reservoir size of Field A is currently estimated to be low and not economically attractive by itself. However, if the development of the nearby Field B turns out to be successful, a more precise estimate of field A can be made: Field B yields valuable information about the reservoir size of Field A. So given the possible information updates, the investor designs a strategic developing plan as displayed in figure 1. If the drilling on B is successful and the reservoir of Field B turns out to be of a sufficiently large volume, the producer may decide to build a new platform on Field A, which allows the productions of both A and B at the same time due to a platform’s larger capacity compared to one single pipeline. Thus Field A can be considered as an extension option on Field B, which should be exercised only when the reservoir of B reveals a sufficiently good state.

The development timeline is illustrated in figure 2. T is the license expiration date for the development of the area covering Field A and B. T_A and T_B are the estimated production durations of A and B, and t_A and t_B are the starting date of their productions, respectively. Interval I is the maximum waiting period before starting B; B needs to be started at the latest at the end of I to allow for both the exploitation of B and A before the overall license expires at T . So the investor could start developing B any time in time interval I , the choice of

†We are indebted to a referee for pointing that out.

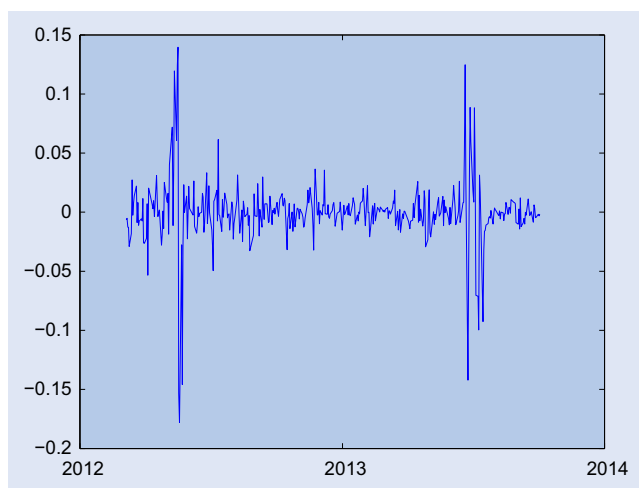


Figure 3. TTF daily logarithmic return data.

timing reflects the exercise time of a waiting option. After B has been brought on stream, information about its reservoir size becomes available and additional information on the reservoir size of A may become available if for example the two reservoir sizes are correlated.

We explore various different correlation scenario's in what follows. Once the information of B has become available (along with new gas price developments), the investor may choose to develop A; Exploitation of B unlocks the option on A. If she starts A, she should do so at the latest before the end of time interval II to stay within the time limit set by the expiry date of the overall license. Our aim is to value Field B accurately, so that the valuation can serve as a reserve price for the acquisition of the area combining both A and B. Since B unlocks the option on A, the project value of B should include the value of managerial flexibility embedded in Prospect A.

3.1. Reservoir distribution

Possibility of success (POS) stands for the probability of a successful drilling. So the probability of a dry well (zero reservoir) is $1 - POS$. Based on a successful drilling, the investor expects a reservoir size R following a certain distribution. Three commonly used assumptions for the distribution of R are a triangular distribution, a lognormal distribution and a variant on the latter, a truncated lognormal distribution.

The triangular distribution is the industry standard for capital budgeting problems due to its simplicity. It considers only three outcomes of R , namely P_{10} , P_{50} , and P_{90} . Naturally, in line with the concept of a Cumulative Density Function, reservoir amount $R_{P_{10}}$ of P_{10} case means the probability that the realization of the gas reservoir size is lower than $R_{P_{10}}$ is 90%. This simple representation is popular among decision-makers, because it provides reasonable and easy proxies for low/medium/high reserve size cases, based on which a scenario analysis can be set up for capital budgeting procedures.

Despite the simplicity of triangular distribution, geophysicists prefer a lognormal distribution because it provides more insights into the reservoir distribution. However, a standard

lognormal distribution ranges from 0 to infinity, which is obviously an unrealistic range for gas reservoir size distributions. Therefore, we use a truncated lognormal distribution for the simulation of the reservoir volume size. To fully approximate the estimations provided by the geophysicists, the reservoir distribution sometimes cannot solely be characterized by a single truncated lognormal one. For example, the reservoir size of Field B here is better described by the sum of two weighted lognormal probability distribution functions. The two lognormal distributions are truncated at 99% quantile, one with parameters $(-0.1772, 0.5336)$ † and a weight of 0.8661, the other with parameters $(-26.6623, 0.0002)$ ‡ and a weight of 0.1339.

3.2. Option characteristics of the valuation problem

The strategic plan followed by the firm is divided into two steps. First, the firm might wait and meanwhile observe the market price of gas to decide whether to start developing B or not. This decision has to be made within three years (before the end of time period I), so as not to exceed the remaining life of the relevant exploration licenses. This set-up means that the firm has a wait-and-see Bermuda-type§ option on B with a maturity of three years. Once this development option is exercised, the firm may build a platform and further develop A based on a not-worse-than-P10 realized reservoir amount of B (figure 1). If the firm has waited the maximum amount of time with B, the option to bring A into development can thus be seen as the unlocking of a European option. If $t_B < T - T_A - T_B$, the second option has Bermuda characteristics too. Thus, Project B has multiple and compound option characteristics with a sequential structure, whose values are calculated in section 5. How we formally price this complicated set of options is explained in section 4.5.

4. Methodology

We first present the econometric approach to analyse gas prices and in particular their volatility structure. We apply both GARCH and GAS models to analyse daily returns and their volatility, using data obtained from TTF, the Dutch gas market. We then introduce the UIP approach and its implementation for the option pricing problem.

4.1. Data

For this study, two sources of data are needed. One is gas price data, i.e. the publicly traded TTF price, which we obtain from Datastream. TTF trades gas in euros per megawatt hour. Statistical descriptions of the price data are shown in table 1. The other source of data concerns the information on reservoir size of the fields under consideration, which has been received from the geophysicists involved in an actual North Sea project.

†(Mean, standard deviation).

‡(Mean, standard deviation).

§A Bermuda option is an American option with a set of predetermined exercise timing possibilities.

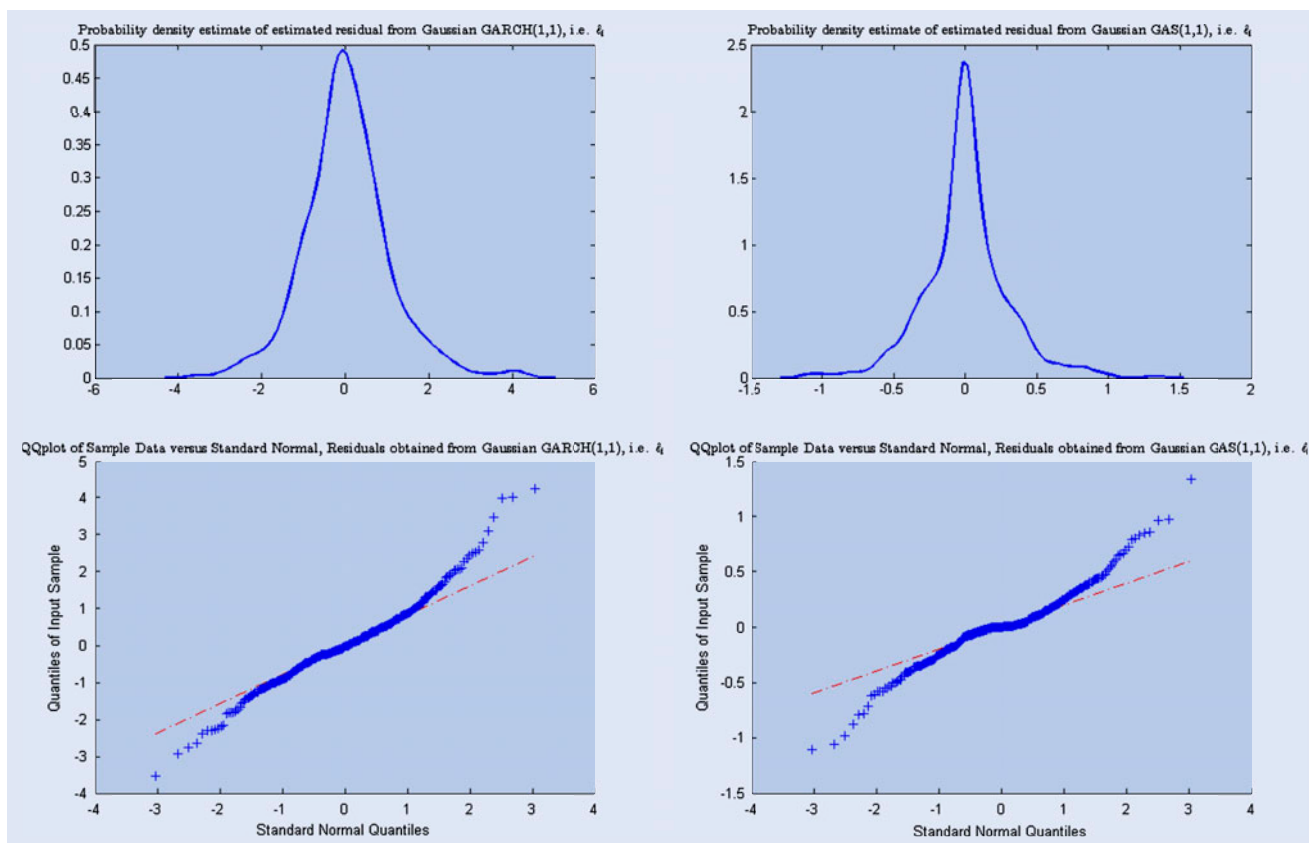


Figure 4. Residual tests.

Geophysical data include the estimated reservoir size distribution, the production profiles and production costs.

The gas daily return series ranges from 5 March 2012 to 27 September 2013, shown in figure 3. The time series is stationary by both Dickey–Fuller test and Phillips–Perron test.

4.2. GAS models

The GAS model follows Creal *et al.* (2013). y_t is the demeaned daily log return of gas on the TTF market and has a probability distribution function $p(y_t|f_t; \theta_t)$, where f_t stands for unobserved time-varying factors and θ_t contains unknown parameters.

$$\begin{aligned}
 y_t &= \sigma_t \varepsilon_t \\
 f_{t+1} &= \omega + A s_t + B f_t \\
 s_t &= S_t \nabla_t \\
 S_t &= -\mathbb{E}_{t-1} [\nabla_t \nabla_t']^{-1} \\
 \nabla_t &= \frac{\partial \log p(y_t|f_t; \theta_t)}{\partial f_t}
 \end{aligned}$$

The scaling matrix S_t equals the Fisher information matrix and ∇_t stands for the ‘score’ as in ‘GAS’. Hence, s_t is also called the scaled score function. We assume that ε_t follows a standard normal (Gaussian) distribution but also investigate the possibility of fatter tails by basing the GAS model on a Student’s t distribution with estimated degrees of freedom, which allows

us to test for normality. The model collapses into a Gaussian GARCH or a t -GARCH one with the appropriate assumptions on the factor f_t as we show in appendix A.1.1; as GARCH turns out to be a special case of GAS, choosing between them can be done on the basis of a simple log-likelihood ratio test.

4.3. Estimation results and diagnostic tests

Our econometric analysis shows that the Gaussian GAS model yields a lower log-likelihood value, -672.36 , comparing to -614.76 for the Gaussian GARCH model. Thus a Gaussian GARCH model outperforms a Gaussian GAS model by 9% in terms of log-likelihood. And the kernel density plots and the QQ-plots in figure 4 imply that the residuals from both Gaussian GARCH and Gaussian GAS models present fat tail leading to a rejection of the normality hypothesis. Therefore we proceed with a Student’s t -based GAS model, to capture the impact of the fat tails embedded in the data.

Table 2 lists the estimation results from all four models considered. The estimated degree of freedom for the Student’s t distribution is 3.95 and 3.91 for Student’s t -GARCH and Student’s t -GAS model respectively, and is significant for both models.† In addition, the log-likelihood from the models with Student’s t distribution is significantly larger than the one from

†Arguably more important, the degrees of freedom parameter is very significantly lower than the number where the difference between t -distribution and the normal becomes negligible (higher than 30).

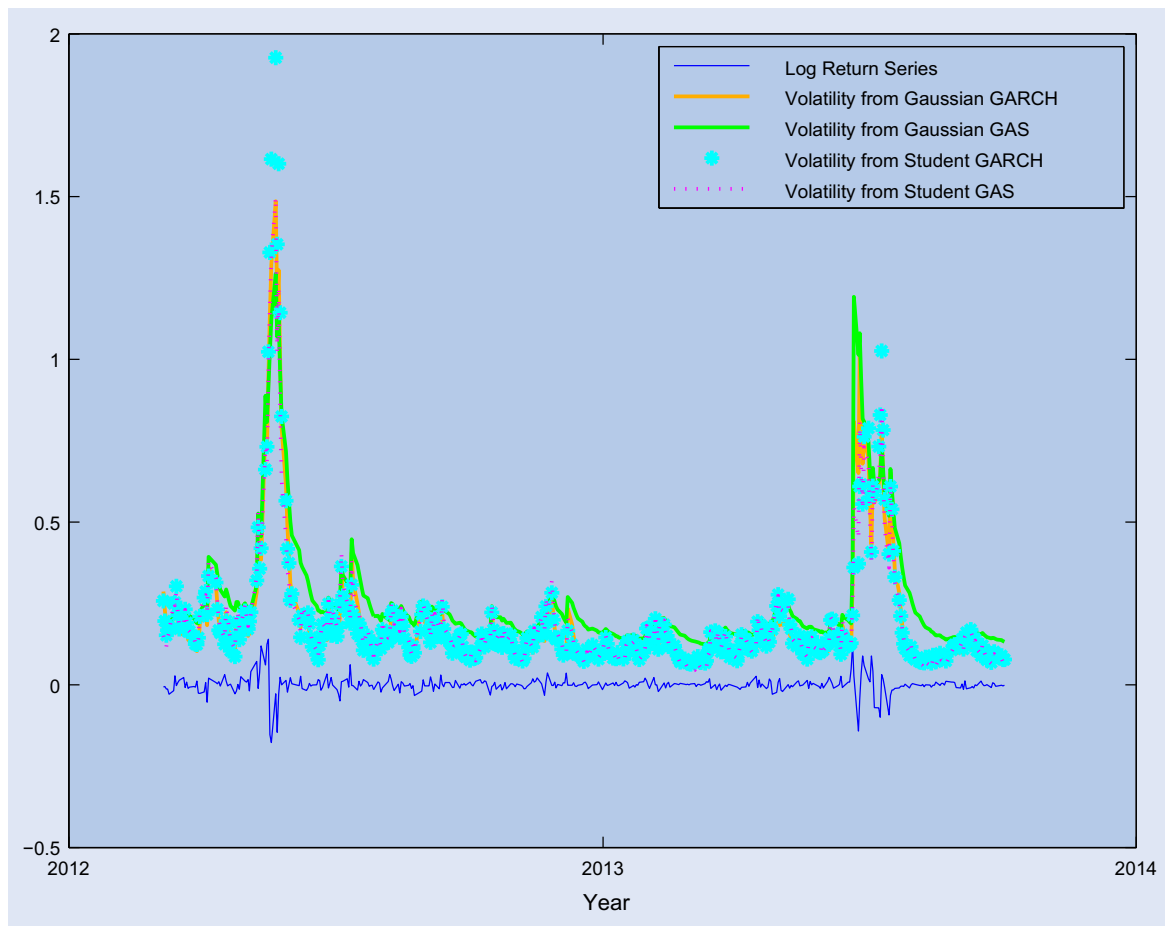


Figure 5. Comparison of estimated volatility with Gaussian/Student's-t GARCH/GAS models.

Table 1. Statistical description of the gas prices.

	Observations	Mean	Standard deviation	Min	Max
Gas price	400	25.678	2.571	20.595	39.391
Gas price return	399	1.955×10^{-4}	0.027	-0.178	0.140

Table 2. Estimation results.

	Gaussian		Student's <i>t</i>	
	GARCH	GAS	GARCH	GAS
ω	4.6086 (1.4039)	0.1823 (0.6241)	3.7312 (1.2269)	0.2447 (0.6235)
A	0.3822*** (9.4001)	0.2245*** (8.7826)	0.2860*** (7.3775)	0.2434*** (7.6351)
B	0.9803*** (57.7331)	0.9329*** (56.8993)	0.9704*** (32.7113)	0.9205*** (35.0938)
ν (Degree of freedom)			3.9511*** (5.0168)	3.9096*** (4.6893)
LogLikelihood	-614.759	-672.361	-595.759	-593.894***

t statistics are reported in parentheses.

***1% significance level.

**5% significance level.

*10% significance level.

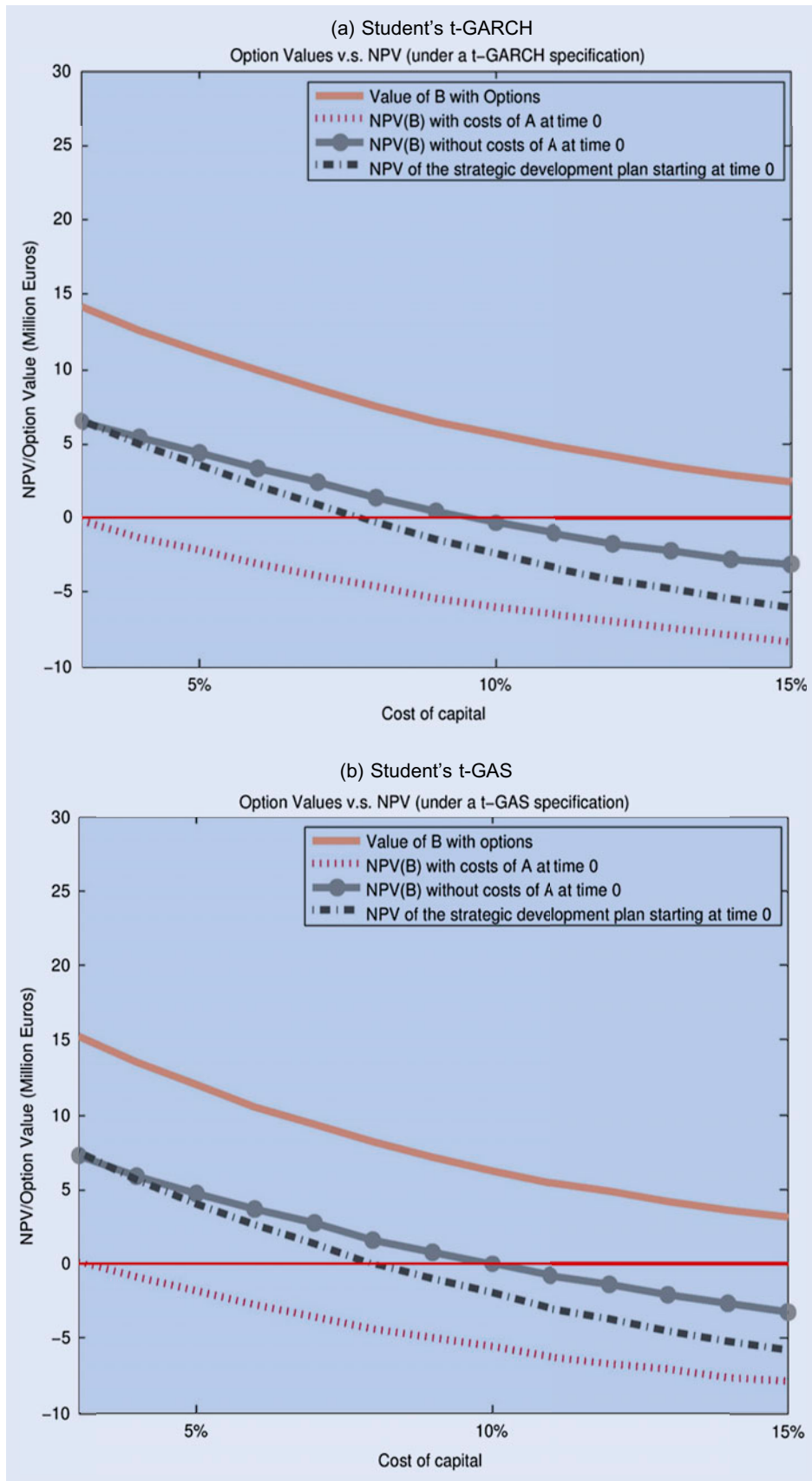


Figure 6. Option pricing results using a cost-of-capital approach.

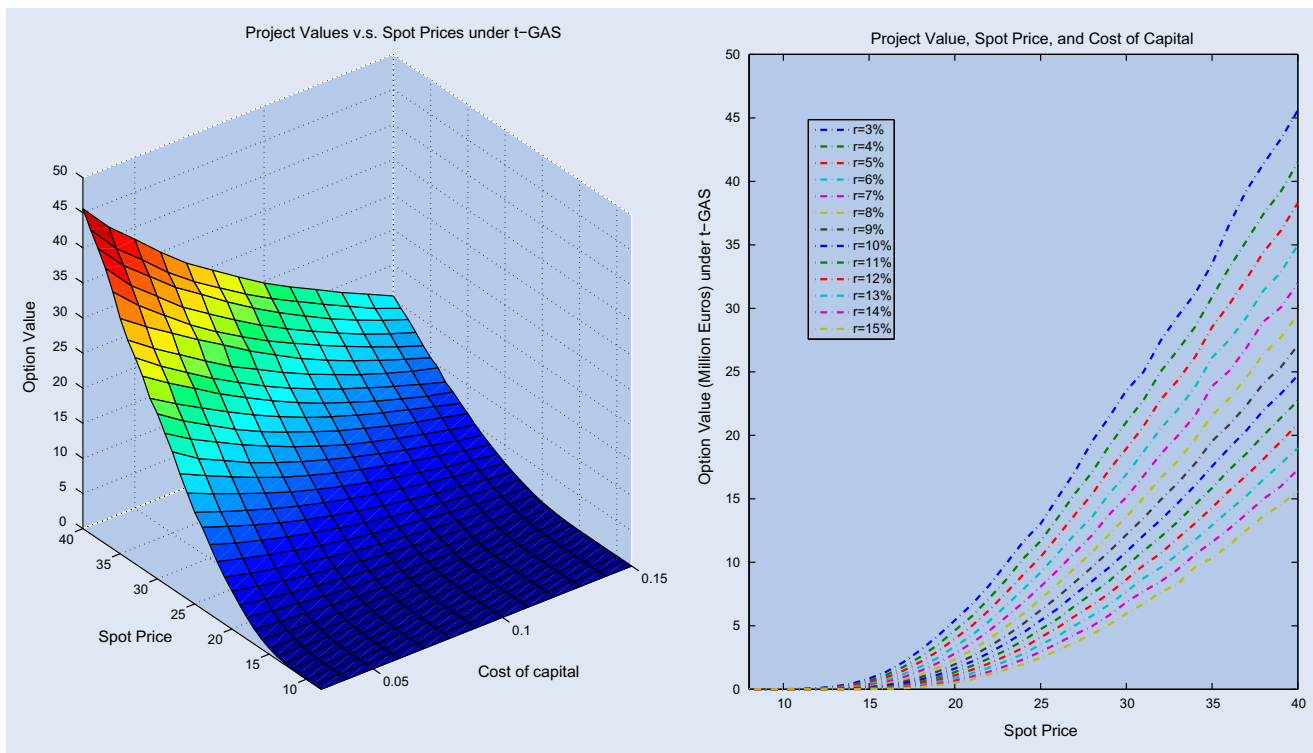


Figure 7. Spot prices vs. option value under a Student’s *t*-GAS specification.

those with Gaussian distributions, which also confirms our findings from the residual plots. The result of a log-likelihood ratio test[†] between Gaussian-GAS and Student’s *t*-GAS is significant, implying the existence of fat-tails in the data series.

Figure 5 demonstrates the variances σ_t^2 estimated from all four models. As can be seen, a model with constant variance assumption is not able to capture all the relevant features of this time series adequately. All Gaussian/Student’s-*t* GARCH/GAS models are able to characterize the high volatility periods, contrary to a constant volatility model. As illustrated in figure 3, two highly volatile periods in mid 2012 and mid 2013 can be easily replicated using the stochastic volatility models, which would obviously be impossible using a constant volatility model.

The Student’s *t*-GAS model produces a slightly more smooth volatility series than the Student’s *t*-GARCH model: during the high volatility periods, e.g. the estimated volatility for mid 2012 given by a Student’s *t*-GAS model is smaller than one based on a Student’s *t*-GARCH model. This observation is attributed to a characteristic feature of Student’s *t*-GAS models, that they adjust quickly to new observations. Of course extreme outcomes of returns do not necessarily stem from high volatility, the occurrence of tail events can occur in low volatility periods also, but will be a much rarer, more unlikely event.

4.4. Evaluating the compound options

We present two methods to evaluate the complicated set of options embedded in project B. First we take the more con-

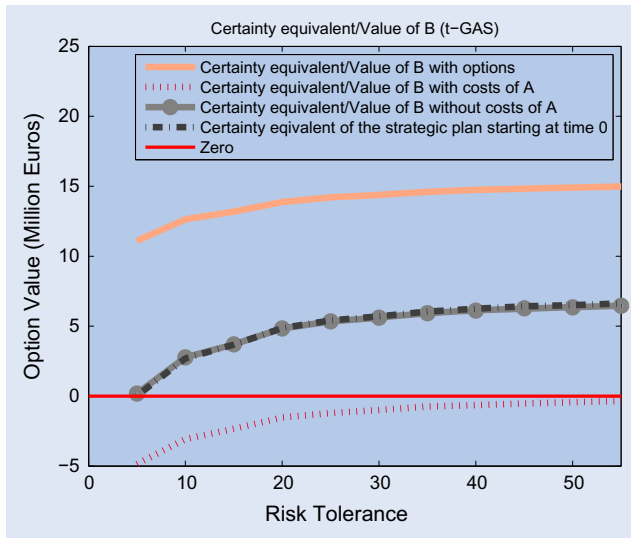


Figure 8. UIP (Student’s *t*-GAS).

ventional approach starting from risk neutral preferences and a constant discount rate: the cost of capital method (CoC). This method is presented only because it provides an intermediate approach between standard scenario analysis and our preferred approach, UIP. The CoC method is easier to explain to practitioners since it is closely related to standard NPV approaches, but improves on it by incorporating the option exercise decisions explicitly. However, the use of a constant discount rate is not appropriate anymore under more general preferences, hence the step to our preferred approach, UIP, where we do not need to make such restrictive assumptions.

[†] $LR = 2(-593.894 + 672.361) = 156.934 > \chi^2(1)$.

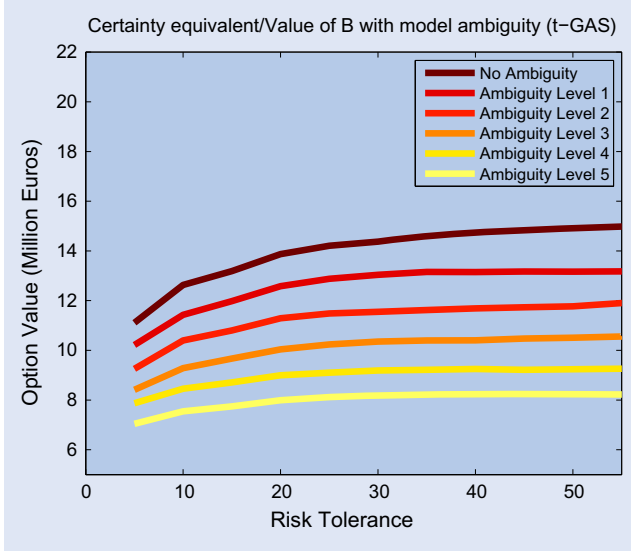
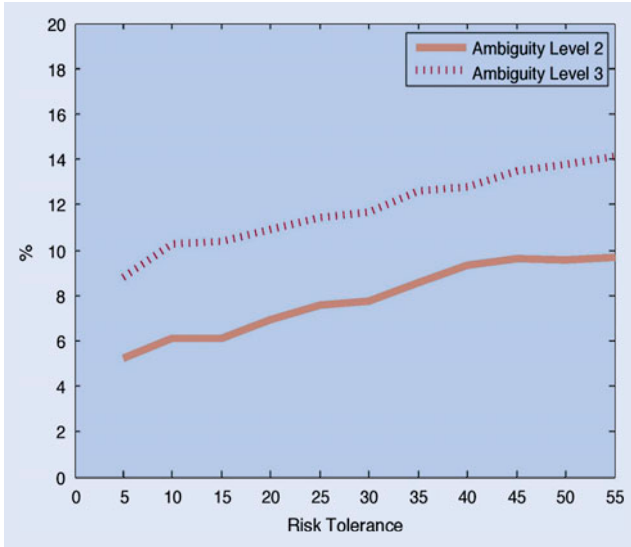


Figure 9. Option values with persistent model ambiguity.

Figure 10. Ambiguity in field A only (Student's t -GAS).

4.4.1. Evaluating the compound options: the CoC. The cost-of-capital method comes down to solving the following optimization problem:

$$\max_{t \in I} \mathbb{E}_0 \left(\exp(-r_f t) \max(X_t + A_t, 0) \right)$$

where:

$$X_t = \mathbb{E}_t \left[NPV_t^B | \mathcal{F}_{t-1} \right]$$

and,

$$A_t = \mathbb{E}_t \left[\max_{t \in II} \mathbb{E}_t \left(\exp(-r_f(t - t_A)) \times \max(NPV_t^{A|B}, 0) \right) | \mathcal{F}_{t-1} \right]$$

\mathcal{F}_t represents the market information and exploration information obtained at t . Given a cost-of-capital value, $NPV_t^{A|B}$ is

the NPV of A based on the information obtained from B. The optimization set-up under UIP is introduced in section 4.4.2 below.

4.4.2. Evaluating the compound options: UIP. Van Wijnbergen and Zhao (2016) apply an integrated approach adjusted from Smith and Nau (1995) and Smith and McCardle (1998) based on the assumption that (A) gas price risk can be hedged so risk neutral valuation can be used in that dimension, but (B) reservoir risk is not hedgeable so a preference based valuation method becomes necessary in that dimension. A problem was that the Gaussian GARCH(1,1) model for gas prices used in Van Wijnbergen and Zhao (2016) violates the constant variance property necessary for the applicability of risk neutral pricing methods, but Duan (1995) has proposed a local variant on risk neutral pricing which can be used in at most a GARCH(1,1) setting, which is what we used in Van Wijnbergen and Zhao (2016). But that mixed approach cannot be used here because the more general and (in our case) statistically preferred GAS models do not satisfy Duan's conditions necessary for the applicability of his local variant on risk-neutral pricing. Since now the gas price volatility risks are unhedgeable too, we take the logical next step and assume that neither risk can be hedged and accordingly adopt multidimensional UIP for both the gas price and reservoir size risks.

We assume the investor has an exponential utility function: $u_t(x_t) = -\exp(-x_t/\rho_t)$, where ρ_t represents the decision-maker's risk tolerance. This form of the preference function is chosen because it implies independence of trading strategies of the investor's wealth, which we consider a desirable property since we refer to a corporate decision analysis. A high ρ_t implies a high tolerance for risk (low risk aversion). Our basic criterion then relies on the discounted value of certainty equivalence cash flows, where the certainty equivalence is calculated using a specific value for ρ . Given the degree of risk tolerance, the certainty equivalent \tilde{x}_t^{CE} then represents the project value, in comparison to the NPVs used above, as in the following formula:

$$\tilde{x}_t^{CE} = -\rho_t \ln \left(\mathbb{E}_t \left[\exp(-\tilde{x}_t/\rho_t) \right] \right)$$

where \tilde{x}_t represents an uncertain cash flow at t .

Suppose a project has future cash flows $\{CF_0, CF_1, \dots, CF_T\}$ and the NPV of the project is

$$v_t = NPV_t(CF_t) = \begin{cases} 0 & \text{if no exercise} \\ \mathbb{E}_t \left(POS \times \sum_{i=t}^{T+t} e^{-(i-t)r_f} CF_i \right) & \text{if exercise} \end{cases}$$

where POS is the possibility of success and r_f is the risk free rate.

Combining the above formulas, we write down the formula for effective certainty equivalent $ECE_{t+1}(\cdot)$, which is defined by taking expectations over period- t 's risk conditional on the information available at time t .

$$ECE_{t+1}[v_{t+1} | \mathcal{F}_t] = -\gamma_{t+1} \ln \left(\mathbb{E}_t \left[e^{-\frac{v_{t+1}}{\gamma_{t+1}}} | \mathcal{F}_t \right] \right)$$

where $\gamma_t = \sum_{\tau=t}^T \frac{\rho_\tau}{(1+r_f)^{\tau-t}}$ is the NPV of the future risk tolerances.

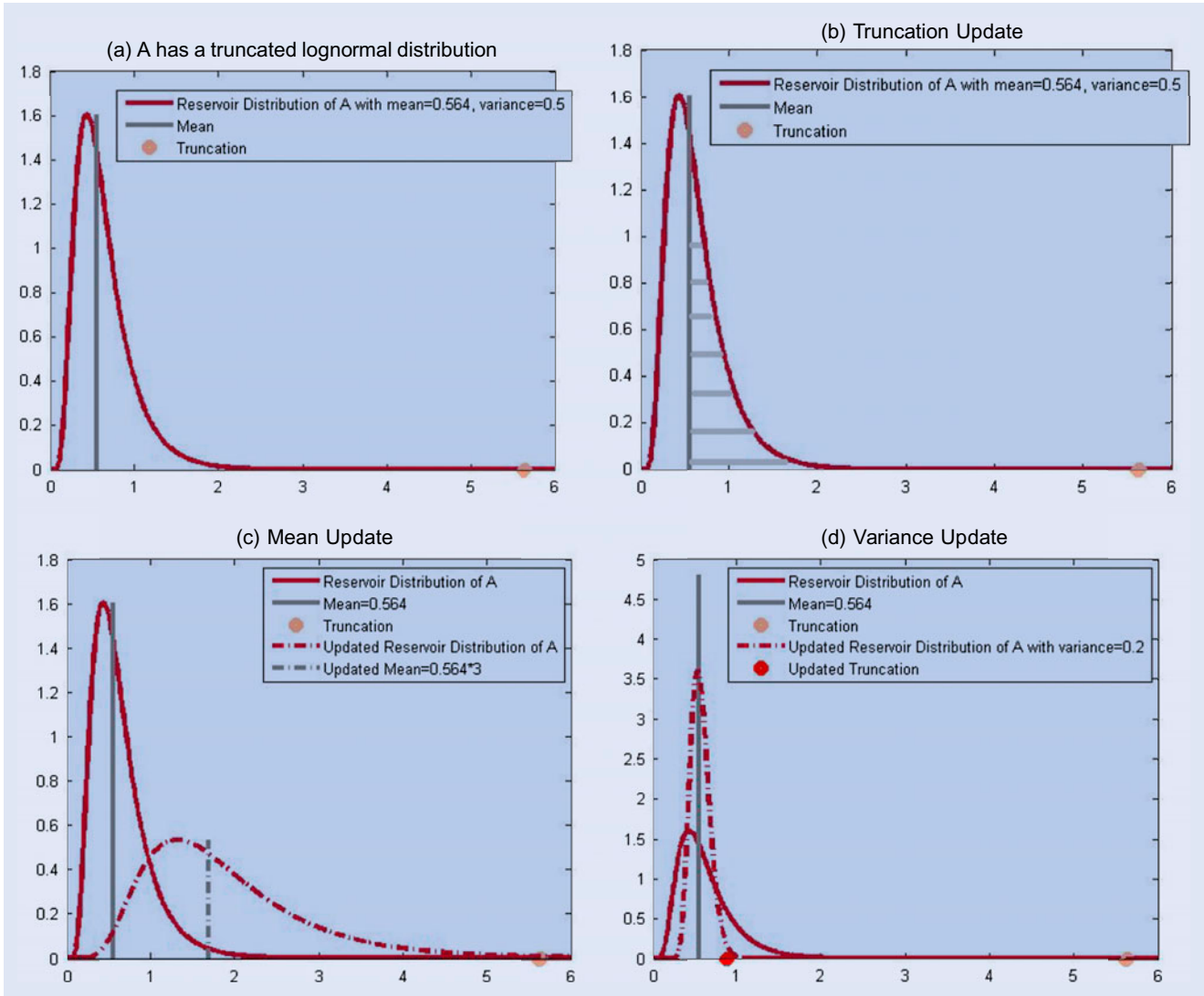


Figure 11. Reservoir correlation examples.

Therefore, under the UIP setting, the option problem comes down to solving maximization problem:

$$\max_{t \in I} \mathbb{E}_0 \left(\exp(-r_f t) \max(X_t + A_t, 0) \right)$$

where:

$$X_t = -\gamma_t \ln \left(\mathbb{E}_t \left[e^{-\frac{v_t^B}{\gamma_t}} \mid \mathcal{F}_{t-1} \right] \right)$$

$$A_t = -\gamma_t \ln \left(\mathbb{E}_t \left[e^{-\frac{\max_{t \in I} \mathbb{E}_t \left(\exp(-r_f(t-t)) \max(v_t^{A|B}, 0) \right)}{\gamma_t}} \mid \mathcal{F}_{t-1} \right] \right).$$

4.5. Implementing UIP: the Least Squares Monte Carlo Method

As indicated earlier, UIP comes down to determining the price at which a risk averse investor becomes indifferent between on the one hand paying that price and receiving an uncertain claim, vs. not paying that price but also not receiving that claim, all the time maintaining an optimal trading strategy.

The ‘maintaining an optimal trading strategy’ condition implies that an explicit Dynamic Stochastic Programming approach is called for, based on an explicit modelling of preferences (i.e. an explicit utility function). There is no closed form solution to the particular optimization problem we have to analyse, since we are adopting the GARCH and GAS frameworks in addition to the presence of unhedgeable reservoir size risks. Another complication is related to the various endogenous exercise moments; no analytical solution exists for such options either (in asset pricing jargon: we are dealing with what would be called American options in continuous time, or, more appropriately given our discrete time framework, Bermudatype options). We solve the valuation problem using SDP, and reduce the dimensionality problem using the Least Squares Monte Carlo approach proposed by Longstaff and Schwartz (2001). SDP sets the current value equal to current utility plus the value of continuing in the future. It is the latter component that triggers an exploding dimensionality problem. Longstaff–Schwartz Monte Carlo (LSMC) approximates the continuation value of the claim as a function of the state variables by repeated application of regression techniques on simulated data.

A flowchart (figure A8) in appendix 1 explains how this algorithm works.

5. Results

The number of Monte Carlo simulations used for LSMC is 100 000. Here, POS for drilling at A is 90% and POS for drilling at B is 30%. We first sketch the cost-of-capital method, as an intermediate step going from scenario based NPV analysis to UIP.

5.1. Cost-of-capital method

5.1.1. NPVs vs. option values. The results are shown over a range of values for the cost-of-capital (3–15%) rates in figure 6. Note that the rates refer to real rates, everything is expressed in period zero prices.

Figure 6(a) gives the results based on assuming a t -GARCH(1,1) process for gas prices, and figure 6(b) shows the same set of results but based on assuming a t -GAS(1,1) structure for the volatility process of gas prices.

In both graphs, option values and NPVs of both fields are declining when the assumed cost-of-capital increases, as one should expect given the time structure of cash flows. The horizontal solid (red) line stands for a break-even project, which sets a standard for accepting and rejecting investment projects. The gray-circled line illustrates the NPV of B at time zero without any option values counted. The dashed gray line stands for the NPV of the strategic plan without the wait-and-see Bermuda option on B, i.e. a fixed starting time at $t = 0$.

The first interesting result stems from comparing the two graphs: the option values assuming t -GAS(1,1) are about 0.5–1 million euros higher than the project values based on assuming a t -GARCH(1,1) specification. Note that the t -GARCH and t -GAS models explain the data with similar power in terms of log-likelihood, therefore this difference of option values results from the different volatility structure predicted by two models.

As is shown in figure 6(b) (and table A2 in appendix 1), with a t -GAS specification, the gray circled line intersects the break-even line at a cost-of-capital of 10%, so on a NPV should be positive criterium, the firm would reject the entire project B for any cost-of-capital higher than (or equal to) 10%. But taking into account the various waiting option values changes that outcome: the discounted net project value with all options incorporated more than doubles for a WACC of 3%, declines with higher WACC but the overall project value with options included stays significantly positive for all values of the WACC considered. It does decline with higher discount rates, obviously, because the high CAPEX come upfront but the revenues come later in time. Note also that a platform may have further uses that we do not incorporate: for example, it can be used for gas storage at a later stage. Nevertheless, it is evident that the strategic development plan including a waiting option is worthwhile because the net project value including option values stays positive given cost-of-capitals varying from 3 to 15%. So a second conclusion is that incorporating the option values is a meaningful exercise: otherwise the wrong investment decision would be taken under a wide range of cost-of-capital estimates.

Similar patterns can be found under the t -GARCH specification with slightly lower option values, as demonstrated in figure 6(a) (and table A2 in appendix 1). For example, the break-even point of NPV of B is at a cost-of-capital of 9.5%, compared to 10% in case of a t -GAS model.

The strategic plan is valued less than the NPV of project B without incurring the costs of project A (and thus also foregoing any of its revenue). Later, we show that the strategic plan becomes more valuable if more information will become available in the future.

From here on, we will not report the GARCH results anymore.† They are obviously qualitatively similar to the GAS-based results, but the GAS specification has a stronger basis in the econometric results of our data analysis.

5.1.2. State dependency. As in regular option pricing theory, the option value in our analysis depends on the current market state, in this case the gas price, since the econometric analysis suggests that the best prediction for the future return is mainly influenced by the current state. Figure 7 shows that the value of the project increases with the spot market price. For example, when the spot price is lower than 15 euros per megawatt hour, the option value is close to zero, so for extremely low spot prices, the project has not only a negative NPV but the embedded options are also almost worthless, with a definite rejection at all discount rates as a result.

5.2. Utility indifference pricing

Figure 8 gives the calculated option values based on UIP for a range of values for the risk tolerance parameter ρ . These results are presented in more detail in table A1 in appendix 1. For both pricing models (cost-of-capital and UIP), the option values range between 10 and 15 million euros. As one should expect, option values are increasing in the investor's risk tolerance.‡ Or, to put it differently, the more risk averse an investor is, the less value she attaches to a risky project. It is evident that taking into account the option values once again leads to higher project values, and to a different outcome in terms of the decision to proceed or not. Note that the valuation increases steadily initially as the risk tolerance of the decision-maker goes up from 5 to 40; however, from a risk tolerance of 40 onwards, the valuation flattens out. These findings are similar to the results obtained by Van Wijnbergen and Zhao (2016).

One interesting observation can be made by comparing figures 6(b) and 8. In figure 8, when the risk tolerance of the investor increases and she/he becomes almost risk neutral, the valuations are similar to the ones obtained from the cost-of-capital method with a cost-of-capital of 3%. Since we assume a risk-free rate of 3%, the valuations obtained using these two methods indeed coincide provided the complete market assumption holds or the investor is risk neutral.

†Corresponding GARCH-based results can be found in appendix 1.
‡This may sound plausible, but it is not trivial: note that such a result is typical for real options (unhedgeable risk) only. For standard risk-neutral option pricing methodology to be applicable, all risks need to be hedgeable; in such a complete market environment, option values do not depend on risk aversion.

As one should expect, the UIP approach leads to a comparison between the outcomes based on the different stochastic specifications of the volatility processes that is similar to what we saw comparing the outcomes under different cost of capital values. For all levels of risk tolerance, the t -GAS-based analysis leads to slightly higher valuations than the t -GARCH-based approach due to higher estimated volatilities.

5.3. Model ambiguity

In the discussion so far we have proceeded on the assumption that specific values for the reserve levels were unknown, but their probability distribution was known with full certainty. Of course that may be overly optimistic: there is likely ambiguity about the distribution itself, model ambiguity in short. In this subsection, we take model uncertainty into consideration and show how it affects the project values. We also assume that the investor is *ambiguity averse*. This implies that she considers the worst-case scenario when facing ambiguity, she follows a maxmin strategy: take the minimum value of the maximized outcomes/valuations over the different distributional possibilities (see Gilboa and Schmeidler (1989)). In this example, we assume ambiguity exists in the mean of the reservoir distribution only, and we take the variance of the reservoir distribution as known from the geological structure of the locations. In particular, no ambiguity implies one single known pdf for the reserve size distribution; ambiguity implies there are two different pdf's, with the means further apart for higher levels of ambiguity. In each case, two pdf's are chosen such that with 50/50 weights their expected values average out to the mean of the no-ambiguity case. Initially, we assume the same ambiguity on the reservoir sizes of both A and B. Of course we can apply similar methods based on the assumption of different ambiguity levels for A and B. Of special interest is the case where ambiguity levels are reduced when information becomes available halfway the project. We consider that possibility explicitly in the next section, section 5.3.1.

The structure of ambiguity introduced in this section also implies that the time consistency issues highlighted by Thijssen (2011) and Hellmann and Thijssen (2015) cannot arise. First of all, the pdfs summarizing gas reservoir size distributions are uncorrelated with the stochastics governing the gas price itself.† Without correlation between gas prices and reservoir uncertainty, the different priors can be ranked monotonically as long as there is positive risk aversion since they have equal variance but different means for any given level of ambiguity. So for any given ambiguity level, the same prior will always represent the worst case at all points in time and no time inconsistency problems arise. We have also verified this numerically. Finally, as a referee has pointed out, the Minimax approach advocated by Gilboa and Schmeidler (1989) requires the set of priors giving rise to ambiguity to be closed and convex, which our two prior assumption clearly does not satisfy. However, allowing any linear combination of the two priors into the

†Note that that is not necessarily true under all extraction cost structures. If marginal extraction costs depend on remaining reservoir volumes, the gas price will have an impact on what is called recoverable reserves. That problem does not arise, however, under the extraction cost structure assumed in this study.

ambiguity set does make it closed and convex, and does not change any of our results precisely because of the strong monotonicity property our specific ambiguity structure implies.‡

Figure 9 shows that the project value decreases with higher ambiguity levels. A higher ambiguity level means the decision-maker is less certain about the mean of the reservoir distribution, which leads to a lower level of valuation as a consequence of the Minimax strategy followed. Figure 9 also shows that for higher levels of ambiguity, the valuation differences between decision-makers with different risk tolerance shrink accordingly.

These results provide interesting implications for insurance. The mirror image (upside down) of these graphs can be interpreted as how much the agent would be willing to pay for insurance against a certain risk the agent faces. It implies that for a given ambiguity level, risk averse agents are more likely to buy insurance than high risk tolerance agents (not surprisingly) because they attach a high value to the insurance. On the other hand, for agents with the same risk tolerance, the decision of purchasing the insurance contract depends on their ambiguity level on the underlying processes. For example, according to our results, agents with higher ambiguity levels will pay more for insurance than those with lower ambiguity levels.

5.3.1. When ambiguity is resolved halfway of the process.

In the preceding section, we introduced persistent ambiguity, i.e. uncertainty about the probability structure that remains constant over time. However, it is more reasonable to assume that once production in B has started, more information about A, and more specifically, about the probability distribution of possible outcomes of A, will become available, since the geological structures of B and A are related. And declining ambiguity again brings in rewards for waiting, in a sense once again real option value. We explore the additional value project B gets if its exploration reduces ambiguity over well A once B is brought in production. In particular, we focus on reservoir A ambiguity only, and assume it gets resolved after starting on reservoir B. In other words, starting on B leads not only to more specific information but also narrows down the range of distributional possibilities.

For simplicity and focus we demonstrate the effect for the case where there is just ambiguity about A, which gets resolved once B is brought into operation. The no-ambiguity case is obviously the same as shown in figure 9. But the interesting results come once we assume that starting on B leads to reduced ambiguity on A, for example, because the fields are contiguous. If the ambiguity level of A is at a particular level at the beginning and we know that ambiguity disappears after the development of B, then the difference between no-ambiguity and the project value at that particular Ambiguity-level should be added to the project value of B. Figure 10 makes the point for the two moderate ambiguity level (Level 2 and Level 3): it shows the option values that resolution of ambiguity leads to as a percentage of the original project value of B with ambiguity persistent, and for different levels of risk tolerance.

It is clear from figure 10 that option values go up with risk tolerance and also increase as the initial ambiguity level

‡See for an extreme example of a similar result Byder and Dew-Becker (2016).

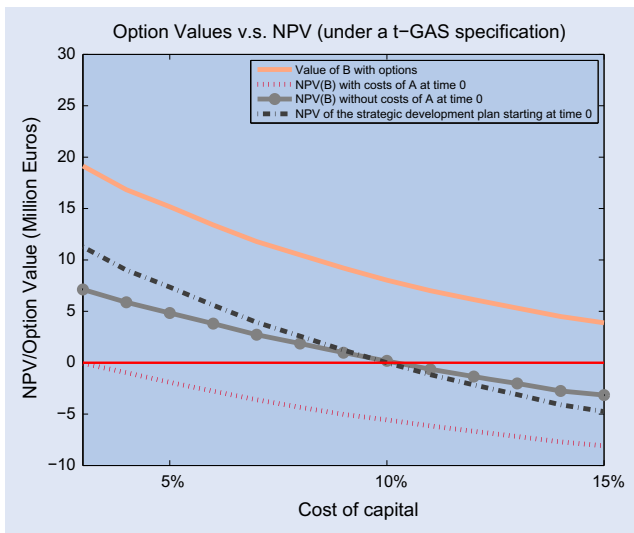


Figure 12. Option pricing results for the general case with reservoir correlation (Student's *t*-GAS).

that gets resolved is higher. And the option value numbers are substantial: in this example, the increase in project value due to the reduction in ambiguity ranges between approximately 5 and 15% of the original project value depending on risk tolerance and level of pre-existing ambiguity.

5.4. Reservoir correlation

Finally, we consider a plausible example of correlated information, shown in figure 11(a). Assume that if B turns out to be a successful development, the information about the reservoir size distribution of A will be updated correspondingly. This can also be interpreted as one example of ambiguity reduction. Figure 11 shows some possible distribution updates for the distribution of A, with the original distribution given in figure 11(a). The following diagrams figure 11(b)–(d) show three different ways the distributional information could change: a truncation point update, a shift in the mean and a reduction in the variance.

In figure 11(b), we show how the distribution changes when the truncation point shifts inwards, i.e. the range of possible outcomes narrows down, as in the shadow area displayed in figure 11(b). Alternatively, the mean could shift; figure 11(c) shows an example where the mean shifts up. Finally, mean and truncation points could be left unchanged but the variance could be reduced as information from B becomes available (figure 11(d)). In what follows we focus on the case where the truncation point shifts inwards once B has started up, the case shown in figure 11(b), to demonstrate how our option technique works. We again present the results using both for the cost-of-capital approach and the UIP approach.

5.4.1. Cost-of-capital method. Comparison of figure 12 with figure 6 shows that reservoir correlation has increased the option value by about 1–5 million over the range of cost-of-capital rates considered (as also shown in figure 13). The reservoir correlation of course does not change the NPV of B,

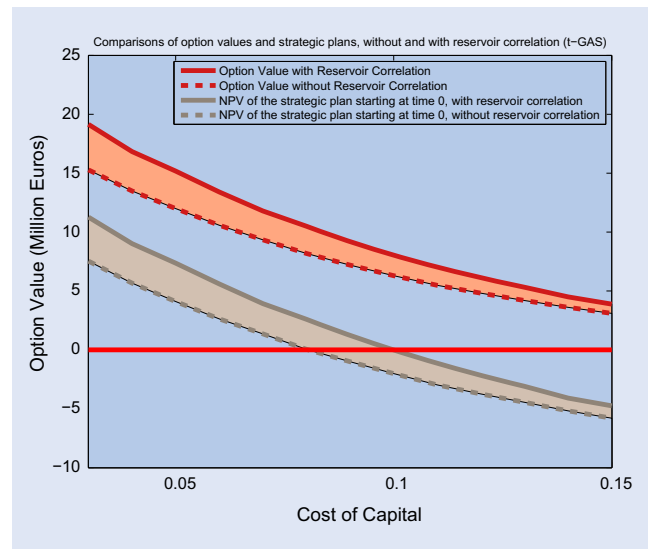


Figure 13. Comparison: option values with and without reservoir correlation (Student's *t*-GAS).

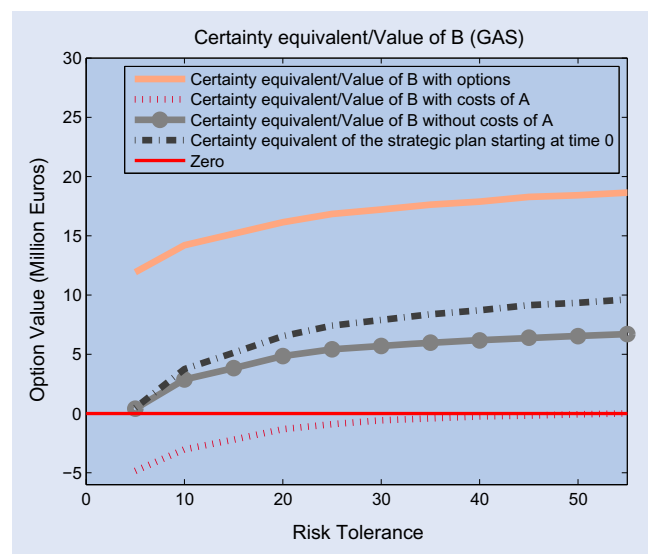


Figure 14. UIP results for the general case with reservoir correlation (Student's *t*-GAS).

therefore the red dashed line and the gray circled line stay the same as in figure 6. Moreover, the strategic plan now outperforms the stand-alone project B for low cost-of-capital estimates; this happens for rates below 10% under a *t*-GAS specification.

5.4.2. Comparison with the case without reservoir correlation. Furthermore, the shadow areas in figure 13 represent the differences of values between the projects with and without reservoir correlation. It is evident that both the option and strategic plan are valued higher when reservoir correlation exists. In other words, the halfway resolution of reservoir distribution ambiguity/correlation increases the project value significantly by adding option value.

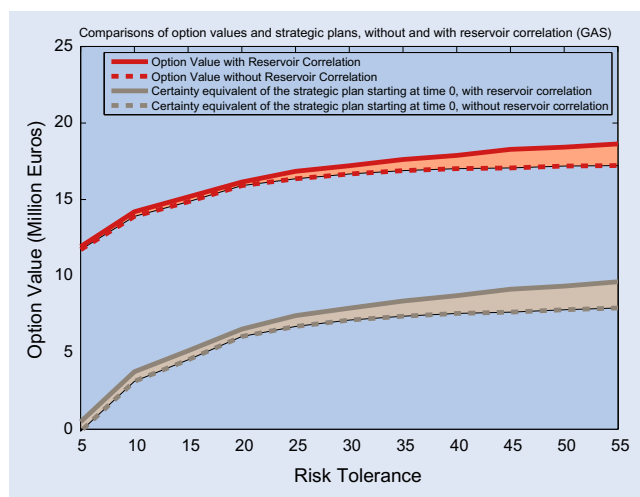


Figure 15. Comparison: option values with and without reservoir correlation (Student's t -GAS).

5.4.3. Utility indifference pricing. When the evaluation is based on UIP instead of on fixed cost-of-capital estimates, similar to the comparison in section 5.4.1, the strategic plan presented brings in more revenues (in NPV terms) than project B on its own, shown in figures 14 and 15. But the more important point is that reduction of uncertainty, this time a narrowing down of the range of possible outcomes, once again leads to substantial option values and correspondingly higher project value. So once again we find that ignoring option values and information acquisition over time leads to overly conservative project valuation and excessively conservative project decisions.

5.4.3.1. Comparison with the case without reservoir correlation. Similar to figure 13, the shadow areas in figure 15 represent the differences in valuation when comparing the projects with and without reservoir correlation. Again we find that ambiguity on A is reduced once B has been brought into operation causes the option values to increase: the more future information gets updated as the project moves ahead, the higher the initial project value is.

6. Conclusion

This paper has focused on the real option approach to solving a contingent claim problem as an alternative method for decision-making under uncertainty. We incorporate many aspects that complicate real-world asset pricing problems, such as incomplete markets and unhedgeable risks, dynamic release of distributional information and non-normal volatility assumptions, all of which invalidate traditional risk neutral approaches to asset pricing. UIP is applied in face of market incompleteness and t -GARCH/ t -GAS models are used for modelling the volatility of gas prices. We show in a real-world example that the Student's t -GARCH/ t -GAS model, with its fatter tails, fits the observed data better than the Gaussian GARCH/GAS model in terms of the associated log-likelihood ratios.

UIP requires the explicit solution of Stochastic Dynamic Programming problems; SDP is widely thought to suffer from the curse of dimensionality to such an extent that it becomes impractical for real-world size problems. But our analysis of a pair of existing gas fields in the North Sea demonstrates that the use of sophisticated simulation and approximation techniques (LSMC) brings problems of real world complexity down to manageable size.

We also take the analysis one step further by introducing deep uncertainty, of the type that cannot be summarized by formulating a probability density function, because it concerns uncertainty about that very density function. In the literature, this sort of uncertainty is referred to as Knightian uncertainty or, the word we prefer, model ambiguity. In our case study, we show that the existence of model ambiguity reduces asset values in a risk averse world and will ceteris paribus lead to more conservative project continuation decisions.

But we also introduce a new angle to this debate by pointing out that for time structured projects with correlated distributions, a new source of option value can emerge. If executing one part of the project leads to reduced model ambiguity concerning the later components of the project, the initial blocks acquire additional option values, which in our case study are shown to be substantial. As the ambiguity level decreases as the project progresses, the initial project becomes more valuable due to the information that will be brought in along with development. The value of projects that allow for that sort of flexibility will be underestimated consistently by more traditional NPV-based valuation approaches. In our real world case study, the biases are shown to be substantial.

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Appendix 1.

A.1. GAS models

A.1.1. Gaussian GARCH model. The model above can be reduced to a Gaussian GARCH model if $f_t = \sigma_t^2$ and $\varepsilon_t \sim N(0, 1)$, i.e.

$$y_t = \sigma_t \varepsilon_t$$

$$\sigma_{t+1}^2 = \omega + A \left(y_t^2 - \sigma_t^2 \right) + B \sigma_t^2$$

where ω , A , and $B - A$ are parameters in a classical Gaussian GARCH model.

A.1.2. Gaussian GAS model. Alternatively, if take $f_t = \log \sigma_t^2$, we obtain a Gaussian GAS(1,1) model, i.e.

$$y_t = \sigma_t \varepsilon_t$$

$$\log \sigma_{t+1}^2 = \omega + A \left(\frac{y_t^2}{\sigma_t^2} - 1 \right) + B \log \sigma_t^2$$

In this model, next period's variance depends in a linear manner on a constant, the current period's variance and the square of the standardized observations, $\frac{y_t^2}{\sigma_t^2}$.

A.1.3. Student's t GARCH model. If the error term ε_t follows a Student's t distribution with degree of freedom ν , then it again becomes a t -GARCH model. Similarly, if we still fit it into a GAS framework, the model can be written as follows:

$$y_t = \sigma_t \varepsilon_t$$

$$\sigma_{t+1}^2 = \omega + A \frac{\nu + 3}{\nu} \left(\left(1 + \frac{y_t^2}{(\nu - 2) \sigma_t^2} \right)^{-1} \frac{\nu + 1}{\nu - 2} y_t^2 - \sigma_t^2 \right) + B \sigma_t^2$$

The estimation results can be found in figures A1–A8 and tables A3–A6.

A.1.4. Student's t GAS model. A Student's t GAS(1,1) model is obtained by choosing $f_t = \log \sigma_t^2$, and $\varepsilon_t \sim t(\nu)$.

$$y_t = \sigma_t \varepsilon_t$$

$$\log \sigma_{t+1}^2 = \omega + A \frac{\nu + 3}{\nu} \left(\left(1 + \frac{y_t^2}{(\nu - 2) \sigma_t^2} \right)^{-1} \frac{(\nu + 1) y_t^2}{(\nu - 2) \sigma_t^2} - 1 \right) + B \log \sigma_t^2$$

A.2. Results under a specification of the gas price volatility process as a Student's *t*-GARCH model

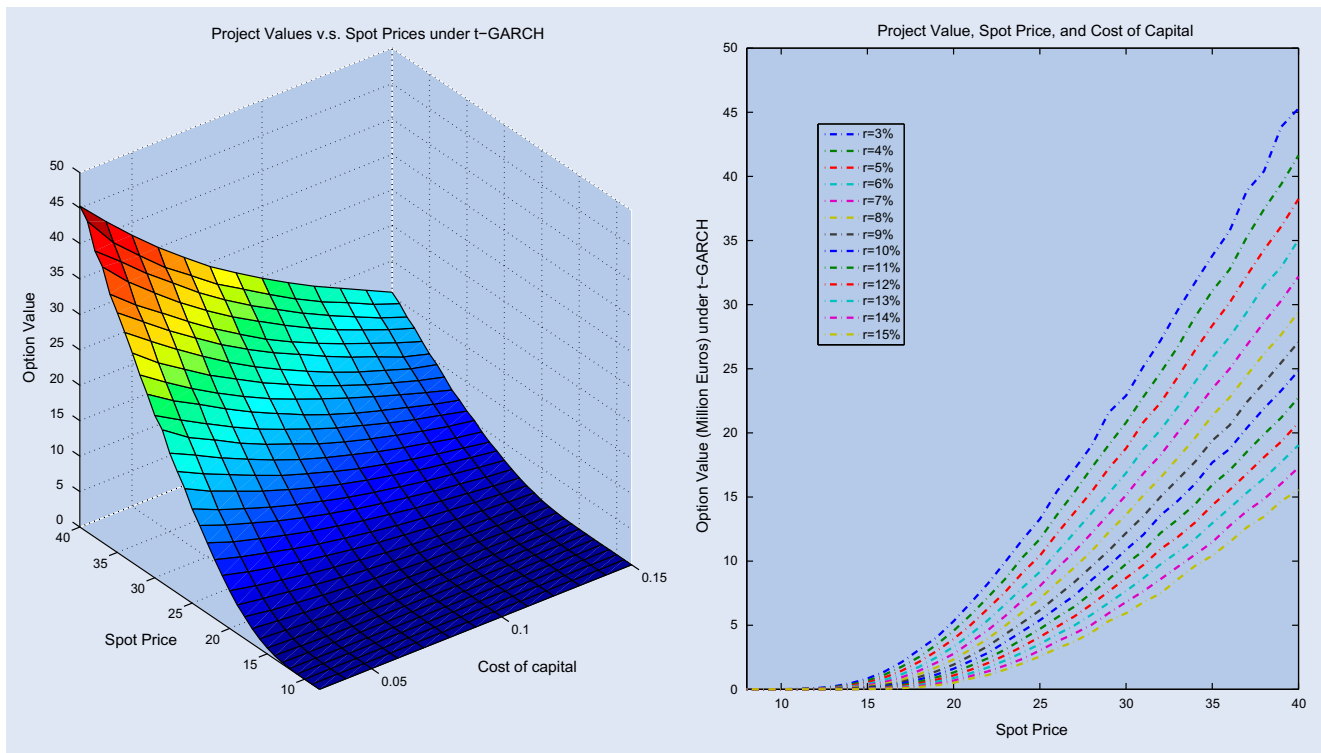


Figure A1. Spot prices vs. option value under a Student's *t*-GARCH specification.

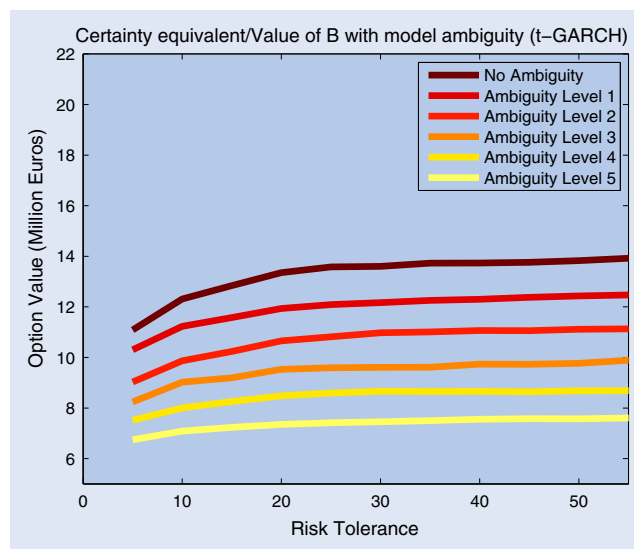


Figure A2. Option values with persistent model ambiguity (*t*-GARCH).

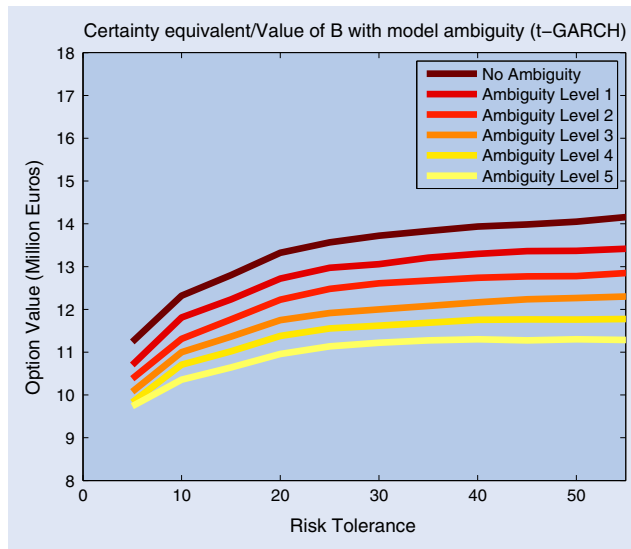


Figure A3. Ambiguity in field a only (*t*-GARCH).

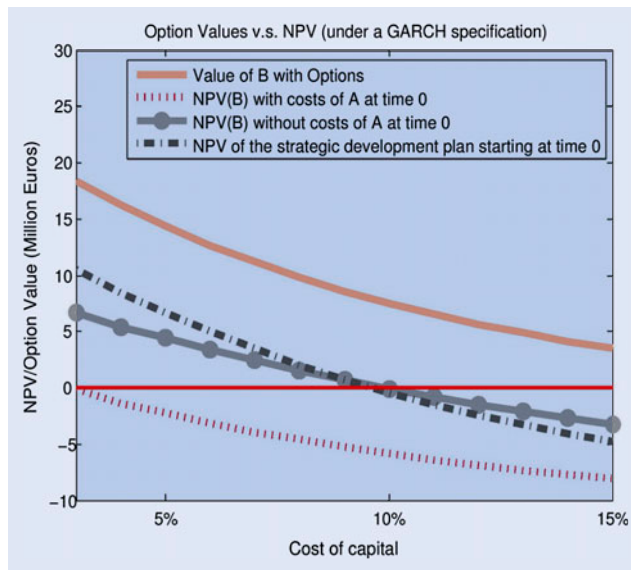


Figure A4. Option pricing results for the general case with reservoir correlation (*t*-GARCH).

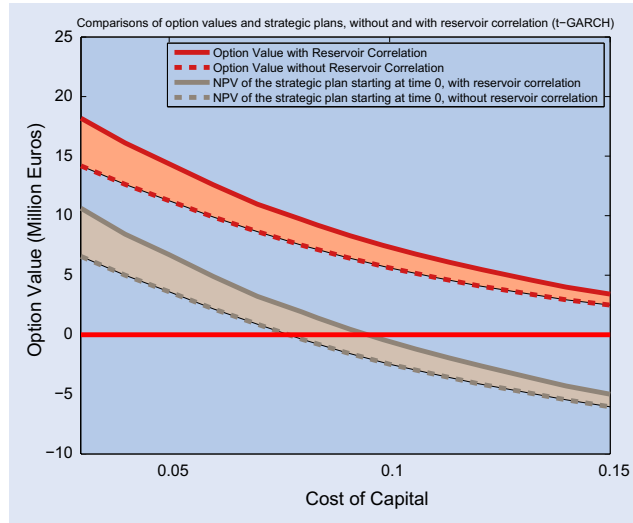


Figure A5. Comparison: option values with and without reservoir correlation (Gaussian GARCH).

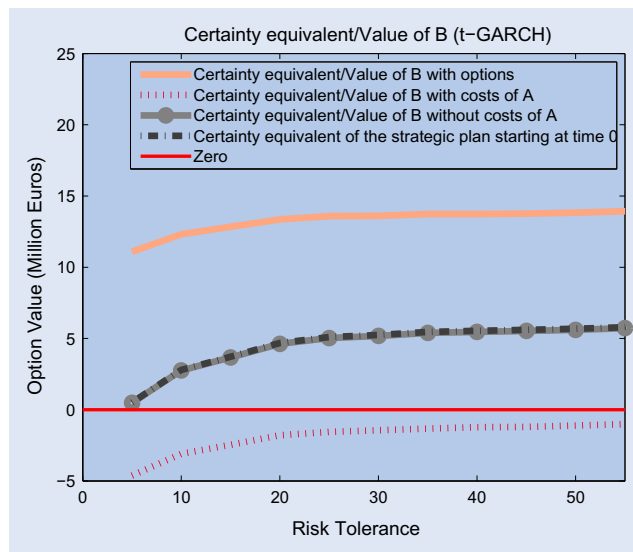


Figure A6. UIP results for the general case with reservoir correlation (t -GARCH).

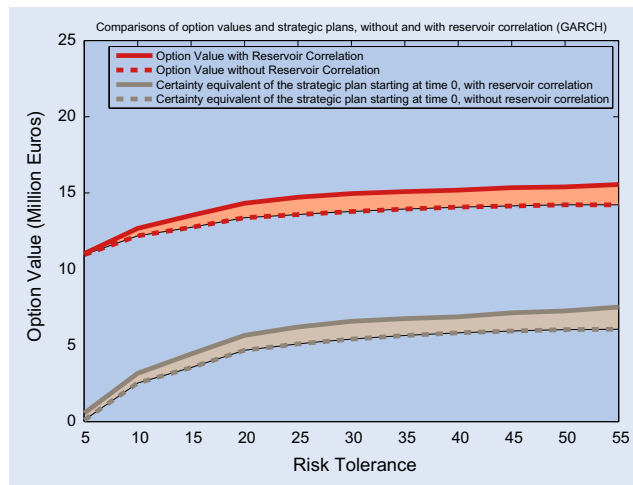


Figure A7. Comparison: option values with and without reservoir correlation (t -GARCH).

A.3. Option results in detail

Table A1. UIP comparison (million euros).

Risk tolerance	Option value (<i>t</i> -GARCH)	Option value (<i>t</i> -GAS)	Difference
5	11.09	11.11	-0.02
10	12.31	12.63	-0.32
15	12.84	13.19	-0.35
20	13.35	13.87	-0.52
25	13.58	14.21	-0.63
30	13.60	14.37	-0.77
35	13.73	14.60	-0.86
40	13.74	14.74	-1.00
45	13.76	14.85	-1.08
50	13.83	14.90	-1.08
55	13.92	14.98	-1.06

Table A2. Option values vs. NPV of B at time 0 (million euros).

Cost-of-capital (%)	Student's <i>t</i> -GARCH			Student's <i>t</i> -GAS		
	Option values	NPV of B at <i>t</i> = 0	Difference	Option values	NPV of B at <i>t</i> = 0	Difference
3	14.18	6.49	7.68	15.28	7.27	8.00
4	12.62	5.41	7.21	13.49	5.90	7.59
5	11.25	4.40	6.85	11.99	4.75	7.25
6	9.89	3.35	6.54	10.57	3.67	6.91
7	8.66	2.37	6.29	9.34	2.72	6.62
8	7.52	1.36	6.17	8.17	1.66	6.51
9	6.51	0.48	6.03	7.19	0.83	6.36
10	5.61	-0.34	5.95	6.30	0.00	6.29
11	4.83	-1.02	5.85	5.47	-0.79	6.26
12	4.12	-1.70	5.82	4.81	-1.43	6.24
13	3.52	-2.22	5.75	4.19	-2.04	6.23
14	2.93	-2.81	5.74	3.61	-2.65	6.25
15	2.48	-3.18	5.66	3.09	-3.28	6.37

Table A3. Model ambiguity.

Risk tolerance	Ambiguity level (left to right, low to high)					
	No ambiguity	Level 1	Level 2	Level 3	Level 4	Level 5
(a) Student's <i>t</i> -GARCH						
5	11.09	10.30	9.03	8.24	7.52	6.74
10	12.31	11.23	9.87	9.02	8.00	7.09
15	12.84	11.58	10.24	9.20	8.26	7.24
20	13.35	11.93	10.65	9.53	8.48	7.35
25	13.58	12.09	10.82	9.59	8.59	7.42
30	13.60	12.17	10.98	9.61	8.67	7.46
35	13.73	12.26	11.01	9.62	8.65	7.50
40	13.74	12.30	11.07	9.74	8.66	7.56
45	13.76	12.38	11.06	9.73	8.64	7.58
50	13.83	12.43	11.11	9.77	8.68	7.57
55	13.92	12.47	11.13	9.89	8.69	7.60
(b) Student's <i>t</i> -GAS						
5	11.11	10.21	9.25	8.41	7.87	7.05
10	12.63	11.43	10.39	9.29	8.46	7.55
15	13.19	11.98	10.80	9.67	8.71	7.74
20	13.87	12.58	11.29	10.04	9.00	7.99
25	14.21	12.88	11.48	10.24	9.10	8.12
30	14.37	13.04	11.55	10.35	9.19	8.19
35	14.60	13.16	11.62	10.40	9.22	8.22
40	14.74	13.15	11.69	10.40	9.25	8.24
45	14.85	13.17	11.73	10.47	9.22	8.23
50	14.90	13.16	11.77	10.51	9.24	8.25
55	14.98	13.18	11.90	10.56	9.26	8.22

Table A4. Model ambiguity of A only.

Risk tolerance	Ambiguity level (left to right, low to high)					
	No ambiguity	Level 1	Level 2	Level 3	Level 4	Level 5
(a) Student's <i>t</i> -GARCH						
5	11.09	10.70	10.38	10.08	9.83	9.74
10	12.31	11.81	11.31	11.00	10.71	10.36
15	12.84	12.23	11.77	11.36	11.02	10.64
20	13.35	12.72	12.23	11.75	11.38	10.96
25	13.58	12.97	12.48	11.92	11.55	11.13
30	13.60	13.06	12.61	12.00	11.62	11.22
35	13.73	13.21	12.67	12.08	11.69	11.27
40	13.74	13.30	12.74	12.16	11.75	11.30
45	13.76	13.36	12.77	12.24	11.76	11.28
50	13.83	13.37	12.78	12.26	11.76	11.30
55	13.92	13.42	12.85	12.30	11.78	11.28
(b) Student's <i>t</i> -GAS						
5	11.11	10.89	10.56	10.21	9.96	9.71
10	12.63	12.16	11.90	11.46	10.98	10.75
15	13.19	12.75	12.43	11.95	11.51	11.22
20	13.87	13.36	12.97	12.51	12.00	11.69
25	14.21	13.66	13.21	12.75	12.27	11.87
30	14.37	13.86	13.34	12.87	12.42	12.01
35	14.60	14.01	13.45	12.96	12.49	12.08
40	14.74	14.11	13.48	13.07	12.49	12.13
45	14.85	14.21	13.54	13.08	12.56	12.17
50	14.90	14.26	13.61	13.10	12.62	12.25
55	14.98	14.35	13.66	13.13	12.70	12.27

Table A5. Option values vs. NPV of B at time 0 (million euros) with reservoir correlation.

Cost-of-capital (%)	Student's <i>t</i> -GARCH			Student's <i>t</i> -GAS		
	Option values	NPV of strategic plan starting at $t = 0$	Difference	Option values	NPV of strategic plan starting at $t = 0$	Difference
3	18.18	10.62	7.57	19.14	11.27	7.87
4	16.11	8.44	7.66	16.83	9.01	7.82
5	14.35	6.72	7.63	15.18	7.36	7.82
6	12.59	4.90	7.69	13.40	5.59	7.81
7	10.96	3.22	7.74	11.79	3.92	7.87
8	9.68	1.92	7.76	10.49	2.59	7.90
9	8.44	0.61	7.83	9.20	1.22	7.98
10	7.35	-0.59	7.94	8.03	-0.01	8.04
11	6.39	-1.62	8.01	7.01	-1.16	8.17
12	5.55	-2.53	8.08	6.14	-2.16	8.30
13	4.75	-3.44	8.19	5.31	-3.10	8.41
14	3.99	-4.30	8.29	4.50	-4.10	8.60
15	3.41	-5.00	8.41	3.88	-4.76	8.64

Table A6. UIP comparison (million euros) with reservoir correlation.

Risk tolerance	Option value under a Student's <i>t</i> -GARCH specification	Option value under a Student's <i>t</i> -GAS specification	<i>Difference</i>
5	11.19	11.33	-0.14
10	12.92	13.11	-0.19
15	13.63	13.81	-0.18
20	14.35	14.76	-0.41
25	14.66	15.19	-0.53
30	14.84	15.40	-0.56
35	14.84	15.72	-0.88
40	15.02	15.92	-0.90
45	15.19	15.87	-0.68
50	15.20	15.94	-0.74
55	15.40	15.94	-0.54

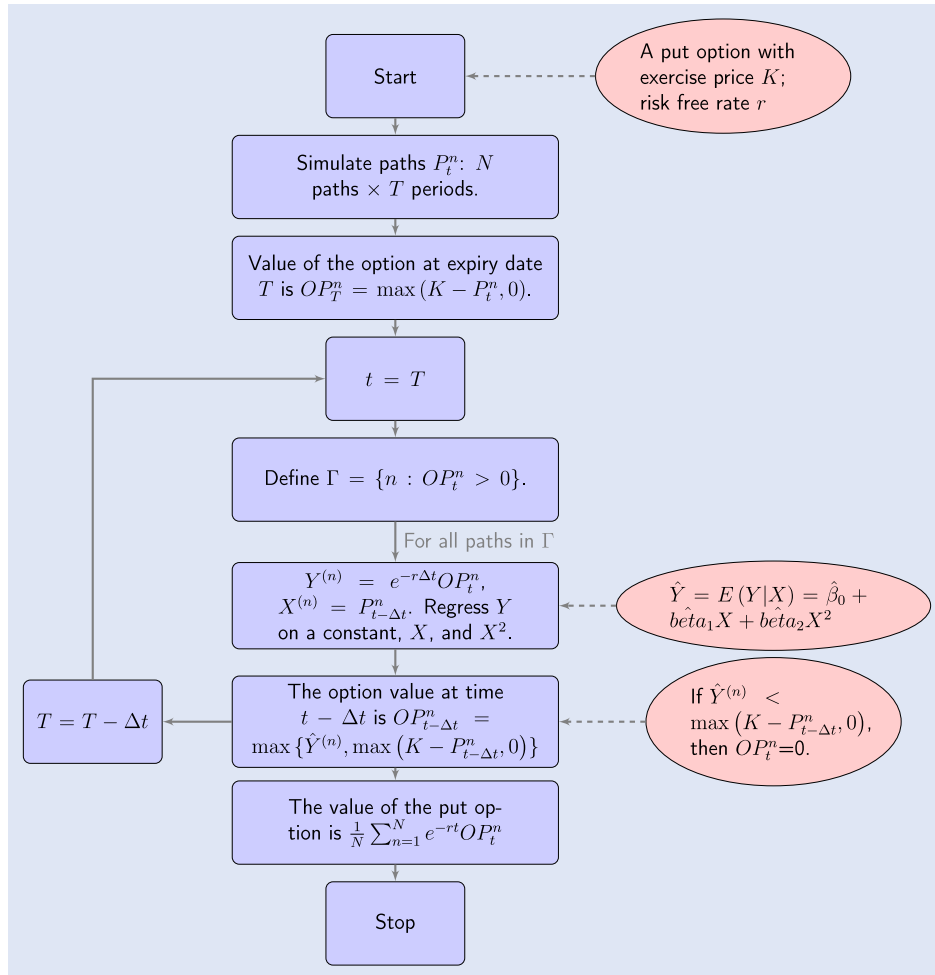


Figure A8. Least Squares Monte Carlo Method (Longstaff and Schwartz 2001).