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Nonthermal correction to black hole spectroscopy

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Abstract Area spectrum of black holes has been obtained via various methods such as quasinormal modes, adiabatic invariance and angular momentum. Among those methods, calculations were done by assuming black holes in thermal equilibrium. Nevertheless, black holes in the asymptotically flat space usually have a negative specific heat and therefore tend to stay away from thermal equilibrium. Even for black holes with a positive specific heat, the temperature may still not be well defined in the process of radiation, due to the back reaction of a decreasing mass. With respect to these facts, it is very likely that Hawking radiation is nonthermal and the area spectrum is no longer equidistant. In this note, we would like to illustrate how the area spectrum of black holes is corrected by this nonthermal effect.

1 Area law and logarithmic correction

A finite size system often displays a discrete energy spectrum as regards quantum fluctuations. It was suggested that since the dynamics of a black hole is uniquely determined by its charge(s), which is closely related to the finite region enclosed by the horizon, one expects the mass or area spectrum to display a similar discreteness [1,2]. There were many proposals to obtain the area spectrum for various black holes since then. Earlier methods of quantizing the horizon area are mostly based on real or imaginary parts of the quasinormal modes [3–12]. Recently the application of an adiabatic invariant action variable did not use the quasinormal modes [13,14] and the idea of quantizing the angular momentum to obtain the area spectrum first appeared in the study of non-extremal RN black holes [15]. The various methods of quantization have settled on a spectrum of equidistant discreteness,

$$\Delta A = c l_p^2. \tag{1}$$

In particular, one obtained $c = 8\pi$ for various kinds of black holes in different spacetime dimensions. Nevertheless, this universal result is closely related to the assumption that the black hole is in the thermal equilibrium state where the Hawking temperature is well defined. Realistic black holes are more likely to be in the nonequilibrium state due to their negative specific heat. Even for black holes with a positive specific heat, the temperature may still be ill defined in the process of radiation, due to the back reaction of the decreasing mass. A universal logarithm correction to the Bekenstein–Hawking area law has been predicted in various theories of quantum gravity and modified general relativity, such that[1] [16]

$$S_{BH} = \frac{A}{4l_p^2} + \alpha \ln\left(\frac{A}{l_p^2}\right), \tag{2}$$

for horizon area A. The above logarithmic correction in (2) can be regarded as the consequence of loop quantum corrections of surface gravity [17,18] where α is the integral of the trace anomaly [19,20]. The corresponding correction to the area spectrum was computed for $\alpha = -\frac{3}{2}$ in the context of an adiabatic invariance approach for constant surface gravity. As a result, an uneven discreteness was observed [13]:

$$\Delta A \simeq 8\pi l_p^2 - \frac{32\pi\alpha l_p^4}{A}. \tag{3}$$

2 Nonthermal correction via back reaction

We are looking for the other correction due to back reaction from the Hawking radiation. Among various models of black hole radiation, the tunneling model proposed by Parikh and Wilczek [21] has provided useful insights in the

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[1] In this paper, we will adopt the units such that Newton constant G, Boltzmann constant k_B, and the speed of light c are all equal to 1. In these units, the Planck constant $\hbar$ has the unit of area l_p^2 and the entropy is dimensionless.

effort to resolve the information loss paradox [22], black hole evolution [23,24], and black hole remnants [25]. The Parikh–Wilczek model regards the Hawking radiation as a tunneling process in some stationary vacuum. The potential barrier is dynamically established due to the back reaction, which observes energy conservation. The emission rate in the tunneling model has a universal result:

$$\Gamma \sim e^{\Delta S_{\mathrm{BH}}}, \tag{4}$$

given the black hole entropy change ΔS_{BH} due to radiation. The back reaction constantly changes the surface gravity during the tunneling process, therefore the black hole is never in thermal equilibrium. In the following, we would like to use the Schwarzschild black hole as an example to argue that the back reaction effect could produce another correction to the area spectrum of order $\mathcal{O}(A^{-1})$.

In the case of Schwarzschild black hole with mass M, we use the logarithmic corrected area law (2) to compute the change of entropy after a particle with mass ω is tunneled out, that is,

$$l_p^2 \Delta S_{\mathrm{BH}} = l_p^2 \left(S_{\mathrm{BH}}\big|_{M-\omega} - S_{\mathrm{BH}}\big|_M \right) = -8\pi M \omega + 4\pi \omega^2$$
$$- \alpha l_p^2 \left(2\frac{\omega}{M} + \frac{\omega^2}{M^2} \right) + \mathcal{O}(\alpha^2), \tag{5}$$

where the first term on the right hand side is nothing but the thermal spectrum if the inverse of the Hawking temperature $T_H^{-1} = 8\pi M$ is identified. In the following we will show that the second term is the nonthermal correction due to back reaction. Those terms with α inside are the series expansion of the logarithmic correction with respect to the large black hole mass and we regard them as the quantum correction to the spectrum. To demonstrate how the area spectrum also receives a correction from those nonthermal and quantum effects, let us recall the derivation of (4) and then divert to the quantization of area. The tunneling process happens at the horizon in the following metric [21]:

$$ds^2 = -\left(1 - \frac{2M}{r}\right)dt^2 + 2\sqrt{\frac{2M}{r}}dt\,dr + dr^2 + r^2 d\Omega_2^2, \tag{6}$$

which can be obtained from the static Schwarzschild black hole metric by a coordinate transformation of the Painlevé-type. The WKB approximation states that the emission rate is $\Gamma \sim e^{-2\mathrm{Im}S/\hbar}$, where the imaginary part of action reads

$$\mathrm{Im}S = \mathrm{Im} \int_{r_{\mathrm{in}}}^{r_{\mathrm{out}}} p_r dr = \mathrm{Im} \int_M^{M-\omega} \int_{2M}^{2(M-\omega)} \frac{dH}{\dot{r}} dr, \tag{7}$$

for differential Hamiltonian $dH = d(M - \omega') = -d\Omega'$ and the trajectories of emission are given by radial null geodesics $\dot{r} = 1 - \sqrt{\frac{2(M-\omega')}{r}}$. We remark that the emitted mass ω has

been subtracted from M such that total energy is conserved. The computation of the above integral showed exactly the result (4) for $\Delta S_{\mathrm{BH}} = S_{\mathrm{BH}}\big|_{M-\omega} - S_{\mathrm{BH}}\big|_M$ [21]. This computation was also carried out in many kinds of black hole backgrounds and the result appears to be quite general. In this paper, we will take it for granted, and we will further apply the Sommerfeld–Bohr quantization rule by demanding that each emission of ω carries away an action quantum h or equivalently one degree of freedom $h/l_p^2 = h/\hbar = 2\pi$ where the unit $\hbar = l_p^2$ is adopted. That is,[2]

$$2\mathrm{Im} \int_{r_{\mathrm{in}}}^{r_{\mathrm{out}}} p_r dr = -\Delta S_{\mathrm{BH}} = 2\pi. \tag{8}$$

This boils down to a simple quadratic equation of ω:

$$\left(2 - \frac{\alpha \hbar}{2\pi M^2}\right)\omega^2 - \left(4M + \frac{\alpha \hbar}{\pi M}\right)\omega + \hbar = 0. \tag{9}$$

Here both nonthermal and quantum effects ignorable, one can drop the ω^2 term and obtain the quantum of mass $\omega = \frac{\hbar}{4M}$ by solving (9). The area discreteness can be computed as

$$\Delta A = 8\pi r \frac{dr}{dM} \Delta M \big|_{r=2M, \Delta M=\omega} = 8\pi \hbar. \tag{10}$$

The universal prefactor 8π agrees with that obtained from previous methods [29]. Now we would like to include the nonthermal and quantum effects by solving (9) honestly and obtain

$$\omega \simeq \frac{\hbar}{4M} + \frac{\hbar^2(\pi - 2\alpha)}{32\pi M^3} + \mathcal{O}\left(\frac{\alpha^2}{M^5}\right), \tag{11}$$

where we choose the smaller root for $\omega < M$ and use the Taylor expansion as long as $M \gg l_p$. Finally, we have the area discreteness

$$\Delta A = 8\pi \hbar + \frac{(16\pi^2 - 32\pi\alpha)\hbar^2}{A} + \cdots. \tag{12}$$

Due to the nonthermal correction, the area spacing gets larger as the horizon area shrinks as $\alpha < \frac{\pi}{2}$ but gets smaller vice versa. This can be regarded as an important signature for the Parikh–Wilczek tunneling model of Hawking radiation if the area discreteness were ever to be detected in the future. The area discreteness can easily be generalized to the Schwarzschild black hole in arbitrary dimension D [30], where

$$\Delta A = 8\pi \hbar^{D/2-1} + \frac{(32\pi^2 - 32\pi(D-2)\alpha)\hbar^{(D-2)}}{(D-2)A} + \cdots, \tag{13}$$

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[2] A similar quantization rule was previously adopted in [26,27] for the Schwarzschild black hole and in [28] for the massless topological black hole.

where $A = r_0^{D-2}\Omega_{D-2}$ for horizon radius r_0. We remark that in $D = 4$ the nonthermal correction is competitive to the quantum correction, however, the former becomes less and less important as D increases.

To obtain a correction to black holes with more charges or different topology, one can in principle solve the following algebraic equation as a consequence of (8)[3]:

$$S_{\mathrm{BH}}(Q_i - q_i) - S_{\mathrm{BH}}(Q_i) + 2\pi = 0, \tag{14}$$

given the change of black hole entropy as a function of black hole charges Q_i and emitted charges q_i. This is the basic assumption in our paper. In the following section, we will show our new results of a nonthermal correction by solving (14) for various kinds of black holes.

3 Nonthermal correction for various black holes

Here we will follow the same approach applied to the Schwarzschild black hole in the previous section and obtain a nonthermal correction for various kinds of black holes. We will assume the limit of large mass, i.e. $M \gg l_p$, to ensure the use of (8), and we will focus only on the nonthermal correction but set $\alpha = 0$ to ignore the quantum correction.

- For a Reissner–Nordström black hole of mass M and electric charge Q, we have the metric

$$\begin{aligned} \mathrm{d}s^2 &= -f(r)\mathrm{d}t^2 + f^{-1}(r)\mathrm{d}r^2 + r^2\mathrm{d}\Omega_2^2, \\ f(r) &= 1 - \frac{2M}{r} + \frac{Q}{r^2}, \end{aligned} \tag{15}$$

where the horizon $r_+ = M + \sqrt{M^2 - Q^2}$. It is convenient to define the extremality $\Gamma = Q/M$ and the charge-mass-ratio of the emitted particle $\gamma = q/\omega$. The area discreteness in general reads

$$\Delta A = 8\pi\hbar(1 + a(\Gamma, \gamma)) + \frac{16\pi^2\hbar^2}{A}(1 + b(\Gamma, \gamma)), \tag{16}$$

where the functions $a(\Gamma, \gamma)$ and $b(\Gamma, \gamma)$ are complicated but can be perturbatively computed. For instance,

$$\begin{aligned} a(\Gamma, \gamma) &\simeq \frac{1}{2}\gamma\Gamma + \mathcal{O}(\Gamma^2), \\ b(\Gamma, \gamma) &\simeq -\frac{\gamma^2}{2} + \frac{3}{2}\gamma\Gamma - \frac{3}{4}\gamma^3\Gamma + \mathcal{O}(\Gamma^2), \end{aligned} \tag{17}$$

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[3] There are many works following the Parikh–Wilczek tunneling model [21] to apply to black holes with more than one charge, such as Reissner–Nordström black holes [31] and BTZ black holes [37], to mention a few. They all agree with the expression (4). It was pointed out in [32] that (4) can be obtained without spacetime but relying on the Hilbert space description of the black hole.

in the near Schwarzschild limit ($\Gamma \ll 1$). On the other hand, in the near extremal limit where $\Gamma, \gamma \to 1$, we obtain

$$\begin{aligned} a(x, \gamma) &\simeq 3 - 12x + \mathcal{O}(x^2), \quad b(x, \gamma) \\ &\simeq 3 - 22x + \mathcal{O}(x^2), \end{aligned} \tag{18}$$

where $\Gamma \equiv 1 - 2x^2$.

- For a BTZ black hole in three dimensions [33,34], the area spectrum has been discussed in [35,36] and the tunneling rate was discussed in [37]. We begin with the metric

$$\begin{aligned} \mathrm{d}s^2 &= -f(r)\mathrm{d}t^2 + f^{-1}(r)\mathrm{d}r^2 + r^2\left(\mathrm{d}\phi - \frac{J}{2r^2}\mathrm{d}t\right)^2, \\ f(r) &= -M + \frac{r^2}{l^2} + \frac{J^2}{4r^2}. \end{aligned} \tag{19}$$

The entropy function is well known to be

$$S_{\mathrm{BH}} = \frac{\pi}{2\hbar}r_+, \tag{20}$$

where $r_+^2 = \frac{1}{2}\left(Ml^2 + \sqrt{M^2l^4 - J^2l^2}\right)$. Following (14), one obtains for nonrotating BTZ ($J = 0$)

$$\Delta A = 8\pi\hbar - \frac{32\pi^2\hbar^2}{A} + \cdots. \tag{21}$$

For $J \neq 0$, the area spacing in general depends on the black hole angular momentum J and the spin of the emitted particle j. If one defines the extremality $\Gamma \equiv J/M$ and the emitted particle's spin–mass ratio $\gamma \equiv j/\omega$, then

$$\Delta A = 8\pi\hbar - \frac{32\pi^2\hbar^2}{A}(1 + a_{\mathrm{BTZ}}(\Gamma, \gamma)) + \cdots, \tag{22}$$

where the function $a_{\mathrm{BTZ}}(\Gamma, \gamma)$ can be solved by Taylor's expansion at small Γ and γ:

$$a_{\mathrm{BTZ}}(\Gamma, \gamma) \simeq \frac{\gamma^2}{l^2} - 2\frac{\gamma\Gamma}{l^2} - \frac{\Gamma^2}{8l^2} + \cdots. \tag{23}$$

The constant leading term in (22) agrees with that found in [35,36], and, moreover, we observe that the nonthermal correction depends on Γ and γ in general.

- For a D-dimensional AdS black hole of different horizon topologies, we have the metric

$$\begin{aligned} \mathrm{d}s^2 &= -f(r)\mathrm{d}t^2 + f^{-1}\mathrm{d}r^2 + r^2\mathrm{d}\Omega_{D-2}^2, \\ f(r) &= k + \frac{r^2}{l^2} - \frac{aM}{r^{D-3}}. \end{aligned} \tag{24}$$

For simplicity, we first examine the one with a planar horizon, that is, $k = 0$. The horizon can be analytically solved as $r_+ = (al^2M)^{\frac{1}{D-1}}$. We find a leading-order equidistant spectrum:

$$\Delta A = 8\pi\hbar^{D/2-1} - \frac{32\pi^2\hbar^{(D-2)}}{(D-2)A} + \cdots . \quad (25)$$

Our finding shows a leading universal factor 8π for any $D > 3$, however, it is different from that obtained in [38]. The nonthermal correction takes the same form as that in the Schwarzschild black hole (13) but with opposite sign. We remark that the correction implicitly depends on the AdS radius of curvature l via the horizon area A. This result cannot be simply compared with (13) in the flat limit $l \to \infty$ due to the different horizon topology chosen here.

For the spherical near-horizon topology, $k = 1$, we find that the correction to the area spectrum explicitly depends on l. In particular, at the limit of large mass and weak curvature (but keeping M/l small), one obtains

$$\Delta A \simeq 8\pi\hbar + \frac{\pi\hbar^2}{M^2} + l^{-2}\left(4M\hbar - \frac{\hbar^3}{8M^3}\right) + \cdots, \quad (26)$$

for $D = 4$. We remark that the result of (13) can be reproduced in the flat limit $l \to \infty$.

- For a D-dimensional Schwarzschild–de Sitter black hole, we have the metric

$$
\begin{aligned}
ds^2 = &-\left(1 - \frac{2M}{r^{D-3}} - \frac{r^2}{l^2}\right)dt^2 \\
&+ \left(1 - \frac{2M}{r^{D-3}} - \frac{r^2}{l^2}\right)^{-1}dr^2 + r^2d\Omega_{D-2}^2. \quad (27)
\end{aligned}
$$

First, we would like to examine the case of $D = 3$, where one obtains the exact solution

$$\Delta A = 8\pi\hbar. \quad (28)$$

Since there is in fact no black hole in three dimensional de Sitter space, this should be identified as the area spectrum of dS_3 space itself.[4]

For $D > 3$, one receives the area spectrum correction. For instance, in $D = 4$ for large M and l:

$$\Delta A \simeq 8\pi\hbar - 2592\pi\hbar2^{2/3}\left(\frac{M}{l}\right)^{8/3} + \cdots . \quad (29)$$

- For a D-dimensional AdS topological black hole, we have the metric [40]

$$
\begin{aligned}
ds^2 = &-\left(-1 - \frac{2M}{r^{D-3}} + \frac{r^2}{l^2}\right)dt^2 \\
&+ \left(-1 - \frac{2M}{r^{D-3}} + \frac{r^2}{l^2}\right)^{-1}dr^2 + r^2d\Omega_{D-2}^2. \quad (30)
\end{aligned}
$$

For $D = 4$, we obtain the area spectrum for large M and l:

$$\Delta A \simeq 8\pi\hbar - 2592\pi\hbar2^{2/3}\left(\frac{M}{l}\right)^{8/3} + \cdots, \quad (31)$$

which is the same as (29). For a massless topological black hole, where $M \to 0$, we obtain the universal result $\Delta A = 8\pi\hbar$.

- For a D-dimensional Gauss–Bonnet black hole, the metric reads

$$
\begin{aligned}
ds^2 &= -f(r)dt^2 + f(r)^{-1}dr^2 + r^2d\Omega_{D-2}^2, \\
f(r) &= 1 + \frac{r^2}{2\alpha}\left[1 - (1 + \frac{4\alpha'a'M}{r^{D-1}})^{1/2}\right], \\
\alpha' &= (D-3)(D-4)\alpha_{GB}, \quad a' = \frac{16\pi G}{(D-2)\Omega_{D-2}}. \quad (32)
\end{aligned}
$$

The tunneling model of the (AdS) Gauss–Bonnet black hole has been studied in [41,42], and the emission rate agrees with that in (4), where the entropy is given by

$$S = \frac{r_+^{D-2}\Omega_{D-2}}{4}\left[1 + 2\left(\frac{D-2}{D-4}\right)\frac{\alpha'}{r_+^2}\right], \quad (33)$$

where r_+ satisfies

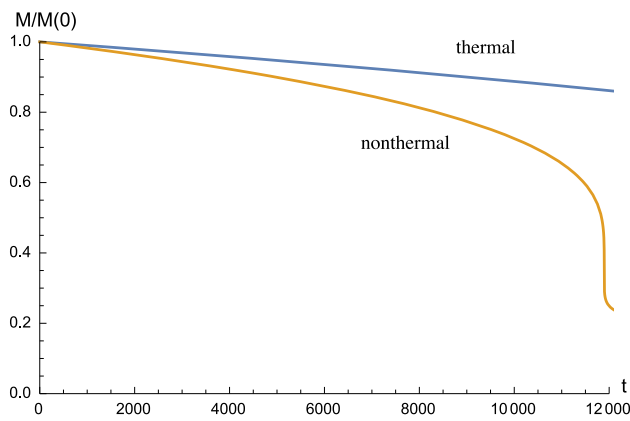$$a'Mr_+^{5-D} = r_+^2 + \alpha'. \quad (34)$$

The area spectrum was discussed in [43], where the conclusion that the entropy spectrum is equally spacing agrees with our assumption (14). In particular, the coefficient α' vanishes for $D = 4$ such that

$$\Delta A = 8\pi\hbar + \frac{16\pi^2\hbar^2}{A} + \cdots, \quad (35)$$

which has no effect from the Gauss–Bonnet term. For $D = 5$, the area spectrum correction can be expressed via Taylor's expansion of α_{GB}/M:

Fig. 1 Time evolution for the usual Hawking radiation as blackbody radiation, and radiation with the nonthermal correction. The latter evaporates faster due to increased spacing of the area spectrum

$$\Delta A = 8\pi\hbar(1 + a_{\mathrm{GB}}(\alpha_{\mathrm{GB}}, M)) + \frac{32\pi^2\hbar^2}{3A}$$
$$\times (1 + b_{\mathrm{GB}}(\alpha_{\mathrm{GB}}, M)) + \cdots,$$
$$a_{\mathrm{GB}}(\alpha_{\mathrm{GB}}, M) = -\frac{3\pi}{2M}\alpha_{\mathrm{GB}} + \frac{9\pi^2}{8M^2}\alpha_{\mathrm{GB}}^2 + \cdots,$$
$$b_{\mathrm{GB}}(\alpha_{\mathrm{GB}}, M) = -\frac{3\pi}{2M}\alpha_{\mathrm{GB}} + \frac{9\pi^2}{16M^2}\alpha_{\mathrm{GB}}^2 + \cdots. \quad (36)$$

4 Discussion

In summary, we have investigated the nonthermal correction to the area spectrum in various kinds of black holes using the quantization rule (8). This semiclassical approximation usually works better for highly excited states, that is, large black hole masses (charges), and the leading term reproduces the universal coefficient 8π. However, if the equidistant spectrum for the entropy, $|\Delta S_{\mathrm{BH}}| = 2\pi$, could persist through the lifetime of a black hole, Eq. (8) predicts an increasing correction to the area spectrum toward the end of evaporation. To estimate the nonthermal correction to the emission rate of a Schwarzschild black holes, we observe in (12) that the nonthermal correction contributes like a quantum correction with $\alpha = -\frac{\pi}{2}$ at the $\mathcal{O}(1/A)$ order. Therefore, the nonthermal effect could be modeled as radiation at an *effective* temperature[5]

$$T_H^{\mathrm{eff}} = \frac{1}{8\pi M}\left(1 - \frac{1}{8M^2}\right)^{-1}. \quad (37)$$

In Fig. 1, we plot both the thermal radiation and the nonthermal radiation for the Schwarzschild black hole. It is expected

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[5] The idea of effective temperature was previously introduced in [26,27] and his definition of effective temperature agrees with our observation here via a quantum correction.

that the black hole speeds up its evaporation in the nonthermal radiation thanks to increasing spacing in area spectrum.

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