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Existence and stability of periodic solutions for impulsive fuzzy BAM Cohen-Grossberg neural networks on time scales

Shaohong Cai[1] and Qianhong Zhang[1,2*]

*Correspondence: zqianhong@163.com
[1]Guizhou Key Laboratory of Economics System Simulation, Guizhou University of Finance and Economics, Huaxi Campus Town, Guiyang, 550025, China
[2]School of Mathematics and Statistics, Guizhou University of Finance and Economics, Huaxi Campus Town, Guiyang, 550025, China

Abstract
This paper is concerned with the existence and global exponential stability of periodic solutions for a kind of impulsive fuzzy Cohen-Grossberg BAM neural networks on time scales. Applying the method of coincidence degree and constructing some suitable Lyapunov functional, we obtain some sufficient conditions for the existence and global exponential stability of periodic solutions for a kind of impulsive fuzzy Cohen-Grossberg BAM neural networks on time scales. Moreover, we give an example to illustrate the results obtained.

Keywords: periodic solutions; fuzzy Cohen-Grossberg BAM neural networks; coincidence degree; impulses; time scales

1 Introduction

In recent years, Cohen and Grossberg BAM neural networks have been extensively studied and applied in many different fields such as associative memory, signal processing, and some optimization problems. They have been widely studied both in theory and applications [1, 2]. Many results for the existence of their periodic solutions and the exponential convergence properties for Cohen-Grossberg neural networks have been reported in the literature (see, *e.g.*, [3–14] and the references therein).

In this paper, we would like to integrate fuzzy operations into Cohen-Grossberg BAM neural networks. Speaking of fuzzy operations, Yang and Yang [15] first introduced fuzzy cellular neural networks (FCNNs) combining those operations with cellular neural networks. So far researchers have found that FCNNs and fuzzy Cohen-Grossberg neural networks are useful in image processing, and some results have been reported on stability, periodicity, and antiperiodicity (see, *e.g.*, [15–23] and the references therein).

In fact, both continuous and discrete systems are very important in implementing and applications. But it is troublesome to study the existence and stability of periodic solutions for continuous and discrete systems, respectively. Therefore, it is meaningful to study that on time scales, which can unify the continuous and discrete situations. In this paper, we consider the following fuzzy Cohen-Grossberg BAM neural networks with impulses on time scales:

$$\begin{cases} x_i^{\Delta}(t) = -a_i(x_i(t))[c_i(x_i(t)) - \bigwedge_{j=1}^{m} \alpha_{ji}(t) f_j(y_j(t-\tau_{ji})) \\ \qquad - \bigvee_{j=1}^{m} \beta_{ji}(t) f_j(y_j(t-\tau_{ji})) + E_i(t)], \quad t \in \mathbb{T}^+, t \neq t_k, \\ \Delta_i(x_i(t_k)) = I_{ik}(x_i(t_k)), \quad k \in \mathbb{N}, i = 1,2,\ldots,n; \\ y_j^{\Delta}(t) = -b_j(y_j(t))[d_j(y_j(t)) - \bigwedge_{i=1}^{n} p_{ij}(t) g_i(x_i(t-\sigma_{ij})) \\ \qquad - \bigvee_{i=1}^{n} q_{ij}(t) g_i(x_i(t-\sigma_{ij})) + F_j(t)], \quad t \in \mathbb{T}^+, t \neq t_k, \\ \Delta_i(y_j(t_k)) = J_{jk}(y_j(t_k)), \quad k \in \mathbb{N}, j = 1,2,\ldots,m. \end{cases} \tag{1}$$

where $x_i(t)$, $y_j(t)$ are the activations of the ith neuron in X-layer and the jth neuron in Y-layer, the functions a_i, b_j represent the abstract amplification functions, whereas the functions c_i, d_j represent the self-excitation rate functions; time delays τ_{ji} and σ_{ij} are positive constants, which correspond to the finite speed of the axonal signal transmission; $\alpha_{ji}(t)$, $\beta_{ji}(t)$, $p_{ij}(t)$, $q_{ij}(t)$ are elements of fuzzy feedback MIN template and fuzzy feedback MAX template in X-layer and Y-layer, respectively; $\bigwedge$ and $\bigvee$ denote the fuzzy AND and fuzzy OR operations, respectively; $E_i(t)$ and $F_j(t)$ denote the ith and jth components of an external input source introduced from outside the network to the cell i in X-layer and the cell j in Y-layer, respectively; $\mathbb{T}$ is an ω-periodic time scale, which has the subspace topology inherited from the standard topology on $\mathbb{R}$. We denote $I_{\mathbb{T}} = I \cap \mathbb{T}$, $\Delta x_i(t_k) = x_i(t_k^+) - x_i(t_k^-)$, $\Delta y_j(t_k) = y_j(t_k^+) - y_j(t_k^-)$, where $x_i(t_k^+)$, $x_i(t_k^-)$, $y_j(t_k^+)$, $y_j(t_k^-)$ ($i = 1,2,\ldots,n$, $j = 1,2,\ldots,m$) represent the right and left limits of $x_i(t_k)$ and $y_j(t_k)$ in the sense of time scales; $\{t_k\}$ is a sequence of real numbers such that $t_1 < t_2 < \cdots$ and $\lim_{k\to+\infty} t_k = +\infty$. There exists a positive integer q such that $t_{k+q} = t_k + \omega$, $I_{ik+q} = I_{ik}$, $J_{jk+q} = J_j k$, $k \in \mathbb{N}$. Without loss of generality, we also assume that $[0,\omega)_{\mathrm{T}} \cap \{t_k, k \in \mathbb{N}\} = \{t_1, t_2, \ldots, t_q\}$. Let $\mathbb{R}^+ = (0,+\infty)$ and $\mathbb{T}^+ = \mathbb{R}^+ \cap \mathbb{T}$.

The initial conditions associated with system (1) are of the form

$$\begin{cases} x_i(t) = \varphi_i(t), \quad t \in [-\tau, 0]_{\mathbb{T}}, \tau = \max_{1\leq i,j\leq n}\{\tau_{ji}\}, \\ y_j(t) = \psi_j(t), \quad t \in [-\sigma, 0]_{\mathbb{T}}, \sigma = \max_{1\leq i,j\leq n}\{\sigma_{ij}\}, \end{cases} \tag{2}$$

where $\varphi_i(t) \in C([-\tau,0]_{\mathbb{T}},\mathbb{R})$, $\psi_j(t) \in C([-\sigma,0]_{\mathbb{T}},\mathbb{R})$. For convenience, we introduce the notation

$$\hat{f} = \frac{1}{\omega}\int_0^{\omega} f(t)\Delta t, \qquad \|f\|_2 = \left(\int_0^{\omega} |f(t)|^2 \Delta t\right)^{1/2},$$

$$\overline{f} = \max_{t\in[0,\omega]_{\mathrm{T}}} |f(t)|, \qquad \|f^{\Delta}\|_2 = \left(\int_0^{\omega} |f^{\Delta}(t)|^2 \Delta t\right)^{1/2},$$

where f is an ω-periodic function.

Throughout this paper, we make the following assumptions:

(A1) $E_i, F_j, \alpha_{ji}, \beta_{ji}, p_{ij}, q_{ij} \in C(\mathbb{T},\mathbb{R})$ are ω-periodic functions, $\tau_{ji}, \sigma_{ij} \in \mathbb{R}^+$, $i = 1,2,\ldots,n$, $j = 1,2,\ldots,m$.

(A2) $a_i, b_j \in C(\mathbb{R},\mathbb{R}^+)$ are bounded functions, namely, there exist positive constants $\underline{a}_i$, $\overline{a}_i$, $\underline{b}_j$, $\overline{b}_j$ such that $\underline{a}_i \leq a_i(\cdot) \leq \overline{a}_i$, $\underline{b}_j \leq b_j(\cdot) \leq \overline{b}_j$, $i = 1,2,\ldots,n$, $j = 1,2,\ldots,m$.

(A3) $c_i, d_j \in C(\mathbb{R},\mathbb{R}^+)$ are delta differentiable, and $0 < \varrho_i \leq c_i^{\Delta} \leq \delta_i$, $0 < \varrho_j' \leq d_j^{\Delta} \leq \delta_j'$, $c_i(0) = 0$, $b_j(0) = 0$, $i = 1,2,\ldots,n$, $j = 1,2,\ldots,m$.

(A4) $f_j, g_i \in C(\mathbb{R},\mathbb{R})$, and there exist M_j, N_i, κ_j, ν_i ($i = 1,2,\ldots,n$, $j = 1,2,\ldots,m$) such that

$$|f_j| \leq M_j, \qquad |g_i| \leq N_i,$$

$$\left|f_j(u)-f_j(v)\right| \le \kappa_j|u-v|, \qquad \left|g_i(u)-g_i(v)\right| \le \nu_i|u-v|.$$

(A5) $I_{ik}, J_{ik} \in C(\mathbb{R},\mathbb{R})$, and there exist positive constants ρ_{ik}, ρ'_{ik} such that $|I_{ik}| \le \rho_{ik}$, $|J_{jk}| \le \rho'_{jk}$, $k \in \mathbb{N}$, $i = 1,2,\dots,n$, $j = 1,2,\dots,m$.

The organization of the paper is as follows. In Section 2, we introduce some definitions and lemmas. In Section 3, by using coincidence degree we establish sufficient conditions for the existence of the periodic solutions of system (1). In Section 4, by constructing Lyapunov functional we derive sufficient conditions for the global exponential stability of periodic solutions of system (1). An example is given to demonstrate the effectiveness of our results in Section 5. Conclusions are drawn in Section 6.

2 Preliminaries

In this section, we shall first recall some basic definitions and lemmas, which are used in what follows.

Let $\mathbb{T}$ be a nonempty closed subset (time scale) of $\mathbb{R}$. The forward and backward jump operators $\sigma, \rho : \mathbb{T} \to \mathbb{T}$ and the graininess $\mu : \mathbb{T} \to \mathbb{R}^+$ are defined, respectively, by

$$\sigma(t) = \inf\{s \in \mathbb{T} : s > t\}, \qquad \rho(t) = \sup\{s \in \mathbb{T} : s < t\}, \qquad \mu(t) = \sigma(t) - t.$$

A point $t \in \mathbb{T}$ is called left-dense if $t > \inf \mathbb{T}$ and $\rho(t) = t$, left-scattered if $\rho(t) < t$, right-dense if $t < \sup \mathbb{T}$ and $\sigma(t) = t$, and right-scattered if $\sigma(t) > t$. If $\mathbb{T}$ has a left-scattered maximum m, then $\mathbb{T}^k = \mathbb{T} \setminus \{m\}$; otherwise, $\mathbb{T}^k = \mathbb{T}$. If $\mathbb{T}$ has a right-scattered minimum m, then $\mathbb{T}_k = \mathbb{T} \setminus \{m\}$; otherwise, $\mathbb{T}_k = \mathbb{T}$.

Let $\omega \in \mathbb{R}^+$; then $\mathbb{T}$ is an ω-periodic time scale if $\mathbb{T}$ is a nonempty closed subset of $\mathbb{R}$ such that $t + \omega \in \mathbb{T}$ and $\mu(t + \omega) = \mu(t)$ for all $t \in \mathbb{T}$.

A function $f : \mathbb{T} \to \mathbb{R}$ is right-dense continuous if it is continuous at right-dense points in $\mathbb{T}$ and its left-side limits exist at left-dense points in $\mathbb{T}$. If f is continuous at each right-dense point and each left-dense point, then f is said to be a continuous function on $\mathbb{T}$.

For $y : \mathbb{T} \to \mathbb{R}$ and $t \in \mathbb{T}^k$, we define the delta derivative $y^\Delta(t)$ of $y(t)$ as the number (if exists) with the property that for given $\varepsilon > 0$, there exists a neighborhood U of t such that $|[y(\sigma(t)) - y(s)] - y^\Delta(t)[\sigma(t) - y(s)]| < \varepsilon|\sigma(t) - s|$ for all $s \in U$. If y is continuous, then y is right-dense continuous, and y is delta differentiable at t, then y is continuous at t. Let y be right-dense continuous. If $Y^\Delta(t) = y(t)$, then we define the delta integral by $\int_a^t y(s)\Delta s = Y(t) - Y(a)$.

Definition 2.1 [24] If $a \in \mathbb{T}$, $\sup \mathbb{T} = \mathbb{R}$, and f is rd-continuous on $[0,\infty)$, then we define the improper integral by

$$\int_a^\infty f(t)\Delta t = \lim_{b\to\infty} \int_a^b f(t)\Delta t,$$

provided that this limit exists, and in this case, we say that the improper integral converges. If this limit does not exist, then we say that the improper integral diverges.

Definition 2.2 [25] For each $t \in \mathbb{T}$, let N be a neighborhood of t. Then, for $V \in C_{\mathrm{rd}}[\mathbb{T} \times \mathbb{R}^n, \mathbb{R}^+)$, we define $D^+V^\Delta(t, x(t))$ so that, for every $\varepsilon > 0$, there exists a right neighborhood

$N_\varepsilon \subset N$ of t such that

$$\frac{V(\sigma(t), x(\sigma(t))) - V(s, x(\sigma(t))) - \mu(t,s)f(t,x(t))}{\mu(t,s)} < D^+V^\Delta(t, x(t)) + \varepsilon$$

for each $s \in N_\varepsilon$, $s > t$, where $\mu(t,s) = \sigma(t) - s$. If t is rd and $V(t, x(t))$ is continuous at t, this reduces to

$$D^+V^\Delta(t, x(t)) = \frac{V(\sigma(t), x(\sigma(t))) - V(t, x(\sigma(t)))}{\sigma(t) - t}.$$

Definition 2.3 [25] Let $\mathbb{T} \neq \mathbb{R}$ be a periodic time scale with periodic p. We say that a function $f : \mathbb{T} \to \mathbb{R}$ is ω periodic if there exists a natural number n such that $\omega = np, f(t + \omega) = f(t)$ for all $t \in \mathbb{T}$, and ω is the least number such that $f(t + \omega) = f(t)$. If $\mathbb{T} = \mathbb{R}$, then we say that f is $\omega > 0$ periodic if ω is the least positive number such that $f(t + \omega) = f(t)$ for all $t \in \mathbb{T}$.

A function $r : \mathbb{T} \to R$ is called regressive if $1 + \mu(t)r(t) \neq 0$ for all $t \in \mathbb{T}^k$.

If r is a regressive function, then the generalized exponential function e_r is defined by

$$e_r(t,s) = \exp\left\{\int_s^t \xi_{\mu(\tau)}\big(r(\tau)\big)\Delta\tau\right\}, \quad s, t \in \mathbb{T},$$

with the cylinder transformation

$$\xi_{h(z)} = \begin{cases} \frac{\log(1+hz)}{h}, & h \neq 0, \\ z, & h = 0. \end{cases}$$

For two regressive functions $p, q : \mathbb{T} \to \mathbb{R}$, we define

$$p \oplus q := p + q + \mu pq; \qquad p \ominus q := p \oplus (\ominus q); \qquad \ominus p := -\frac{p}{1 + \mu p}.$$

Lemma 2.1 [26] *Let p, q be regressive functions on* $\mathbb{T}$. *Then*

(i) $e_0(t,s) = 1$ *and* $e_p(t,t) = 1$;
(ii) $e_p(\sigma(t), s) = (1 + \mu(t)p(t))e_p(t,s)$;
(iii) $e_p(t,s)e_p(s,r) = e_p(t,r)$;
(iv) $e_p^\Delta(\cdot, s) = pe_p(\cdot, s)$.

Lemma 2.2 [27] *Assume that* $f, g : \mathbb{T} \to \mathbb{R}$ *are delta differentiable at* $t \in \mathbb{T}^k$. *Then*

$$(fg)^\Delta(t) = f^\Delta(t)g(t) + f(\sigma(t))g^\Delta(t) = f(t)g^\Delta(t) + f^\Delta(t)g(\sigma(t)).$$

Lemma 2.3 [28] *Let* $t_1, t_2 \in [0, \omega]_{\mathbb{T}}$. *If* $x : \mathbb{T} \to \mathbb{R}$ *is* ω *periodic, then*

$$x(t) \le x(t_1) + \int_0^\omega \big|x^\Delta(s)\big|\Delta s \quad \text{and} \quad x(t) \ge x(t_2) - \int_0^\omega \big|x^\Delta(s)\big|\Delta s.$$

Lemma 2.4 [29] *Let* $a, b \in \mathbb{T}$. *For rd-continuous functions* $f, g : [a,b]_{\mathbb{T}} \to \mathbb{R}$, *we have*

$$\int_a^b \big|f(t)\big|\big|g(t)\big|\Delta t \le \left(\int_a^b \big|f(t)\big|^2\Delta t\right)^{1/2}\left(\int_a^b \big|g(t)\big|^2\Delta t\right)^{1/2}.$$

Lemma 2.5 [29] *Let $\mathbb{T}$ be an ω-periodic time scale. Then $\sigma(t+\omega)=\sigma(t)+\omega$ for all $t\in\mathbb{T}$.*

Lemma 2.6 [29] *Let f be a continuous function on $[a,b]_{\mathbb{T}}$ that is Δ differentiable on $[a,b)_{\mathbb{T}}$. Then there exist $\xi,\tau\in[a,b)_{\mathbb{T}}$ such that*

$$f^{\Delta}(\xi)(b-a)\leq f(b)-f(a)\leq f^{\Delta}(\tau)(b-a).$$

Lemma 2.7 [15] *Let x and y be two states of system* (1). *Then we have*

$$\left|\bigwedge_{j=1}^{n}\alpha_{ij}(t)g_j(x)-\bigwedge_{j=1}^{n}\alpha_{ij}(t)g_j(y)\right|\leq\sum_{j=1}^{n}\left|\alpha_{ij}(t)\right|\left|g_j(x)-g_j(y)\right|$$

and

$$\left|\bigvee_{j=1}^{n}\beta_{ij}(t)g_j(x)-\bigvee_{j=1}^{n}\beta_{ij}(t)g_j(y)\right|\leq\sum_{j=1}^{n}\left|\beta_{ij}(t)\right|\left|g_j(x)-g_j(y)\right|.$$

Definition 2.4 The periodic solution $u^*(t)=(x_1^*(t),\ldots,x_n^*(t),y_1(t),\ldots,y_m(t))^T$ of system (1) with initial value $(\varphi^*(t),\psi^*(t))^T=(\varphi_1^*(t),\ldots,\varphi_n^*(t),\psi_1^*(t),\ldots,\psi_m^*(t))^T$ is said to be globally exponentially stable if there exists a constant $M\geq 1$ and $\varepsilon>0$ such that, for every $\eta\in\mathbb{T}$,

$$\left\|u(t)-u^*(t)\right\|\leq Me_{\ominus\varepsilon}(t,\eta)\left(\sum_{i=1}^{n}\left|\varphi_i(\eta)-x_i^*(\eta)\right|+\sum_{j=1}^{m}\left|\psi_j(\eta)-y_j^*(\eta)\right|\right),$$

where $\eta\in[-\max\{\tau,\sigma\},0]_{\mathbb{T}}$.

3 Existence of periodic solution

In this section, based on Mawhin's continuation theorem, we study the existence of at least one periodic solution of (1). To do so, we shall make some preparations.

Let $\mathbb{X}$, $\mathbb{Y}$ be two Banach space, $L:\operatorname{Dom}L\subset\mathbb{X}\to\mathbb{Y}$ be a linear mapping, and $N:\mathbb{X}\to\mathbb{Y}$ be a continuous mapping. Then L is called a Fredholm mapping of index zero if $\dim\operatorname{Ker}L=\operatorname{codim}\operatorname{Im}L<+\infty$ and $\operatorname{Im}L$ is closed in $\mathbb{Y}$. If L is a Fredholm mapping of index zero and there exist continuous projectors $P:\mathbb{X}\to\mathbb{X}$ and $Q:\mathbb{Y}\to\mathbb{Y}$ such that $\operatorname{Im}P=\operatorname{Ker}L$, $\operatorname{Ker}Q=\operatorname{Im}(I-Q)$, then the mapping $L|_{\operatorname{Dom}L\cap\operatorname{Ker}P}:(I-P)\mathbb{X}\to\operatorname{Im}L$ is invertible. We denote its inverse by K_p. If Ω is an open bounded subset of $\mathbb{X}$, then the mapping N is called L-compact on $\overline{\Omega}$ if $QN(\overline{\Omega})$ is bounded and $K_p(I-Q)N:\overline{\Omega}\to\mathbb{X}$ is compact. Since $\operatorname{Im}Q$ is isomorphic to $\operatorname{Ker}L$, there exists an isomorphism $J:\operatorname{Im}Q\to\operatorname{Ker}L$.

Lemma 3.1 [30] *Let $\mathbb{X}$, $\mathbb{Y}$ be two Banach spaces, and let $\Omega\subset\mathbb{X}$ be open bounded. Suppose that $L:\operatorname{Dom}L\subset\mathbb{X}\to\mathbb{Y}$ is a linear Fredholm operator of index zero with $\operatorname{Dom}L\cap\overline{\Omega}\neq\phi$ and $N:\overline{\Omega}\to\mathbb{Y}$ is L-compact. Furthermore, suppose that*:

(a) *for each $\lambda\in(0,1)$, $x\in\partial\Omega\cap\operatorname{Dom}L$, $Lx\neq\lambda Nx$*;

(b) *for each $x\in\partial\Omega\cap\operatorname{Ker}L$, $QNx\neq 0$*;

(c) $\deg\{JQNx,\Omega\cap\operatorname{Ker}L,0\}\neq 0$.

Then the equation $Lx=Nx$ has at least one solution in $\overline{\Omega}\cap\operatorname{Dom}L$, where $\overline{\Omega}$ is the closure of Ω, and $\partial\Omega$ is the boundary of Ω.

Definition 3.1 A real matrix $A = (a_{ij})_{n\times n}$ is said to be a nonsingular M-matrix if $a_{ij} \le 0$, $i,j = 1,2,\dots,n$ and all successive principal minors of A are positive.

Theorem 3.1 *Under conditions* (A1)-(A5), *let H be a nonsingular M-matrix of the form*

$$H = \begin{pmatrix} H_1 & H_2 \\ H_3 & H_4 \end{pmatrix},$$

where

$$\begin{aligned}
&H_1 = \operatorname{diag}\{\underline{a}_1 - \underline{a}_1\omega\overline{a}_1\delta_1,\dots,\underline{a}_n - \underline{a}_n\omega\overline{a}_n\delta_n\},\\
&H_2 = (h_{ij})_{m\times n}, \qquad H_3 = \bigl(h'_{ji}\bigr)_{n\times m},\\
&h_{ij} = -\biggl(\frac{1}{\varrho_i} + \underline{a}_i\omega\biggr)\frac{\omega}{\sqrt{2}}\overline{a}_i(\overline{\alpha}_{ji} + \overline{\beta}_{ji})\kappa_j\overline{b}_j\delta'_j,\\
&h'_{ji} = -\biggl(\frac{1}{\varrho'_i} + \underline{b}_j\omega\biggr)\frac{\omega}{\sqrt{2}}\overline{b}_j(\overline{p}_{ij} + \overline{q}_{ij})\nu_i\overline{a}_i\delta_i,\\
&H_4 = \operatorname{diag}\bigl\{\underline{b}_1 - \underline{b}_1\omega\overline{b}_1\delta'_1,\dots,\underline{b}_m - \underline{b}_m\omega\overline{b}_m\delta'_m\bigr\}.
\end{aligned}$$

Then system (1) *has at least one ω-periodic solution.*

Proof Let $C[0,\omega;t_1,\dots,t_q]_{\mathbb{T}} = \{u : [0,\omega]_{\mathbb{T}} \to R^{n+m}$ is a piecewise continuous map with first-class discontinuity points in $[0,\omega]_{\mathbb{T}} \cap \{t_k\}$, and at each discontinuity point, it is continuous on the left$\}$. Take

$$\mathbb{X} = \bigl\{u \in C[0,\omega;t_1,\dots,t_q]_{\mathbb{T}} | u(t+\omega) = u(t)\bigr\}, \qquad \mathbb{Y} = \mathbb{X} \times \mathbb{R}^{(n+m)\times(q+1)}$$

with the norm $\|u\|_{\mathbb{X}} = \sum_{i=1}^{n} |x_i|_0 + \sum_{j=1}^{m} |y_j|_0$, where $|x_i|_0 = \max_{t\in[0,\omega]_T} |x_i(t)|$ and $|y_j|_0 = \max_{t\in[0,\omega]_T} |y_j(t)|$. Then $\mathbb{X}$ is a Banach space.

Set

$$L : \operatorname{Dom} L \cap \mathbb{X} \to \mathbb{Y}, \qquad u \to \bigl(u^{\Delta}, \Delta u(t_1),\dots,\Delta u(t_q), 0\bigr) \quad \text{and} \quad N : \mathbb{X} \to \mathbb{X},$$

where

$$Nu = \left(\begin{pmatrix} A_1(t) \\ \vdots \\ \vdots \\ \vdots \\ A_{n+m}(t) \end{pmatrix}, \begin{pmatrix} I_{11}(x_1(t_1)) \\ \vdots \\ I_{n1}(x_n(t_1)) \\ J_{11}(y_1(t_1)) \\ \vdots \\ J_{m1}(y_m(t_1)) \end{pmatrix}, \dots, \begin{pmatrix} I_{1q}(x_1(t_q)) \\ \vdots \\ I_{nq}(x_n(t_q)) \\ J_{1q}(y_1(t_q)) \\ \vdots \\ J_{mq}(y_m(t_q)) \end{pmatrix}, \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}\right),$$

$$\begin{aligned}
A_i(t) &= -a_i\bigl(x_i(t)\bigr)\Biggl[c_i\bigl(x_i(t)\bigr) - \bigwedge_{j=1}^{m}\alpha_{ji}(t)f_j\bigl(y_j(t-\tau_{ji})\bigr) - \bigvee_{j=1}^{m}\beta_{ji}(t)f_j\bigl(y_j(t-\tau_{ji})\bigr) + E_i(t)\Biggr],\\
A_{n+j}(t) &= -b_j\bigl(y_j(t)\bigr)\Biggl[d_j\bigl(y_j(t)\bigr) - \bigwedge_{i=1}^{n}p_{ij}(t)g_i\bigl(x_i(t-\sigma_{ij})\bigr) - \bigvee_{i=1}^{n}q_{ij}(t)g_i\bigl(x_i(t-\sigma_{ij})\bigr) + F_j(t)\Biggr].
\end{aligned}$$

It is easy to see that

$$\operatorname{Ker} L = \{x \in \mathbb{X} : x = h \in \mathbb{R}^{n+m}\},$$
$$\operatorname{Im} L = \left\{ z = (f, c_1, \ldots, c_q, d) \in \mathbb{Y} : \int_0^\omega f(s)\Delta s + \sum_{k=1}^{q} C_k + d = 0 \right\}.$$

Thus, $\dim \operatorname{Ker} L = \operatorname{codim} \operatorname{Im} L = n + m$. So, $\operatorname{Im} L$ is closed in $\mathbb{Y}$, and L is a Fredholm mapping of index zero. Define the project operators P and Q as

$$Px = \frac{1}{\omega}\int_0^\omega u(t)\Delta t, \quad x \in \mathbb{X},$$
$$Qz = Q(f, C_1, \ldots, C_q, d) = \left(\frac{1}{\omega}\left[\int_0^\omega f(s)\Delta s + \sum_{k=1}^{q} C_k + d \right], 0, \ldots, 0, 0 \right).$$

Obviously, P and Q are continuous projectors and satisfy

$$\operatorname{Im} P = \operatorname{Ker} L, \qquad \operatorname{Im} L = \operatorname{Ker} Q = \operatorname{Im}(I - Q).$$

Denoting $L_p^{-1} = L|_{\operatorname{Dom} L \cap \operatorname{Ker} P}$ and generalized inverse by $K_p = L_P^{-1}$, we have

$$(K_p z)(t) = \int_0^t f(s)\Delta s + \sum_{t > t_k} C_k - \frac{1}{\omega}\int_0^\omega \int_0^t f(s)\Delta s \Delta t - \sum_{k=1}^{q} C_k.$$

Similarly to [31], it is not difficult to show that $QN(\overline{\Omega})$ and $K_P(I - Q)N(\overline{\Omega})$ are relatively compact for any open bounded set $\Omega \subset \mathbb{X}$. Therefore, N is L-compact on $\overline{\Omega}$ for any open bounded set $\Omega \subset \mathbb{X}$.

Now, to apply Lemma 3.1, we only need to look for an appropriate open bounded subset Ω. Correspondingly to the operator equation $Lx = \lambda Nx$, $\lambda \in (0, 1)$, we have

$$\begin{cases} x_i^\Delta(t) = \lambda\{-a_i(x_i(t))[c_i(x_i(t)) - \bigwedge_{j=1}^{m} \alpha_{ji}(t) f_j(y_j(t - \tau_{ji})) \\ \qquad - \bigvee_{j=1}^{m} \beta_{ji}(t) f_j(y_j(t - \tau_{ji})) + E_i(t)]\}, \quad t \in \mathbb{T}^+, t \neq t_k, \\ \Delta_i(x_i(t_k)) = \lambda I_{ik}(x_i(t_k)), \quad k \in \mathbb{N}, i = 1, 2, \ldots, n; \\ y_j^\Delta(t) = \lambda\{-b_j(y_j(t))[d_j(y_j(t)) - \bigwedge_{i=1}^{n} p_{ij}(t) g_i(x_i(t - \sigma_{ij})) \\ \qquad - \bigvee_{i=1}^{n} q_{ij}(t) g_i(x_i(t - \sigma_{ij})) + F_j(t)]\}, \quad t \in \mathbb{T}^+, t \neq t_k, \\ \Delta_j(y_j(t_k)) = \lambda J_{jk}(y_j(t_k)), \quad k \in \mathbb{N}, j = 1, 2, \ldots, m. \end{cases} \tag{3}$$

Suppose that $u = (x_1, \ldots, x_n, y_1, \ldots, y_m)^T$ is a solution of system (3) for a certain $\lambda \in (0, 1)$. Multiplying both sides of the first and third equations in system (3) by x_i^Δ and y_j^Δ, respectively, and integrating over $[0, \omega]_{\mathbb{T}}$, we get

$$\int_0^\omega \left|x_i^\Delta(t)\right|^2 \Delta t$$
$$\leq \overline{a}_i \int_0^\omega \left| c_i\big(x_i(t)\big) - \bigwedge_{j=1}^{m} \alpha_{ji}(t) f_j\big(y_j(t - \tau_{ji})\big)\right.$$

$$
\begin{aligned}
&-\bigvee_{j=1}^{m}\beta_{ji}(t)f_j\bigl(y_j(t-\tau_{ji})\bigr)+E_i(t)\Bigg|x_i^{\Delta}(t)\Delta t\\
&\leq\overline{a}_i\Bigg[\int_0^{\omega}\bigl|c_i\bigl(x_i(t)\bigr)-c_i(0)\bigr|x_i^{\Delta}(t)\Delta t\\
&\quad+\int_0^{\omega}\Bigg|\bigwedge_{j=1}^{m}\alpha_{ji}(t)f_j\bigl(y_j(t-\tau_{ji})\bigr)-\bigwedge_{j=1}^{m}\alpha_{ji}(t)g_j(0)\Bigg|x_i^{\Delta}(t)\Delta t\\
&\quad+\int_0^{\omega}\Bigg|\bigvee_{j=1}^{m}\beta_{ji}(t)f_j\bigl(y_j(t-\tau_{ji})\bigr)-\bigvee_{j=1}^{m}\beta_{ji}(t)g_j(0)\Bigg|x_i^{\Delta}(t)\Delta t\\
&\quad+\overline{E}_i\int_0^{\omega}\bigl|x_i^{\Delta}(t)\bigr|\Delta t\Bigg].
\end{aligned}
$$

In view of Lemma 2.4, Lemma 2.7, and (A2)-(A4), we have

$$
\begin{aligned}
&\int_0^{\omega}\bigl|x_i^{\Delta}(t)\bigr|^2\Delta t\\
&\leq\overline{a}_i\Bigg[\delta_i\int_0^{\omega}\bigl|x_i(t)\bigr|\bigl|x_i^{\Delta}(t)\bigr|\Delta t+\sum_{j=1}^{m}(\overline{\alpha}_{ji}+\overline{\beta}_{ji})\biggl(\int_0^{\omega}\bigl|f_j\bigl(y_j(t-\tau_{ji})\bigr)\bigr|^2\Delta t\biggr)^{1/2}\\
&\quad\times\biggl(\int_0^{\omega}\bigl|x_i^{\Delta}(t)\bigr|^2\Delta t\biggr)^{1/2}+\overline{E}_i\int_0^{\omega}\bigl|x_i^{\Delta}(t)\bigr|\Delta t\Bigg]\\
&\leq\overline{a}_i\Bigg[\delta_i\|x_i\|_2\bigl\|x_i^{\Delta}\bigr\|_2+\sum_{j=1}^{m}(\overline{\alpha}_{ji}+\overline{\beta}_{ji})\sqrt{\omega}M_j\bigl\|x_i^{\Delta}\bigr\|_2+\overline{E}_i\sqrt{\omega}\bigl\|x_i^{\Delta}\bigr\|_2\Bigg],
\end{aligned}
$$

namely,

$$
\bigl\|x_i^{\Delta}\bigr\|_2\leq\overline{a}_i\Bigg[\delta_i\|x_i\|_2+\sum_{j=1}^{m}(\overline{\alpha}_{ji}+\overline{\beta}_{ji})\sqrt{\omega}M_j+\overline{E}_i\sqrt{\omega}\Bigg]:=\overline{a}_i\delta_i\|x_i\|_2+B_i, \tag{4}
$$

where $B_i=\overline{a}_i[\sum_{j=1}^{m}(\overline{\alpha}_{ji}+\overline{\beta}_{ji})\sqrt{\omega}M_j+\overline{E}_i\sqrt{\omega}]$. Similarly, we obtain that

$$
\bigl\|y_j^{\Delta}\bigr\|_2\leq\overline{b}_j\Bigg[\delta_j'\|y_j\|_2+\sum_{i=1}^{n}(\overline{p}_{ij}+\overline{q}_{ij})\sqrt{\omega}N_i+\overline{F}_j\sqrt{\omega}\Bigg]:=\overline{b}_j\delta_j'\|y_j\|_2+B_j', \tag{5}
$$

where $B_j'=\overline{b}_j[\sum_{i=1}^{n}(\overline{p}_{ij}+\overline{q}_{ij})\sqrt{\omega}N_i+\overline{F}_j\sqrt{\omega}]$. Setting $t_0=t_0^+=0$ and $t_{q+1}=\omega$, in view of (3), (A2)-(A5), Lemma 2.4, and Lemma 2.7, we have

$$
\begin{aligned}
&\int_0^{\omega}\bigl|x_i^{\Delta}(t)\bigr|\Delta t\\
&=\sum_{k=1}^{q+1}\int_{t_{k-1}^+}^{t_k}\bigl|x_i^{\Delta}(t)\bigr|\Delta t+\sum_{k=1}^{q}\bigl|I_k\bigl(x_i(t_k)\bigr)\bigr|\\
&\leq\overline{a}_i\Bigg[\int_0^{\omega}\bigl|c_i\bigl(x_i(t)\bigr)\bigr|\Delta t+\int_0^{\omega}\Bigg|\bigwedge_{j=1}^{m}\alpha_{ji}(t)f_j\bigl(y_j(t)\bigr)-\bigwedge_{j=1}^{m}\alpha_{ji}(t)f_j(0)\Bigg|\Delta t
\end{aligned}
$$

$$
\begin{aligned}
&+ \int_0^{\omega} \left| \bigwedge_{j=1}^{m} \alpha_{ji}(t) f_j\big(y_j(t-\tau_{ji})\big) - \bigwedge_{j=1}^{m} \alpha_{ji}(t) f_j(t) \right| \Delta t \\
&+ \int_0^{\omega} \left| \bigvee_{j=1}^{m} \beta_{ji}(t) f_j\big(y_j(t)\big) - \bigvee_{j=1}^{m} \beta_{ji}(t) f_j(0) \right| \Delta t \\
&+ \int_0^{\omega} \left| \bigvee_{j=1}^{m} \beta_{ji}(t) f_j\big(y_j(t-\tau_{ji})\big) - \bigvee_{j=1}^{m} \beta_{ji}(t) f_j(t) \right| \Delta t + \overline{E}_i \omega \Bigg] + q\bar{I}_k \\
\le{}& \overline{a}_i \Bigg[\int_0^{\omega} \big| c_i\big(x_i(t)\big) \big| \Delta t + \sum_{j=1}^{m} (\overline{\alpha}_{ji} + \overline{\beta}_{ji}) \int_0^{\omega} \big| f_j\big(y_j(t)\big) \big| \Delta t \\
&+ \sum_{j=1}^{m} (\overline{\alpha}_{ji} + \overline{\beta}_{ji}) \int_0^{\omega} \big| f_j\big(y_j(t-\tau_{ji})\big) - f_j\big(y_j(t)\big) \big| \Delta t + \overline{E}_i \omega \Bigg] + q\bar{I}_k \\
\le{}& \overline{a}_i \Bigg[\delta_i \sqrt{\omega} \|x_i\|_2 + \sum_{j=1}^{m} (\overline{\alpha}_{ji} + \overline{\beta}_{ji}) \omega M_j \\
&+ \sum_{j=1}^{m} (\overline{\alpha}_{ji} + \overline{\beta}_{ji}) \frac{\kappa_j}{\sqrt{2}} \omega^{3/2} \big\| y_j^{\Delta} \big\|_2 + \overline{E}_i \omega \Bigg] + q\bar{I}_k \\
\le{}& \overline{a}_i \Bigg[\delta_i \sqrt{\omega} \|x_i\|_2 + \sum_{j=1}^{m} (\overline{\alpha}_{ji} + \overline{\beta}_{ji}) \omega M_j \\
&+ \sum_{j=1}^{m} (\overline{\alpha}_{ji} + \overline{\beta}_{ji}) \frac{\kappa_j}{\sqrt{2}} \omega^{3/2} \big(\overline{b}_j \delta_j' \|y_j\|_2 + B_j' \big) + \overline{E}_i \omega \Bigg] + q\bar{I}_k
\end{aligned} \tag{6}
$$

and

$$
\begin{aligned}
&\int_0^{\omega} \big| y_j^{\Delta}(t) \big| \Delta t \\
&\le \overline{b}_j \Bigg[\delta_j' \sqrt{\omega} \|y_j\|_2 + \sum_{i=1}^{n} (\overline{p}_{ij} + \overline{q}_{ij}) \omega N_i \\
&\quad + \sum_{i=1}^{n} (\overline{p}_{ij} + \overline{q}_{ij}) \frac{\nu_i}{\sqrt{2}} \omega^{3/2} \big(\overline{a}_i \delta_i \|x_i\|_2 + B_i \big) + \overline{F}_j \omega \Bigg] + q\bar{J}_k.
\end{aligned} \tag{7}
$$

Integrating both sides of (3) from 0 to ω, we have

$$
\begin{aligned}
&\left| \int_0^{\omega} a_i\big(x_i(t)\big) c_i\big(x_i(t)\big) \Delta t \right| \\
&= \Bigg| \int_0^{\omega} a_i\big(x_i(t)\big) \Bigg[\bigwedge_{j=1}^{m} \alpha_{ji}(t) f_j\big(y_j(t-\tau_{ji})\big) \\
&\quad + \bigwedge_{j=1}^{m} \beta_{ji}(t) f_j\big(y_j(t-\tau_{ji})\big) + E_i(t) \Bigg] \Delta t + \sum_{k=1}^{q} I_k\big(x_i(t_k)\big) \Bigg| \\
&\le \overline{a}_i \int_0^{\omega} \Bigg[\left| \bigwedge_{j=1}^{m} \alpha_{ji}(t) f_j\big(y_j(t)\big) - \bigwedge_{j=1}^{m} \alpha_{ji}(t) f_j(0) \right|
\end{aligned}
$$

$$
\begin{aligned}
&+ \left|\bigwedge_{j=1}^{m} \alpha_{ji}(t) f_j\big(y_j(t-\tau_{ji})\big) - \bigwedge_{j=1}^{m} \alpha_{ji}(t) f_j\big(y_j(t)\big)\right| \\
&+ \left|\bigvee_{j=1}^{m} \beta_{ji}(t) f_j\big(y_j(t)\big) - \bigvee_{j=1}^{m} \beta_{ji}(t) f_j(0)\right| \\
&+ \left|\bigvee_{j=1}^{m} \beta_{ji}(t) f_j\big(y_j(t-\tau_{ji})\big) - \bigvee_{j=1}^{m} \beta_{ji}(t) f_j\big(y_j(t)\big)\right| + E_i(t)\Bigg]\Delta t + \sum_{k=1}^{q}\big|I_k\big(x_i(t_k)\big)\big| \\
\le{}& \overline{a}_i\Bigg[\sum_{j=1}^{m}(\overline{\alpha}_{ji} + \overline{\beta}_{ji}) \int_0^{\omega} \big|f_j\big(y_j(t)\big)\big|\Delta t \\
&+ \sum_{j=1}^{m}(\overline{\alpha}_{ji} + \overline{\beta}_{ji}) \int_0^{\omega} \big|f_j\big(y_j(t-\tau_{ji})\big) - f_j\big(y_j(t)\big)\big|\Delta t + \overline{E}_i\omega\Bigg] + \sum_{k=1}^{q}\big|I_k\big(x_i(t_k)\big)\big| \\
\le{}& \overline{a}_i\Bigg[\sum_{j=1}^{m}(\overline{\alpha}_{ji} + \overline{\beta}_{ji})\omega M_j + \sum_{j=1}^{m}(\overline{\alpha}_{ji} + \overline{\beta}_{ji})\frac{\kappa_j}{\sqrt{2}}\omega^{3/2}\big(\overline{b}_j\delta_j'\|y_j\|_2 + B_j'\big) + \overline{E}_i\omega\Bigg] + q\rho_k
\end{aligned}
$$

and

$$
\begin{aligned}
&\left|\int_0^{\omega} b_i\big(y_j(t)\big) d_j\big(y_j(t)\big)\Delta t\right| \\
&\quad\le \overline{b}_j\Bigg[\sum_{i=1}^{n}(\overline{p}_{ij} + \overline{q}_{ij})\omega N_i + \sum_{i=1}^{n}(\overline{p}_{ij} + \overline{q}_{ij})\frac{\nu_i}{\sqrt{2}}\omega^{3/2}\big(\overline{a}_i\delta_i\|x_i\|_2 + B_i\big) + \overline{F}_j\omega\Bigg] + q\rho_k'.
\end{aligned}
$$

Applying Lemma 2.6 and (A3), we obtain

$$
\begin{aligned}
&\left|\int_0^{\omega} a_i\big(x_i(t)\big) x_i(t)\Delta t\right| \\
&\quad\le \frac{\overline{a}_i}{\varrho_i}\Bigg[\sum_{j=1}^{m}(\overline{\alpha}_{ji} + \overline{\beta}_{ji})\omega M_j + \sum_{j=1}^{m}(\overline{\alpha}_{ji} + \overline{\beta}_{ji})\frac{\kappa_j}{\sqrt{2}}\omega^{3/2}\big(\overline{b}_j\delta_j'\|y_j\|_2 + B_j'\big) + \overline{E}_i\omega\Bigg] + \frac{1}{\varrho_i} q\rho_k
\end{aligned}
\tag{8}
$$

and

$$
\begin{aligned}
&\left|\int_0^{\omega} b_i\big(y_j(t)\big) y_j(t)\Delta t\right| \\
&\quad\le \frac{\overline{b}_j}{\varrho_j'}\Bigg[\sum_{i=1}^{n}(\overline{p}_{ij} + \overline{q}_{ij})\omega N_i + \sum_{i=1}^{n}(\overline{p}_{ij} + \overline{q}_{ij})\frac{\nu_i}{\sqrt{2}}\omega^{3/2}\big(\overline{a}_i\delta_i\|x_i\|_2 + B_i\big) + \overline{F}_j\omega\Bigg] + \frac{1}{\varrho_j'} q\rho_k'.
\end{aligned}
\tag{9}
$$

From Lemma 2.3, for any $t_1^i, t_2^i, t_3^j, t_4^j \in [0,\omega]_{\mathbb{T}}$, $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, m$, we have

$$
\begin{cases}
\int_0^{\omega} a_i(x_i(t))x_i(t)\Delta t \le \int_0^{\omega} a_i(x_i(t))x_i(t_1^i)\Delta t \\
\qquad\qquad + \int_0^{\omega} a_i(x_i(t))(\int_0^{\omega} |x_i^{\Delta}(t)|\Delta t)\Delta t, \\
\int_0^{\omega} a_i(x_i(t))x_i(t)\Delta t \le \int_0^{\omega} a_i(x_i(t))x_i(t_2^i)\Delta t \\
\qquad\qquad - \int_0^{\omega} a_i(x_i(t))(\int_0^{\omega} |x_i^{\Delta}(t)|\Delta t)\Delta t
\end{cases}
\tag{10}
$$

and

$$\begin{cases} \int_0^\omega b_j(y_j(t))y_j(t)\Delta t \le \int_0^\omega b_j(y_j(t))y_j(t_3^j)\Delta t \\ \qquad\qquad + \int_0^\omega b_j(y_j(t))(\int_0^\omega |y_j^\Delta(t)|\Delta t)\Delta t, \\ \int_0^\omega b_j(y_j(t))y_j(t)\Delta t \le \int_0^\omega b_j(y_j(t))y_j(t_4^j)\Delta t \\ \qquad\qquad - \int_0^\omega b_j(y_j(t))(\int_0^\omega |y_j^\Delta(t)|\Delta t)\Delta t. \end{cases} \tag{11}$$

Dividing by $\int_0^\omega a_i(x_i(t))\Delta t$ and $\int_0^\omega b_j(y_j(t))\Delta t$ both sides of (10) and (11), respectively, we obtain, for $i = 1, 2, \ldots, n$,

$$\begin{cases} x_i(t_1^i) \ge \frac{1}{\int_0^\omega a_i(x_i(t))\Delta t} \int_0^\omega a_i(x_i(t))x_i(t)\Delta t - \int_0^\omega |x_i^\Delta(t)|\Delta t, \\ x_i(t_2^i) \le \frac{1}{\int_0^\omega a_i(x_i(t))\Delta t} \int_0^\omega a_i(x_i(t))x_i(t)\Delta t + \int_0^\omega |x_i^\Delta(t)|\Delta t, \end{cases} \tag{12}$$

and, for $j = 1, 2, \ldots, m$,

$$\begin{cases} y_j(t_3^j) \ge \frac{1}{\int_0^\omega b_j(y_j(t))\Delta t} \int_0^\omega b_j(y_j(t))y_j(t)\Delta t - \int_0^\omega |y_j^\Delta(t)|\Delta t, \\ y_j(t_4^j) \le \frac{1}{\int_0^\omega b_j(y_j(t))\Delta t} \int_0^\omega b_j(y_j(t))y_j(t)\Delta t + \int_0^\omega |y_j^\Delta(t)|\Delta t. \end{cases} \tag{13}$$

Let $\overline{t}_i, \underline{t}_i, \overline{t}_j', \underline{t}_j' \in [0,\omega]_{\mathbb{T}}$ be such that $x_i(\overline{t}_i) = \max_{t\in[0,\omega]_{\mathbb{T}}} x_i(t)$, $x_i(\underline{t}_i) = \min_{t\in[0,\omega]_{\mathbb{T}}} x_i(t)$, $y_j(\overline{t}_j') = \max_{t\in[0,\omega]_{\mathbb{T}}} y_j(t)$, $y_j(\underline{t}_j') = \min_{t\in[0,\omega]_{\mathbb{T}}} y_j(t)$. From (12) we have, for $i = 1, 2, \ldots, n$,

$$\begin{aligned} x_i(\underline{t}_i) &\ge -\frac{1}{\underline{a}_i\omega}\left|\int_0^\omega a_i\bigl(x_i(t)\bigr)x_i(t)\Delta t\right| - \int_0^\omega \bigl|x_i^\Delta(t)\bigr|\Delta t \\ &\ge -\frac{1}{\underline{a}_i\varrho_i\omega}\Biggl[q\rho_k + \overline{a}_i\Biggl(\sum_{j=1}^m(\overline{\alpha}_{ji} + \overline{\beta}_{ji})\omega M_j + \sum_{j=1}^m(\overline{\alpha}_{ji} + \overline{\beta}_{ji}) \\ &\quad \times \frac{\kappa_j}{\sqrt{2}}\omega^{3/2}\bigl(\overline{b}_j\delta_j'\|y_j\|_2 + B_j'\bigr) + \overline{E}_i\omega\Biggr)\Biggr] \\ &\quad - \overline{a}_i\Biggl[\delta_i\sqrt{\omega}\|x_i\|_2 + \sum_{j=1}^m(\overline{\alpha}_{ji} + \overline{\beta}_{ji})\omega M_j + \sum_{j=1}^m(\overline{\alpha}_{ji} + \overline{\beta}_{ji}) \\ &\quad \times \frac{\kappa_j}{\sqrt{2}}\omega^{3/2}\bigl(\overline{b}_j\delta_j'\|y_j\|_2 + B_j'\bigr) + \overline{E}_i\omega\Biggr] - q\rho_k \end{aligned}$$

and

$$\begin{aligned} x_i(\overline{t}_i) &\le \frac{1}{\underline{a}_i\omega}\left|\int_0^\omega a_i\bigl(x_i(t)\bigr)x_i(t)\Delta t\right| - \int_0^\omega \bigl|x_i^\Delta(t)\bigr|\Delta t \\ &\le \frac{1}{\underline{a}_i\varrho_i\omega}\Biggl[q\rho_k + \overline{a}_i\Biggl(\sum_{j=1}^m(\overline{\alpha}_{ji} + \overline{\beta}_{ji})\omega M_j + \sum_{j=1}^m(\overline{\alpha}_{ji} + \overline{\beta}_{ji}) \\ &\quad \times \frac{\kappa_j}{\sqrt{2}}\omega^{3/2}\bigl(\overline{b}_j\delta_j'\|y_j\|_2 + B_j'\bigr) + \overline{E}_i\omega\Biggr)\Biggr] \\ &\quad + \overline{a}_i\Biggl[\delta_i\sqrt{\omega}\|x_i\|_2 + \sum_{j=1}^m(\overline{\alpha}_{ji} + \overline{\beta}_{ji})\omega M_j + \sum_{j=1}^m(\overline{\alpha}_{ji} + \overline{\beta}_{ji}) \\ &\quad \times \frac{\kappa_j}{\sqrt{2}}\omega^{3/2}\bigl(\overline{b}_j\delta_j'\|y_j\|_2 + B_j'\bigr) + \overline{E}_i\omega\Biggr] + q\rho_k. \end{aligned}$$

Similarly, from (13) we obtain, for $j = 1, 2, \ldots, m$,

$$\begin{aligned} y_j(\underline{t}_j') &\geq -\frac{1}{\underline{b}_j\omega}\left|\int_0^\omega b_j(y_j(t))y_j(t)\Delta t\right| - \int_0^\omega \left|y_j^\Delta(t)\right|\Delta t \\ &\geq -\frac{1}{\underline{b}_j\varrho_j'\omega}\left[q\rho_k' + \overline{b}_j\left(\sum_{i=1}^n(\overline{p}_{ij}+\overline{q}_{ij})\omega N_i + \sum_{i=1}^n(\overline{p}_{ij}+\overline{q}_{ij})\right.\right. \\ &\quad \left.\left.\times \frac{\nu_i}{\sqrt{2}}\omega^{3/2}\left(\overline{a}_i\delta_i\|x_i\|_2 + B_i\right) + \overline{F}_j\omega\right)\right] \\ &\quad - \overline{b}_j\left[\delta_j'\sqrt{\omega}\|y_j\|_2 + \sum_{i=1}^n(\overline{p}_{ij}+\overline{q}_{ij})\omega N_i + \sum_{i=1}^n(\overline{p}_{ij}+\overline{q}_{ij})\right. \\ &\quad \left.\times \frac{\nu_i}{\sqrt{2}}\omega^{3/2}\left(\overline{a}_i\delta_i\|x_i\|_2 + B_i\right) + \overline{F}_j\omega\right] - q\rho_k' \end{aligned}$$

and

$$\begin{aligned} y_j(\underline{t}_j') &\leq \frac{1}{\underline{b}_j\omega}\left|\int_0^\omega b_j(y_j(t))y_j(t)\Delta t\right| - \int_0^\omega \left|y_j^\Delta(t)\right|\Delta t \\ &\leq \frac{1}{\underline{b}_j\varrho_j'\omega}\left[q\rho_k' + \overline{b}_j\left(\sum_{i=1}^n(\overline{p}_{ij}+\overline{q}_{ij})\omega N_i + \sum_{i=1}^n(\overline{p}_{ij}+\overline{q}_{ij})\right.\right. \\ &\quad \left.\left.\times \frac{\nu_i}{\sqrt{2}}\omega^{3/2}\left(\overline{a}_i\delta_i\|x_i\|_2 + B_i\right) + \overline{F}_j\omega\right)\right] \\ &\quad + \overline{b}_j\left[\delta_j'\sqrt{\omega}\|y_j\|_2 + \sum_{i=1}^n(\overline{p}_{ij}+\overline{q}_{ij})\omega N_i + \sum_{i=1}^n(\overline{p}_{ij}+\overline{q}_{ij})\right. \\ &\quad \left.\times \frac{\nu_i}{\sqrt{2}}\omega^{3/2}\left(\overline{a}_i\delta_i\|x_i\|_2 + B_i\right) + \overline{F}_j\omega\right] + q\rho_k'. \end{aligned}$$

Therefore, we obtain that, for $i = 1, 2, \ldots, n$,

$$\begin{aligned} \max_{t\in[0,\omega]_{\mathbb{T}}}\left|x_i(t)\right| &\leq \frac{1}{\underline{a}_i\varrho_i\omega}\left[q\rho_k + \overline{a}_i\left(\sum_{j=1}^m(\overline{\alpha}_{ji}+\overline{\beta}_{ji})\omega M_j + \sum_{j=1}^m(\overline{\alpha}_{ji}+\overline{\beta}_{ji})\right.\right. \\ &\quad \left.\left.\times \frac{\kappa_j}{\sqrt{2}}\omega^{3/2}\left(\overline{b}_j\delta_j'\|y_j\|_2 + B_j'\right) + \overline{E}_i\omega\right)\right] \\ &\quad + \overline{a}_i\left[\delta_i\sqrt{\omega}\|x_i\|_2 + \sum_{j=1}^m(\overline{\alpha}_{ji}+\overline{\beta}_{ji})\omega M_j + \sum_{j=1}^m(\overline{\alpha}_{ji}+\overline{\beta}_{ji})\right. \\ &\quad \left.\times \frac{\kappa_j}{\sqrt{2}}\omega^{3/2}\left(\overline{b}_j\delta_j'\|y_j\|_2 + B_j'\right) + \overline{E}_i\omega\right] + q\rho_k \end{aligned} \tag{14}$$

and, for $j = 1, 2, \ldots, m$,

$$\begin{aligned} \max_{t\in[0,\omega]_{\mathbb{T}}}\left|y_j(t)\right| &\leq \frac{1}{\underline{b}_j\varrho_j'\omega}\left[q\rho_k' + \overline{b}_j\left(\sum_{i=1}^n(\overline{p}_{ij}+\overline{q}_{ij})\omega N_i + \sum_{i=1}^n(\overline{p}_{ij}+\overline{q}_{ij})\right.\right. \\ &\quad \left.\left.\times \frac{\nu_i}{\sqrt{2}}\omega^{3/2}\left(\overline{a}_i\delta_i\|x_i\|_2 + B_i\right) + \overline{F}_j\omega\right)\right] \end{aligned}$$

$$+ \overline{b}_j \Bigg[\delta'_j \sqrt{\omega} \|y_j\|_2 + \sum_{i=1}^{n} (\overline{p}_{ij} + \overline{q}_{ij}) \omega N_i + \sum_{i=1}^{n} (\overline{p}_{ij} + \overline{q}_{ij}) \times \frac{\nu_i}{\sqrt{2}} \omega^{3/2} \big(\overline{a}_i \delta_i \|x_i\|_2 + B_i \big) + \overline{F}_j \omega \Bigg] + q\rho'_k. \tag{15}$$

In addition, we have that

$$\|x_i\|_2 = \left(\int_0^{\omega} \big| x_i(s) \big|^2 \Delta s \right)^{1/2} \le \sqrt{\omega} \max_{t \in [0,\omega]_{\mathbb{T}}} \big| x_i(t) \big|, \quad i = 1, 2, \ldots, n,$$
$$\|y_j\|_2 = \left(\int_0^{\omega} \big| y_j(s) \big|^2 \Delta s \right)^{1/2} \le \sqrt{\omega} \max_{t \in [0,\omega]_{\mathbb{T}}} \big| y_j(t) \big|, \quad j = 1, 2, \ldots, m.$$

By (14) we obtain, for $i = 1, 2, \ldots, n$,

$$\begin{aligned} \underline{a}_i \sqrt{\omega} \|x_i\|_2 \le{}& \frac{1}{\varrho_i} \Bigg[q\rho_k + \overline{a}_i \Bigg(\sum_{j=1}^{m} (\overline{\alpha}_{ji} + \overline{\beta}_{ji}) \omega M_j + \sum_{j=1}^{m} (\overline{\alpha}_{ji} + \overline{\beta}_{ji}) \\ &\times \frac{\kappa_j}{\sqrt{2}} \omega^{3/2} \big(\overline{b}_j \delta'_j \|y_j\|_2 + B'_j \big) + \overline{E}_i \omega \Bigg) \Bigg] \\ &+ \underline{a}_i \omega \overline{a}_i \Bigg[\delta_i \sqrt{\omega} \|x_i\|_2 + \sum_{j=1}^{m} (\overline{\alpha}_{ji} + \overline{\beta}_{ji}) \omega M_j + \sum_{j=1}^{m} (\overline{\alpha}_{ji} + \overline{\beta}_{ji}) \\ &\times \frac{\kappa_j}{\sqrt{2}} \omega^{3/2} \big(\overline{b}_j \delta'_j \|y_j\|_2 + B'_j \big) + \overline{E}_i \omega \Bigg] + \underline{a}_i \omega q \rho_k, \end{aligned}$$

that is,

$$(\underline{a}_i - \underline{a}_i \omega \overline{a}_i \delta_i) \|x_i\|_2 - \overline{a}_i \left(\overline{a}_i \omega + \frac{1}{\varrho_i} \right) \sum_{j=1}^{m} (\overline{\alpha}_{ji} + \overline{\beta}_{ji}) \frac{\kappa_j}{\sqrt{2}} \omega \overline{b}_j \delta'_j \|y_j\|_2 \le \frac{1}{\sqrt{\omega}} \Upsilon_i, \tag{16}$$

where, for $i = 1, 2, \ldots, n$,

$$\Upsilon_i = \left(\underline{a}_i \omega + \frac{1}{\varrho_i} \right) \Bigg[q\varrho_k + \overline{a}_i \Bigg(\sum_{j=1}^{m} (\overline{\alpha}_{ji} + \overline{\beta}_{ji}) \omega M_j + \sum_{j=1}^{m} (\overline{\alpha}_{ji} + \overline{\beta}_{ji}) \frac{\kappa_j}{\sqrt{2}} \omega^{3/2} B'_j + \overline{E}_i \omega \Bigg) \Bigg].$$

Similarly, we have

$$\big(\underline{b}_j - \underline{b}_j \omega \overline{b}_j \delta'_j \big) \|y_j\|_2 - \overline{b}_j \left(\overline{b}_j \omega + \frac{1}{\varrho'_j} \right) \sum_{i=1}^{n} (\overline{p}_{ij} + \overline{q}_{ij}) \frac{\nu_i}{\sqrt{2}} \omega \overline{a}_i \delta_i \|x_i\|_2 \le \frac{1}{\sqrt{\omega}} \Upsilon'_j, \tag{17}$$

where, for $j = 1, 2, \ldots, m$,

$$\Upsilon'_j = \left(\underline{b}_j \omega + \frac{1}{\varrho'_j} \right) \Bigg[q\varrho'_k + \overline{b}_j \Bigg(\sum_{i=1}^{n} (\overline{p}_{ij} + \overline{q}_{ij}) \omega N_i + \sum_{i=1}^{n} (\overline{p}_{ij} + \overline{q}_{ij}) \frac{\nu_i}{\sqrt{2}} \omega^{3/2} B_i + \overline{F}_j \omega \Bigg) \Bigg].$$

Denote $\|u\|_2 = (\|x_1\|_2, \ldots, \|x_n\|_2, \|y_1\|_2, \ldots, \|y_m\|_2)^T$ and

$$\Upsilon = \frac{1}{\sqrt{\omega}} \big(\Upsilon_1, \ldots, \Upsilon_n, \Upsilon'_1, \ldots, \Upsilon'_m \big)^T.$$

Then (16) and (17) can be written in the matrix form

$$H\|u\|_2 \leq \Upsilon.$$

From the conditions of Theorem 3.1 we have that H is a nonsingular M-matrix, so that

$$\|u\|_2 \leq H^{-1}\Upsilon := \bigl(D_1, \ldots, D_n, D'_1, \ldots, D'_m\bigr)^T, \tag{18}$$

that is, $\|x_i\|_2 \leq D_i$, $i = 1, 2, \ldots, n$, and $|y_j\|_2 \leq D'_j$, $j = 1, 2, \ldots, m$.

From (14) and (15) we have, for $i = 1, 2, \ldots, n$,

$$\begin{aligned}
\max_{t\in[0,\omega]_{\mathbb{T}}} \bigl|x_i(t)\bigr| \leq{}& \frac{1}{\underline{a}_i\varrho_i\omega}\Biggl[q\rho_k + \overline{a}_i\Biggl(\sum_{j=1}^{m}(\overline{\alpha}_{ji} + \overline{\beta}_{ji})\omega M_j + \sum_{j=1}^{m}(\overline{\alpha}_{ji} + \overline{\beta}_{ji}) \\
&\times \frac{\kappa_j}{\sqrt{2}}\omega^{3/2}\bigl(\overline{b}_j\delta'_j D'_j + B'_j\bigr) + \overline{E}_i\omega\Biggr)\Biggr] \\
&+ \overline{a}_i\Biggl[\delta_i\sqrt{\omega}D_i + \sum_{j=1}^{m}(\overline{\alpha}_{ji} + \overline{\beta}_{ji})\omega M_j + \sum_{j=1}^{m}(\overline{\alpha}_{ji} + \overline{\beta}_{ji}) \\
&\times \frac{\kappa_j}{\sqrt{2}}\omega^{3/2}\bigl(\overline{b}_j\delta'_j D'_j + B'_j\bigr) + \overline{E}_i\omega\Biggr] + q\rho_k := G_i
\end{aligned}$$

and, for $j = 1, 2, \ldots, m$,

$$\begin{aligned}
\max_{t\in[0,\omega]_{\mathbb{T}}} \bigl|y_j(t)\bigr| \leq{}& \frac{1}{\underline{b}_j\varrho'_j\omega}\Biggl[q\rho'_k + \overline{b}_j\Biggl(\sum_{i=1}^{n}(\overline{p}_{ij} + \overline{q}_{ij})\omega N_i + \sum_{i=1}^{n}(\overline{p}_{ij} + \overline{q}_{ij}) \\
&\times \frac{\nu_i}{\sqrt{2}}\omega^{3/2}(\overline{a}_i\delta_i D_i + B_i) + \overline{F}_j\omega\Biggr)\Biggr] \\
&+ \overline{b}_j\Biggl[\delta'_j\sqrt{\omega}D'_j + \sum_{i=1}^{n}(\overline{p}_{ij} + \overline{q}_{ij})\omega N_i + \sum_{i=1}^{n}(\overline{p}_{ij} + \overline{q}_{ij}) \\
&\times \frac{\nu_i}{\sqrt{2}}\omega^{3/2}(\overline{a}_i\delta_i D_i + B_i) + \overline{F}_j\omega\Biggr] + q\rho'_k := G'_j.
\end{aligned}$$

Let $G = \sum_{i=1}^{n} G_i + \sum_{j=1}^{m} G'_j + G_0$, where G_0 is a positive constant. Clearly, F is independent of λ. Take $\Omega = \{u \in \mathbb{X} | \|u\|_{\mathbb{X}} < G\}$. Obviously, Ω satisfies condition (a) of Lemma 3.1.

When $u(t) \in \partial\Omega \cap \operatorname{Ker} L = \partial\Omega \cap \mathbb{R}^{n+m}$, u is a constant vector with $\|u\| = G$. Furthermore, take $J : \operatorname{Im} Q \to \operatorname{Ker} L$. Then

$$JQN(x_i) = -a_i(x_i)\Biggl[c_i(x_i) - \bigwedge_{j=1}^{m}\hat{\alpha}_{ji}f_j(y_j) - \bigvee_{j=1}^{m}\hat{\beta}_{ji}f_j(y_j) + \hat{E}_i\Biggr] + \frac{1}{\omega}\sum_{k=1}^{q} I_{ik}(x_i)$$

for $i = 1, 2, \ldots, n$, and

$$JQN(y_j) = -b_j(y_j)\Biggl[d_j(y_j) - \bigwedge_{i=1}^{n}\hat{p}_{ij}g_i(x_i) - \bigvee_{i=1}^{n}\hat{q}_{ij}g_i(x_i) + \hat{F}_j\Biggr] + \frac{1}{\omega}\sum_{k=1}^{q} J_{jk}(y_j)$$

for $j = 1, 2, \ldots, m$.

We can take G large enough such that

$$
\begin{aligned}
u^T JQNu \le \sum_{i=1}^{n} \Bigg\{ & -x_i a_i(x_i) \Bigg[c_i(x_i) - \bigwedge_{j=1}^{m} \hat{\alpha}_{ji} f_j(y_j) - \bigvee_{j=1}^{m} \hat{\beta}_{ij} f_j(y_j) + \hat{E}_i \Bigg] \\
& + \frac{1}{\omega} x_i \sum_{k=1}^{q} I_{ik}(x_i) \Bigg\} \\
& + \sum_{j=1}^{m} \Bigg\{ -y_j b_j(y_j) \Bigg[d_j(y_j) - \bigwedge_{i=1}^{n} \hat{p}_{ij} g_i(x_i) - \bigvee_{i=1}^{n} \hat{q}_{ij} g_i(x_i) + \hat{F}_j \Bigg] \\
& + \frac{1}{\omega} y_j \sum_{k=1}^{q} J_{jk}(y_j) \Bigg\} \\
& < 0.
\end{aligned}
$$

Hence, for any $x \in \partial\Omega \cap \operatorname{Ker} L$, $QNu \neq 0$, namely, condition (b) in Lemma 3.1 is satisfied.

Furthermore, let $\Psi(\gamma; u) = -\gamma u + (1-\gamma)QNu$. Then, for any $u \in \partial\Omega \cap \operatorname{Ker} L$, $u^T \Psi(\gamma; u) < 0$, and we get

$$
\deg\{JQN, \Omega \cap \operatorname{Ker} L, 0\} = \deg\{-u, \Omega \cap \operatorname{Ker} L, 0\} \neq 0.
$$

This shows that condition (c) in Lemma 3.1 is satisfied. Thus, by Lemma 3.1 we conclude that $Lu = Nu$ has at least one solution in $\mathbb{X}$, that is, system (1) has at least one ω-periodic solution. This completes the proof. □

4 Global exponential stability of periodic solutions

Suppose that $u^*(t) = (x_1^*(t),, \ldots, x_n^*(t), y_1(t), \ldots, y_m(t))^T$ is an ω-periodic solution of system (1). We will construct some suitable Lyapunov functions to prove the global exponential stability of this periodic solution.

Theorem 4.1 *Assume that all conditions of Theorem* 3.1 *are satisfied. Suppose further that*:

(A6) *The impulsive operators* $I_{ik}(x_i(t_k)), J_{jk}(y_j(t_k))$ *satisfy*

$$
\begin{cases}
I_{ik}(x_i(t_k)) = -\gamma_{ik}(x_i(t_k)), & 0 < \gamma_{ik} < 2, i = 1, 2, \ldots, n, k \in \mathbb{N}, \\
J_{jk}(y_j(t_k)) = -\overline{\gamma}_{jk}(y_j(t_k)), & 0 < \overline{\gamma}_{jk} < 2, j = 1, 2, \ldots, m, k \in \mathbb{N}.
\end{cases}
$$

(A7)

$$
\begin{cases}
\underline{a}_i \varrho_i - \sum_{j=1}^{m} (\overline{p}_{ij} + \overline{q}_{ij}) \nu_i \overline{b}_j > 0, & i = 1, 2, \ldots, n, \\
\underline{b}_j \varrho_j' - \sum_{i=1}^{n} (\overline{\alpha}_{ji} + \overline{\beta}_{ji}) \kappa_j \overline{a}_i > 0, & j = 1, 2, \ldots, m.
\end{cases}
$$

Then the ω-periodic solution of (1) *is globally exponentially stable*.

Proof According to Theorem 3.1, we know that system (1) has an ω-periodic solution $u^*(t) = (x_1^*(t), \ldots, x_n^*(t), y_1^*(t), \ldots, y_m^*(t))^T$.

Let $u(t) = (x_1(t), \dots, x_n(t), y_1(t), \dots, y_m(t))^T$ be an arbitrary solution of system (1). In view of (A6), from (1) we have

$$\begin{cases} (x_i(t) - x_i^*(t))^\Delta = -[a_i(x_i(t))c_i(x_i(t)) - a_i(x_i^*(t))c_i(x_i^*(t))] \\ \qquad + [a_i(x_i(t)) \bigwedge_{j=1}^m \alpha_{ji}(t)f_j(y_j(t-\tau_{ji})) \\ \qquad - a_i(x_i^*(t)) \bigwedge_{j=1}^m \alpha_{ji}(t)f_j(y_j^*(t-\tau_{ji}))] \\ \qquad + [a_i(x_i(t)) \bigvee_{j=1}^m \beta_{ji}(t)f_j(y_j(t-\tau_{ji})) \\ \qquad - a_i(x_i^*(t)) \bigvee_{j=1}^m \beta_{ji}(t)f_j(y_j^*(t-\tau_{ji}))], \\ \qquad t > 0, t \neq t_k, k = 1,2,\dots, \\ \Delta x_i((t_k) - x_i^*(t_k)) = -\gamma_{ik}(x_i(t_k) - x_i^*(t_k)), \quad i = 1,2,\dots,n; \\ (y_j(t) - y_j^*(t))^\Delta = -[b_j(y_j(t))d_j(y_j(t)) - b_j(y_j^*(t))d_j(y_j^*(t))] \\ \qquad + [b_j(y_j(t)) \bigwedge_{i=1}^n p_{ij}(t)g_i(x_i(t-\sigma_{ij})) \\ \qquad - b_j(y_j^*(t)) \bigwedge_{i=1}^n p_{ij}(t)g_i(x_i^*(t-\sigma_{ij}))] \\ \qquad + [b_j(y_j(t)) \bigvee_{i=1}^n q_{ij}(t)g_i(x_i(t-\sigma_{ij})) \\ \qquad - b_j(y_j^*(t)) \bigvee_{i=1}^n q_{ij}(t)g_i(x_i^*(t-\sigma_{ij}))] \\ \qquad t > 0, t \neq t_k, k = 1,2,\dots, \\ \Delta y_j((t_k) - y_j^*(t_k)) = -\overline{\gamma}_{jk}(y_j(t_k) - y_j^*(t_k)), \quad j = 1,2,\dots,m. \end{cases}$$

In view of this system, for $t > 0$, $t \neq t_k$, $k \in \mathbb{N}$, we have

$$\begin{cases} D^+|x_i(t) - x_i^*(t)|^\Delta \leq -\underline{a}_i\varrho_i|x_i(t) - x_i^*(t)| + \overline{a}_i \sum_{j=1}^m (\overline{\alpha}_{ji} + \overline{\beta}_{ji}) \\ \qquad \times \kappa_j|y_j(t-\tau_{ji}) - y_j^*(t-\tau_{ji})|, \\ D^+|y_j(t) - y_j^*(t)|^\Delta \leq -\underline{b}_j\varrho_j'|y_j(t) - y_j^*(t)| + \overline{b}_j \sum_{i=1}^n (\overline{p}_{ij} + \overline{q}_{ij}) \\ \qquad \times \nu_i|x_i(t-\sigma_{ij}) - x_i^*(t-\sigma_{ij})| \end{cases} \tag{19}$$

for $i = 1,2,\dots,n$, $j = 1,2,\dots,m$. From (A6) we have that

$$\begin{cases} |x_i(t_k^+) - x_i^*(t_k^+)| = |1-\gamma_{ik}||x_i(t_k) - x_i^*(t_k)| \leq |x_i(t_k) - x_i^*(t_k)|, \\ |y_j(t_k^+) - y_j^*(t_k^+)| = |1-\overline{\gamma}_{jk}||y_j(t_k) - y_j^*(t_k)| \leq |y_j(t_k) - y_j^*(t_k)| \end{cases} \tag{20}$$

for $i = 1,2,\dots,n$, $j = 1,2,\dots,m$, $k \in \mathbb{N}$.

Let F_i and G_j be defined by

$$\begin{cases} F_i(\theta_i) = \underline{a}_i\varrho_i - \theta_i - \sum_{j=1}^m (\overline{p}_{ij} + \overline{q}_{ij})\nu_i\overline{b}_j e_{\theta_i}(\sigma(t), t-\sigma_{ij}), \\ G_j(\xi_j) = \underline{b}_j\varrho_j' - \xi_j - \sum_{i=1}^n (\overline{\alpha}_{ji} + \overline{\beta}_{ji})\kappa_j\overline{a}_i e_{\xi_j}(\tau(t), t-\tau_{ji}), \end{cases}$$

where $\theta_i, \xi_j \in [0,\infty)$, $i = 1,2,\dots,n$, $j = 1,2,\dots,m$. It is clear that

$$F_i(0) = \underline{a}_i\varrho_i - \sum_{j=1}^m (\overline{p}_{ij} + \overline{q}_{ij})\nu_i\overline{b}_j > 0,$$

$$G_j(0) = \underline{b}_j\varrho_j' - \sum_{i=1}^n (\overline{\alpha}_{ji} + \overline{\beta}_{ji})\kappa_j\overline{a}_i > 0$$

for $i = 1,2,\dots,n$, $j = 1,2,\dots,m$.

Since F_i, G_j are continuous on $[0,\infty)$ and $F_i(\theta_i) \to -\infty$, $G_j(\xi_j) \to -\infty$ as $\theta_i \to +\infty$, $\xi_j \to +\infty$, there exist $\theta_i^*, \xi_j^* > 0$ such that $F_i(\theta_i^*) = 0$, $G_j(\xi_j^*) = 0$ and $F_i(\theta_i) > 0$, $G_j(\xi_j) > 0$ for

$\theta_i \in (0, \theta_i^*)$, $\xi_j \in (0, \xi_j^*)$. By choosing $\varepsilon = \min_{1\le i\le n, 1\le j\le m}\{\theta_i^*, \xi_j^*\}$ we have

$$\begin{cases} F_i(\varepsilon) = \underline{a}_i\varrho_i - \varepsilon - \sum_{j=1}^m (\overline{p}_{ij} + \overline{q}_{ij})\nu_i\overline{b}_j e_\varepsilon(\sigma(t), t-\sigma_{ij}) \ge 0, \\ G_j(\varepsilon) = \underline{b}_j\varrho_j' - \varepsilon - \sum_{i=1}^n (\overline{\alpha}_{ji} + \overline{\beta}_{ji})\kappa_j\overline{a}_i e_\varepsilon(\tau(t), t-\tau_{ji}) \ge 0 \end{cases}$$

for $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, m$.

Now let us define

$$\begin{cases} \mu_i(t) = e_\varepsilon(t,\delta)|x_i(t) - x_i^*(t)|, & t \in [-\tau, \infty), i = 1, 2, \ldots, n, \\ \omega_j(t) = e_\varepsilon(t,\delta)|y_j(t) - y_j^*(t)|, & t \in [-\sigma, \infty), j = 1, 2, \ldots, m, \end{cases} \tag{21}$$

where $\delta \in [-\max\{\tau, \sigma\}, 0]$, for $t > 0$, $t \neq t_k$, $k \in \mathbb{N}$, $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, m$.

It follows from (19) and (21) that

$$\begin{aligned} D^+\mu_i^\Delta(t) &\le \varepsilon e_\varepsilon(t,\delta)\bigl|x_i(t) - x_i^*(t)\bigr| \\ &\quad + e_\varepsilon\bigl(\sigma(t),\delta\bigr)\Biggl(-\underline{a}_i\varrho_i\bigl|x_i(t) - x_i^*(t)\bigr| \\ &\quad + \overline{a}_i\sum_{j=1}^m (\overline{\alpha}_{ji} + \overline{\beta}_{ji})\kappa_j\bigl|y_j(t-\tau_{ji}) - y_j^*(t-\tau_{ji})\bigr|\Biggr) \\ &\le -(\underline{a}_i\varrho_i - \varepsilon)\mu_i(t) \\ &\quad + \overline{a}_i\sum_{j=1}^m (\overline{\alpha}_{ji} + \overline{\beta}_{ji})\kappa_j e_\varepsilon\bigl(\sigma(t), t-\tau_{ji}\bigr)\omega_j(t-\tau_{ji}), \end{aligned} \tag{22}$$

$$\begin{aligned} D^+\omega_j^\Delta(t) &\le -\bigl(\underline{b}_j\varrho_j' - \varepsilon\bigr)\omega_j(t) \\ &\quad + \overline{b}_j\sum_{i=1}^n (\overline{p}_{ij} + \overline{q}_{ij})\nu_i e_\varepsilon\bigl(\sigma(t), t-\sigma_{ij}\bigr)\mu_i(t-\sigma_{ij}). \end{aligned} \tag{23}$$

Also, we have

$$\begin{cases} \mu_i(t_k^+) = |1 - \gamma_{ik}|\mu_i(t_k) \le \mu_i(t_k), & i = 1, 2, \ldots, n, k \in \mathbb{N}, \\ \omega_j(t_k^+) = |1 - \overline{\gamma}_{jk}|\omega_j(t_k) \le \omega_j(t_k), & j = 1, 2, \ldots, m, k \in \mathbb{N}. \end{cases}$$

Consider the Lyapunov functional

$$\begin{aligned} V(t) &= \sum_{i=1}^n \Biggl(\mu_i(t) + \overline{a}_i\sum_{j=1}^m \bigl(\bigl|\alpha_{ji}(t)\bigr| + \bigl|\beta_{ji}(t)\bigr|\bigr) \\ &\quad \times \kappa_j e_\varepsilon\bigl(\sigma(t), t-\tau_{ji}\bigr)\int_{t-\tau_{ji}}^t \omega_j(s)\Delta s\Biggr) \\ &\quad + \sum_{j=1}^m \Biggl(\omega_j(t) + \overline{b}_j\sum_{i=1}^n \bigl(\bigl|p_{ij}(t)\bigr| + \bigl|q_{ij}(t)\bigr|\bigr) \\ &\quad \times \nu_i e_\varepsilon\bigl(\sigma(t), t-\sigma_{ij}\bigr)\int_{t-\sigma_{ij}}^t \mu_i(s)\Delta s\Biggr). \end{aligned} \tag{24}$$

Calculating the Δ-derivatives of V along (22) and (23), we get

$$\begin{aligned}
D^+V^\Delta(t) \le{}& \sum_{i=1}^{n}\Bigg[-(\underline{a}_i\varrho_i-\varepsilon)\mu_i(t)+\overline{a}_i\sum_{j=1}^{m}(\overline{\alpha}_{ji}+\overline{\beta}_{ji}) \\
&\times \kappa_j e_\varepsilon\big(\sigma(t),t-\tau_{ji}\big)\omega_j(t)\Bigg] \\
&+\sum_{j=1}^{m}\Bigg[-\big(\underline{b}_j\varrho'_j-\varepsilon\big)\omega_j(t)+\overline{b}_j\sum_{i=1}^{n}(\overline{p}_{ij}+\overline{q}_{ij}) \\
&\times \nu_i e_\varepsilon\big(\sigma(t),t-\sigma_{ij}\big)\mu_i(t)\Bigg] \\
\le{}& -\sum_{i=1}^{n}\Bigg(\underline{a}_i\varrho_i-\varepsilon-\sum_{j=1}^{m}\overline{b}_j(\overline{p}_{ij}+\overline{q}_{ij})\nu_i e_\varepsilon\big(\sigma(t),t-\sigma_{ij}\big)\Bigg)\mu_i(t) \\
&-\sum_{j=1}^{m}\Bigg(\underline{b}_j\varrho'_j-\varepsilon-\sum_{i=1}^{n}\overline{a}_i(\overline{\alpha}_{ji}+\overline{\beta}_{ji})\kappa_j e_\varepsilon\big(\sigma(t),t-\tau_{ji}\big)\Bigg)\omega_j(t) \\
\le{}& -\sum_{i=1}^{n}F_i(\varepsilon)\mu_i(t)-\sum_{j=1}^{m}G_j(\varepsilon)\omega_j(t) \\
\le{}& 0, \quad t>0, t\neq t_k, k\in\mathbb{N}.
\end{aligned}$$

Also,

$$\begin{aligned}
V\big(t_k^+\big) ={}& \sum_{i=1}^{n}\Bigg(\mu_i\big(t_k^+\big)+\overline{a}_i\sum_{j=1}^{m}\big(\big|\alpha_{ji}\big(t_k^+\big)\big|+\big|\beta_{ji}\big(t_k^+\big)\big|\big) \\
&\times \kappa_j e_\varepsilon\big(\sigma\big(t_k^+\big),t_k^+-\tau_{ji}\big)\int_{t_{k^+}-\tau_{ji}}^{t_k^+}\omega_j(s)\Delta s\Bigg) \\
&+\sum_{j=1}^{m}\Bigg(\omega_j\big(t_k^+\big)+\overline{b}_j\sum_{i=1}^{n}\big(\big|p_{ij}\big(t_k^+\big)\big|+\big|q_{ij}\big(t_k^+\big)\big|\big) \\
&\times \nu_i e_\varepsilon\big(\sigma\big(t_k^+\big),t_k^+-\sigma_{ij}\big)\int_{t_k^+-\sigma_{ij}}^{t_k^+}\mu_i(s)\Delta s\Bigg) \\
\le{}& \sum_{i=1}^{n}\Bigg(\mu_i(t_k)+\overline{a}_i\sum_{j=1}^{m}\big(\big|\alpha_{ji}(t_k)\big|+\big|\beta_{ji}(t_k)\big|\big) \\
&\times \kappa_j e_\varepsilon\big(\sigma(t_k),t_k-\tau_{ji}\big)\int_{t_k-\tau_{ji}}^{t_k}\omega_j(s)\Delta s\Bigg) \\
&+\sum_{j=1}^{m}\Bigg(\omega_j(t_k)+\overline{b}_j\sum_{i=1}^{n}\big(\big|p_{ij}(t_k)\big|+\big|q_{ij}(t_k)\big|\big) \\
&\times \nu_i e_\varepsilon\big(\sigma(t_k),t_k-\sigma_{ij}\big)\int_{t_k-\sigma_{ij}}^{t_k}\mu_i(s)\Delta s\Bigg) \\
\le{}& V(t_k), \quad k\in\mathbb{N}.
\end{aligned}$$

On other hand, we note that $V(t) > 0$ for $t > 0$ and $V(0)$ is positive and finite. Therefore, it follows that $V(t) \leq V(0)$ for $t > 0$. From this and from (24) we obtain

$$\begin{aligned}\sum_{i=1}^{n} \nu_i(t) + \sum_{j=1}^{m} \omega_j(t) \leq{}& \sum_{i=1}^{n} \left(\mu_i(0) + \overline{a}_i \sum_{j=1}^{m} (\overline{\alpha}_{ji} + \overline{\beta}_{ji}) \kappa_j e_\varepsilon\bigl(\sigma(0), -\tau_{ji}\bigr) \int_{-\tau_{ji}}^{0} \omega_j(s) \Delta s \right) \\ &+ \sum_{j=1}^{m} \left(\omega_j(0) + \overline{b}_j \sum_{i=1}^{n} (\overline{p}_{ij} + \overline{q}_{ij}) s \nu_i e_\varepsilon\bigl(\sigma(0), -\sigma_{ij}\bigr) \int_{-\sigma_{ij}}^{0} \mu_i(s) \Delta s \right)\end{aligned}$$

for $t > 0$. In view of (21) and the last inequality, we have, for $t > 0$,

$$\begin{aligned}&\sum_{i=1}^{n} \bigl| x_i(t) - x_i^*(t) \bigr| + \sum_{j=1}^{m} \bigl| y_j(t) - y_j^*(t) \bigr| \\ &\quad\leq e_{\Theta\varepsilon}(t, \delta) \Biggl[\sum_{i=1}^{n} \left(1 + \sum_{j=1}^{m} \overline{b}_j (\overline{p}_{ij} + \overline{q}_{ij}) \nu_i e_\varepsilon\bigl(\sigma(0), -\sigma_{ij}\bigr) \sigma_{ij} \right) \\ &\qquad \times \max_{\delta \in [-\tau, 0]_{\mathbb{T}}} \bigl| \varphi_i(\delta) - x_i^*(\delta) \bigr| \\ &\qquad + \sum_{j=1}^{m} \left(1 + \sum_{i=1}^{n} \overline{a}_i (\overline{\alpha}_{ji} + \overline{\beta}_{ji}) \kappa_j e_\varepsilon\bigl(\sigma(0), -\tau_{ji}\bigr) \tau_{ji} \right) \\ &\qquad \times \max_{\delta \in [-\tau, 0]_{\mathbb{T}}} \bigl| \psi_j(\delta) - y_j^*(\delta) \bigr| \Biggr] \\ &\quad\leq M e_{\Theta\varepsilon}(t, \delta) \Biggl[\sum_{i=1}^{n} \max_{\delta \in [-\tau, 0]_{\mathbb{T}}} \bigl| \varphi_i(\delta) - x_i^*(\delta) \bigr| + \sum_{j=1}^{m} \max_{\delta \in [-\tau, 0]_{\mathbb{T}}} \bigl| \psi_j(\delta) - y_j^*(\delta) \bigr| \Biggr],\end{aligned}$$

where

$$\begin{aligned}M = \max_{1 \leq i \leq n, 1 \leq j \leq m} \Biggl\{ & 1 + \sum_{j=1}^{m} \overline{b}_j (\overline{p}_{ij} + \overline{q}_{ij}) \nu_i e_\varepsilon\bigl(\sigma(0), -\sigma_{ij}\bigr) \sigma_{ij}, \\ & 1 + \sum_{i=1}^{n} \overline{a}_i (\overline{\alpha}_{ji} + \overline{\beta}_{ji}) \kappa_j e_\varepsilon\bigl(\sigma(0), -\tau_{ji}\bigr) \tau_{ji} \Biggr\} \geq 1.\end{aligned}$$

By Definition 2.4 the periodic solution of system (1) is globally exponentially stable. This completes the proof. □

5 An example

Example 5.1 Consider the following fuzzy Cohen-Grossberg BAM neural networks with impulses:

$$\begin{cases} x_i^\Delta(t) = -a_i(x_i(t))[c_i(x_i(t)) + \bigwedge_{j=1}^{2} \alpha_{ji}(t) f_j(y_j(t - \tau_{ji})) \\ \qquad + \bigvee_{j=1}^{2} \beta_{ji}(t) f_j(y_j(t - \tau_{ji})) - E_i(t)], \quad t \in \mathbb{T}, t > 0, \\ \Delta x_i(t_k) = -0.1 x_i(t_k), \quad t = t_k = 2k, i = 1, 2; \\ y_j^\Delta(t) = -b_j(y_j(t))[d_j(y_j(t)) + \bigwedge_{i=1}^{2} p_{ij}(t) g_i(x_i(t - \sigma_{ij})) \\ \qquad + \bigvee_{i=1}^{2} q_{ij}(t) g_i(x_i(t - \sigma_{ij})) - F_j(t)], \quad t \in \mathbb{T}, t > 0, \\ \Delta y_j(t_k) = -0.2 y_j(t_k), \quad t = t_k = 2k, j = 1, 2, \end{cases} \tag{25}$$

where $\mathbb{T}$ is a 2π-periodic time scale, $a_1(u) = \frac{1}{9\pi}(2 + \cos u)$, $a_2(u) = \frac{1}{9\pi}(2 - \cos u)$, $b_1(u) = \frac{1}{12\pi}(2 + \sin u)$, $b_2(u) = \frac{1}{12\pi}(2 - \sin u)$, $c_1(u) = \frac{1}{3}u$, $c_2(u) = \frac{2}{3}u$, $d_1(u) = \frac{3}{2}u$, $d_2(u) = \frac{1}{2}u$, $f_i(u) = g_i(u) = \frac{1}{2}(|u + 1| - |u - 1|)$, $\alpha_{11}(t) = \frac{1}{20}\cos t$, $\alpha_{12}(t) = \alpha_{21}(t) = 0$, $\alpha_{22}(t) = \frac{1}{20}\sin t$, $\beta_{11}(t) = \frac{1}{20}\sin t$, $\beta_{12}(t) = \beta_{21}(t) = 0$, $\beta_{22}(t) = \frac{1}{20}\sin t$, $p_{11}(t) = \frac{1}{18}\sin t$, $p_{12}(t) = p_{21}(t) = 0$, $p_{22}(t) = \frac{1}{18}\sin t$, $q_{11}(t) = \frac{1}{18}\cos t$, $q_{12}(t) = q_{21}(t) = 0$, $q_{22}(t) = \frac{1}{18}\sin u$, $\tau_{ji}(t) = \frac{1}{2}\sin t\sigma_{ij}(t) = \frac{2}{3}\cos t$, $\kappa_j = \nu_i = 1$ $(i,j = 1,2)$. By calculating we have $\overline{a}_1 = \overline{a}_2 = \frac{1}{3\pi}$, $\underline{a}_1 = \underline{a}_2 = \frac{1}{9\pi}$, $\overline{b}_1 = \overline{b}_2 = \frac{1}{4\pi}$, $\underline{b}_1 = \underline{b}_2 = \frac{1}{12\pi}$, $\overline{\alpha}_{11} = \frac{1}{20}$, $\overline{\alpha}_{22} = \frac{1}{20}$, $\overline{\alpha}_{12} = \overline{\alpha}_{21} = 0$, $\overline{\beta}_{11} = \frac{1}{20}$, $\overline{\beta}_{22} = \frac{1}{20}$, $\overline{\beta}_{12} = \overline{\beta}_{21} = 0$, $\overline{p}_{11} = \overline{p}_{22} = \frac{1}{18}$, $\overline{p}_{12} = \overline{p}_{21} = 0$, $\overline{q}_{11} = \overline{q}_{22} = \frac{1}{18}$, $\overline{q}_{12} = \overline{q}_{21} = 0$.

It is easy to compute

$$E = \begin{pmatrix} \frac{7}{81\pi} & 0 & -\frac{7}{90\sqrt{2}\pi} & 0 \\ 0 & \frac{5}{81\pi} & 0 & -\frac{29}{540\sqrt{2}\pi} \\ -\frac{1}{972\sqrt{2}\pi} & 0 & \frac{1}{48\pi} & 0 \\ 0 & -\frac{2}{243\sqrt{2}\pi} & 0 & \frac{1}{16\pi} \end{pmatrix}$$

and

$$\underline{a}_1\varrho_1 - \sum_{j=1}^{2}(\overline{p}_{1j} + \overline{q}_{1j})\nu_1\overline{b}_1 = \frac{1}{108\pi} > 0,$$

$$\underline{a}_2\varrho_2 - \sum_{j=1}^{2}(\overline{p}_{2j} + \overline{q}_{2j})\nu_2\overline{b}_2 = \frac{5}{108\pi} > 0,$$

$$\underline{b}_1\varrho_1' - \sum_{i=1}^{2}(\overline{\alpha}_{1i} + \overline{\beta}_{1i})\kappa_1\overline{a}_1 = \frac{11}{120\pi} > 0,$$

$$\underline{b}_2\varrho_2' - \sum_{i=1}^{2}(\overline{\alpha}_{2i} + \overline{\beta}_{2i})\kappa_2\overline{a}_2 = \frac{1}{120\pi} > 0.$$

Hence, we have that $E = (e_{ij})_{4\times 4}$ is a nonsingular M-matrix. From Theorem 3.1 and Theorem 4.1 we know that system (25) has at least one 2π-periodic solution, which is globally exponentially stable.

6 Conclusions

In this paper, we have studied the existence and globally exponential stability of the periodic solution for fuzzy Cohen-Grossberg BAM neural networks with impulses on time scales. Some sufficient conditions set up here are easily verified, and these conditions are correlated with parameters of system (1). The obtained criteria can be applied to design globally exponential stability of periodic continuous and discrete fuzzy Cohen-Grossberg BAM neural networks.

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

The authors indicated in parentheses made substantial contributions to the following tasks of research: drafting the manuscript (SC); participating in the design of the study (SC, QZ); writing and revision of the paper (QZ). Both authors read and approved the final manuscript.

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